

3. Confidence Level for Favourite Algorithm Outperforming Decision Tree

We are comparing two algorithms using

10-fold cross-validation repeated 10 times (paired t-test is appropriate for this setup, but the table only shows the results of the 10 final folds). The comparison involves the mean difference in error rates.

U^d between the two algorithms over the $k = 10$ folds.

Totals:

$$d_i = E^{\text{(Fav)}} - E^{\text{(DT)}} = -0.0109$$

$$(d_i - U^d)^2 = 2.299 \times 10^{-4}$$

- Number of folds: $k = 10$.
- Mean difference: $U^d = \text{Sum}(d_i) / k = -0.0109 / 10 = -0.00109$.
 - This negative mean difference suggests the Decision Tree (DT) algorithm has a lower average error rate (a higher accuracy) than the Favourite Algorithm (Fav), since $d_i = E^{\text{(Fav)}} - E^{\text{(DT)}}$.

The standard deviation of the differences is:

$$\begin{aligned} S(d) &= \text{Root}(1 / (10 - 1) * 2.299 * 10^{-4}) \\ &= \text{Root}(2.554 * 10^{-5}) \\ &= 0.005054 \end{aligned}$$

The t-statistic for comparing U^d to 0 (where U^d is the true difference) is:

$$\begin{aligned} T &= U^d - 0 / S(d) * \text{Root}(k) \\ &= -0.00109 / (0.005054 / \text{Root}(10)) \\ &= -0.00109 / 0.00160 \\ &= -0.681 \end{aligned}$$

We have $k - 1 = 9$ degrees of freedom. We are looking for the confidence level that the Favourite Algorithm will outperform the Decision Tree. Outperforming means $E(\text{Fav}) < E(\text{DT})$, so the difference $U(d) = E(\text{Fav}) - E(\text{DT})$ must be negative ($U(d) < 0$).

The t-statistic is $t = -0.681$. Since $U^*(d)$ is already negative, we need to find the probability of a t-score being this low or lower (one-tailed test). We look up the absolute value

$|t| = 0.681$ in the t-table (Table 4) for

$N = 9$ degrees of freedom.

The smallest value for $v = 9$ in the table is $t = 0.985$ at a 65% confidence level (two-sided).

- For a 65% two-sided confidence interval, the t-value is $t_{65,9} = 0.985$.
- This corresponds to a one-tailed probability of $1 - 0.65 = 0.175$ in the right tail, and $1 - 0.175 = 0.825$ in the left tail (confidence of being worse).

Since our t-statistic (0.985) is bigger than 0.825 but smaller than 1.383, we can be **at least 80% confident** (but not quite 90%) that your favorite algorithm will outperform the decision tree induction algorithm in this situation.