Python Problem Set for Semester-I, 2022

1. Write a Python function for factorial, where an interger n is the argument. Use this function to calculate factorial for a user given integer (maximum value 10). Print the value of the factorial.

2. User has given a DNA sequence: a string consisting of characters A, T, G and C. Your task is to print the character with longest repetition in the sequence. Also print the number of repetation.
   Example
   User given sequence $\rightarrow$ ATGGGTCCG. Answer $\rightarrow$ G

3. Take an integer n as user input. If n is even, then n will be devided by 2. If n is odd, then n will be multiplied by 3 and adds 1. This process will continue till n becomes 1. Print all the intermediate numbers. Take input numbers as $n = 201, 537$.
   Example
   $3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1, \quad 12 \rightarrow 6 \rightarrow 3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$

4. Get all the intermediate numbers obtained from above program as an list. Print the list.

5. Obtain a string with at least 9 characters as user input. Print the string in reverse order.

6. Print numbers in the following manner

```
1                   1
1 2                 2  3
1 2 3               4  5  6
1 2 3 4             7  8  9  10
1 2 3 4 5           11 12 13 14 15

15 14 13 12 11
10 9  8  7
6  5  4
3  2
1
```

7. Construct the number in reverse order of a user given number. *Example : If the given number is* 98452 *then you have to construct the number* 25489.

8. Write a Pyhton function for calculating the factorial of a user given integer. Test your function writing a main program for calculating the factorial of $7, 9, 12$.

9. Write a Python function to evalute the following function

$$f(x) = a + bx^2$$

   Write a three column data file separated by commas in the following manner. In first column the x-values to be written for $-2 \leq x \leq 2$ with increment 0.1. In 2nd column write the y-values corresponding to each of the x-values for $a = 0.2, b = -0.5$. In 3rd column write the y-values for $a = -2.2, b = -0.5$.

10. Read the data file created in the above problem and store the three columns in three different lists. Let us say the 1st column x-values list is x, 2nd column y-values list is y_1 and 3rd column y-values list is y_2. Now, write another data file with the columns x and $y_1 + y_2$.

11. Create a list with elements $1, 2, ..., 10$ using

    (a) list comprehension.

(b) range function.
(c) for loop starting from a blank list.

12. Create a list with elements *'p', 'r', 'o', 'g', 'r', 'a', 'm', 'i', 'z'*.
    (a) Print elements with indices $3, 5, 7$.
    (b) Print all elements beyond index 4.
    (c) Print elements from index 2 to index 5.
    (d) Print all elements in reverse order.

13. Create two lists with elements $1, 3, 5, 7, 9$ and $2, 4, 6, 8, 10$. Print the list created with the concatenation of these two lists
    (a) using the list method `extend`.
    (b) using the operator `+`.

14. Create a list with elements $-1, 2, -2, 4, -3, 6, -4, 8$.
    (a) Print the number of elements of the list.
    (b) Modify the list with addition of another element -5. Print the modified list.
    (c) Print the index of the element 6.
    (d) Modify the list removing the element -2. Print the modified list.
    (e) Print the sum of the elements of the list using the list method `sum`.
    (f) Get the maximum and minimum of the list using the list method `max` and `min`.

15. Print the list $[[-1, 2, 5], [2, -6, 9], [12, -11, -25]]$ in the following way

$$\begin{matrix} -1 & 2 & 5 \\ 2 & -6 & 9 \\ 12 & -11 & -25 \end{matrix}$$

16. Write a Python program to sort a given set of numbers. Check your program for the following set of numbers, $2.07, -3.29, 5.83, 7.29, -2.63, -8.28, 4.61, 0.94, -8.29, -1.38, -4.69$.

17. For a given matrices $A_{m\times n}$ write a Python function to obtain $kA_{m\times n}$. Where k is a scalar. Check your function with the following values.

$$A = \begin{pmatrix} -12.4 & 3.37 & -22.83 \\ 14.94 & -26.28 & -41.28 \end{pmatrix} \quad \text{and} \quad k = 2.95$$

18. For two given matrices $A_{m\times n}$ and $B_{m\times n}$ write a Python function for matrix addition and substraction. Check your programs for

$$A = \begin{pmatrix} -2.4 & 3.27 & -2.86 \\ 1.64 & -6.28 & -4.28 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4.72 & 2.39 & -0.89 \\ -7.27 & -1.67 & 7.92 \end{pmatrix}$$

19. For two given matrices $A_{m\times p}$ and $B_{p\times n}$ write a Python function for matrix multiplication. Check your function for these matrices

$$A = \begin{pmatrix} -2.4 & 3.27 & -2.86 \\ 1.64 & -6.28 & -4.28 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4.72 & 2.39 \\ -7.27 & -1.67 \\ -2.81 & 1.56 \end{pmatrix}$$

20. Use the above functions to calculate

$$1.2A^2, \quad 3.2A^3, \quad 2A^4$$

for the matrix

$$A = \begin{pmatrix} 4.23 & -2.39 \\ -5.27 & -1.67 \end{pmatrix}$$

21. Use the Python functions 17, 18, 19 to calculate the following series

$$I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \frac{A^5}{5!}$$

for

$$A = \begin{pmatrix} -2.4 & 3.27 \\ 1.64 & -6.28 \end{pmatrix}$$

22. Using the Python functions 18, 19 calculate

$$AB - BA \ , \quad AB + BA \ , AB^T + BA^T, \quad \mathrm{Tr}(B)A^T \text{ and } A + B + 5I$$

for

$$A = \begin{pmatrix} -2.4 & 3.27 \\ 1.64 & -6.28 \end{pmatrix} \text{ and } B = \begin{pmatrix} -0.4 & 7.27 \\ 5.64 & -3.48 \end{pmatrix}$$

23. Write two Python functions to calculate permutation and combination defined as

$$^nP_k = \frac{n!}{(n-k)!}$$

$$^nC_k = \frac{n!}{k!(n-k)!}$$

Here, take n and k are the arguments of those Python functions. Check your functions by calculating $^5P_3$ and $^5C_3$.

24. Verify the following finite series relations for $n = 100$.

(a)

$$1 + 2 + 3 + 4 + ... = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

(b)

$$1 + 2^2 + 3^2 + 4^2 + ... = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(c)

$$1 + 2^3 + 3^3 + 4^3 + ... = \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

25. Get the sum of the following series correct upto 5 decimal places.

(a)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... = \sum_{n=0}^{\infty} \frac{1}{2^n}$$

Comapare your result with the exact value 2.0.

(b)

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + ... = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

Comapare your result with the exact value $\ln 2$.

(c)

$$\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} + ... = \sum_{n=0}^{\infty} \frac{1}{(2n+1)(2n+2)} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)2n}$$

Comapare your result with the exact value $\ln 2$.

(d)

$$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+...=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)(n+2)}$$

Comapare your result with the exact value $2\ln 2-1$.

(e)

$$\frac{1}{1.2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}+\frac{1}{4.2^4}+...=\sum_{n=1}^{\infty}\frac{1}{n2^n}$$

Comapare your result with the exact value $\ln 2$.

(f)

$$1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+...=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}$$

Comapare your result with the exact value $\frac{1}{e}$.

(g)

$$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...=\sum_{n=0}^{\infty}\frac{1}{n!}$$

Comapare your result with the exact value e.

26. Determine the sum of the following series correct upto 5 decimal places. From that value determine the value of π correct upto 5 decimal palces.

(a)

$$-1+\frac{1}{3}-\frac{1}{5}+\frac{1}{7}-\frac{1}{9}+...=\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}=-\frac{\pi}{4}$$

(b)

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+...=\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$$

(c)

$$1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+...=\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$$

(d)

$$1+\frac{1}{2^6}+\frac{1}{3^6}+\frac{1}{4^6}+\frac{1}{5^6}+...=\sum_{n=1}^{\infty}\frac{1}{n^6}=\frac{\pi^6}{945}$$

27. Write Python functions to calculate the sum of the following series correct upto 5 decimal places. Verify your result for $x=0.75$ with value calculated from the appropriate mathematical function from the `math` module.

(a)

$$e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}$$

(b)

$$xe^x=\sum_{k=0}^{\infty}k\frac{x^k}{k!}$$

(c)

$$\sin x=\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

(d)

$$\sinh x=\sum_{k=0}^{\infty}\frac{x^{2k+1}}{(2k+1)!}$$

(e)

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

(f)

$$\cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

(g)

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^{k}}{k}$$

(h)

$$\sin^{-1} x = \sum_{k=0}^{\infty} \frac{(2k)! x^{2k+1}}{2^{2k}(k!)^2(2k+1)}$$

(i)

$$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$$

28. Using the above Python functions calculate

    (a) $1 - e^{0.6x^2}$ for $x = 1.42$
    (b) $e^{\sin x}$ for $x = \frac{\pi}{6}$
    (c) $\cos(e^x)$ for $x = 1.62$
    (d) $1 + xe^x + \sin(2x)$ for $x = 0.63$
    (e) $\sin^{-1}(e^{-x})$ for $x = 2.52$

29. Create a data file to store the following data in two columns (upper row is x-values and lower row is y-values).

1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
0.718	0.525	0.406	0.205	0.038	-0.147	-0.300	-0.471	-0.645	-0.887

Read the data file and create two arrays. Fit the data with $a + bx$. Obtain the values of a and b.

30. Create a data file to store the following data in two columns (upper row is x-values and lower row is y-values).

3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50
-5.156	-6.869	-8.037	-10.337	-12.435	-13.699	-17.196	-18.771	-21.267	-24.301

Read the data file and create two arrays. Fit the data with $a + bx^2$. Obtain the values of a and b.

31. Create a data file to store the following data in two columns (upper row is x-values and lower row is y-values).

0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.087	0.046	0.019	0.009	0.005	0.002	0.001	0.000	0.000	0.000

Read the data file and create two arrays. Fit the data with ae^{bx}. Obtain the values of a and b.

32. Create a data file to store the following data in two columns (upper row is x-values and lower row is y-values).

0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.522	5.473	18.110	47.656	96.828	193.936	310.719	438.571	644.893	1003.880

Read the data file and create two arrays. Fit the data with ax^b. Obtain the values of a and b.

33. Create a data file to store the following data in two columns (upper row is x-values and lower row is y-values).

0.50	0.70	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30
2.946	4.033	4.810	5.650	5.938	6.650	7.158	7.526	7.786	7.733

Read the data file and create two arrays. Fit the data with $a + b\ln x$. Obtain the values of a and b.

34. Create a data file to store the following data in two columns (upper row is x-values and lower row is y-values).

-2.29	-1.78	-1.27	-0.76	-0.25	0.25	0.76	1.27	1.78	2.29
0.000	1.115	1.533	1.742	1.805	1.737	1.714	1.534	1.161	0.000

Read the data file and create two arrays. Fit the data with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Obtain the values of a and b.

35. An equation with one unknown is defined as

$$f(x) = 0$$

One root within the bound $[a, b]$ of this equation could be obtained with a tolerance ϵ by bisection method. Write a Python function to solve a algebraic equation by bisection method taking f, a, b, ϵ as arguments. Verify your function by determining roots with tolerance 10^{-5} of the following equations within the ranges given.

(a)
$$f(x) = x^2 - 1 = 0 \quad \text{within the ranges} \quad [-5, 0] \text{ and } [0, 5]$$

(b)
$$f(x) = x^3 - 3x^2 - 6x + 8 = 0 \quad \text{within the ranges} \quad [-5, 0], [0, 3] \text{ and } [3, 5]$$

(c)
$$\sin(2x) - 2\sin\left(x - \frac{\pi}{4}\right) - 1 = 0 \quad \text{within the ranges} \quad [-2\pi, 0] \text{ and } [0, 2\pi]$$

(d)
$$\tan^2 x - 3 = 0 \quad \text{within the ranges} \quad [-5, 0] \text{ and } [0, 5]$$

(e)
$$\tan x - x = 0 \Rightarrow \sin x - x\cos x = 0 \quad \text{within the ranges} \quad [-\pi, \pi], [\pi, 2\pi] \text{ and } [2\pi, 3\pi]$$

36. One root the equation with one unknown
$$f(x) = 0$$
be determined with tolerance ϵ by Newton-Raphson method for a given starting point x_0. Newton-Raphson method requires derivative of $f(x)$, i.e, $f'(x)$. Write a Python function with arguments f, f', x_0, ϵ to determine the root of the equation $f(x) = 0$. Use this Python function to determine the roots of the following equation with the given x_0.

(a)
$$x^2 - 9x + 8 = 0 \quad \text{for} \quad x_0 = \; 0.0 \text{ and } 10.0$$

(b)
$$\sin 2x = 0 \quad \text{for} \quad x_0 = \; -1.82,\; 0.45 \text{ and } 1.2$$

(c)
$$\sin^2 x - \sin x - 2 = 0 \quad \text{for} \quad x_0 = \; \pi \text{ and } 3\pi$$

(d)
$$\tan 2x - 1 = 0 \quad \text{for} \quad x_0 = \frac{3\pi}{16} \text{ and } \frac{11\pi}{16}$$

37. The first order initial value problem is defined as

$$\frac{dy}{dx} = f(x, y) \quad \text{with } y(x_0) = y_0.$$

Numerically the solution could be obtained by Euler method. Numerical solution is given by discrete y-values $y_0, y_1, ..., y_n$ for dicrete x-values $x_0, x_1, ...x_n$. Write a Python function for solving the above differential equation by Euler method. In this Python function put $f(x, y), x_0, y_0$ and $x_0, x_1, ...x_n$ as arguments. Obviously, the return is the array $y_0, y_1, ..., y_n$. Use the above Python function to obtain the solution of the following initial value problems. Write a data-file containing the x-values and y-values in two columns.

(a)
$$\frac{dy}{dx} = x^2 + 1 \quad \text{with } y(1) = 4$$

(b)
$$\frac{dy}{dx} = \frac{x}{y^2} \quad \text{with } y(0) = 0$$

(c)
$$\frac{dy}{dx} = \frac{x^2 + 2}{y} \quad \text{with } y(0) = 0$$

(d)
$$\frac{dy}{dx} = 2x(y^2 + 9) \quad \text{with } y(0) = 0$$

(e)
$$\frac{dy}{dx} = y^2 - 2x + 2 \quad \text{with } y(0) = 0$$

(f)
$$\frac{dy}{dx} = \frac{e^x}{y} \quad \text{with } y(0) = 1$$

(g)
$$\frac{dy}{dx} = -\frac{x \cos x}{1 - 6y^5} \quad \text{with } y(\pi) = 0$$