Problem Set 8

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Abstract

In this problem set, we explore the uses of the discrete Fourier transform and Python's differential equation solvers by considering musical signals and the Lorenz equations that model deterministic chaos.

1 Introduction

The discrete Fourier transform (DFT) is used to determine the frequencies present in a discrete signal such as a musical note. If we represent the discrete signal as the sequence $\{x_n\}_{n=1}^{N}$ then the $k-th$ DFT mode is given by the following formula:

$$c_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N}$$

k is the Fourier index and is related to the frequency (Hertz) by the following formula:

$$f = \frac{kR}{N}$$

where R is the sampling rate. A plot of the magnitudes of these DFT modes versus the frequency will show some large peaks at some frequencies. The frequencies associated with these dominant peaks reveal the frequencies present in the signal. For the first exercise, the discrete signals will be signals made by a piano and a trumpet. Since these signals have very large sample sizes, implementing the previous summation formula for the DFTs in a computer will be inefficient. Therefore, we resort to fast Fourier transforms found in Python's `scipy` module.

The second exercise involves a system of first order differential equations known as the Lorenz equations. These equations have the following form:

$$\frac{dx}{dt} = \sigma(y-x) \qquad \frac{dy}{dt} = rx - y - xz \qquad \frac{dz}{dt} = xy - bz$$

where $x(t), y(t)$, and $z(t)$ are functions of time and σ, r, and b are constants. These equations exemplify deterministic chaos since the functions will show random spikes even when no randomness is built into the system of equations.

2 Methods

2.1 Exercise 1:

First we implement a function called `readAndDetermineFFT` that reads the signal from the input file name and plots the signal. The function then uses fast Fourier transforms found in Python's `scipy` module to determine the Fourier modes c_k and their magnitudes. The function then plots the magnitudes of the first 10,000 modes. Finally, the function determines the frequencies from the sampling rate of $44,100$ samples per second by calling the function `fftfreq`() from the `scipy` module. The function then plots the mode magnitudes and frequencies. This function is called twice, first for the piano signal stored in the file `piano.txt` and then for the trumpet signal stored in the file `trumpet.txt`.

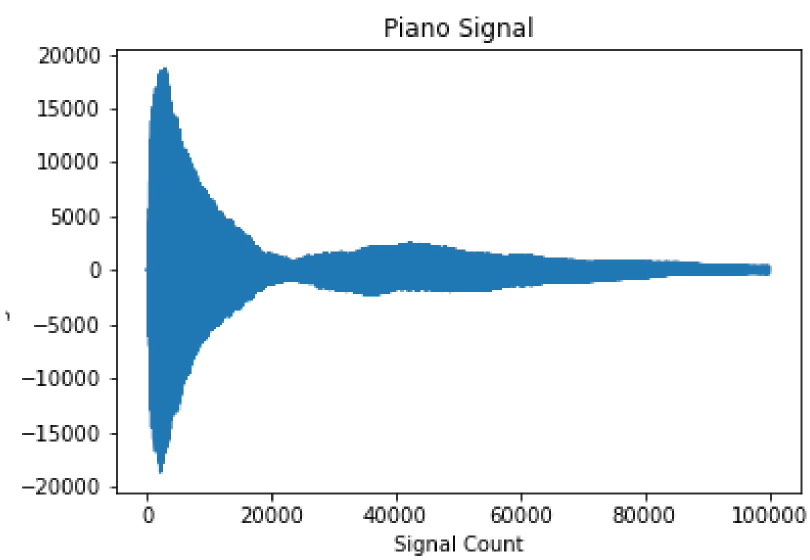
2.2 Exercise 2:

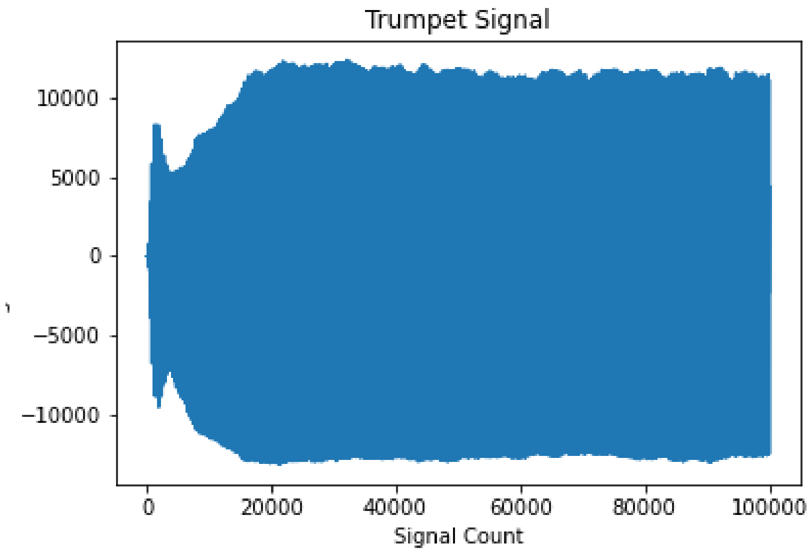
We implement the system of differential equations by using the `odeint` function from the `scipy.integrate` module. We set the constants as follows: $\sigma = 10$, $r = 28$, and $b = \frac{8}{3}$. The initial conditions are as follows: $x(0) = 0, y(0) = 1$, and $z(0) = 0$. We set the time to range from 0 to 50 seconds and make a plot of $y(t)$ versus t and $z(t)$ versus $x(t)$.

3 Results

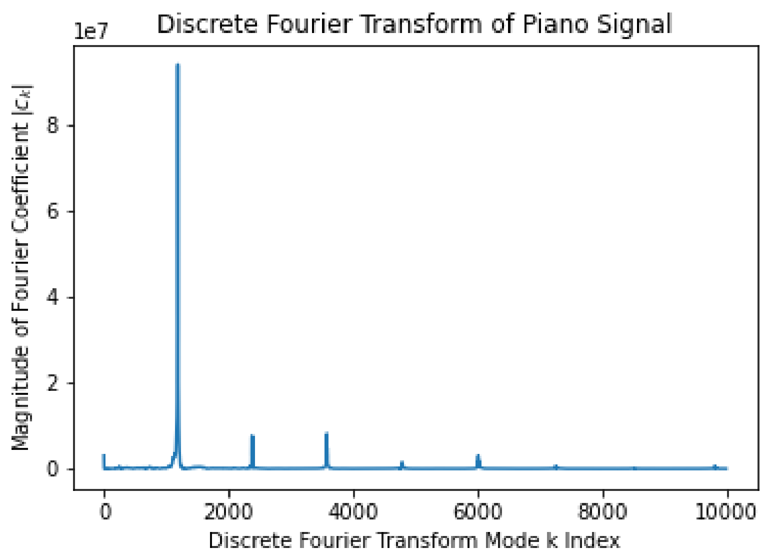
3.1 Exercise 1:

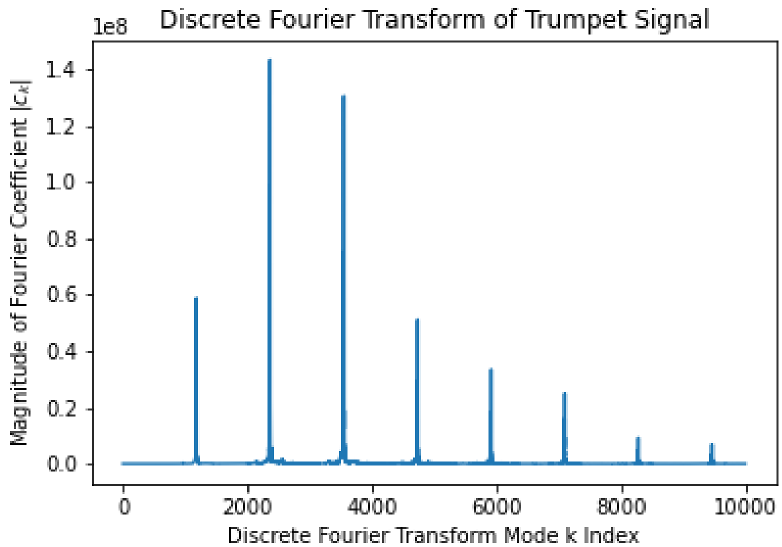
Here are the plots of the piano and trumpet signals.





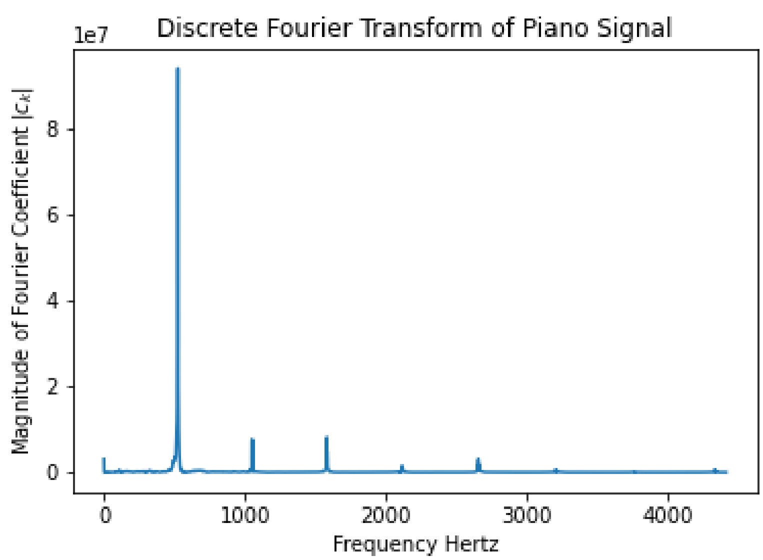
Here are the plots of their DFTs.

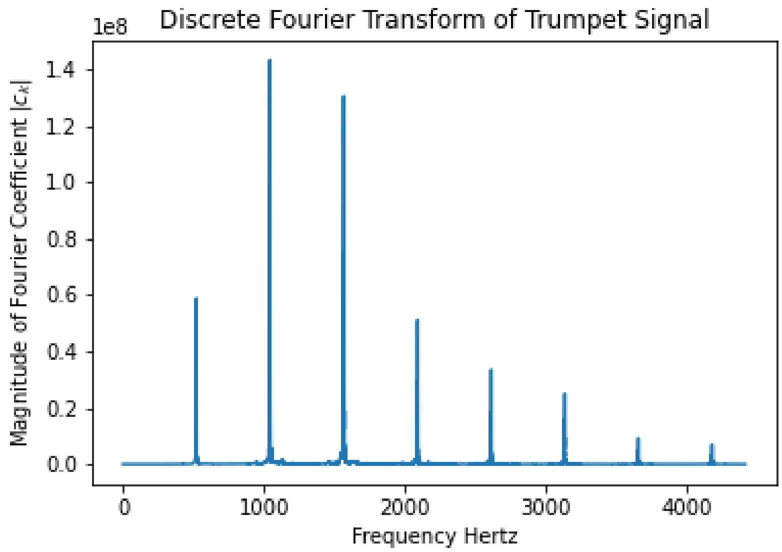




We see that the piano signal's DFT contains one major peak at relatively small frequency while the trumpet signal's DFT contains several major peaks for many frequencies, many of which are larger than the frequency of the piano spectrum's major peak. The piano produces a low sound while the trumpet produces a higher sound due to its broader range of larger frequencies.

Here are the plots of the DFTs with the frequencies. These plots allows us to estimate where the dominant peaks are.





Here are the frequencies associated with the largest peak in each DFT plot.

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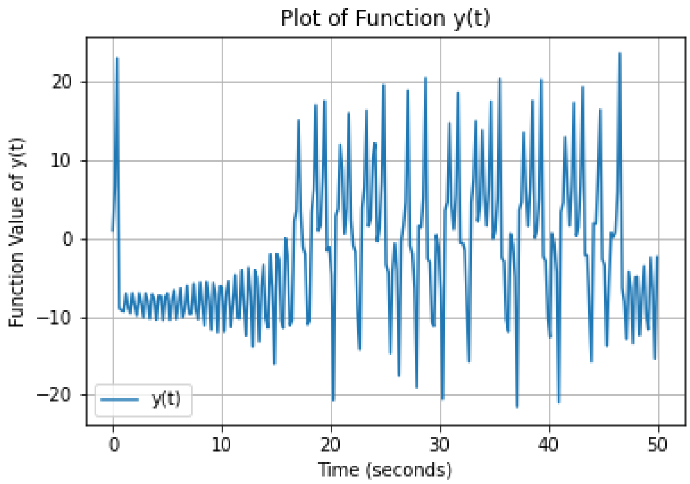
The main peak of the piano signal corresponds to a frequency of 524.7952479524795
Hertz.
The main peak of the trumpet signal corresponds to a frequency of 1043.8574385743857
Hertz.
```

The piano signal's main FFT peak corresponds to a frequency of 524.8 Hz, which corresponds to the C5 note (523.25 Hz). The trumpet signal's main peak corresponds to a frequency of 1043.9 Hz, or the C6 note (1046.50 Hz), but we see that the prior peak has a frequency near the C5 note. Information on musical notes and their frequencies were taken from https://pages.mtu.edu/s̃uits/notefreqs.html

3.2 Exercise 2:

Here is the plot of $y(t)$ versus time. We can see the unpredictable behavior of this function.



Here is the plot of $z(t)$ and $x(t)$. This is the strange attractor.