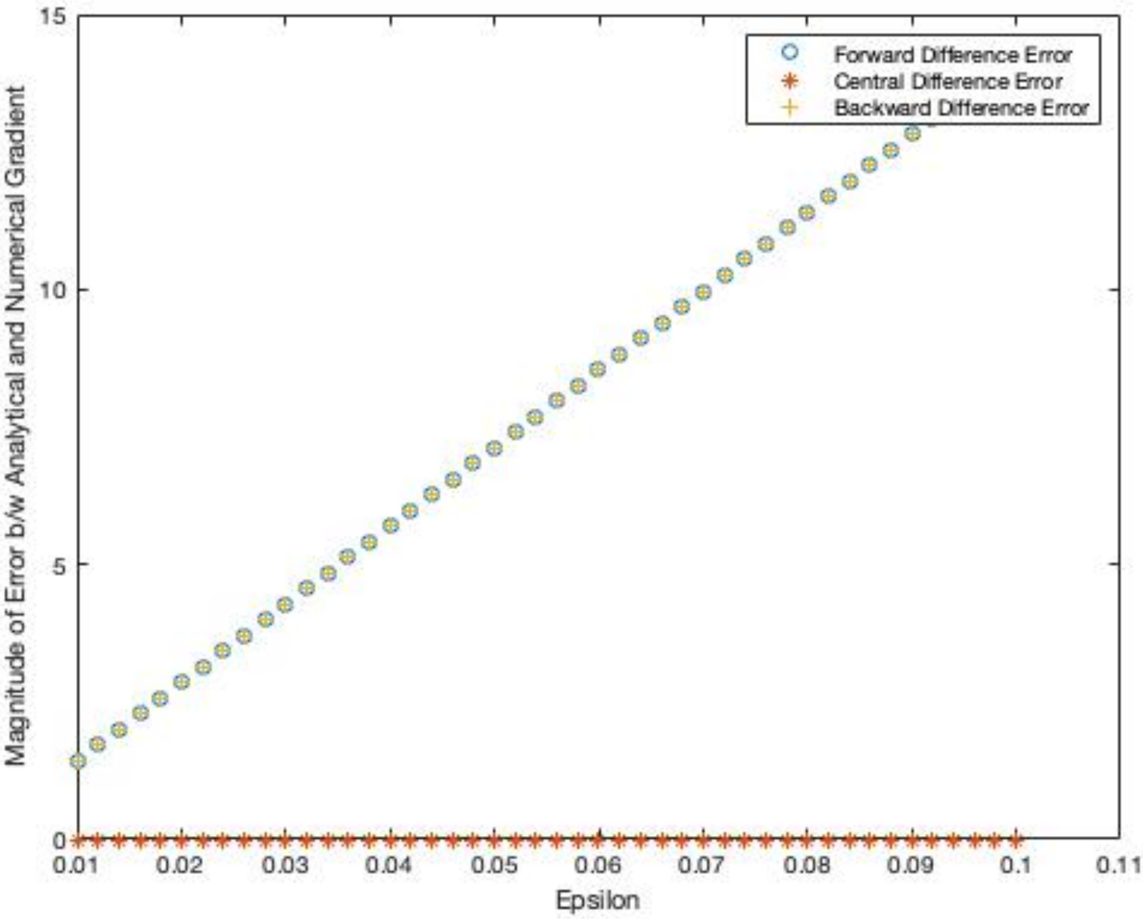
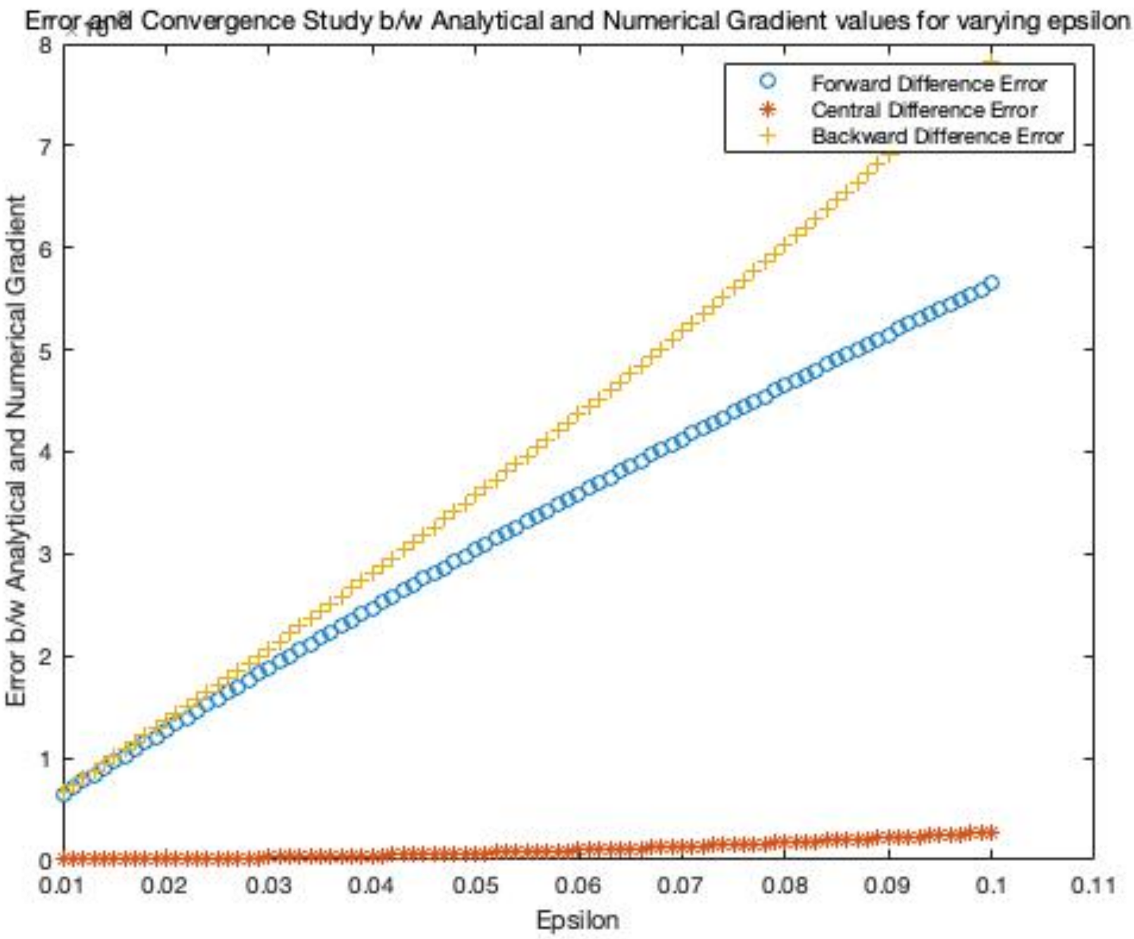
1. | Analytical Gradient | Forw. Diff. | Backw. Diff. | Central Diff. |
|---|---|---|---|
| 1.1896E+02 | 1.1956E+02 | 1.1835E+02 | 1.1896E+02 |
| 2.5705E+02 | 2.5834E+02 | 2.5576E+02 | 2.5705E+02 |



2. Analytical Gradient Forw. Diff. Backw. Diff. Central Diff.

Analytical Gradient	Forw. Diff.	Backw. Diff.	Central Diff.
-3.4200E-02	-3.3553E-02	-3.4868E-02	-3.4203E-02
2.6000E-03	2.6089E-03	2.5909E-03	2.6000E-03

<The error values are in range 10E-3 that is being covered by the title of the graph>



3. The Directional Derivate of Given function through forward difference is -2.3801E+03.
COMPARING WITH ANALYTICAL VALUE OF DIRECTIONAL DERIVATIVE
The Directional Derivate calculated through analytical forumla is -2.4020E+03.
<MATLAB Output>

<iii> Given $f = 100(x_2 - x_1^2)^2 + (1-x_1)^2$ $\quad \underline{x}^0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $d = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

<a> write exp^n of $f(\alpha) = f(\underline{x}(\alpha))$ along $\underline{d}$ from x^0

<b> use forward difference to estimate directional derivative of f along $\underline{d}$ at $x^0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Hint : $D_{\underline{d}} f(x^0) = \left.\frac{\partial f}{\partial \alpha}\right|_{\alpha=0}$ and $f(\alpha) = f(\underline{x}^0 + \alpha \underline{d})$

Solution:

<a> ~~f(α) = 100~~ $\underline{x}(\alpha) = (\underline{x}^0 + \alpha \underline{d})$

$$\therefore \underline{x}(\alpha) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \alpha \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2-\alpha \\ 1 \end{bmatrix} \begin{matrix} x_1(\alpha) \\ x_2(\alpha) \end{matrix}$$

. Substituting values of $x_1(\alpha)$ & $x_2(\alpha)$:
and $f(x(\alpha)) = f(\alpha) = 100(1-(2-\alpha)^2)^2 + (1-(2-\alpha))^2$

$$\therefore f(\alpha) = 100\left[1-(2-\alpha)^2\right]^2 + \left[1-(2-\alpha)\right]^2$$

<b> Directional Derivative:

$$D_{\underline{d}} f(x^0) = \left.\frac{d f(\alpha)}{d\alpha}\right|_{\alpha=0}$$

→ If we have to use forward difference the calculation is going to look ~~lif~~ like:

$$\lim_{\substack{\epsilon \to 0 \\ \alpha=0}} \frac{f(\alpha+\epsilon) - f(\alpha)}{\epsilon} = \left.\frac{d f(\alpha)}{d\alpha}\right|_{\alpha=0} = D_{\underline{d}} f(\underline{x}^0)$$

<Rest is done in MATLAB code>

4.

⟨IV⟩ Determine ~~we~~ whether the following function positive definite or not :

$$f = 2x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 4x_2x_3 \qquad \underline{x} \in \mathbb{R}^3$$

Solⁿ

a) In order to write this in form $x^T A x$ where $x = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \rightarrow (A)$$

· Here $\underline{A}$ is a symmetric matrix w/o loss of generality.

∴ On multiplication of expression (A) we get :

$$A_{11}x_1^2 + A_{22}x_2^2 + A_{33}x_3^2 + (A_{12}+A_{21})x_1x_2 + (A_{31}+A_{13})x_1x_3 + (A_{23}+A_{32})x_2x_3.$$

→ Comparing with our original quadratic expression we get :

$$\left.\begin{aligned} &\bullet\ A_{11} = 2 \qquad A_{22} = 5 \qquad A_{33} = 3. \\ &\bullet\ A_{21} = A_{12} = -1 \\ &\bullet\ A_{23} = A_{32} = -2 \\ &\bullet\ A_{13} = A_{31} = 0 \end{aligned}\right\}$$

$$\therefore \underline{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -2 & 3 \end{bmatrix}$$

Plugging this ~~eqn~~ value of $\underline{A}$ in MATLAB Code for Q4 we try to determine that whether $\underline{A}$ is positive-definite by Sylvester's test. Also if it's positive definite all eigen values must be positive {Additional Check}

5. Completely solved in Code.