NAME : DEBOJIT DAS
ROLL : 22110067
DEPT. : MECHANICAL ENGINEERING
ASSIGNMENT 3 → ME 639

1. Review the discussion on singularities, decoupling of singularities, and various examples of singularities and singular configurations in the textbook. Describe in 3-4 sentences in your own words what is a singular configuration and how do you find singular configurations. Also, can you detect if a particular configuration is close to a singular configuration using the Manipulator Jacobian?

Definition of Singular Configuration

- Singular configurations occur when the Jacobian matrix J of the manipulator has a rank less than its maximum possible values. This reduced rank implies a loss of independence in the manipulator's movement or forces.

- At a singular configuration, certain directions of motion become unattainable, meaning the end-effector cannot move in specific directions despite joint movements

Finding Singular Configurations

- Singular configurations are identified by analyzing the Jacobian matrix of the manipulator Specially, a configuration is singular if the determinant of the Jacobian matrix is zero which correspons to the Jacobian losing rank.

- For a manipulator with a spherical wrist, singular configurations can include wrist singularities (related to the Euler angle parameterization of orientation) and arm singularities, which can be found by solving specific equations related to the Jacobian matrix.

Detecting Proximity to Singular Configurations Using the Manipulator Jacobian

- The proximity to a singular configuration can be interferred by observing the condition of the Jacobian matrix. As a manipulator approaches a singularity, certain columns of the Jacobian may become linearly dependent, or the determinant of the Jacobian may approach zero.

- Additionally, near singular configurations the manipulator may exhibit characteristics like unbounded joint velocities for bounded end effector velocities, or unbounded end effector forces.

5. Solve problem 3-7 in the textbook and also verify your hand-derived answers using the code in Task 3.

7. Consider the three-link cartesian manipulator of Figure 3.28. Derive the forward kinematic equations using the DH-convention.

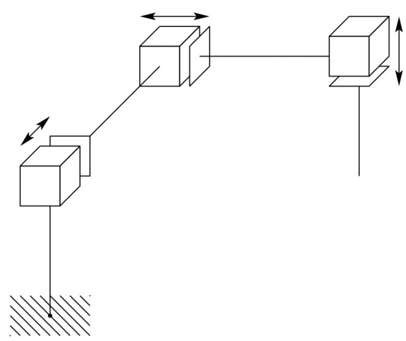


Fig. 3.28 Three-link cartesian robot

DH Parameter Table

link	d	θ	a	α
1	d_1	0	0	$-\pi/2$
2	d_2	0	0	$-\pi/2$
3	d_3	0	0	0

General Forward Kine matic Matrix

$$A_{n+1} = \begin{bmatrix} \cos\theta_n & -\sin\theta_n \cos\alpha_n & \sin\theta_n \sin\alpha_n & r_n \cos\theta_n \\ \sin\theta_n & \cos\theta_n \cos\alpha_n & -\cos\theta_n \sin\alpha_n & r_n \sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & r_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = A_0 \times A_1 \times A_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_1 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Further Solved in Code in Jupyter Notebook

6. Solve problem 3-8 in the textbook and also verify your hand-derived answers using the code in Task 3.

8. Attach a spherical wrist to the three-link articulated manipulator of Problem 3-6. as shown in Figure 3.29. Derive the forward kinematic equations for this manipulator.

6. Consider the three-link articulated robot of Figure 3.27. Derive the forward kinematics equation using DH convention

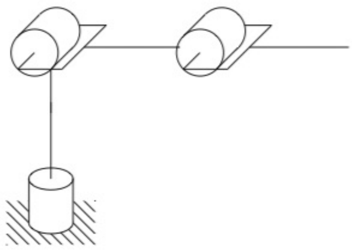


Fig. 3.27 Three-link articulated robot

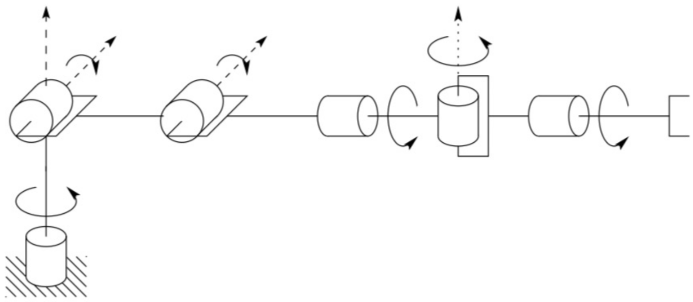


Fig. 3.29 Elbow manipulator with spherical wrist

DH Parameter Table

link	d	θ	a	α
1	d_1	0	$\pi/2$	θ^*
2	0	a_2	0	θ^*
3	0	a_3	0	θ^*
4	0	$-\pi/2$	0	θ^*
5	0	$-\pi/2$	0	θ^*
6	d_6	0	0	θ^*

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_1 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6^0 = A_1 \cdots A_6$$

$$A_6^6 = \begin{bmatrix} R_{3\times3} & t_{1\times3} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Further Solved in Code in Jupyter Notebook

7. Compare the three different configurations for 2R manipulator (direct drive, remotely-driven, and 5-bar parallelogram arrangement) and explain the key differences and advantages of each arrangement.

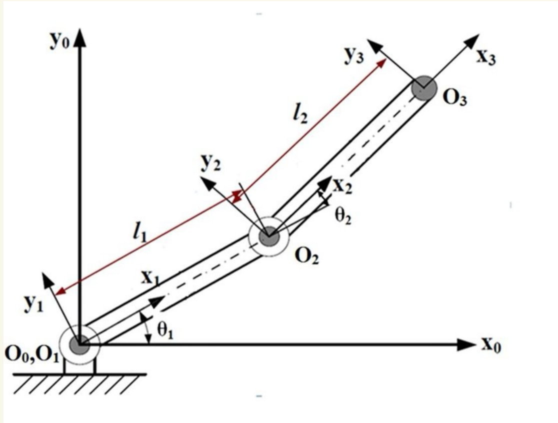
DIRECT DRIVE CONFIGURATION

In a direct drive configuration the motors are directly connected to the joints. This means no transmission system gears or belts

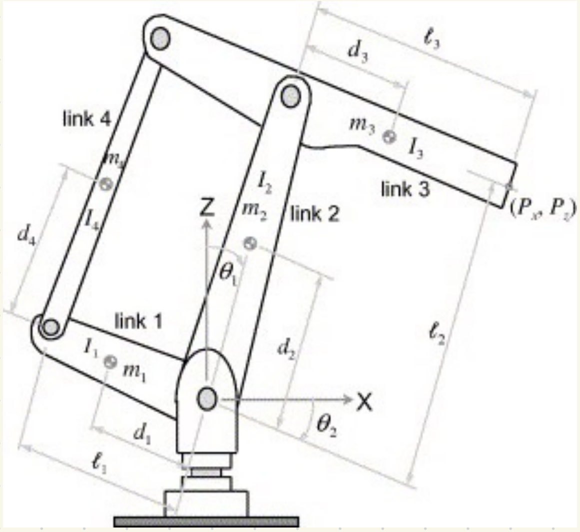
Advantages

→ Direct coupling allows for precise control and quick response

→ The absense of additional components simplifies the mechanical design



• 2R manipulator (Direct Drive)



• 5 bar Parallelogram arrangemt for 2R Manipulator

Remotely driven 2R manipulator is very similar to the direct drive, the difference is the motors sometimes (single or both) would be based somewhere else and a belt would be driving the joints.

REMOTELY DRIVEN CONFIGURATION

In a remotely driven configuration the motors are not located at the joints. Instead, power is transmitted to the joints through mechanisms like shafts, gears or belts.

Advantages

→ Reduced load on the arm

→ Heat and vibration isolation of the motors

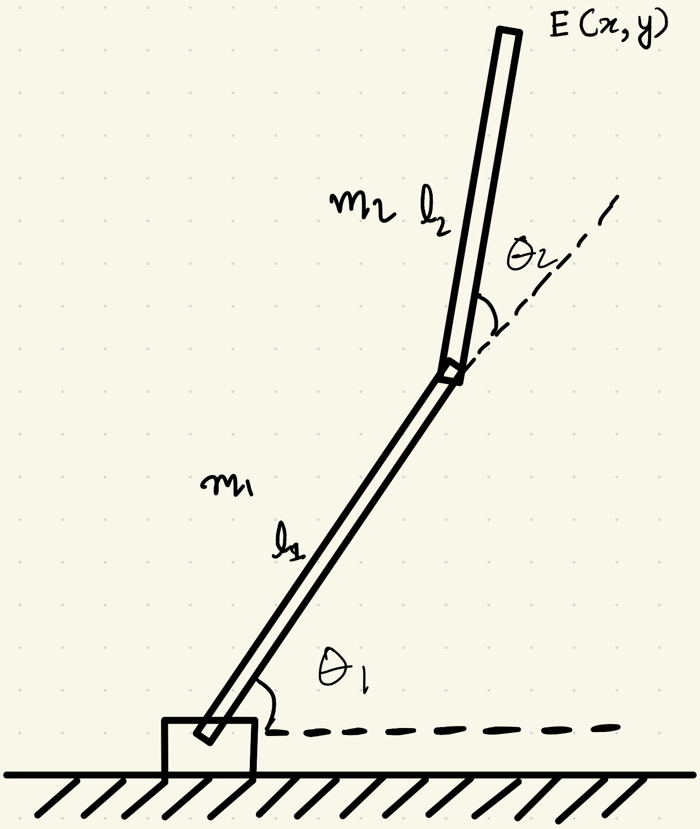
5- Bar Parallelogram Configuration

The configuration uses parallelogram linkage system to control the movement of the arm. It typically involves two parallel bar that keep the orientation of the end effector constant

Advantages

→ The parallelogram arrangement keeps the end effector's orientation constant relative to the base

8. Complete the derivation of the dynamic equations of 2R manipulator discussed in class and compare your results with those in the miniproject. Remark on any discrepancies or observations.



θ_1, θ_2 : joint angles

l_1 l_2 : lengths of the two links

m_1 m_2 : masses of the links

I_1 I_2 : moment of inertia of each link

First kinetic Energy

$$\boxed{k_i = \frac{1}{2} m_i v_{ci}^2 + \frac{1}{2} I_c \dot{\theta}_1^2}$$

using this the kinetic energy for link 1 & 2 can be calculated

$\therefore$ $k_i = k_1 + k_2$

Potential Energy

$V = V_1 + V_2$

$$\boxed{\text{Lagrangian} \quad L = K - V}$$

Lagrangian Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i}\right) = \tau_n \quad \text{———} \; (1)$$

Finally solving ① we get

$$\sum_i d_{in}(\theta)\ddot{\theta}_i + \sum_{i,j} \frac{\partial}{\partial \theta_i} d_{in}(\theta)\dot{\theta}_i \dot{\theta}_j - \frac{1}{2}\sum_{i,j} \frac{\partial_{ij}(\theta_i)}{\partial \theta_n} \theta_i \theta_j + \frac{\partial V(\theta)}{\partial \theta_n} = \tau_n$$

$$\boxed{C_{ijn} = \frac{1}{2}\left[\frac{\partial d_{in}(\theta)}{\partial \theta_j} + \frac{\partial d_{nj}(\theta)}{\partial \theta_i} - \frac{\partial d_{ij}(\theta)}{\partial \theta_n} \right.} \rightarrow \text{Christoffels symbols}$$

$$\therefore \sum d_{in}(\theta)\ddot{\theta}_i + \sum C_{ijn}(\theta,\dot{\theta})\,\dot{\theta}_i\dot{\theta}_j + d_n(\theta) = \tau_n$$

$$\boxed{D(\theta)\ddot{\theta} + C(\theta\,\dot{\theta}) + \phi(q) = \tau} \rightarrow \text{Hence Derived}$$

- In the miniproject brute force method was used without having information about the Christoffel symbols making calculations very difficult.