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Assignment #3

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1. $f(x) = x^2 + 4\cos(x)$ $\quad x \in \mathbb{R}$ $\quad x^*$ {minimiser} between $[1, 2]$

Uncertainty Range = 0.2

(a) Plot $f(x)$ vs x in given domain $[1,2]$ can be seen as generated by MATLAB. {at end of this solution}

(b)

Iteration (k)	a_k	b_k	$f(a_k)$	$f(b_k)$	Uncertainty Level
0	1	2	3.1612	2.3354	[1, 2]
1	1.381	1.618	2.662	2.4292	[1.381, 2]
2	1.618	1.7635	2.4292	2.3439	[1.618, 2]
3	1.7635	1.854	2.3439	2.3196	[1.7635, 2]
4	1.854	1.9096	2.3196	2.3171	[1.854, 2]
5	1.9096	1.944	2.3171	2.3207	[1.854, 1.944]

{Final Uncertainty range = $0.09 \leq 0.2$}

Solution: Uncertainty range = 0.2

∴ According to GSM, if after 'N' iterations, the uncertainty range is to reduce to 0.2 then,

$$2(0.61803)^N \leq 0.2$$

$$\therefore N \log(0.61803) \leq \log(0.1) \quad (\because \log(0.61803) < 0)$$

$$\therefore N \geq 4.79$$

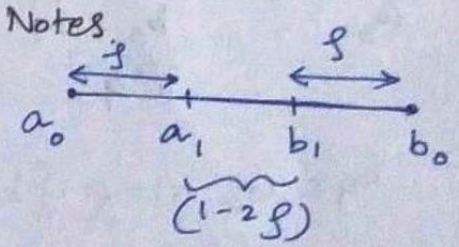
$$\therefore N \approx 5$$

 # Iteration 1:

$a_0 = 1$

$b_0 = 2$

$\rho = \frac{3-\sqrt{5}}{2} = 0.382$ → Derived in Notes



$\therefore a_1 = a_0 + \rho(b_0 - a_0) = 1.381$

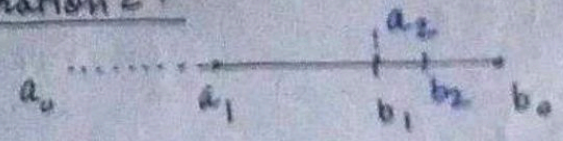
$b_1 = a_0 + (1-\rho)(b_0 - a_0) = 1.618$

$f(a_1) = 2.662$; $f(b_1) = 2.4292$

$\therefore f(a_1) > f(b_1)$ ∴ Minimiser lies b/w $[a_1, b_0]$

∴ New uncertainty level $[1.381, 2]$

Iteration 2:



- We choose a_2 such that it coincides with b_1.

$\therefore a_2 = 1.618$ and $f(a_2) = 2.4292$

Now $b_2 = a_1 + (1-\beta)(b_0 - a_1)$

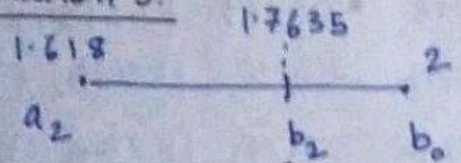
$= 1.381 + (0.618)(2 - 1.381) = 1.7635$

AND $f(b_2) = 2.3439$.

$\therefore f(a_2) > f(b_2)$

$\therefore$ Minimiser lies b/w $[a_2, b_0] = [1.618, 2]$

Iteration 3:



Similarly $a_3 = b_2$

$a_3 = 1.7635$ & $f(a_3) = 2.3439$.

$b_3 = a_2 + (1-\beta)(b_0 - a_2)$

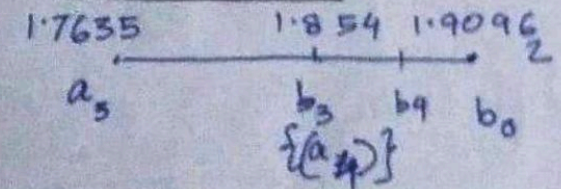
$= 1.618 + (0.618)\{2 - 1.618\} = 1.854$

AND, $f(b_3) = 2.3196$

$\therefore f(a_3) > f(b_3)$

$\therefore$ Minimiser lies b/w $[a_3, b_0] = [1.7635, 2]$

Iteration 4



$a_4 = 1.854$

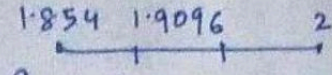
$f(a_4) = 2.3196$.

$b_4 = a_3 + (1-\beta)\{b_0 - a_3\} = 1.7635 + 0.618\{2 - 1.7635\} = 1.9096$

$\therefore f(b_4) = 2.3171$

$\therefore f(a_4) > f(b_4)$ $\therefore$ Minimiser lies b/w $[1.854, 2]$

Iteration 5 {AND LAST}



a_5 will coincide with b_4

$\therefore a_5 = 1.9096$

$f(a_5) = 2.3171$

and $b_5 = a_4 + (1-\varphi)\{b_0 - a_4\}$

$= 1.854 + 0.618(2 - 1.854) = 1.944$

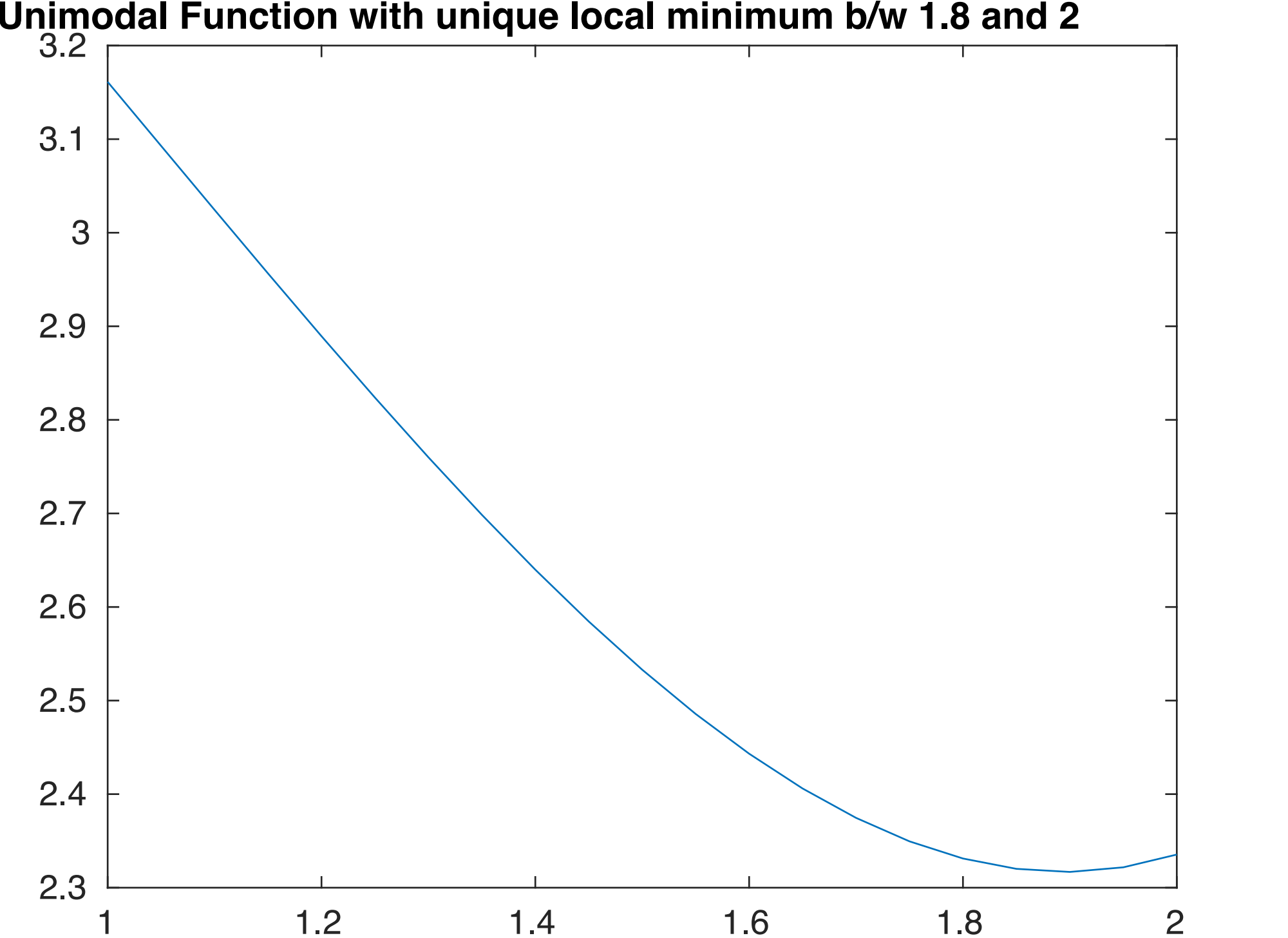
AND, $f(b_5) = 2.3207$

$\therefore f(b_5) > f(a_5)$

$\therefore$ Minimiser must lie b/w $[a_4, b_5] \approx [1.854, 1.944]$

& Uncertainty Range = $0.09 \leq 0.2$

{On checking via MATLAB function 'fminbnd', the result is shown 1.8955 ~~wd~~ which is inside our interval}


Unimodal Function with unique local minimum b/w 1.8 and 2
3.2
3.1
3
2.9
2.8
2.7
2.6
2.5
2.4
2.3
1
1.2
1.4
1.6
1.8
2

2. Output of MATLAB Code included in ZIP file

```
Left point of initial range,  a0: 1
Right point of initial range, b0: 2
Give Final Interval Length DLT/ Final uncertainty range: 0.23


 Current Interval after Iteration 1 is:
     1.3820E+00      2.0000E+00

 Current Interval after Iteration 2 is:
     1.3820E+00      1.7639E+00

 Current Interval after Iteration 3 is:
     1.5279E+00      1.7639E+00

 Current Interval after Iteration 4 is:
     1.5279E+00      1.6738E+00

 Current Interval after Iteration 5 is:
     1.5836E+00      1.6738E+00


Number of Function Evaluations = 5

 Value of X closest to minimiser, Value of function F at X =
     1.6180E+00      7.6805E+00

Value of minimum X calculated by MATLAB function fminbnd and Value of function F at X =
     1.6094E+00      7.6804E+00
```