Perfect Squares

I/p: n = 12

O/p: 3

$2^2 = 4$

(4 + 4 + 4)

1^2

1 + 1 + 1 + 1

4 +

4

6

→ write 'n' as a sum of Perfect square

⇒ ∴ and the no. of perfect square must be minimum

I/p: n = 13

O/p: 2

(4 + 9)

2

3

Now Logical concept

$\Rightarrow$ $\boxed{n = 12}$

$\hookrightarrow$ Sum of Perfect Square $= n$

$4+4+9+16+1$

(this should be in min number)

or, $n - \left(\text{sum of perfect squares} \right) = 0$

* we have many options of Perfect Squares to Subtract

17

$1+1+1+1+$

17

$N = 17$

$17 - 9 = 6$

$6 - 4 = 2$

$2 - 1 = 1$

$1 - 1 = 0$

4

Logic

Ex $n = 18$

$\therefore$ we want $\boxed{\text{minimum}}$ no. of Perfect squares

n - sum of p.s $= 0$

↳ we have many options to subtract perfect squares to 'n' and make it '0'

↳ $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$

• If you are thinking, that choosing the largest one (16) at 1st step would work

see,

$$18 - (4^2) = 2$$

$$2 - (1^2) = 1$$

$$1 - (1^2) = 0$$

Future steps

3 step

But, optimal solution is to pick $\boxed{9}$ first

$$18 - (3^2) = 9$$

$$9 - (3^2) = 0$$

Future steps

2 steps

optimal

$n = 18$

$n = 18 - 16 = 2$] 1

$n = 2$

$n = 2 - 1^2 = 1$] 1

$n = 1$

$n = 1 - 1^2 = 0$] 1

3 step

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

→ we are not sure, if subtracting 1, 4, 9 or 16 first would be optimal as we don't know the future result.

~~#~~ DP Rule : So, whenever, you are not sure which step would be optimal, no need to stress on guessing the optimal one.

TRY ALL OPTIONS, AND CHOOSE THE OPTIMAL ONE.

$$\begin{cases} \text{Solve}(18) \Rightarrow 18 - (1^2) \Rightarrow 17 \Rightarrow \text{Solve}(17) \\ \text{Solve}(18) \Rightarrow 18 - (2^2) \Rightarrow 14 \Rightarrow \text{Solve}(14) \\ \text{Solve}(18) \Rightarrow 18 - (3^2) \Rightarrow 9 \Rightarrow \text{Solve}(9) \\ \text{Solve}(18) \Rightarrow 18 - (4^2) \Rightarrow 2 \Rightarrow \text{Solve}(2) \end{cases}$$

these are the subproblems

$$\to \text{solve}(18) = 1 + \min\left[\left(\text{solve}(17), \text{solve}(14), \text{solve}(9), \text{solve}(2)\right)\right]$$

Recursive Solution

I/p

'n' → A number

Magical function

O/p → No. of min Perfect Square to get sum = n

I/p n=12 → Solve( ) → O/p 3

4+4+4

Logical Part ⟹

TRY ALL OPTIONS, AND CHOOSE THE OPTIMAL ONE.

Solve (18) ⟹ 18 − (1^2) ⟹ 17 ⟹ Solve (17)

Solve (18) ⟹ 18 − (2^2) ⟹ 14 ⟹ Solve (14)

Solve (18) ⟹ 18 − (3^2) ⟹ 9 ⟹ Solve (9)

Solve (18) ⟹ 18 − (4^2) ⟹ 2 ⟹ Solve (2)

these are the subproblems

~~18 − 5^2~~

$$\text{solve}(18) = 1 + \min(\text{solve}(17),\ \text{solve}(14),\ \text{solve}(9),\ \text{solve}(2))$$

Base Condition ⟶ int solve (int n)

smallest valid +ve Input

n = 0, 1, 2, 3 . . .

o/p : 0

if (n <= 0) return 0;

Recursive Code

n=18

```
int solve (int n)
{
    if (n <= 0) return 0;      ] Base condition
    int ans = INT_MAX;
    for (int i=1; i*i <= n; i++)
    {
        int sqNum = i*i;   // 1, 4, 9, 16
        int count = 1 + solve(n - sqNum);
        ans = min(ans, count);
    }
    return ans;
}
```

18 - 1

Try all option and choose the minimum one

i=1   1 < 18
i=2   4 < 18
i=3   9 < 18
i=4   16 < 18

18 - 1 = 17
4 = 14
9 = 9
16 = 2

$$
\begin{tikzcd}
\text{Recursive code} \arrow[rr] \arrow[dr] & & \text{DP code} \\
& \text{memoization} \arrow[ur] &
\end{tikzcd}
$$

$n \leftarrow 10^4$

```
vector<int> dp(10001, -1);
```

DP Code

vector<int> dp(10001, -1);

```
int solve(int n) **
{
    if(n<=0) return 0;          ] Base condition
    if(dp[n] != -1) return dp[n]
    int ans = INT_MAX;
    for(int i=1; i*i <=n; i++)
    {
        int sqNum = i*i;
        int count = 1 + solve(n-sqNum);
        ans = min(ans, count);
    }
    return dp[n] = ans;
}
```