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DEPT. : MECHANICAL ENGINEERING

ASSIGNMENT 2 → ME 639

Q1. Show that $RS(a)R^T = S(Ra)$, where R is a rotation matrix.

<u>To Prove</u> $RS(a)R^T = S(Ra)$

We know $R(a \times b) = Ra \times Rb$ — (i)

Also, $S(a)p = a \times p$ — (ii)

Now from (ii)

$RS(a)R^T = R(a \times R^T)$

Now from (i)

$R(a \times R^T) = Ra \times RR^T$

We know for Rotation Matrices $RR^T = I$

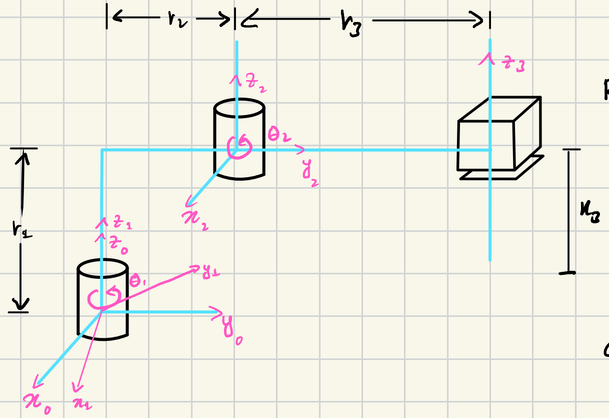
$\therefore Ra \times RR^T = (Ra \times I)$

Now again from (ii)

$S(Ra)$

Hence Proved $RS(a)R^T = S(Ra)$

Q2. Work out the various coordinate frames (show them on a clearly marked figure) and work out p_0 using a composition of homogeneous transformations for the RRP SCARA configuration.



• Schematic Representation OF RRP SCARA

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

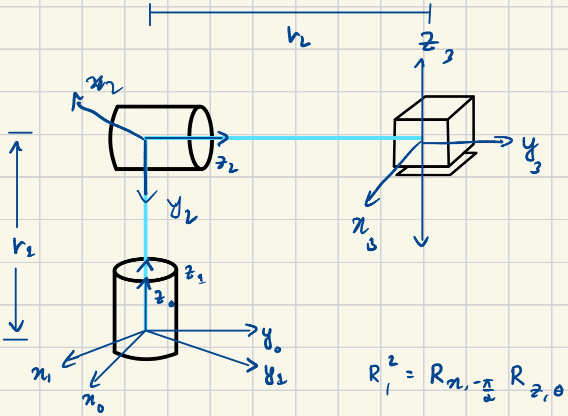
$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ r_2 \\ r_1 \end{bmatrix} \quad R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} 0 \\ r_3 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ -H_3 \end{bmatrix} \quad H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & r_2 \\ 0 & 0 & 1 & r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^1 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}}$$

Q4 Repeat the above exercise for the Stanford-type RRP configuration, again write a python code that can return the position vector of the end effector for any given configuration of joint variables (angles and extension).



$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = R_{x,-\frac{\pi}{2}} R_{z,\theta} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ r_1 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & 0 & 1 \\ -\sin\theta_2 & -\cos\theta_2 & 0 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ r_2 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ -r_3 \end{bmatrix}$$

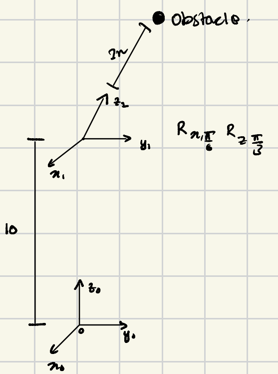
$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & r_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}}$$

Q5.

A drone took off from a base station and traveled 10m straight up. If you consider an inertial frame attached at the base station with the z axis pointing straight up and x and y axes along the ground forming a right-hand system, then this would be 10m in the z direction. At this hover point, the drone orientation is as if it completed a 30-degree rotation about the x-axis followed by a 60-degree rotation about the resulting new (current) z-axis. Further, it is then observed using a lidar installed on the drone that an obstacle is 3m exactly above the drone (in the drone frame). Find the position vector of the obstacle with respect to the base coordinate frame using a composition of homogeneous transformations. Also, show the choice of coordinate frames using a neat sketch.



$Ox_0y_0z_0$: Base Frame
Inertial Frame of Reference

Assumption: Rotations are counterclockwise

$$R_0^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^0 = R_{x\frac{\pi}{6}} R_{z\frac{\pi}{3}}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{1}{2} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$d_1^0 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$P_n = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$H_0^0 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{1}{2} & 0 \\ \frac{\sqrt{3}}{4} & \frac{1}{4} & \frac{\sqrt{3}}{2} & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_1^0 \begin{bmatrix} P_n \\ 1 \end{bmatrix}$$

Q6. Read about a few different types of gearboxes typically used with motors in a robotic application and explain in 2-3 sentences in your own words some key pros and cons of each gearbox type and where it is typically used. Further, explain if you would typically see a gearbox used along with a motor in a drone application. Explain the reasons.

Planetary Gearbox:

- Pros: Planetary gearboxes offer high torque output, good efficiency, and compact size due to their coaxial design. They provide precise motion control and are ideal for applications where space is limited.

- Cons: They can be more complex and expensive than other gearbox types, and they may produce more heat due to friction. Lubrication and maintenance are essential.

- Typical Use: Planetary gearboxes are often used in robotic arms and industrial automation systems that require precise positioning and high torque.

Worm Gearbox:

- Pros: Worm gearboxes provide high reduction ratios in a compact form factor. They exhibit excellent self-locking characteristics, preventing back-driving when the motor is turned off.

- Cons: They tend to have lower efficiency and can generate more heat due to friction. They may not be suitable for applications requiring rapid changes in direction.

- Typical Use: Worm gearboxes are commonly found in applications like conveyor systems, where slow and steady movement is required.

Harmonic Drive Gearbox:

- Pros: Harmonic drive gearboxes offer zero-backlash, high precision, and compact design. They excel in applications that demand precise positioning and lightweight components.

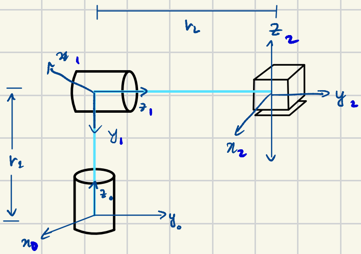
- Cons: They can be relatively expensive and may not handle high torque loads as effectively as planetary gearboxes. They also require careful installation and maintenance.

- Typical Use: These gearboxes are used in robotic joints and precision instruments due to their precision and compactness.

Regarding the use of gearboxes in drone applications, it's less common to see traditional gearboxes used with electric motors in drones. Drones typically employ brushless DC (BLDC) motors directly connected to propellers, which provide a more direct and efficient power transfer. The main reasons for this are weight considerations and the need for high-speed rotation in drone propellers. Adding a gearbox would introduce complexity, weight, and potential points of failure, which are generally undesirable in drone design. Instead, drones often rely on lightweight, high-efficiency motors to achieve the desired thrust and agility.

Q7. Derive the Manipulator Jacobian for the RRP SCARA configuration.

	PRISMATIC	REVOLUTE
LINEAR	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_n^0 - d_{i-1}^0)$
ROTATIONAL	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$



$$R_0^1 = \begin{bmatrix}\cos\theta_1 & -\sin\theta_1 & 0\\ \sin\theta_1 & \cos\theta_1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad d_0^1 = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R_1^2 = R_{x,-\frac{\pi}{2}} R_{z,\theta} \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}\cos\theta_2 & -\sin\theta_2 & 0\\ \sin\theta_2 & \cos\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix} \quad d_1^2 = \begin{bmatrix}0\\0\\r_1\end{bmatrix}$$

$$= \begin{bmatrix}\cos\theta_2 & -\sin\theta_2 & 0\\ 0 & 0 & 1\\ -\sin\theta_2 & -\cos\theta_2 & 0\end{bmatrix} \quad d_2^3 = \begin{bmatrix}0\\0\\r_2\end{bmatrix}$$

$$R_2^3 = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix} \quad P_3 = \begin{bmatrix}0\\0\\-x_3\end{bmatrix}$$

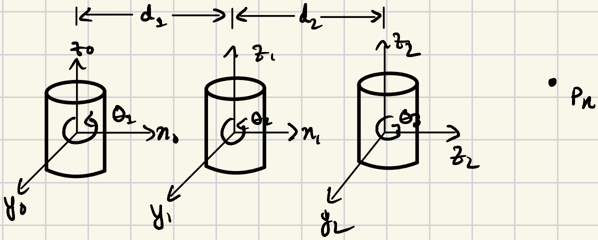
$$H_0^1 = \begin{bmatrix}\cos\theta_1 & -\sin\theta_1 & 0 & 0\\ \sin\theta_1 & \cos\theta_1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad H_1^2 = \begin{bmatrix}\cos\theta_2 & -\sin\theta_2 & 0 & 0\\ 0 & 0 & 1 & 0\\ -\sin\theta_2 & -\cos\theta_2 & 0 & r_1\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$H_2^3 = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & r_2\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Jacobian

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_3^0 - d_0^0) & R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times d_3^0 - d_1^0 & R_2^0\begin{bmatrix}0\\0\\1\end{bmatrix}\\ R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix} & R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{bmatrix}0\\0\\0\end{bmatrix}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{d}_3\end{bmatrix}$$

Q9 Derive the Manipulator Jacobian for the RRR configuration with all rotation axis parallel to each other (entire robot is planar like the elbow manipulator).



	Prismatic	Revolute
Linear	$R_{i-1}^0 \begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times d_n^0 - d_{i-1}^0$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix}0\\0\\1\end{bmatrix}$

$$R_0^0 = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad d_0^0 = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R_1^0 = \begin{bmatrix}\cos\theta_1 & -\sin\theta_1 & 0\\ \sin\theta_1 & \cos\theta_1 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad d_1^0 = \begin{bmatrix}d_1\\0\\0\end{bmatrix}$$

$$R_2^1 = \begin{bmatrix}\cos\theta_2 & -\sin\theta_2 & 0\\ \sin\theta_2 & \cos\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad d_2^1 = \begin{bmatrix}d_2\\0\\0\end{bmatrix}$$

d_3^0, d_2^0 can be found from the homogeneous transformation matrices H_3^0 and H_2^0 respectively

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

$$H_2^0 = H_1^0 H_2^1$$

Jacobian.

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} R_0^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_0^0) & R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times d_3^0 - d_1^0 & R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times d_3^0 - d_2^0 \\ R_0^0 \begin{bmatrix}0\\0\\1\end{bmatrix} & R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} & R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix} \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}$$