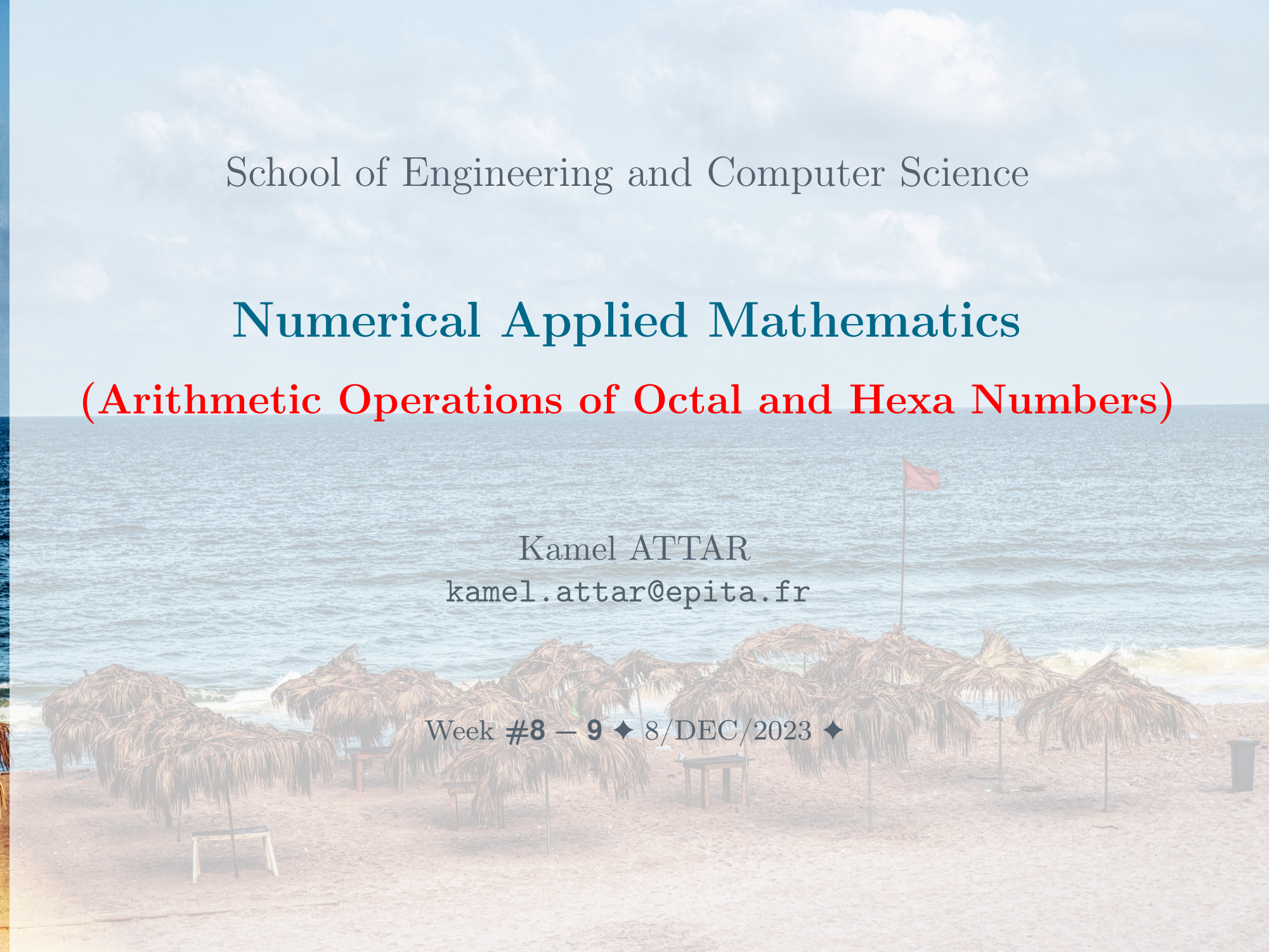

School of Engineering and Computer Science

Numerical Applied Mathematics

(Arithmetic Operations of Octal and Hexa Numbers)

Kamel ATTAR
kamel.attar@epita.fr

Week **#8 — 9** ✦ 8/DEC/2023 ✦

★ Addition of Octal Numbers ★

First Method

- Addition of octal numbers is carried out by the same principle as that of decimal or binary numbers.
- There are two methods of adding octal numbers. I will solve each example by using methods for better understanding.

In this method you should remember these points:

- Think each number as a decimal number and add them as decimal numbers.
- After adding each column if the sum of a column exceeds to **7** divide the result by **8** to evaluate equivalent octal value.
- The remainder is going to be the part of the answer (sub-sum) while the quotient is going to become carry

Example

$$5_8 + 7_8 = (\underbrace{5+7}_{12=8\times 1+4})_8 = 14_8 \quad ; \quad 4_8 + 5_8 = (\underbrace{4+5}_{9=8\times 1+1})_8 = 11_8$$

$$6_8 + 7_8 = (\underbrace{6+7}_{13=8\times 1+5})_8 = 15_8 \quad ; \quad 7_8 + 7_8 = (\underbrace{7+7}_{14=8\times 1+6})_8 = 16_8$$

★ Addition of Octal Numbers ★

First Method

Example

a. $(162)_8 + (537)_8$ b. $(136)_8 + (636)_8$ c. $(25.27)_8 + (13.2)_8$ d. $(67.5)_8 + (45.6)_8$

Solution

a.

Carry	1	1	
	1	6	2
+	5	3	7
Result	7	2	1

$$\begin{cases} \underbrace{2_8 + 7_8}_{9} = 8 \times 1 + 1 = 11_8 \\ \underbrace{6_8 + 3_8 + 1_8}_{10} = 8 \times 1 + 2 = 12_8 \end{cases}$$

b.

Carry		1	
	1	3	6
+	6	3	6
Result	7	7	4

$$\begin{cases} \underbrace{6_8 + 6_8}_{12} = 8 \times 1 + 4 = 14_8 \end{cases}$$

★ Addition of Octal Numbers ★

First Method

Solution

c. +

Carry	1				
	2	5	·	2	7
	1	3	·	2	0
Result	4	0	·	4	7

$$\begin{cases} \underbrace{5_8 + 3_8}_{8} = 8 \times 1 + 0 = 10_8 \end{cases}$$

d. +

Carry		1	1		
		6	7	·	5
		4	5	·	6
Result	1	3	5	·	3

$$\begin{cases} \underbrace{5_8 + 6_8}_{11} = 8 \times 1 + 3 = 11_8 \\ \underbrace{7_8 + 5_8 + 1_8}_{13} = 8 \times 1 + 5 = 15_8 \\ \underbrace{6_8 + 4_8 + 1_8}_{11} = 8 \times 1 + 3 = 13_8 \end{cases}$$

✎ **Exercise 1.** Evaluate

a. $(167)_8 + (765)_8$ b. $(123)_8 + (7651)_8$ c. $(246.57)_8 + (357.1)_8$ d. $(374)_8 + (2705)_8 + (5502)_8$

★ Addition of Octal Numbers ★

Second Method

In this method octal addition table is given. How this table is helpful in addition of octal numbers. Let's understand.

- Look at first row (red colored row), you can call it as X
- Look at first column (blue colored column) , you can call it as Y
- The rest of the table is black. This is the sum of X and Y

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

★ Addition of Octal Numbers ★

Second Method

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example

	Carry	**1**	
+		**2**	**3**
		4	**5**
	Result	**7**	**0**

- First you have to add 1st column numbers. Assign $X = 3$, $Y = 5$. Search in table you will get your result. The intersection of column **3** and row **5** is **10**.
- For 2nd column again assign $X = 2 + 1$, $Y = 4$ The intersection of column **3** and row **4** is **7**.

Question Verify the previous answers by using the octal table

Subtraction of Octal Numbers

First Method : Straight Forward subtraction (Case: Minuend > Subtrahend)

- All the rules we follow remains same as we do in other number systems.
- Borrow is equal to the base of number system.
  - If we are working with base **2** we borrow **2**.
  - If we are working with base **8** we borrow **8**.

Example

Carry		**8**	**8**
	~~**3**~~**2**	~~**4**~~**3**	**5**
−	**1**	**4**	**6**
Result	**1**	**7**	**7**

- 1st column (units column) subtraction. **5** − **6** This is not possible. You have to borrow from tens column. The number you borrow is **8**. So units column subtraction is **5** + **8** − **6**
- 2nd column (tens column) subtraction. **4** becomes **3** after borrow. **3** − **4**. To proceed this subtraction you have to borrow from hundreds column. So tens column subtraction is **3** + **8** − **4**

✎ **Exercise 2.**

a. $(757.76)_8 - (451.77)_8$ b. $(565.67)_8 - (234.34)_8$ c. $(2046)_8 - (653)_8$.

★ Subtraction of Octal Numbers ★

Second Method: 7's Complement

- This method is same as that discussed in binary subtraction.

Minuend − subtrahend=difference Or Minuend + [7's complement]=difference

- First, confirm that the digits in the subtrahend and minuends should be equal.
- A 7's complement of a number can be achieved by complementing each digit of the number to 7 see the following table:

7's complement							
0	1	2	3	4	5	6	7
↓	↓	↓	↓	↓	↓	↓	↓
7	6	5	4	3	2	1	0

$$\Longrightarrow \left[(1357)_8\right]^{7s} = (6420)_8$$

- Finally, we add this with the minuend.

Case ❶: If the result of addition has a carry over then it is dropped and an **1** is added in the last bit.

Case ❷: If there is no carry over, then **7**'s complement of the result of addition is obtained to get the final result and it is negative.

Subtraction of Octal Numbers

Second Method: **7**'s Complement (Case-**1**: When Carry bit **1** Minuend > Subtrahend)

Example (Case-1: When Carry bit 1)

Subtract **146** from **345** by using **7**'s complement.

Solution

Octal	7's complement
146	**631**

$$\begin{array}{r|ccc} \text{Carry} & & & \\ \hline & 3 & 4 & 5 \\ - & 1 & 4 & 6 \\ \hline & & & \end{array} \iff \begin{array}{r|ccc} \text{Carry} & \mathbf{1} & & \\ \hline & 3 & 4 & 5 \\ + & 6 & 3 & 1 \\ \hline \text{Result} & 1 & 7 & 6 \end{array}$$

The result of addition has a carry over then it is dropped and an **1** is added in the last bit. Thus the required difference is $\mathbf{176 + 1 = 177}$.

Subtraction of Octal Numbers

Second Method: **7**'s Complement (Case-**2**: When no Carry bit Minuend < Subtrahend)

Example (Case-2: When no Carry bit)

Subtract **765** from **257** by using **7**'s complement.

Solution

First we find the **7**'s complement of subtrahend **765**.

Binary	7's complement
765	**012**

$$
-\begin{array}{|c|ccc|}
\hline
\text{Carry} & & & \\
\hline
 & 2 & 5 & 7 \\
 & 7 & 6 & 5 \\
\hline
\text{Result} & 1 & 7 & 7 \\
\hline
\end{array}
\quad\Longrightarrow\quad
+\begin{array}{|c|ccc|}
\hline
\text{Carry} & & \mathbf{1} & \\
\hline
 & 2 & 5 & 7 \\
 & 0 & 1 & 2 \\
\hline
\text{Result} & 2 & 7 & 1 \\
\hline
\end{array}
$$

There is no carry over, then **7**'s complement of the result of addition is obtained to get the final result and it is negative. Thus the difference is $-\left[(271)_8\right]^{7s} = -(506)_8$.

★ Subtraction of Octal Numbers ★

Third Method: **8**'s Complement

- This method is same as that discussed in binary subtraction.

Minuend − subtrahend=difference Or Minuend + [**8**'s complement]=difference

- First, confirm that the digits in the subtrahend and minuends should be equal.
- A **8**'s complement of a number can be achieved by complementing each digit of the number to **7** then we add **1** see the following:

$$(\blacksquare\ \blacksquare\ \blacksquare\ \blacksquare)^{8's} = (\blacksquare\ \blacksquare\ \blacksquare\ \blacksquare)^{7's} + \mathbf{0001}$$

- Finally, we add this with the minuend.

Case ❶: If the result of addition has a carry over then it is dropped and the result is positive.
Case ❷: If there is no carry over, the **8**'s complement of the sum will be the result and it is negative.

Subtraction of Octal Numbers

Third Method: **8**'s Complement (Case-**1**: When Carry bit **1** Minuend > Subtrahend)

Example (Case-1: When Carry bit 1)

Subtract **146** from **345** by using **8**'s complement.

Solution

$$(146)^{8's} = (146)^{7's} + 001 = 631 + 001 = 632\,.$$

$$\begin{array}{r|ccc} \text{Carry} & & & \\ \hline & 3 & 4 & 5 \\ - & 1 & 4 & 6 \\ \hline & & & \end{array} \quad \Longleftrightarrow \quad + \begin{array}{r|ccc} \text{Carry} & 1 & & \\ \hline & 3 & 4 & 5 \\ & 6 & 3 & 2 \\ \hline \text{Result} & 1 & 7 & 7 \end{array}$$

The result of addition has a carry over then it is dropped and the result is **177**.

Subtraction of Octal Numbers

Third Method: **8**'s Complement (Case-**2**: When no Carry bit Minuend < Subtrahend)

Example (Case-2: When no Carry bit)

Subtract **765** from **257** by using **8**'s complement.

Solution

First we find the **8**'s complement of subtrahend **765**.

$$(765)^{8's} = (765)^{7's} + 001 = 012 + 001 = 013\,.$$

$$-\begin{array}{|c|ccc|}\hline \text{Carry} & & & \\ \hline & 2 & 5 & 7 \\ & 7 & 6 & 5 \\ \hline \text{Result} & 1 & 7 & 7 \\ \hline\end{array} \quad\Longrightarrow\quad +\begin{array}{|c|ccc|}\hline \text{Carry} & & \mathbf{1} & \\ \hline & 2 & 5 & 7 \\ & 0 & 1 & 3 \\ \hline \text{Result} & 2 & 7 & 2 \\ \hline\end{array}$$

There is no carry over, then **8**'s complement of the result of addition is obtained to get the final result and it is negative. Thus the difference is $-\left[(272)_8\right]^{8s} = -(506)_8$.

✎ **Exercise 3.** Find the answer of the following operations (using **7**'s and **8**'s complement):
a. $(675)_8 - (722)_8$ b. $(757.76)_8 - (451.77)_8$ c. $(565.67)_8 - (234.34)_8$.

★ Multiplication of Octal Numbers ★

In multiplication of octal numbers a simple rule for multiplication of two digits in any radix is to multiply them in decimal.

- If the product is less than the radix, then we take it as the result.
- If the product is greater than the radix we divide it by the radix and take the remainder as the least significant digit. The quotient is taken as carry in the next significant digit.

Example

For example, $(3)_8 \times (2)_8 = (6)_8$ but $(3)_8 \times (4)_8 = (14)_8$ since $3 \times 4 = 12$ is decimal and division of **12** by **8** has the remainder **4** and quotient **1**.

Example

Let's find $6_8 \times 23_8$. We have $6 \times 3 = 18$ in decimal, which when divided by **8** gives a remainder **2** and carry **2**. Again $6 \times 2 = 12$ in decimal, and $12 + 2 = 14$. This when divided by **8** gives a remainder **6** and a carry **1**. Hence $6_8 \times 23_8 = 162_8$

★ Multiplicationn of Octal Numbers ★

✎ **Exercise 4.** Complete the following table

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6				
3	0	3	6					
4	0	4						
5	0	5						
6	0	6						
7	0	7						

✎ **Exercise 5.** Evaluate a. $15_8 \times 44_8$ b. $614_8 \times 25_8$

★ Division of Octal Numbers ★

The method followed in binary division is also similar to that adopted in decimal system. First we make a table for **4** and it's multiples

	Decimal	Octal
4×1	4	4
4×2	8	10
4×3	12	14
4×4	16	20
4×5	20	24
4×6	24	30
4×7	28	34

$\Longrightarrow$

$$\begin{array}{r|l} 7\ 1\ 2 & 4 \\ \hline -\ 4\ \downarrow & \\ \hline 3\ 1 & \\ -\ 3\ 0\ \downarrow & \\ \hline 0\ 1\ 2 & \\ -\ 1\ 0 & \\ \hline 0\ 0\ 2 & \end{array} \qquad 162$$

Then $(712)_8 = (162)_8 \times 4_8 + 2_8$

★ Division of Octal Numbers ★

✎ **Exercise 6.** Evaluate

a. $(6573)_8 \div (16)_8$ b. $(457.43)_8 \div (7)_8$ c. $(737.72)_8 \div (1.2)_8$

★ Addition of Hexa Numbers ★

- Hex numbers are used extensively in machine-language computer programming and in conjunction with computer memories. When working in these areas, there will be situations where hex numbers have to be added or subtracted.
- It was shown how binary addition (base **2**) with its two digits, **1** and **0**, and Octal addition (base **8**) with its eight digits, **0**, **1**, **2**, **3**, **4**, **5**, **6** and **7**, are performed the same way decimal addition (base **10**) is with its ten digits, **0**, **1**, **2**, **3**, **4**, **5**, **6**, **7**, **8** and **9**.
- The only difference is the limitation placed on the addition by the number of digits. In binary, the addition of two or three ones results in a carry since the result goes beyond **1**, the largest binary digit. Octal doesn't require a carry until the result goes beyond **7** and Decimal doesn't require a carry until the result goes beyond **9**.
- The addition of Hexadecimal can be done in the same manner as binary, octal and decimal addition.

Example

$$9_{16} + 7_{16} = (\underbrace{9+7}_{16=16\times 1+0})_{16} = 10_{16} \quad ; \quad D_{16} + 5_{16} = (\underbrace{13+5}_{18=16\times 1+2})_{16} = 12_{16}$$

Example

a. $(58)_{16} + (24)_{16}$ b. $(3AF)_{16} + (23C)_{16}$ c. $(3DA32)_{16} + (4292F)_{16}$.

Solution

Example

a. $(58)_{16} + (24)_{16}$ b. $(3AF)_{16} + (23C)_{16}$ c. $(3DA32)_{16} + (4292F)_{16}$.

Solution

a. b.

Carry				Carry		1	
+	5	8		+	3	A	F
	2	4			2	3	C
Result	7	C		Result	5	E	B

$$\begin{cases} \underbrace{F_{16} + C_{16}}_{27} = 16 \times 1 + 11 = 1B_{16} \end{cases}$$

c.

Carry	1	1		1	
+	3	D	A	3	2
	4	2	9	2	F
Result	8	0	3	6	1

$$\begin{cases} \underbrace{2_{16} + F_{16}}_{17} = 16 \times 1 + 1 = 11_{16} \\ \underbrace{A_{16} + 9_{16}}_{19} = 16 \times 1 + 3 = 13_{16} \\ \underbrace{D_{16} + 2_{16} + 1}_{16} = 16 \times 1 + 0 = 10_{16} \end{cases}$$

✎ **Exercise 8.** Add the following hexadecimal numbers.

a. $(AE465)_{16} + (E536EF)_{16}$ b. $(93453F)_{16} + (F60D.C)_{16}$ c. $(D.3761)_{16} + (4.D)_{16}$.

Subtraction of Hexa Numbers

First Method : Straight Forward subtraction (Case: Minuend > Subtrahend)

- All the rules we follow remains same as we do in other number systems.
- Borrow is equal to the base of number system.
  - If we are working with base **2** we borrow **2**.
  - If we are working with base **8** we borrow **8**.
  - If we are working with base **16** we borrow **16**.

Example

Carry		**16**	
	~~D~~ C	3	A
−		F	4
Result	C	4	6

- 1st column (units column) subtraction. $A - 4 = 10 - 4 = 6$.
- 2nd column (tens column) subtraction. $3 - F$, this is not possible. We have to borrow from hundreds column. The number we borrow is **16**. So tens column subtraction is $16 + 3 - F = 4$
- 3rd column (Hundreds column) subtraction. D becomes C after borrow. So hundreds column subtraction is $C - 0 = C$.

✎ Exercise 9.

a. $(2AB5)_{16} - (CF3)_{16}$ b. $(DE.21C)_{16} - (1E.D0C)_{16}$ c. $(7B6CD)_{16} - (CA \cdot D)_{16}$ d. $(CB85.11)_{16} - (11C.A)_{16}$.

★ Subtraction of Hexa Numbers ★

Second Method: **15**'s Complement

- This method is same as that discussed in Octal and binary subtractions.

Minuend − subtrahend=difference Or Minuend + [**15**'s complement]=difference

- First, confirm that the digits in the subtrahend and minuends should be equal.
- A **15**'s complement of a number can be achieved by complementing each digit of the number to **15** (subtract each hex digit from **15**) see the following table:

15's complement															
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	***A***	***B***	***C***	***D***	***E***	***F***
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
F	***E***	***D***	***C***	***B***	***A***	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**0**

$$\rightarrow \left[(73A)_{16}\right]^{15s} = (8C5)_{16}$$

- Finally, we add this with the minuend.

Case ❶: If the result of addition has a carry over then it is dropped and an **1** is added in the last bit.

Case ❷: If there is no carry over, then **15**'s complement of the result of addition is obtained to get the final result and it is negative.

Subtraction of Hexa Numbers

Second Method: **15**'s Complement (Case-**1**: When Carry bit **1** Minuend > Subtrahend)

Example (Case-1: When Carry bit 1)

Subtract **11*CA*** from ***CB*85** by using **15**'s complement.

Solution

Hexa	**15**'s complement
11*CA*	***EE*35**

$$\begin{array}{r|cccc} \text{Carry} & & & & \\ \hline & C & B & 8 & 5 \\ - & 1 & 1 & C & A \\ \hline & & & & \end{array} \iff \begin{array}{r|cccc} \text{Carry} & 1 & 1 & & \\ \hline & & C & B & 8 & 5 \\ + & & E & E & 3 & 5 \\ \hline \text{Result} & & B & 9 & B & A \end{array}$$

The result of addition has a carry over then it is dropped and an **1** is added in the last bit. Thus the required difference is $B9BA + 1 = B9BB$.

Subtraction of Hexa Numbers

Second Method: **15**'s Complement (Case-**2**: When no Carry bit Minuend < Subtrahend)

Example (Case-2: When no Carry bit)

Subtract ***D1CA*** from ***CB85*** by using **15**'s complement.

Solution

Hexa	**15**'s complement
D1CA	**2*E*35**

$$
\begin{array}{lcccc}
\text{Carry} & & & & \\
 & C & B & 8 & 5 \\
- & 1 & 1 & C & A \\
\hline
 & & & &
\end{array}
\quad \Longleftrightarrow \quad
\begin{array}{lcccc}
\text{Carry} & 1 & & & \\
 & C & B & 8 & 5 \\
+ & 2 & E & 3 & 5 \\
\hline
\text{Result} & F & 9 & B & A
\end{array}
$$

There is no carry over, then **15**'s complement of the result of addition is obtained to get the final result and it is negative. Thus the difference is $-\left[(F9BA)_{16}\right]^{15s} = -(0645)_{16}$.

★ Subtraction of Hexa Numbers ★

Third Method: **16**'s Complement

- This method is same as that discussed in octal and binary subtraction.

Minuend − subtrahend=difference Or Minuend + [**16**'s complement]=difference

- First, confirm that the digits in the subtrahend and minuends should be equal.
- A **16**'s complement of a number can be achieved by complementing each digit of the number to **15** then we add **1** see the following:

$$(\blacksquare\,\blacksquare\,\blacksquare\,\blacksquare)^{16's} = (\blacksquare\,\blacksquare\,\blacksquare\,\blacksquare)^{15's} + \mathbf{0001}$$

- Finally, we add this with the minuend.

Case ❶: If the result of addition has a carry over then it is dropped and the result is positive.
Case ❷: If there is no carry over, the **16**'s complement of the sum will be the result and it is negative.

Subtraction of Hexa Numbers

Third Method: **16**'s Complement (Case-**1**: When Carry bit **1** Minuend > Subtrahend)

Example (Case-1: When Carry bit 1)

Subtract **63*B*** from ***EC*1** by using **16**'s complement.

Solution

$$(63B)^{16's} = (63B)^{15's} + 001 = 9C4 + 001 = 9C5\,.$$

	Carry					Carry	**1**	**1**		
		E	***C***	**1**				***E***	***C***	**1**
−		**6**	**3**	***B***	⟺	+		**9**	***C***	**5**
						Result		**8**	**8**	**6**

The result of addition has a carry over then it is dropped and the result is **886**.

Subtraction of Hexa Numbers

Third Method: **16**'s Complement (Case-**2**: When no Carry bit Minuend < Subtrahend)

Example (Case-2: When no Carry bit)

Subtract ***F3B*** from ***EC*1** by using **16**'s complement.

Solution

First we find the **8**'s complement of subtrahend **765**.

$$(F3B)^{16's} = (F3B)^{15's} + 001 = 0C4 + 001 = 0C5\,.$$

	Carry					Carry		1	
−		*E*	*C*	1	$\Longrightarrow$	+	*E*	*C*	1
		F	3	*B*			0	*C*	5
	Result	1	7	7		Result	*F*	8	6

There is no carry over, then **16**'s complement of the result of addition is obtained to get the final result and it is negative. Thus the difference is $-\left[(F86)_{16}\right]^{16s} = -(07A)_{16}$.

✎ **Exercise 10.** Find the answer of the following operations (using three different ways):

a. $(6D0)_{16} - (45A)_{16}$ b. $(666.AB)_{16} - (556.F8)_{16}$ c. $(6C95.DD67)_{16} - (77A9.3A61)_{16}$.

★ Multiplication of Hexa Numbers ★

- In the multiplication of hexadecimal numbers, if the product is less than radix of hexadecimal (i.e, **16**). Then we take it as the result,
- else divide it by radix of hexadecimal (i.e., **16**) and take the remainder as the LSB (the least significant bit). The quotient is taken as carry in the next significant digit.

Example

$$
\begin{array}{cccccc}
 & & & \textcircled{4} & & \\
 & & & \textcircled{3} & & \\
 & & & 7 & 5 & \\
\times & & & D & A & \\
\hline
 & & 4 & 9 & 2 & \\
+ & 5 & F & 1 & 0 & \\
\hline
 & 6 & 3 & A & 2 &
\end{array}
$$

$$
\begin{cases}
\underbrace{5_{16} \times A_{16}}_{50} = 16 \times 3 + 2 = 32_{16} \\
\underbrace{7_{16} \times A_{16}}_{70} = 16 \times 4 + 6 = 46_{16} \\
\underbrace{5_{16} \times D_{16}}_{65} = 16 \times 4 + 1 = 41_{16} \\
\underbrace{7_{16} \times D_{16}}_{91} = 16 \times 5 + 11 = 5B_{16}
\end{cases}
$$

★ Multiplication of Hexa Numbers ★

Example

Evaluate $\mathbf{6}\boldsymbol{AB} \times \boldsymbol{A}\mathbf{9}$.

Solution

★ Multiplication of Hexa Numbers ★

Example

Evaluate $6AB \times A9$.

Solution

$$\begin{array}{rcccc} & & \textcircled{6} & \textcircled{6} & \\ & & 6 & A & B \\ \times & & & A & 9 \\ \hline & 3 & C & 0 & 3 \end{array}$$

$$\begin{cases} \underbrace{9_{16} \times B_{16}}_{99} & = & 16 \times 6 + 3 & = 63_{16} \\ \underbrace{9_{16} \times A_{16}}_{90} + \textcircled{6} & = & \underbrace{16 \times 5 + 10}_{5A_{16}} + \textcircled{6} & = 60_{16} \\ \underbrace{9_{16} \times 6_{16}}_{54} + \textcircled{6} & = & \underbrace{16 \times 3 + 6}_{36_{16}} + \textcircled{6} & = 3C_{16} \end{cases}$$

★ Multiplication of Hexa Numbers ★

Solution

$$
\begin{array}{cccccc}
 & & & \textcircled{6} & \textcircled{6} & \\
 & & & \textcircled{6} & \textcircled{6} & \\
 & & & 6 & A & B \\
\times & & & & A & 9 \\
\hline
 & & 3 & C & 0 & 3 \\
+ & 4 & 2 & A & E & 0 \\
\hline
\end{array}
$$

$$
\begin{cases}
\underbrace{A_{16} \times B_{16}}_{110} & = 16 \times 6 + 14 & = 6E_{16} \\
\underbrace{A_{16} \times A_{16}}_{100} + \textcircled{6} & = \underbrace{16 \times 6 + 4}_{64_{16}} + \textcircled{6} & = 6A_{16} \\
\underbrace{A_{16} \times 6_{16}}_{60} + \textcircled{6} & = \underbrace{16 \times 3 + 12}_{3C_{16}} + \textcircled{6} & = 42_{16}
\end{cases}
$$

★ Multiplication of Hexa Numbers ★

Solution

$$
\begin{array}{cccccc}
 & & & \textcircled{6} & \textcircled{6} & \\
 & & & \textcircled{6} & \textcircled{6} & \\
 & & & 6 & A & B \\
\times & & & & A & 9 \\
\hline
 & & 3 & C & 0 & 3 \\
+ & 4 & 2 & A & E & 0 \\
\hline
 & 4 & 6 & 6 & E & 3
\end{array}
$$

$$
\begin{cases}
\underbrace{A_{16} \times B_{16}}_{110} = 16 \times 6 + 14 = 6E_{16} \\
\underbrace{A_{16} \times A_{16}}_{100} + \textcircled{6} = \underbrace{16 \times 6 + 4}_{64_{16}} + \textcircled{6} = 6A_{16} \\
\underbrace{A_{16} \times 6_{16}}_{60} + \textcircled{6} = \underbrace{16 \times 3 + 12}_{3C_{16}} + \textcircled{6} = 42_{16}
\end{cases}
$$

Example

Evaluate $\mathbf{769.}\boldsymbol{E2} \times \mathbf{8.}\boldsymbol{A}$.

Solution

Example

Evaluate $769.E2 \times 8.A$.

Solution

$$
\begin{array}{cccccccc}
 & & \textcircled{4} & \textcircled{6} & \textcircled{8} & & \textcircled{1} & \\
 & & 7 & 6 & 9 & . & E & 2 \\
\times & & & & & 8 & . & A \\
\hline
 & & 4 & A & 2 & 2 & D & 4
\end{array}
$$

$$
\begin{cases}
\underbrace{A_{16} \times 2_{16}}_{20} & = 16 \times 1 + 4 & = 14_{16} \\
\underbrace{A_{16} \times E_{16}}_{140} + \textcircled{1} & = \underbrace{16 \times 8 + 12}_{8C_{16}} + \textcircled{1} & = 8D_{16} \\
\underbrace{A_{16} \times 9_{16}}_{90} + \textcircled{8} & = \underbrace{16 \times 5 + 10}_{5A_{16}} + \textcircled{8} & = 62_{16} \\
\underbrace{A_{16} \times 6_{16}}_{60} + \textcircled{6} & = \underbrace{16 \times 3 + 12}_{3C_{16}} + \textcircled{6} & = 42_{16} \\
\underbrace{A_{16} \times 7_{16}}_{70} + \textcircled{4} & = \underbrace{16 \times 4 + 6}_{46_{16}} + \textcircled{4} & = 4A_{16}
\end{cases}
$$

Solution

$$
\begin{array}{cccccccc}
 & & \textcircled{3} & \textcircled{4} & \textcircled{7} & & \textcircled{1} & \\
 & & \textcircled{4} & \textcircled{6} & \textcircled{8} & & \textcircled{1} & \\
 & & 7 & 6 & 9 & . & E & 2 \\
\times & & & & & 8 & . & A \\
\hline
 & & 4 & A & 2 & 2 & D & 4 \\
+ & 3 & B & 4 & F & 1 & 0 & \mathbf{0} \\
\hline
\end{array}
$$

$$
\begin{cases}
\underbrace{8_{16} \times 2_{16}}_{112} & = 16 \times 1 + 0 & = 10_{16} \\
\underbrace{8_{16} \times E_{16}}_{112} + \textcircled{1} & = \underbrace{16 \times 7 + 0}_{70_{16}} + \textcircled{1} & = 71_{16} \\
\underbrace{8_{16} \times 9_{16}}_{72} + \textcircled{7} & = \underbrace{16 \times 4 + 8}_{48_{16}} + \textcircled{7} & = 4F_{16} \\
\underbrace{8_{16} \times 6_{16}}_{48} + \textcircled{4} & = \underbrace{16 \times 3 + 0}_{30_{16}} + \textcircled{4} & = 34_{16} \\
\underbrace{8_{16} \times 7_{16}}_{56} + \textcircled{3} & = \underbrace{16 \times 3 + 8}_{38_{16}} + \textcircled{3} & = 3B_{16}
\end{cases}
$$

Solution

$$
\begin{array}{cccccccc}
 & & \textcircled{3} & \textcircled{4} & \textcircled{7} & & \textcircled{1} & \\
 & & \textcircled{4} & \textcircled{6} & \textcircled{8} & & \textcircled{1} & \\
 & & 7 & 6 & 9 & . & E & 2 \\
\times & & & & & 8 & . & A \\
\hline
 & & 4 & A & 2 & 2 & D & 4 \\
+ & 3 & B & 4 & F & 1 & 0 & 0 \\
\hline
 & 3 & F & F & 1 & 3 & D & 4 \\
\hline
\multicolumn{8}{|c|}{3FF1 \cdot 3D4}
\end{array}
$$

$$
\begin{cases}
\underbrace{8_{16} \times 2_{16}}_{112} & = 16 \times 1 + 0 & = 10_{16} \\
\underbrace{8_{16} \times E_{16}}_{112} + \textcircled{1} & = \underbrace{16 \times 7 + 0}_{70_{16}} + \textcircled{1} & = 71_{16} \\
\underbrace{8_{16} \times 9_{16}}_{72} + \textcircled{7} & = \underbrace{16 \times 4 + 8}_{48_{16}} + \textcircled{7} & = 4F_{16} \\
\underbrace{8_{16} \times 6_{16}}_{48} + \textcircled{4} & = \underbrace{16 \times 3 + 0}_{30_{16}} + \textcircled{4} & = 34_{16} \\
\underbrace{8_{16} \times 7_{16}}_{56} + \textcircled{3} & = \underbrace{16 \times 3 + 8}_{38_{16}} + \textcircled{3} & = 3B_{16}
\end{cases}
$$

★ Division of Hexa Numbers ★

The hexadecimal numbers division is same as the division performs in other number systems. All the rules and principles are same.

$3A \times 1$	$3A$
$3A \times 2$	74
$3A \times 3$	AE
$3A \times 4$	$3A$
$3A \times 4$	$E8$
$3A \times 5$	122
$3A \times 6$	$15C$
$3A \times 7$	196
$3A \times 8$	$1D0$
$3A \times 9$	$20A$
$3A \times A$	244
$3A \times B$	$27E$

$\Longrightarrow$

$$
\begin{array}{rcccc|l}
 & A & E & 8 & 8 \quad 7 & 3A \\
- & & & \downarrow & \downarrow \quad | & \hline \\
 & A & E & \downarrow & \downarrow \quad | & 3025.A3 \\
\hline
 & 0 & 0 & 8 & 8 \quad | & \\
- & & & & \quad \downarrow & \\
 & & & 7 & 4 & \\
\hline
 & & & 1 & 4 \quad 7 & \\
- & & & 1 & 2 \quad 2 & \\
\hline
 & & & 0 & 2 \quad 50 & \\
- & & & & 2 \quad 24 & \\
\hline
 & & & & B0 & \\
- & & & & AE & \\
\hline
 & & & & 2 &
\end{array}
$$

★ Division of Hexa Numbers ★

12×1	12
12×2	24
12×3	36
12×4	48
12×5	5*A*
12×6	6*C*
12×7	7*E*
12×8	90
12×9	*A*2
$12 \times A$	*B*4
$12 \times B$	*C*6
$12 \times C$	*D*8
$12 \times D$	*EA*

$\Longrightarrow$

	E	*C*	*E*	·	4		6
−			↓				
	E	*A*					
	0	2	*E*				
−					↓		
		2	4				
			A		4		
−			*A*		2		↓
			0		2		6
−					2		4
					0		2

12

*D*2.92

★ Division of Hexa Numbers ★

$257 \cdot 8A_{16} \div 5 \cdot 1_{16} \Longleftrightarrow 2578 \cdot A_{16} \div 51_{16}$

51×1	51
51×2	$A2$
51×3	$F3$
51×4	144
51×5	195
51×6	$1E6$
51×7	237
51×8	288
51×9	$2D9$
$51 \times A$	$32A$
$51 \times B$	$37B$
$51 \times C$	$3CC$
$51 \times D$	$41D$

$\Longrightarrow$

$$
\begin{array}{rcccccc|c}
 & 2 & 5 & 7 & 8 & \cdot & A & 51 \\
- & & & & \downarrow & & & \hline \\
 & 2 & 3 & 7 & \downarrow & & & 76 \cdot 6DA \\
\hline
 & 0 & 2 & 0 & 8 & & \downarrow & \\
- & & & & & & & \\
 & & 1 & E & 6 & & \downarrow & \\
\hline
 & & & 2 & 2 & & A & \\
- & & & 1 & E & & 6 & \\
\hline
 & & & & 4 & & 40 & \\
- & & & & 4 & & 1D & \\
\hline
 & & & & & & 330 & \\
- & & & & & & 32A & \\
\hline
 & & & & & & 6 &
\end{array}
$$


Thank you! Questions?