20/10/2021

Assignment 5-6

1

1 (iii) RosenBrock Function $= 100(x_2 - x_1^2)^2 + (1-x_1)^2$

Prove (1,1) is unique global minimiser of f over $\mathbb{R}^2$

· First Order Necessary Condition :

$$\nabla f(x) = \begin{bmatrix} 400x_1^3 - 400x_1x_2 + 2x_1 - 2 \\ 200(x_2 - x_1^2) \end{bmatrix}$$

$$\therefore \nabla f(x)_{[1,1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \langle \text{Satisfied} \rangle$$

· Second Order Necessary Condition:

$$H_f(x) = \begin{bmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}$$

$$H_f(x)_{[1,1]} = \begin{bmatrix} 802 & -400 \\ -400 & 200 \end{bmatrix}$$

→ Checking all the principal minors of Hessian Matrix:

· $H_f(x)(1) = 802 > 0$

· $H_f(x)(2) = 802 \times 200 - (400 \times 400) = 400 > 0 \quad \{\det(H_f(x))\}$

∴ [1,1] satisfies both the FONC and SONC. Hence, it is a unique global minimiser of Rosenbrock Function.

Also, the fact that $f(x) > f((1,1)^T)$ for all $x \neq [1,1]^T$

(IV) Initial Guess = $[0, 0]^T = x^{(0)}$

Hint = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\therefore H_f(x)^{-1} = \frac{1}{1200(x_1^2 - 400x_2 + 2)\times 200 - 1600x_1^2}\begin{bmatrix} 200 & 400x_1 \\ 400x_1 & 1200x_1^2 - 400x_2 + 2 \end{bmatrix}$$

$$= \frac{1}{80{,}000(x_1^2 - x_2) + 400}\begin{bmatrix} 200 & 400x_1 \\ 400x_1 & 1200x_1^2 - 400x_2 + 2 \end{bmatrix}$$

$\therefore$ According to Newton's Algorithm:

$$x^{k+1} = x^k - H_f(x_k)^{-1} g_k$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{400}\begin{bmatrix} 200 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \end{bmatrix} \rightarrow \nabla f(x^{(0)}) \text{ or } g(x^0)$$

$$\therefore x^{(1)} = \frac{-1}{400}\begin{bmatrix} -400 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore H_f(x^{(1)})^{-1} = \frac{1}{80400}\begin{bmatrix} 200 & 400 \\ 400 & 1202 \end{bmatrix}$$

$$\therefore x^{(2)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{1}{80400}\begin{bmatrix} 200 & 400 \\ 400 & 1202 \end{bmatrix}\begin{bmatrix} 400 \\ -200 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

As expected 'Newton's method' converges to minimiser in just two steps.

3

(v) $\alpha_k = 0.05$ * Gradient Descent Algorithm : $x^{k+1} = x^k - \alpha_k \nabla f(x^k)$

$\therefore$ $x^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 0.05 \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$

$x^{(2)} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} - 0.05 \begin{bmatrix} 400(0.1)^3 + 2(0.1) - 2 \\ 200(0 - 0.1^2) \end{bmatrix}$

$= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} - 0.05 \begin{bmatrix} -1.4 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} 0.17 \\ 0.1 \end{bmatrix}$ $\rightarrow$ Not nearly close to original minimiser in 2 iterations.

3 (ii) The DFP algorithm converges to $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ with an initial guess of $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and converges to $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ for initial guess of $\begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$. Therefore the algorithm steps towards the closest local minimiser, depending on the choice of initial guess.