

# Perfect Squares

I/p:  $n=12$

o/p: 3 ( $4+4+4$ )

$$2^2 = 4$$



$\Rightarrow$  write 'n' as a  
 sum of perfect square  
 $\Rightarrow \therefore$  and the no. of perfect  
 square must be minimum

I/p:  $n=13$

o/p: 2

$(4+9)$   
2      3

Now Logical concept

$$\Rightarrow \boxed{n=12} \rightarrow \text{Sum of Perfect Square} = \textcircled{n}$$

(this should be in min number)

$$\frac{4+4+9+16+1}{}$$

or,  $\underline{n} - (\text{sum of perfect squares}) = \underline{0}$

we have many options of perfect squares to subtract

$$\begin{array}{r} \textcircled{17} \\ \textcircled{1+1+1+1+} \\ \hline \textcircled{17} \end{array}$$

$$\begin{array}{r} N = \textcircled{17} \\ \textcircled{17} - \textcircled{9} = \textcircled{6} \\ \textcircled{6} - \textcircled{4} = \textcircled{2} \\ \textcircled{2} - \textcircled{1} = \textcircled{1} \\ \textcircled{1} - \textcircled{1} = \textcircled{0} \\ \textcircled{7} \end{array}$$

Logic

Ex

$n = 18$  →

∴ we want minimum no. of Perfect Square

↳ we have many options to subtract perfect squares to 'n' and make it '0'

$n$  - sum of p.s = 0

↳  $1^2 = 1$ ,  $2^2 = 4$ ,  $3^2 = 9$ ,  $4^2 = 16$  ✓

•> If you are thinking, that choosing the largest one (16) at 1<sup>st</sup> step would work

see,  $18 - (4^2) = 2$  ✓  
 $2 - (1^2) = 1$   
 $1 - (1^2) = 0$  ] Future steps ] 3 step

But, optimal solution is to pick 9 first

$18 - (3^2) = 9$  ✓  
 $9 - (3^2) = 0$  ] Future steps ] 2 steps ] 0/1 step

$$\begin{aligned} n &= 18 \\ n &= 18 - 16 = 2 \quad \text{①} \\ n &= 2 \end{aligned}$$

$$\begin{aligned} 1^2 &= 1 \\ 2^2 &= 4 \\ 3^2 &= 9 \\ 4^2 &= 16 \end{aligned}$$

$$\begin{aligned} n &= 2 - 1^2 = 1 \quad \text{①} \\ n &= 1 \end{aligned}$$

$$n = 1 - 1^2 = 0 \quad \text{①}$$

3 step

→ we are not sure, if subtracting 1, 4, 9 or 16 first would be optimal as we don't know the future result.

DP Rule : So, whenever, you are not sure which step would be optimal, no need to stress on guessing the optimal one.

TRY ALL OPTIONS, AND CHOOSE THE OPTIMAL ONE.

Solve (18)  $\Rightarrow$  18 - (1<sup>2</sup>)  $\Rightarrow$  17  $\Rightarrow$  Solve (17)

Solve (18)  $\Rightarrow$  18 - (2<sup>2</sup>)  $\Rightarrow$  14  $\Rightarrow$  Solve (14)

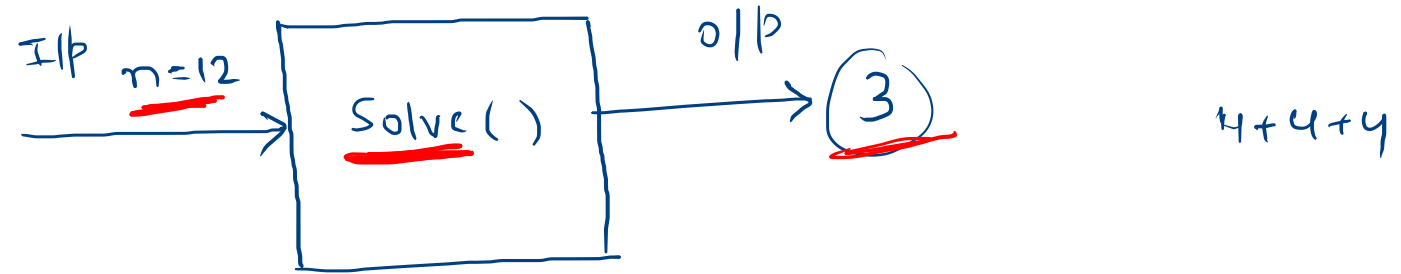
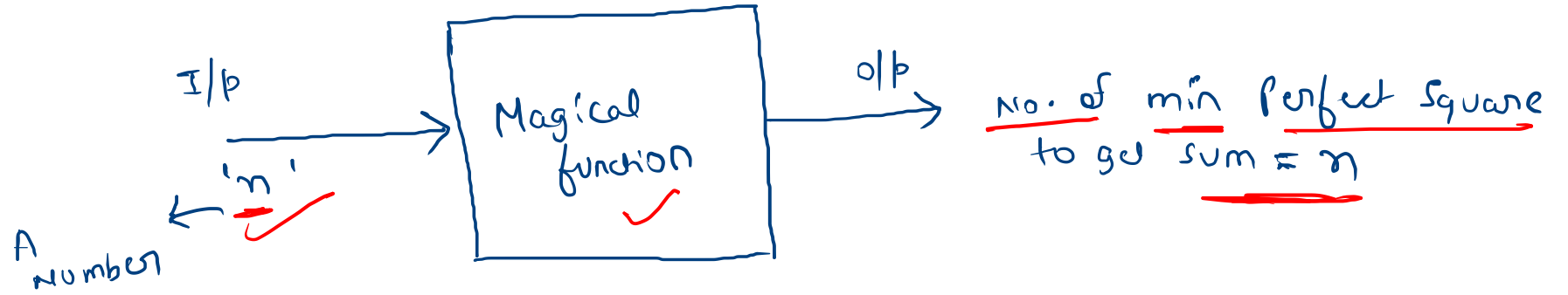
Solve (18)  $\Rightarrow$  18 - (3<sup>2</sup>)  $\Rightarrow$  9  $\Rightarrow$  Solve (9)

Solve (18)  $\Rightarrow$  18 - (4<sup>2</sup>)  $\Rightarrow$  2  $\Rightarrow$  Solve (2)

these are the subproblems

$\rightarrow \text{solve}(18) = 1 + \min \left( \text{solve}(17), \text{solve}(14), \text{solve}(9), \text{solve}(2) \right)$

# Recursive Solution



Logical  
part  
⇒

TRY ALL OPTIONS, AND CHOOSE THE OPTIMAL ONE.

Solve (18) ⇒  $18 - (1^2) ⇒ 17 ⇒$

Solve (18) ⇒  $18 - (2^2) ⇒ 14 ⇒$

Solve (18) ⇒  $18 - (3^2) ⇒ 9 ⇒$

Solve (18) ⇒  $18 - (4^2) ⇒ 2 ⇒$

Solve (17)

Solve (14)

Solve (9)

Solve (2)

these are  
the subproblems

18 - 5<sup>2</sup> X  
→ solve(18) = 1 + min ( solve(17), solve(14), solve(9), solve(2) )

Base  
condition

int solve (int n)

smallest valid +ve Input

n = 0, 1, 2, 3 ...

if (n ≤ 0) return 0

o/b: 0

# Recursive Code

```
int Solve (int n)
```

```
{
```

```
if (n <= 0) return 0;    ] Base condition
```

```
int ans = INT_MAX;
```

```
for (int i=1 ; i*i <= n ; i++)
```

```
{
```

```
int SqHum = i*i;    // 1, 4, 9, 16
```

```
int count = 1 + Solve(n - SqHum);
```

```
ans = min(ans, count);
```

```
return ans;
```

```
}
```

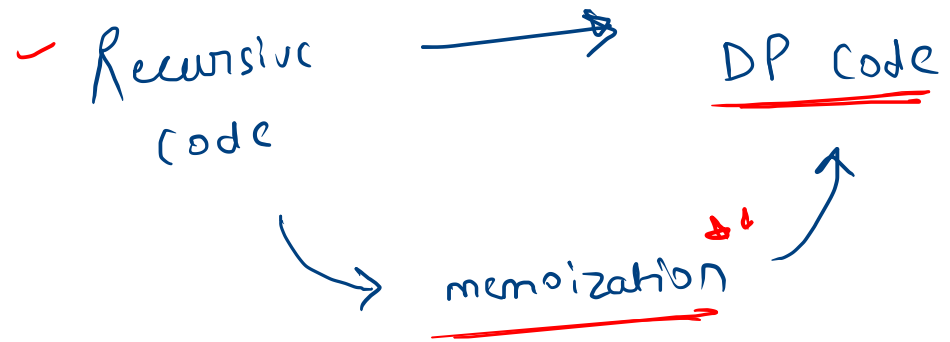
Try all  
option and  
choose the  
minimum one

n=18

i=1	1 < 18
i=2	4 < 18
i=3	9 < 18
i=4	16 < 18

✓ 18 - 1 = 17  
4 = 14  
9 = 9  
16 = 2





vector<int> dp (10001, -1);

$n \leq \underline{\underline{10^4}}$

DP  
Code

int solve (int n)<sup>★★</sup>

vector<int> dp (10001, -1);

{ if (n <= 0) return 0; } Base condition

✓ if (dp[n] != -1) return dp[n] ✓

int ans = INT\_MAX;

for (int i=1; i\*i <= n; i++)

{

int sqHum = i\*i;

int count = 1 + solve(n - sqHum);

ans = min(ans, count);

}

return dp[n] = ans; ✓

}