EE2703 : Applied Programming Lab
Assignment 10
Linear and Circular Convolution

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0.1 Abstract

Aim of this assignment is to know how to get the linear convolution and circular convolution of two signals using python.

0.2 Introduction

We have 3 ways to convolve two signals in python and they are as follows.

`Linear Convolution:`
We can do a linear convolution of two signals $y[n]$ and $h[n]$ using the direct summation algorithm which would include two for loops.

```
for n corresponding to x: # iterate over indices in x
  y[n]=0;                 # initialize y to zero
  for k in range(K):      # iterate over indices in h
    y[n] += x[n-k]*h[k]   # compute the convolution sum
  #end for
#end for
```

We can do this linear convolution using just one for loop by using the python function *np.convolve*.

`Circular Convolution:`
To do a circular convolution we first make the non-periodic signals periodic.
let $x[n]$ be a sequence of N values.Then

$$\tilde{x}[n] = \begin{cases} x[n] & 0 \leq n < N \\ x[n-N] & N \leq n < 2N \\ x[n+N] & -N \leq n < 0 \end{cases}$$

This way we convert the two non-periodic signals $x[n]$ and $h[n]$ to periodic signals.
Now we use normal convolution on these periodic signals to obtain the circular convolution of the non-periodic signals.

$$\tilde{y}[n] = \sum_{m=0}^{N-1} \tilde{x}[n-m]\tilde{h}[m]$$

`Circular Convolution as linear convolution using aliasing:`
To do this we use the following algorithm.

- Suppose $h[]$ fits into a 2^m window. We zero pad $h[k]$ if required.
- Divide $x[]$ into sections 2^m long.
- Apply circular convolution to each section of $x[]$.
- Stitch the outputs back together.

0.3 Assignment

0.3.1 Question 1 and Question 2

Here, we are going to plot the magnitude and phase response of the signal given in h.csv.
To do so we first read the h.csv file and put the values into a list. To do that we first download the file and then write the following code to read it.

```
def Q1(printing=False, filename="h.csv"):
    f = open(filename)

    b = f.readlines()
    for i in range(len(b)):
        b[i] = float(b[i].strip("\n"))
    f.close()

    b = np.array(b)

    if printing:
        print("The coefficients are :\n")
        print(b)

    return b
```

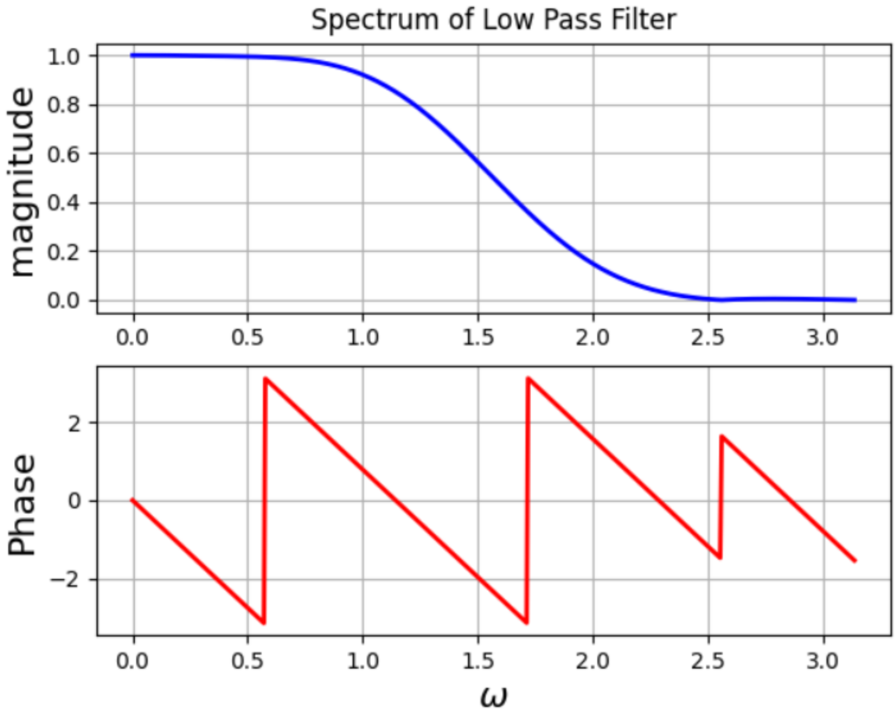
Then we use $scipy.signal.freqz()$ to obtain the spectrum and the phase of the signal.
So we have to write just one line of code to obtain it.

```
w, h = sig.freqz(Q1())
```

Now we have the code to plot it.

```
figure()
subplot(2, 1, 1)
plot(w, abs(h), 'b', lw=2)
ylabel(r"magnitude", size=16)
title(r"Spectrum of Low Pass Filter")
grid(True)
subplot(2, 1, 2)
plot(w, angle(h), 'r', lw=2)
ylabel(r"Phase", size=16)
xlabel(r"$\omega$", size=16)
grid(True)
show()
```

Which when run we get the following plot.



0.3.2 Question 3

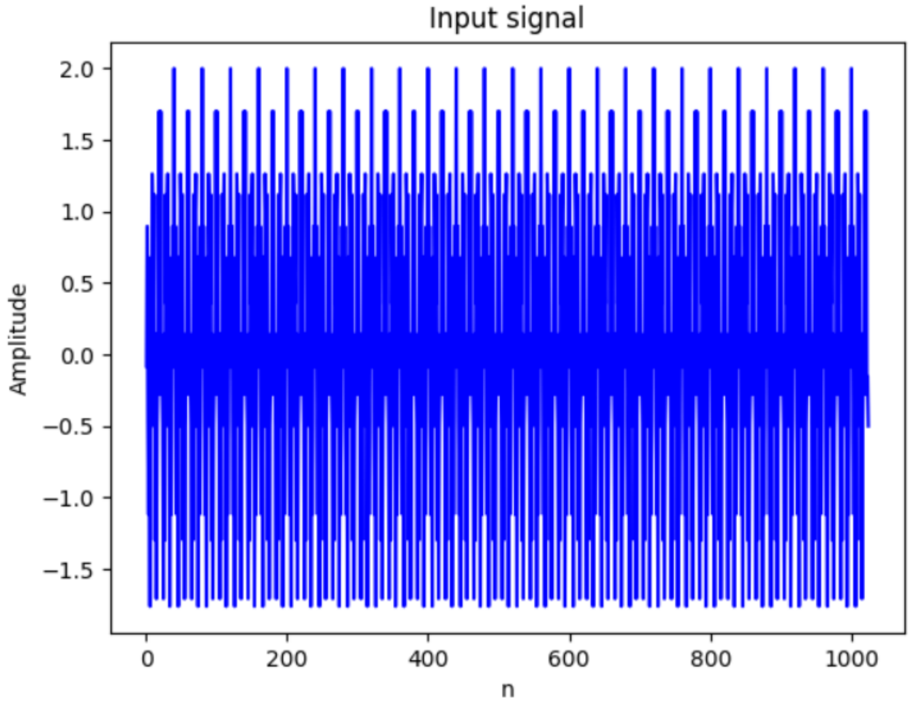
Now we generate the following sequence.

$$x = cos(0.2\pi n) + cos(0.85\pi n)$$

Where n goes from 1 to 2^{10}.
To do so we write the following function.

```
def Q3(plotting=False):
    n = np.arange(1, (2**10)+1)
    x = np.cos(0.2*pi*n) + np.cos(0.85*pi*n)

    if plotting:
        title("Input signal")
        plot(n, x, "r")
        show()

    return x, n
```

Running the above code with plotting=True we get the following as the input signal.

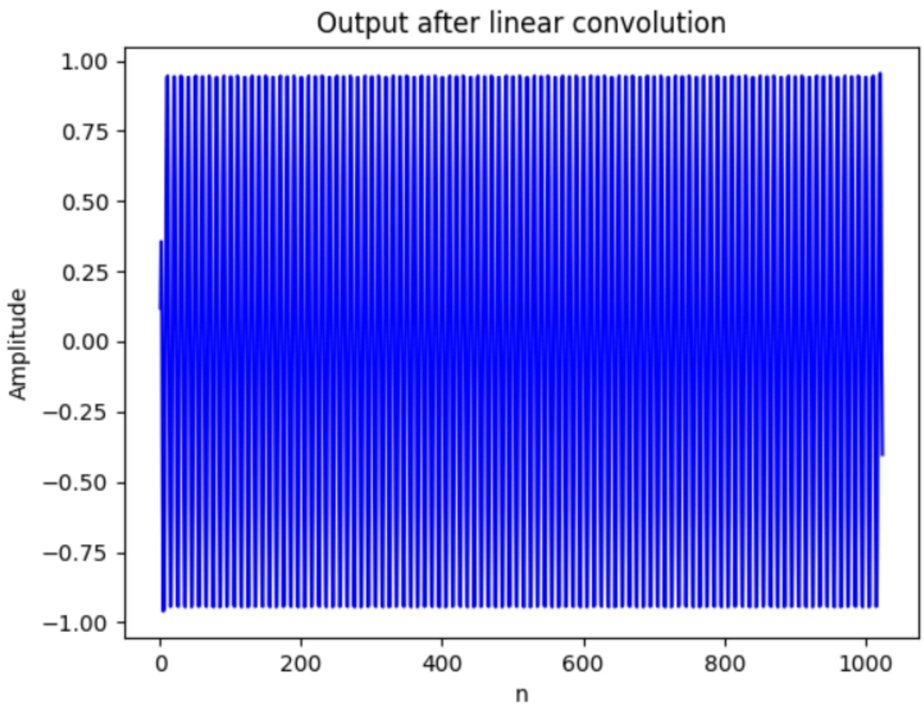


0.3.3 Question 4

Here we do a linear convolution between $x[n]$, generated from the function written in Question 3, and $h[n]$.
To do that we write the following code.

```
def Q4(plotting=False):

    b = Q1()
    x, n = Q3()

    y = np.convolve(x, b, mode="same")

    if plotting:
        title("Output after linear convolution")
        plot(n, real(y), "b")
        show()

    return y
```

Running the above function with $plotting = True$, we have the following plot.



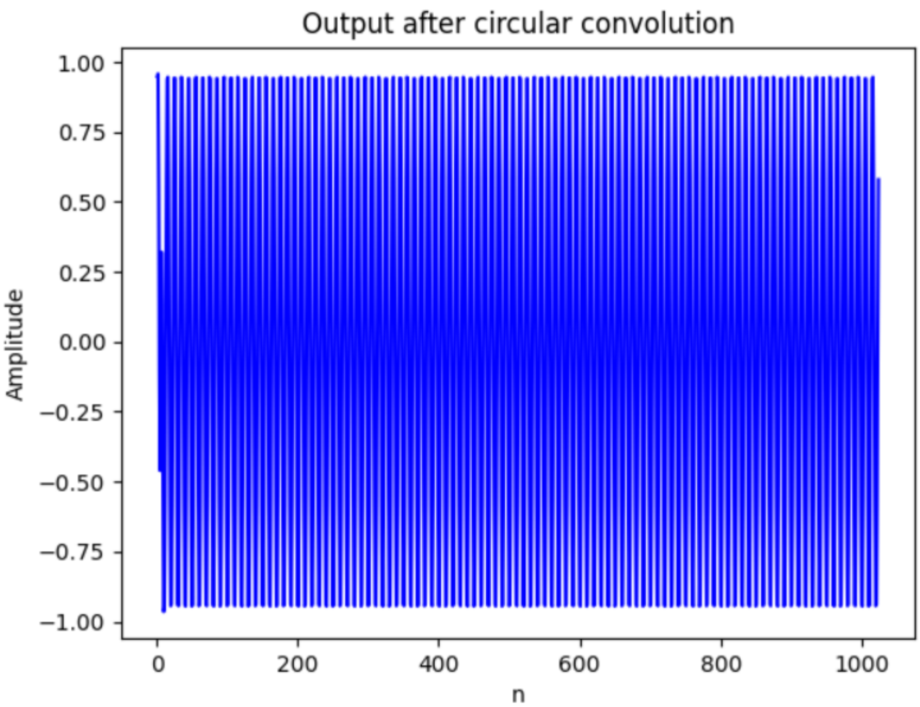
0.3.4 Question 6

Now we find the convolution of $x[n]$ and $h[n]$ using circular convolution.

To find it we write the following code.

```
# circular convolution
def Q5(plotting=False):

    x, n = Q3()
    w, h = Q2()
    b = Q1()

    y = ifft(fft(x)*fft(concatenate((b, zeros(len(x)-len(b))))))

    if plotting:
        title("Output after circular convolution")
        plot(n, real(y), "b")
        show()

    return y
```

Running the above code with $plotting = True$ we get the following plot.



0.3.5 Question 6

Now we find the linear convolution using the circular convolution. The steps to do so are as follows.

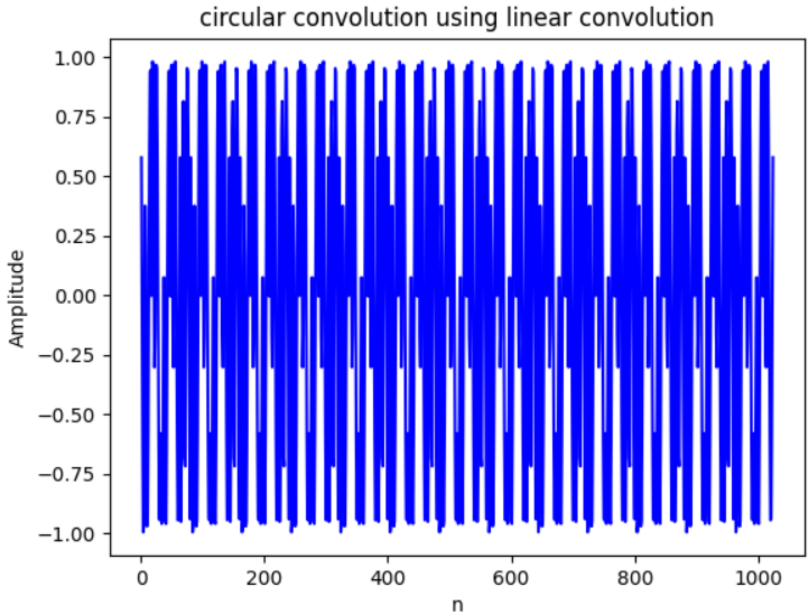
- Suppose $h[]$ fits into a 2^m window. We zero pad the $h[k]$ if required.
- Divide $x[]$ into sections 2^m long.
- Apply circular convolution to each section of $x[]$.
- Stitch the outputs back together.

Following the steps as given in the question we write the following code.

```
# circular convolution using linear convolution
def Q6(plotting=False):

    x, n = Q3()
    b = Q1()

    P = len(b)
    m = int(ceil(log2(P)))

    b_padded = np.concatenate((b, zeros((2**m)-P)))

    len_of_zero_array = (int(ceil(len(x)/2**m)))*(int(2**m))-len(x)
    x_padded = np.concatenate((x, np.zeros(len_of_zero_array)))
```

```
    y = []

    for i in range(int(len(x_padded)/(2**m))):
        x_i = np.concatenate((x_padded[i*(2**m):(i+1)*(2**m)], np.zeros(P-1)))
        x_i = x_padded[i*(2**m):(i+1)*(2**m)]
        y_i = ifft(
            fft(x_i)*fft(b_padded))
        y = np.concatenate((y, y_i))

    if plotting:
        title("some outout")
        plot(n, real(y), "b")
        show()

    return y
```

Running the above code with $plotting = True$ we get the following plot.



0.3.6 Question 7

In this question we will be using the Zadoff-Chu sequence.

We will be plotting the correlation of the Zadoff-Chu sequence.
To do we first need to write the code for reading the file.

```
def file_reading_for_Q7(filename="x1.csv", printing=False):
    f = open(filename)

    b = f.readlines()
    for i in range(len(b)):
```

```python
        b[i] = complex(b[i].strip(" ").strip("i\n")+"j")
    f.close()

    b = np.array(b)

    if printing:
        print("The coefficients are :\n")
        print(b)

    return b
```

Then we write the following code to find and plot the correlation of the Zadoff-Chu sequence.

```python
# Circular correlation
def Q7():
    x = file_reading_for_Q7()

    x2 = np.roll(x, 5)

    cor = np.fft.ifftshift(np.correlate(x2, x, 'full'))
    n = linspace(0, len(cor)-1, len(cor))

    title("Circular correlation")
    plot(n, abs(cor), "b")
    xlim(0, 20)
    show()
```

Running the above code we get the following plot as the correlation.

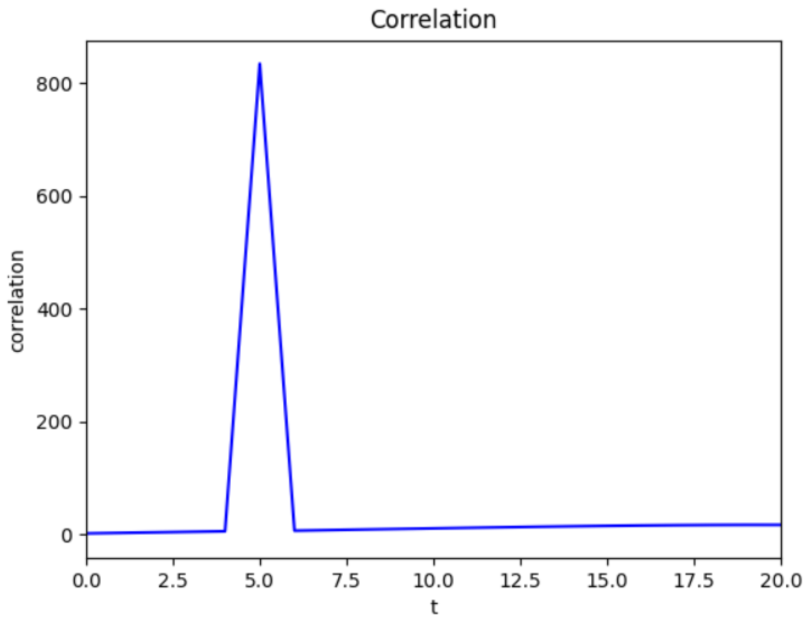


Figure 1: time-frequency plot of the DFT of chirped signal

We see that there is a peak in the correlation at t=5 which is the delay. Thus, we have verified the property of the Zadoff-Chu Sequence.

0.4 Conclusions

In this assignment we have explored different algorithms for convolution. We explored Linear Convolution, Circular convolution and a hybrid between the two. After that we verified the properties of the given Zadoff-Chu Sequence.