EE2703 : Applied Programming Lab
End Semester Examination-2022

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Abstract

The objective of the given question is the following :

- Calculate the magnetic field due to a half-wave dipole antenna at a point by using Ampere's law.
- Calculate the magnetic field due to a half-wave dipole antenna at a point by using the vector potential at that point.
- Calculate the current in the half-wave dipole antenna.
- To find whether the given approximate formula is a good approximation.

Theoretical concepts required

Magnetic field due to current in the dipole is given by the following formula.

$$2\pi a H_\phi(z) = I$$

Vector potential due to current in the dipole is given by the following formula.

$$\vec{A}(r,z) = \frac{\mu_0}{4\pi}\int \frac{I(z')\hat{z}e^{-jkR}dz'}{R}$$

Where $\vec{R} = \vec{r} - \vec{r'} = r\hat{r} + z\hat{z} - z'\hat{z}$ and k=ω/c. Here $\vec{r}$ is the point where we want the field and $\vec{r'} = z'\vec{z'}$ is the point on the wire.

We will be using the above two formulas to calculate I flowing through the wire.

The approximate equation for the current I flowing through the half-wave dipole is the following.

$$I = \left\{ \begin{array}{ll} I_m \sin\left(k(l-z)\right) & 0 \leq z \leq l \\ I_m \sin\left(k(l+z)\right) & -l \leq z < 0 \end{array} \right\}$$

Algorithm to find the current flowing throught the dipole

- Find the vector z which stores the locations of the currents.
- Find the matrix M in the matrix formula used to find the magnetic field using Ampere's law.
- Find the matrix Q and Q_B calculated using the vector potential method of finding the magnetic field.
- Calculate the vector J and I using M , Q and Q_B

0.1 Find the vector z which stores the locations of the currents.

We divide the wire into $2N + 1$ elements of length dz each. Then we define the vector z as an array from $-N \times dz$ to $N \times dz$.
We use the following code to get it.

```
z = numpy.arange(-N, N+1, 1)*dz
```

Now we also find the vector u which contains the locations on the wire where the currents are unknown. We do that by writing the following code.

```
u = np.concatenate((z[1:N], z[N+1:-1]), axis=None)
```

We also initialise the I vector containing the current values of $2N + 1$ locations. In that we know that the current value in the end and beginning is 0 and at the N^{th} location the value is Im (feed current).
Knowing this we initialize the I variable by the following code.

```
I = np.zeros((2*N)+1)
I[0] = 0
I[-1] = 0
I[N] = Im
```

We also initialise the J vector which contains the current values which are unknown.

```
J = np.zeros((2*N)-2)
```

0.2 Find the matrix M in the matrix formula used to find the magnetic field using Ampere's law

The Ampere's law for the given dipole is

$$2\pi a H_\phi(z_i) = I_i$$

Where z_i is the i^{th} value of the z vector and I_i is the current value in that location of the wire used to make the antenna.
We now write it in the matrix form to get the values of magnetic fields at all the locations.

$$\begin{pmatrix} H_\phi[z_1] \\ \dots \\ H_\phi[z_{N-1}] \\ H_\phi[z_{N+1}] \\ \dots \\ H_\phi[z_{2N-1}] \end{pmatrix} = \frac{1}{2\pi a} \begin{pmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} J_1 \\ \dots \\ J_{N-1} \\ J_{N+1} \\ \dots \\ J_{2N-1} \end{pmatrix}$$

Which can be written as

$$H_\phi(r) = M * J$$

Here r=a (the radius of the wire). The matrix M is a constant($\frac{1}{2\pi a}$) multiplied by a $(2N-2)$ by $(2N-2)$ identity matrix.
We write the following function to compute M.

```
def compute_M(N):
    M = np.identity((2*N)-2, dtype=float)*(1/(2*pi*a))
    return M
```

0.3 Find the matrix Q and Q_B calculated using the vector potential method of finding the magnetic field.

The formula to find the vector potential is

$$\vec{A}(r,z) = \frac{\mu_0}{4\pi} \int \frac{I(z')\hat{z}e^{-jkR}dz'}{R}$$

This integral can be approximated to a summation as shown below.

$$A_{z,i} \quad = \quad \frac{\mu_0}{4\pi} \sum_j \frac{I_j \exp\left(-jkR_{ij}\right) dz'_j}{R_{ij}}$$

$$
\begin{aligned}
&= \sum_j I_j \left(\frac{\mu_0}{4\pi} \frac{\exp(-jkR_{ij})}{R_{ij}} dz'_j \right) \\
&= \sum_j P_{ij} I_j + P_B I_N
\end{aligned}
$$

Here $[P_{ij}]=P$ and P_B is a column vector.

$$
P_B = \frac{\mu_0}{4\pi} \frac{\exp\left(-jkR_{iN}\right)}{R_{iN}} dz'_j
$$

To find P and P_B we write the following code.

```
# Initialising R ad RiN
x = np.linspace(-N+1, N-1, num=(2*N)-1, dtype=float)     # column index range
y = np.linspace(-N+1, N-1, num=(2*N)-1, dtype=float)     # row index range
# column indexes without 0
x = np.delete(x, N-1, 0)
y = np.delete(y, N-1, 0)                                  # row indexes without 0
Y, X = np.meshgrid(y, x, sparse=False)
R = np.sqrt(np.square(a)+np.square((X-Y)*dz))
RiN = (a**2 + u**2)**0.5

# Matrix P calculated from matrix R
P = (mu0/(4*pi))*(np.exp(-1j*k*R))*(dz/R)

# Matrix PB calculated from matrix RiN
PB = (mu0/(4*pi))*(np.exp(-1j*k*RiN))*(dz/RiN)
```

We also need to compute Rz and Ru vectors. To do so we write the following code.

```
j = sp.symbols("j")
Rz = (np.square(a)+np.square(z-j*dz))**0.5
Ru = (np.square(a)+np.square(u-j*dz))**0.5
```

To find the magnetic field from the vector potential, we have the following differential equation (here we are interested in the ϕ component of $\vec{H}$ and $\vec{A}$ has only $\hat{z}$ component). So we get the following formula.

$$
H_\phi(r,z) = -\frac{1}{\mu}\frac{\partial A_z}{\partial r} = -\sum_j \frac{\mu_0}{4\pi} \frac{dz'_j}{\mu} \frac{\partial}{\partial r} \left(\frac{\exp\left(-jkR_{ij}\right)}{R_{ij}} \right) I_j
$$

Here we can write the following

$$
\frac{\partial}{\partial r} R_{ij} = \frac{1}{2R_{ij}} 2r = \frac{r}{R_{ij}}
$$

So we can further write,

$$\begin{aligned} H_\phi(r,z_i) &= -\sum_j \frac{dz'_j}{4\pi}\left(\frac{-jk}{R_{ij}} - \frac{1}{R_{ij}^2}\right)\exp\left(-jkR_{ij}\right)\frac{rI_j}{R_{ij}} \\ &= -\sum_j P_{ij}\frac{r}{\mu_0}\left(\frac{-jk}{R_{ij}} - \frac{1}{R_{ij}^2}\right)I_j + P_B\frac{r}{\mu_0}\left(\frac{-jk}{R_{iN}} - \frac{1}{R_{iN}^2}\right)I_m \\ &= \sum_j Q'_{ij}I_j \end{aligned}$$

So now we can further write it as,

$$\begin{aligned} H_\phi(r,z_i) &= \sum_j Q'_{ij}I_j \\ &= \sum_j Q_{ij}J_j + Q_{Bi}I_m \end{aligned}$$

Where $[Q_{ij}]$ is equal to Q matrix and $[Q_{Bi}]$ is the Q_B column vector.
Now from the formula we can say

$$Q_{ij} = P_{ij}\frac{r}{\mu_0}\left(\frac{jk}{R_{ij}} + \frac{1}{R_{ij}^2}\right)$$

$$Q_{Bi} = P_{Bi}\frac{r}{\mu_0}\left(\frac{jk}{R_{iN}} + \frac{1}{R_{iN}^2}\right)I_m$$

The code to compute Q and Q_B is given below.

```
# Matrix Q calculated from P
Q = P*(a/mu0)*((1j*k/R)+(1/(R**2)))

# Matrix QB calculated from PB
QB = np.transpose(PB*(a/mu0)*((1j*k/RiN)+(1/(RiN**2))))
```

0.4 Calculate the vector J and I using M , Q and Q_B

Now we have two formulas for finding the magnetic field. Equating them results in the following formula.

$$MJ = QJ + Q_B I_m$$

$$(M - Q)J = Q_B I_m$$

So now we can re-write this as,

$$J = (M - Q)^{-1} Q_B I_m$$

So as we know the value of M , Q , Q_B and I_m, we can find J.

We write the following code to find J.

```
J = np.dot(np.linalg.inv(M-Q), QB)*Im
```

Now we can find the unknown values of the vector I.

```
I[1:N] = J[:N-1]
I[N+1:-1] = J[N-1:]
```

Note that only the real part of J will be stored in I.

0.5 Compare the obtained value with that calculated from the approximate equation

The approximate equation for the current in the half-wave dipole antenna is

$$I = \left\{ \begin{array}{ll} I_m \sin\left(k(l-z)\right) & 0 \leq z \leq l \\ I_m \sin\left(k(l+z)\right) & -l \leq z < 0 \end{array} \right\}$$

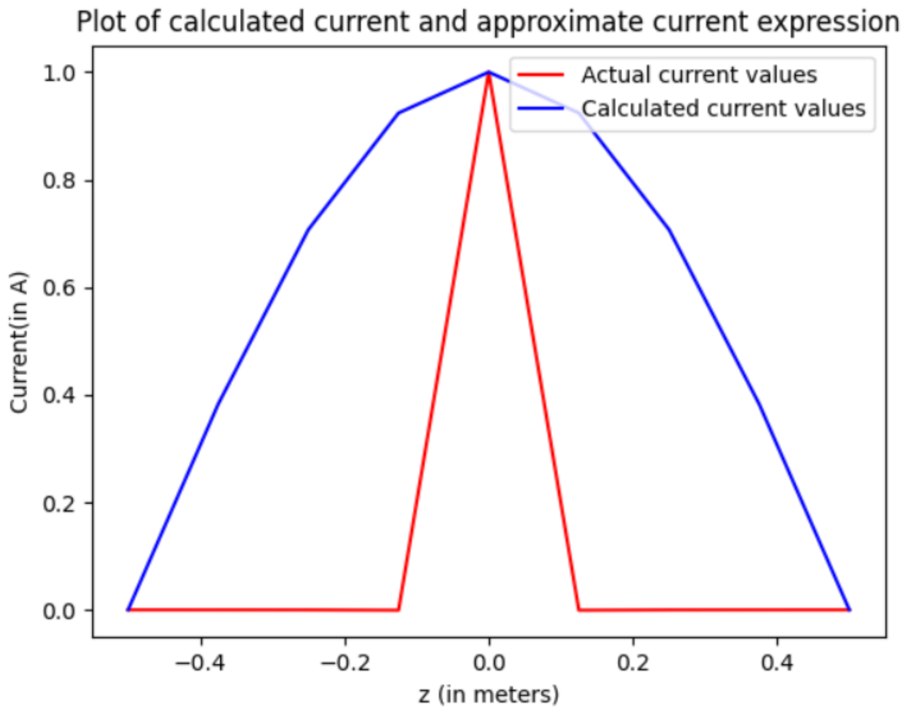
so we write the below code to calculate the approximate current values.

```
I_approx = Im*np.sin(k*(l - abs(z)))
```

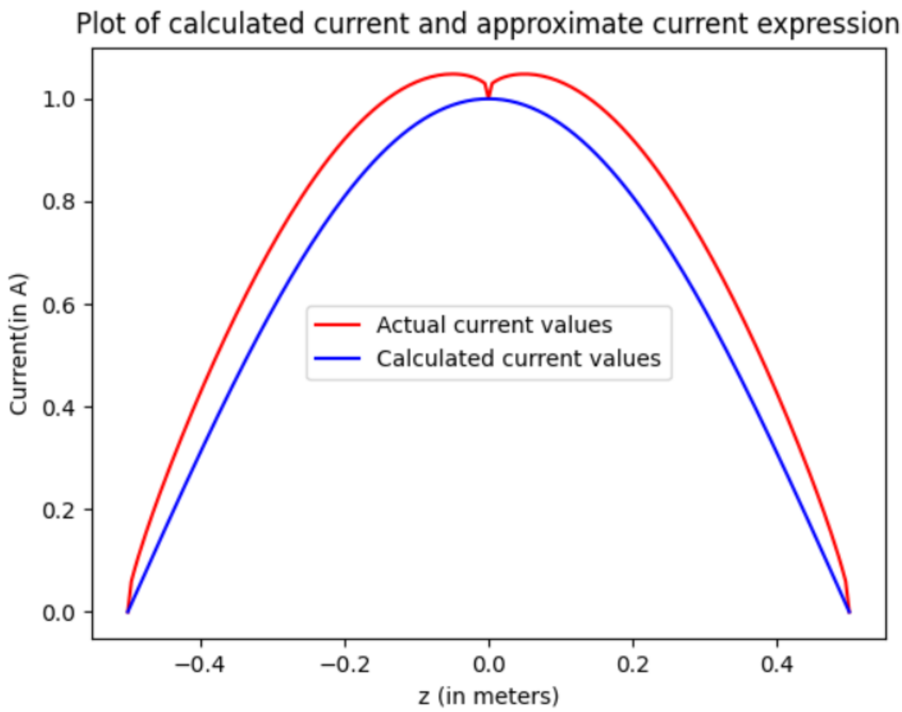
Now we write the below code to plot the to functions.

```
plot(z, I, "r", label="Actual current values")
plot(z, I_approx, "b", label="Calculated current values")
ylabel("Current(in A)")
xlabel("z (in meters)")
legend(loc="center")
show()
```

We get the following graph as the output for N=4.



We get the following graph as the output for N=100 and changing the location of the legend.



0.6 Conclusion

We see that there is a slight divergence between the calculated values and the values obtained through the given algorithm. This is because of factors like dz is not sufficiently small, fringing effects and the finite thickness of the antenna. Approximating the integral as a sum does not give precise results. But on the whole this expression is a very good approximation of the current flowing through the half-wave dipole antenna.