

1. Calculate the 95% confidence interval for the true error E

The hypothesis h_1 was tested on $N=45$ instances. The observed error rate $E^{\wedge} = 6.67\%$.

The number of errors is $k = N \times E^{\wedge} = 45 \times 0.0667 = 3$ errors. Since N is relatively small ($N=45$) and k is small ($k=3$), we use the Normal Approximation to estimate the confidence interval for the true error E , assuming the errors follow a Binomial distribution.

The $N\%$ confidence interval for the true error ϵ is given by:

$$E^{\wedge} \pm Z(N) * \sqrt{E^{\wedge}(1-E^{\wedge})/N}$$

For a 95% confidence interval, the two-sided $Z(N)$ value is $Z(95) = 1.96$.

- $E^{\wedge} = 0.0667$
- $1 - E^{\wedge} = 1 - 0.0667 = 0.9333$
- $N = 45$

The standard error is:

$$\begin{aligned} \text{Root}(E^{\wedge}(1 - E^{\wedge})/N) &= \text{Root}(0.0667 * 0.9333 / 45) \\ &= \text{Root}(0.0622 / 45) \\ &= \text{Root}(0.001382) \\ &= 0.03718 \end{aligned}$$

The margin of error is:

$$\begin{aligned} Z(95) \times \text{Standard Error} &= 1.96 \times 0.03718 \\ &= 0.07287 \end{aligned}$$

The 95% confidence interval for the true error $E = 0.0667 \pm 0.07287$

The interval is from max (0, $0.0667 - 0.07287$) to min (1, $0.0667 + 0.07287$), because an error rate must be between 0 and 1.

Confidence Interval (CI): **[0%, 13.96%]**