Lab 4

Beam problems: Generation of element stiffness matrix and load vector from the shape function of a beam element of length "*a*" using numerical integration and verify it with the closed form solution.

$$w^e(x) = \boldsymbol{N}^e \boldsymbol{d}^e$$

where; $$\boldsymbol{N}^e = \left[1 - \frac{3x^2}{a^2} + \frac{2x^3}{a^3} \quad x - \frac{2x^2}{a} + \frac{x^3}{a^2} \quad \frac{3x^2}{a^2} - \frac{2x^3}{a^3} \quad -\frac{x^2}{a} + \frac{x^3}{a^2}\right]$$

$$\boldsymbol{d}^e = [w_1 \quad \theta_1 \quad w_2 \quad \theta_2]^T$$

$$\boldsymbol{K}^e = \int_0^a \boldsymbol{B}^{eT}\, EI\, \boldsymbol{B}^e\, dx, \qquad \boldsymbol{B}^e = \frac{d^2}{dx^2}[\boldsymbol{N}^e]$$

$$\boldsymbol{f}^e = \int_0^a \boldsymbol{N}^{eT} q(x)\ dx, \qquad x = \left(\frac{1+\xi}{2}\right)a, \xi \in [-1,1]$$

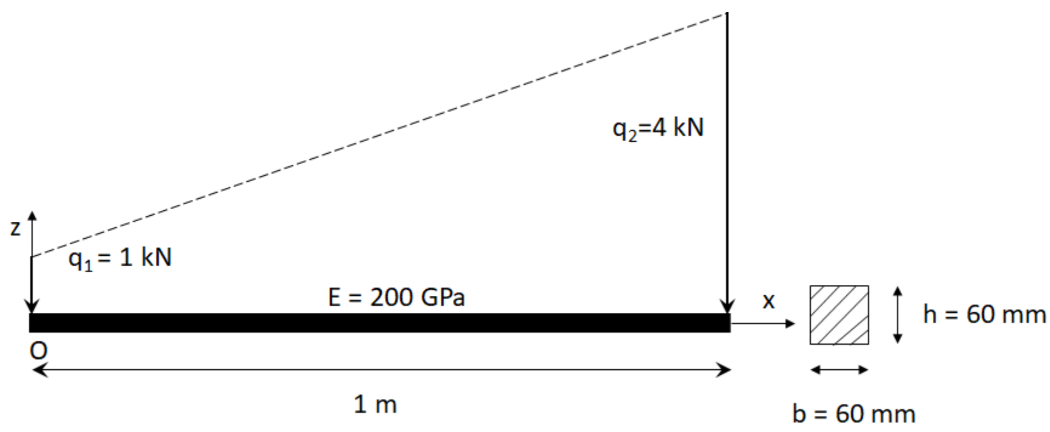

Table: Gauss Quadrature

NIP	ξ_k	W_k
2	$\xi_1 = -0.5773502691896258$ $\xi_2 = +0.5773502691896258$	$W_1 = 1.0000000000000000$ $W_2 = 1.0000000000000000$
3	$\xi_1 = -0.7745966692414834$ $\xi_2 = 0.0000000000000000$ $\xi_3 = +0.7745966692414834$	$W_1 = 0.5555555555555556$ $W_2 = 0.8888888888888889$ $W_3 = 0.5555555555555556$
4	$\xi_1 = -0.8611363115940526$ $\xi_2 = -0.3399810435848563$ $\xi_3 = +0.3399810435848563$ $\xi_4 = +0.8611363115940526$	$W_1 = 0.3478548451374539$ $W_2 = 0.6521451548625461$ $W_3 = 0.6521451548625461$ $W_4 = 0.3478548451374539$