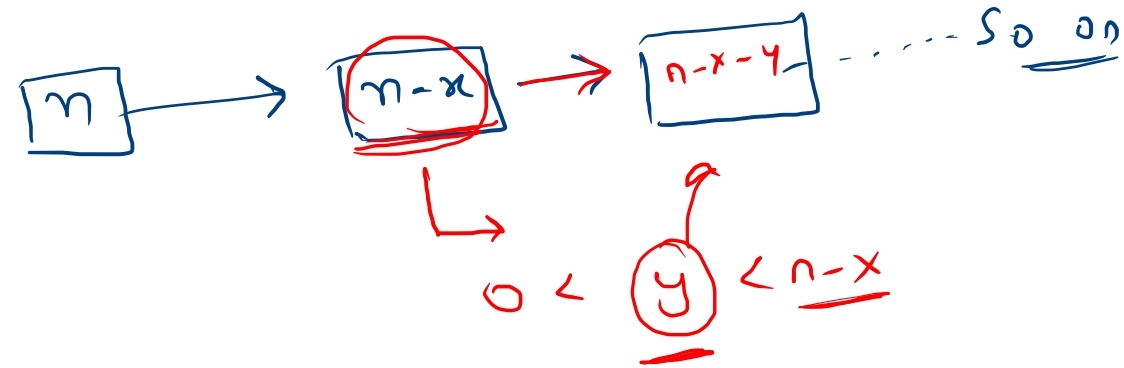
1) Divisor Game

Alice
Bob

n → integer

-> We want, Alice win ✓
-> First move is Alice

-> Given Integer n
   ↳ choose integer b/w 0 < x < n
   such that
   (n % x == 0)



IIb : $n = 2$

Soln : Alice → $x = 1$

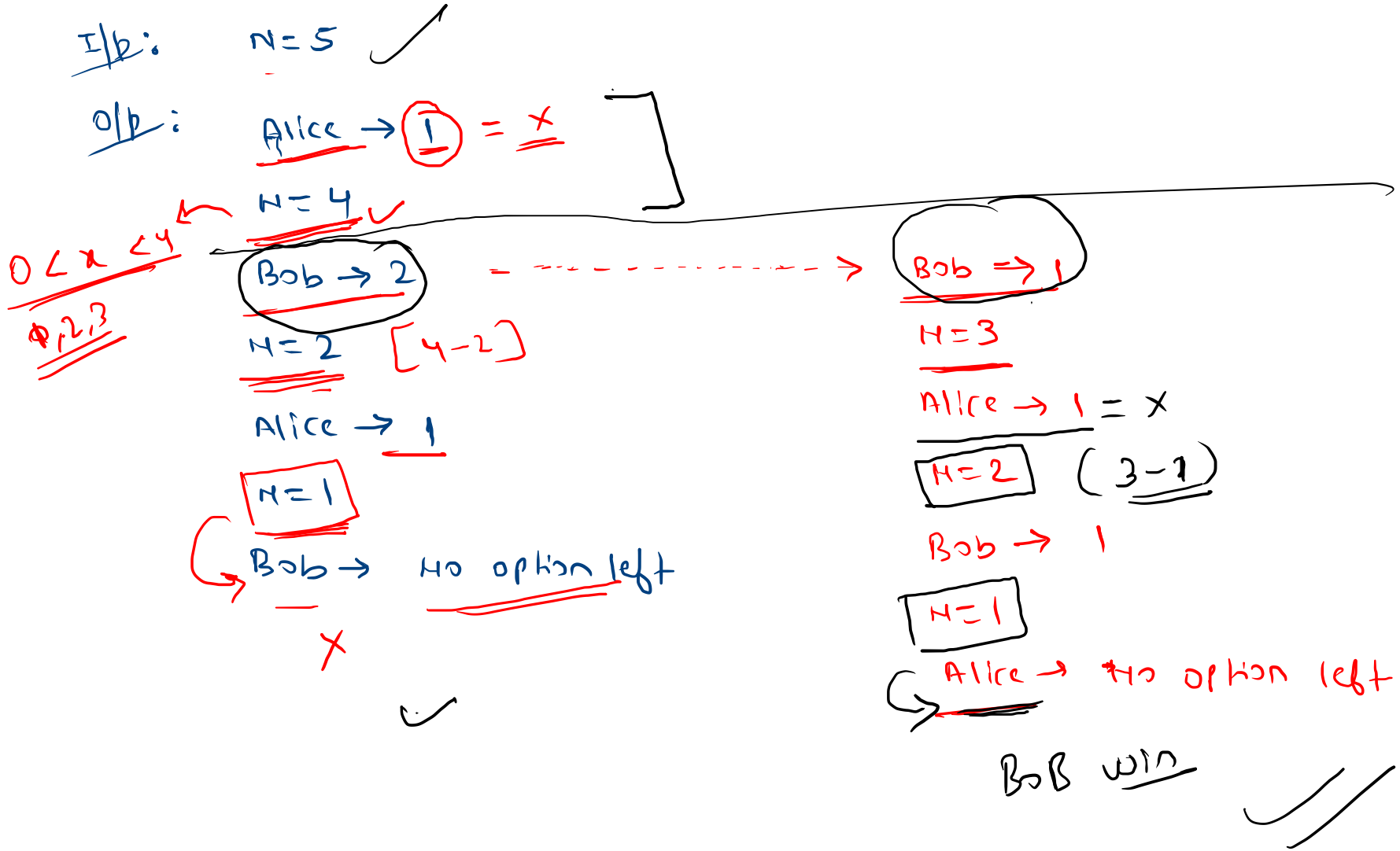
$n = 1$ $[2 - 1]$ →

Bob → No option left to choose no.

∴ Alice win

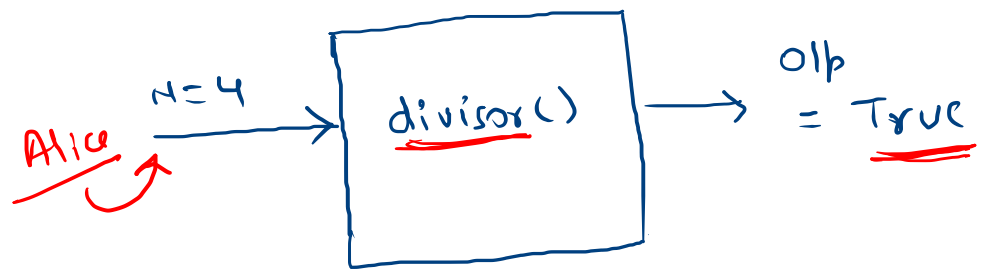
$n = 1$

$0 < x < 1$


I/p: N=5
O/p:
Alice → 1 = x
N=4
0 < x < 4
1,2,3
Bob → 2
N=2 [4-2]
Alice → 1
N=1
Bob → No option left
X
Bob → 1
N=3
Alice → 1 = x
N=2 (3-1)
Bob → 1
N=1
Alice → No option left
Bob win

Recursive solution

whoever gives I/p to magical function, It will give whether He will win or not



Sub problem

Logic Part

N = 4

↳ we have to choose divisor of N = 4 from x=1 to x=3 (1, 2, 3)

$0 < x < N$

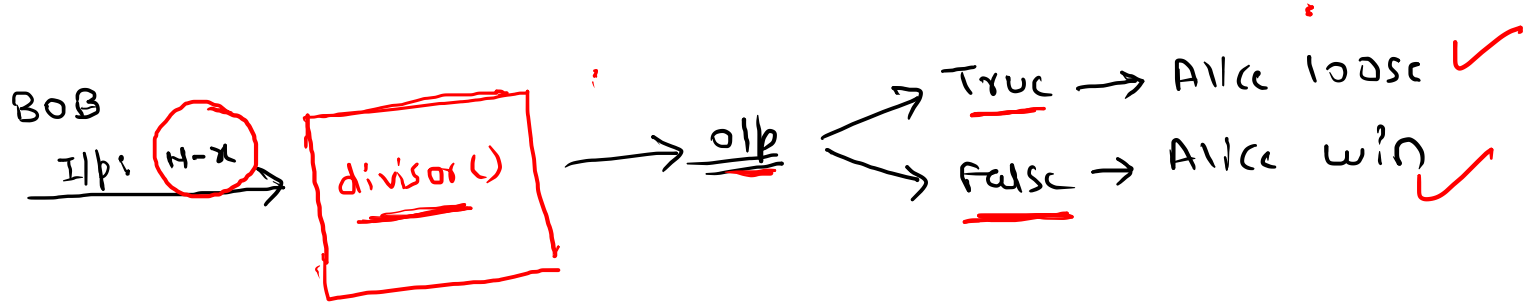
↳ 1st move → Alice

Let's suppose if we found "x" which can divide N

↳ so, N → N-x* → BoB

Alice

this No. is taken care by BoB and BoB will win or not ?



Recursive code

```
bool divisor (int N)
{
    // Base condition
    for (int x=1 ; x<N ; x++)
    {
        if (N % x == 0)
        {
            ! divisor (N-x) ;
        }
    }
    return false;
}
```

Alice

$0 < x < N$

1 to N-1

Alice

N

N-x → BoB

magical / Recursive

Base condition

Check o/p → Minimum Valid I/p

bool divisor (int N)

↳ N = 0, 1, 2, 3 ...

0 < x < N

0 < x < 1

Alice Loose

false

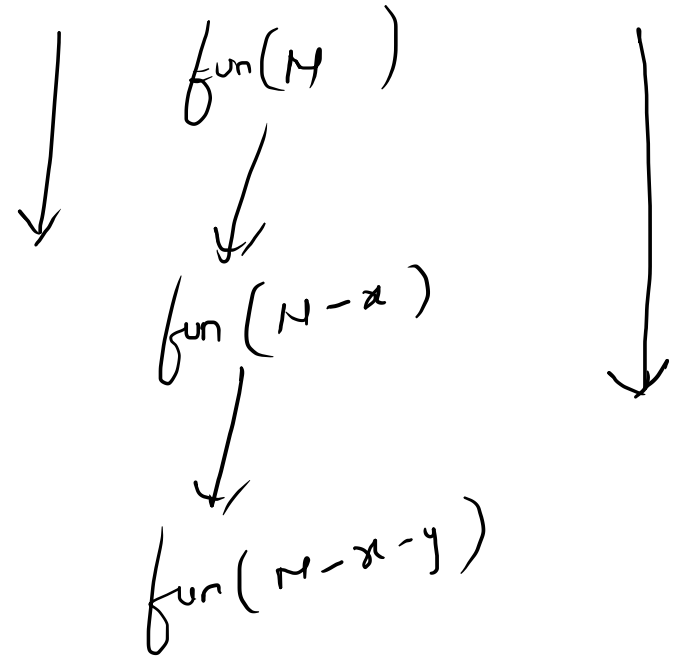
Alice Loose

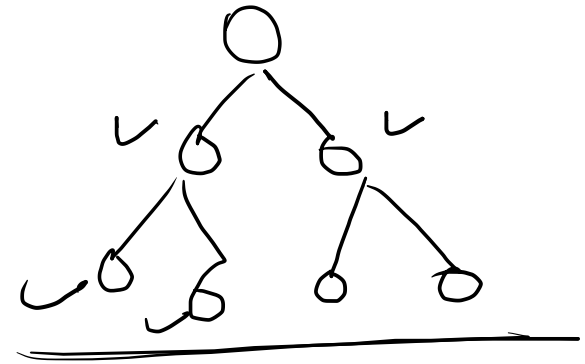
Recursive code

```
bool divisor (int N)
{
    if (N<=1) return false;
    # for (int x=1 ; x<N ; x++)
    {
        if (N%x == 0)
        {
            ! divisor (N-x);
        }
    }
    return false;
}
```

← Alice

Logic


fun(N)
fun(N-x)
fun(N-x-y)



How
Mathematical
Solution

•> N = 4

A → N = 2

→ N = 1

N = 1

→ Bob ✗

Some facts

•> Anyone has value N = 1 will loose

•> Anyone has value N = 2 will definetely win

•> For N >= 2, N will definetely reduced to 2

See the pattern,

✓ # 3 X=1

↳ 3-1 = 2

✓ # 5 X=1

↳ 5-1 = 4 ⇒ 4-1 ⇒ 3

↳ X=1

10

↳ 10-5 = 5

↳ X=5

Ultimately the Fight is for '2'

Whoever has '2' at the end will win

Common observation ⇒

```
if (N % 2 == 0)
    return True
else
    return False
```

N → even

Alice

N → odd

Alice X

Try Example

N = 61
23
14
7
6
3