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#Matematyka Konkretna
#Laboratorium 7
#Senecki Daniel https://github.com/Debenter/MKLab7
#Wariant 1
import numpy as np
import matplotlib.pyplot as plt
from mpl toolkits.mplot3d import Axes3D
# Modified function to minimize
def funkcja(x, y):
   return (x + 3*y)**3 + 2*x
# Modified gradient descent function
def gradient descent (learning rate, iterations):
   x = np.random.uniform(1, 100)
   y = np.random.uniform(1, 100)
   history = []
   for _ in range(iterations):
        # Numerical gradients
       df dx = 3 * (x + 3*y)**2 + 2 # Partial derivative with respect
to x
       df dy = 9 * (x + 3*y)**2 # Partial derivative with respect
to y
       x = x - learning rate * df dx
       y = y - learning rate * df dy
       history.append([x, y, funkcja(x, y)])
   return np.array(history)
# Visualization of the function
x_{vals} = np.linspace(1, 100, 100)
y vals = np.linspace(1, 100, 100)
X, Y = np.meshgrid(x vals, y vals)
Z = funkcja(X, Y)
# 3D plot initialization
fig = plt.figure()
ax = fig.add subplot(111, projection='3d')
ax.plot surface(X, Y, Z, cmap='viridis', alpha=0.8, edgecolor='k')
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# Initial point
ax.scatter(1, 1, funkcja(1, 1), color='red', marker='o', s=100,
label='Start')
# Optimization
learning rate = 0.0001 # Adjust the learning rate to a smaller value
iterations = 100
history = gradient descent(learning rate, iterations)
# Trajectory
ax.plot(history[:, 0], history[:, 1], history[:, 2], color='blue',
marker='o', label='Minimization')
# Final minimum point
ax.scatter(history[-1, 0], history[-1, 1], history[-1, 2],
color='green', marker='o', s=100, label='Minimum')
ax.set xlabel('X')
ax.set ylabel('Y')
ax.set zlabel('f(X, Y)')
ax.legend()
plt.show()
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