

# Power of Numbers

I/p : N=2

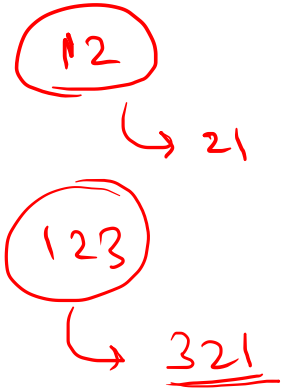
o/p : (N)<sup>R</sup> Reverse of N is 2

$$\underline{(2)^2 = 4}$$

I/p : N=12

o/p : (N)<sup>R</sup> Reverse of N is 21

$$\underline{(12)^{21} = 864354781}$$



✓ NAIVE SOLUTION

Ex: (12, 21)  $(N)^R \rightarrow$  Reverse

long long ans = 1;

Solution:

for( int i=1 ; i<=21 ; i++ )

{

ans = ans \* 12;

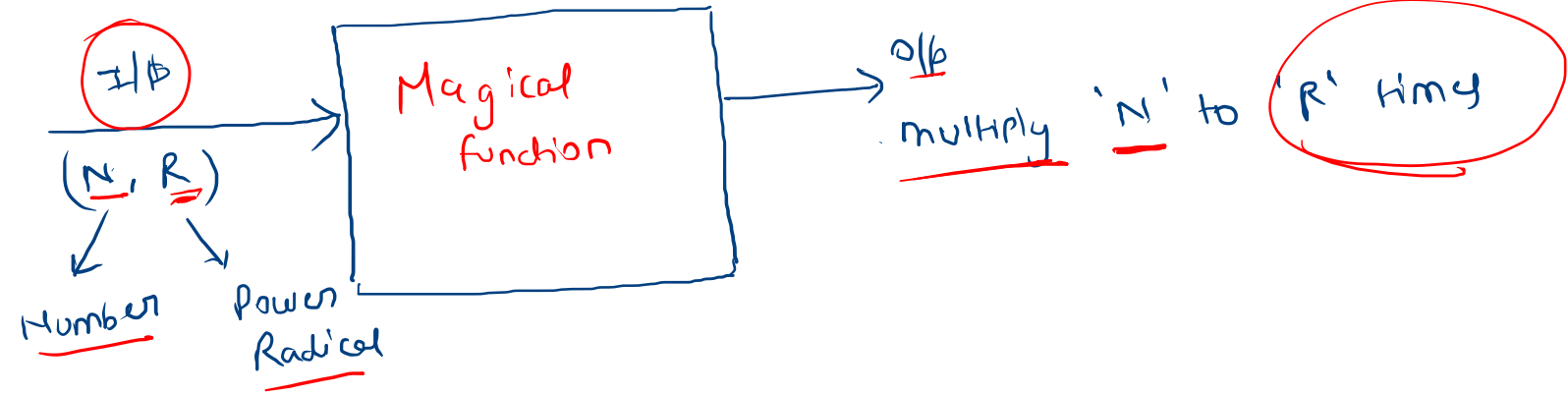
}

cout << ans << endl;

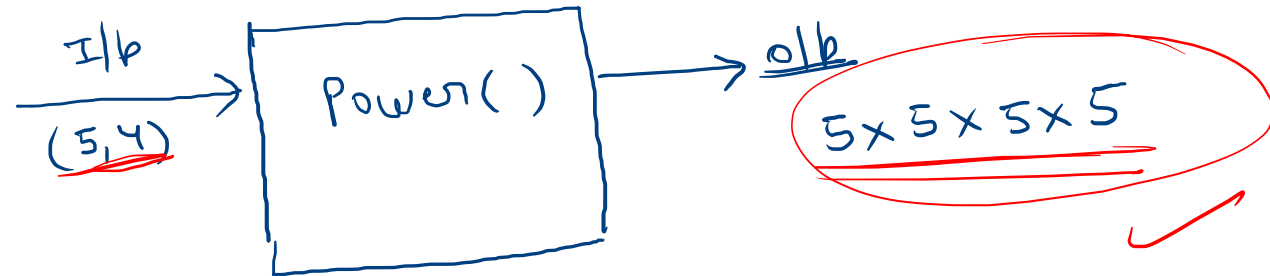
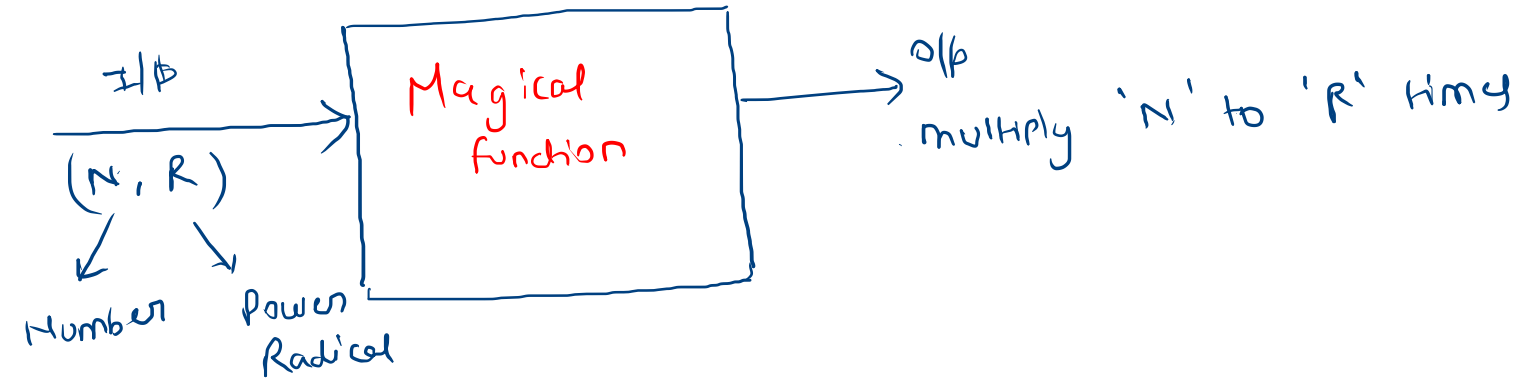
$O(N)$   
Time

R

Recursive  
solution



Recursive  
solution



Now  
Think of  
Recursive  
function

Now  
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Recursive  
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Ex:  $(5)^4$



$5 \times 5 \times 5 \times 5$

$5 \times \text{pow}(5, 3)$

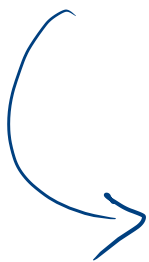
Recursive fun<sup>n</sup>

Now  
think of  
Recursive  
function

Ex:  $(5)^4$

$$\rightarrow 5 \times \underline{5 \times 5 \times 5}$$

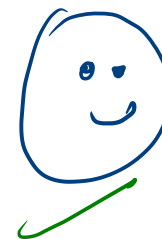
$$5 \times \text{pow}(5, \underline{3})$$



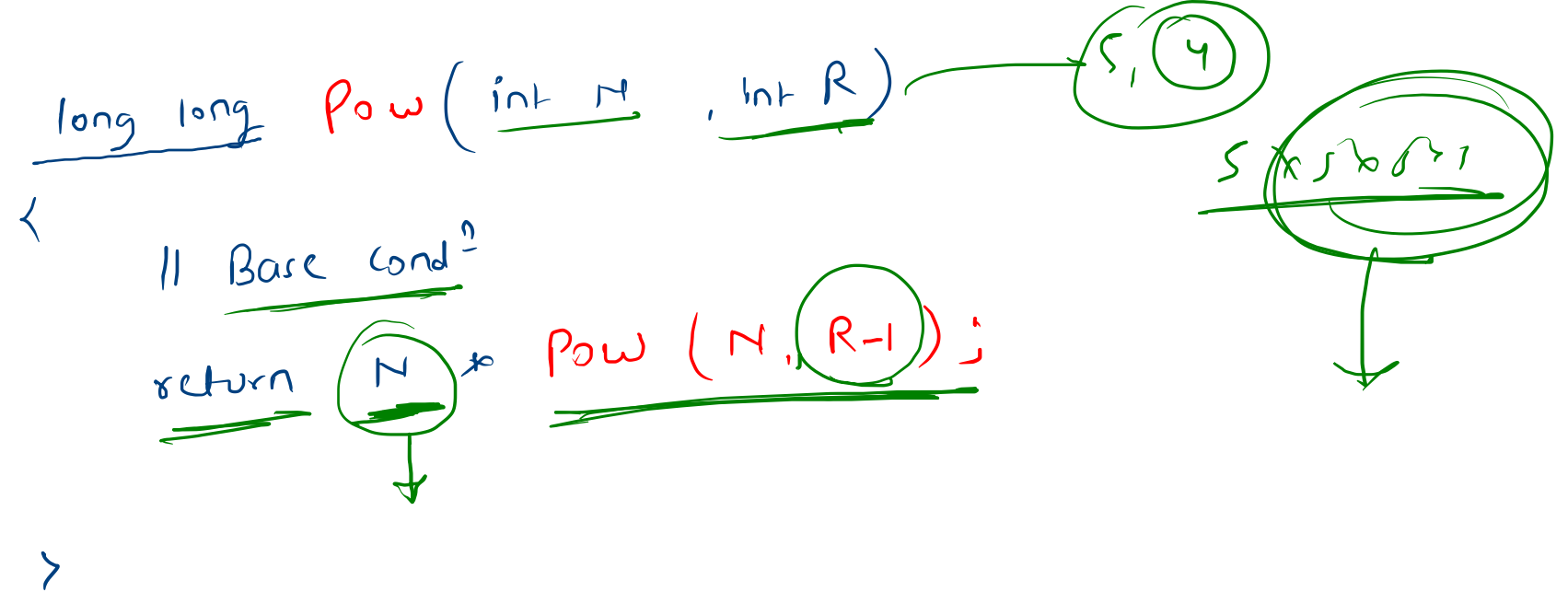
pow(5, 4)  $\rightarrow 5^4$

↓ Break down

5  $\times$  pow(5, 3)



Recursive  
function





Base  
cond<sup>n</sup>:

Smallest valid I/p → check  
output

I/p: N, R → R = 0, 1, 2, 3  
↓  
if N = 0, 1, 2, 3, ...  
↓  
Smallest  
↓  
Smallest

if (N == 0) return 0;  
if (R == 0) return 1;  
Base  
cond<sup>n</sup>

25 0  
25 3

0<sup>5</sup>, 0<sup>6</sup>  
0<sup>1</sup>

25<sup>10</sup>

Recursive  
function

```
long long Pow(int N, int R)
{
    if (N == 0) return 0;
    if (R == 0) return 1;
    return N * Pow(N, R-1);
}
```

Recursive  
Tree

Prob:  $(5)^4$

↳

$\text{pow}(5, 4)$

o/k 625

5  $\times$   $\text{pow}(5, 3)$

if ( $n == 0$ )  $\rightarrow 0$   
 $r == 0$   $\rightarrow 1$

5  $\times$   $\text{pow}(5, 2)$   
125

5  $\times$   $\text{pow}(5, 1)$   
25

5  $\times$   $\text{pow}(5, 0)$   
1  
5

$\text{pow}(5, 4)$

$\downarrow$   
 $5 \times \text{pow}(5, 3)$

Now

Given a number and its reverse. Find that number raised to the power of its own reverse.

**Note:** As answers can be very large, print the result modulo  $10^9 + 7$ .

↳

Answers are very large

↳

$$\text{Ans} \% 10^9 + 7$$

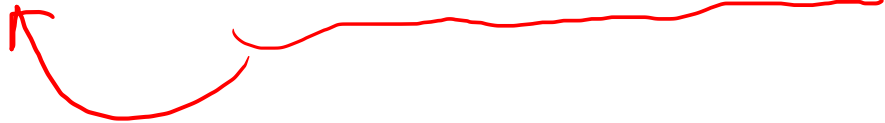
↓

$$\text{int MOD} = 10^9 + 7; \\ (1000000007);$$

Modular  
Equation

$$\begin{aligned} \checkmark \quad \underline{(a * b)} \underline{\cdot m} &= \underline{\left( \underline{(a \cdot m)} * \underline{(b \cdot m)} \right) \cdot m} \\ \checkmark \quad \underline{(a + b)} \underline{\cdot m} &= \underline{\left( \underline{(a \cdot m)} + \underline{(b \cdot m)} \right) \cdot m} \end{aligned}$$

}



Modular  
Equation

$$(a * b) \% m = ((a \% m) * (b \% m)) \% m$$

$$(a + b) \% m = ((a \% m) + (b \% m)) \% m$$

Info ✓

Below equations are valid

$$(a * b) \% m = ((a \% m) * (b \% m)) \% m$$

$$(a + b) \% m = ((a \% m) + (b \% m)) \% m$$

// m is added to handle negative numbers

$$(a - b + m) \% m = ((a \% m) - (b \% m) + m) \% m$$

But,

$$(a / b) \% m \text{ may NOT be same as } ((a \% m) / (b \% m)) \% m$$

For example,  $a = 10$ ,  $b = 5$ ,  $m = 5$ .

$(a / b) \% m$  is 2, but  $((a \% m) / (b \% m)) \% m$   
is not defined.

Recursive function

$$\begin{aligned} & \underline{(a * b) \% m} \\ & \downarrow \\ & \underline{(a \% m * b \% m) \% m} \end{aligned}$$

long long **Pow**(int N, int R)

< if (N == 0) return 0;

if (R == 0) return 1;

return

N \* Pow(N, R-1)

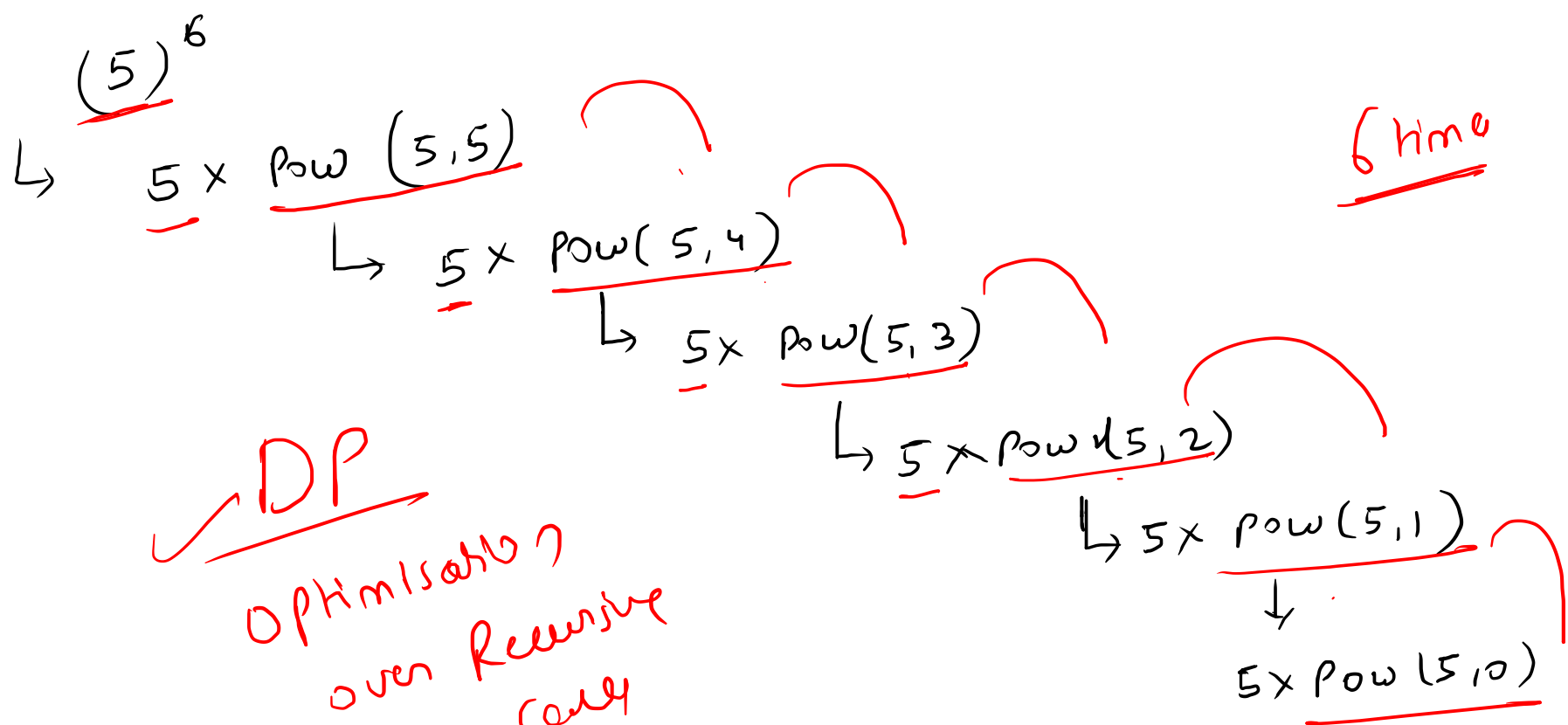
Ans  
a \* b

Ans % MOD ✓

(a \* b) % MOD

(a % MOD \* b % MOD) % MOD

Example:



6 time

✓ DP  
Optimisation  
over Recursive  
call



$5^6$  →  
↪

$5^3$  → ans

ans \* ans

$5^6$  ↪  $(6) \times$

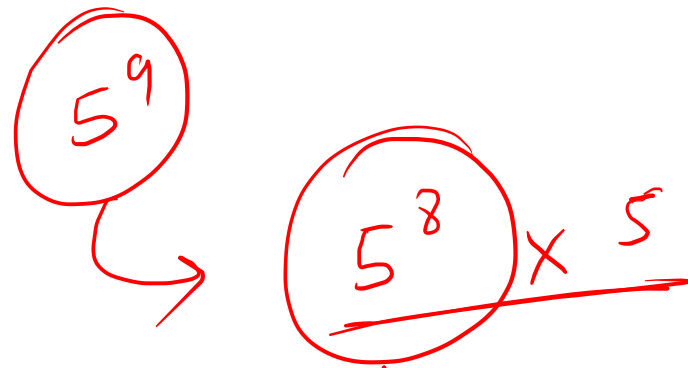
$(5^3) \rightarrow$   $5 \times 5 \times 5$   
↪ ans

]

3 times

RC

↪ return ans \* ans



ans  $\rightarrow$   $5^4$

$\hookrightarrow$  ans  $\times 5$



Now, we  
will optimise  
solution ✓

Ex:  $(5)^6$

$$\rightarrow \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$$

$\rightarrow$  if ( $R \% 2 == 0$ )

$\rightarrow$  long long ans =  $(5)^3 \rightarrow$

return ans \* ans ;

Optim  
 $(N)^{R/2}$

Ex  $(5)^9$

$\rightarrow$  if ( $R \% 2 \neq 0$ )

$\rightarrow$  long long ans =  $(5)^4 \rightarrow$

return  $\boxed{\text{ans} \times \text{ans} \times 5}$

(N)  $(R-1)/2$