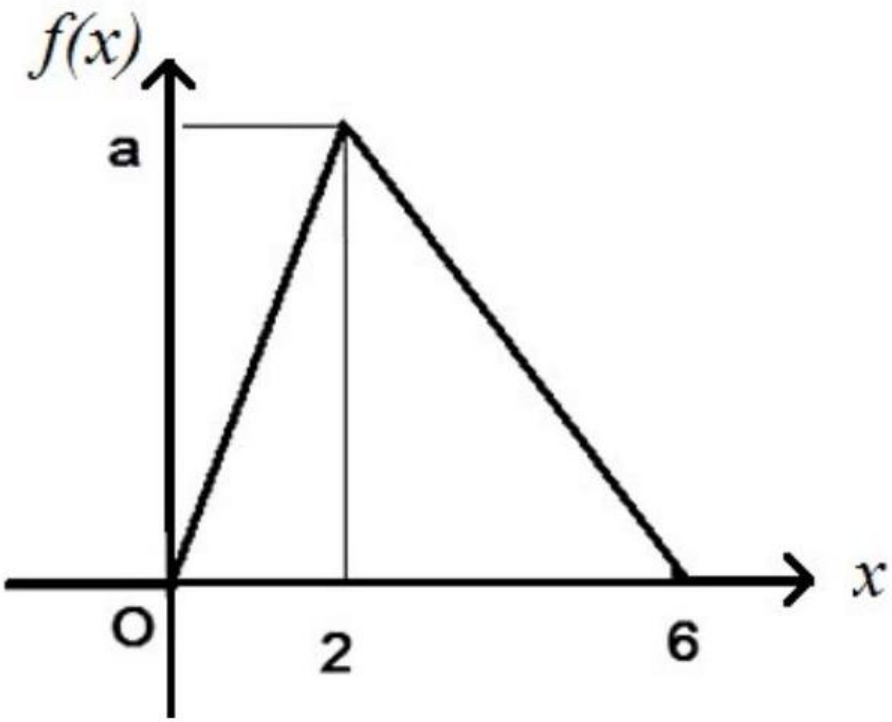
Exercises

✓ Exercise 1 [*]

A continuous random variable X admits for probability density function f, a piece-wise affine function whose plot is shown below



a) What is the value of $a = f(2)$?
b) Compute $P(X = 1), P(X < 1), P(X < 2)$ and $P(2 < X \leq 6)$.
c) What is the expected value of X ?

Exercise 2[*]

A continuous random variable X admits for probability density function

$$f(x) = \begin{cases} \frac{k}{x} \text{ if } x \in [1,4] \\ 0 \text{ otherwise} \end{cases}$$

a) Find the value of the constant k.
b) Compute $P(1 \leq X \leq 2 \mid 1 \leq X \leq 3)$.
c) Determine $E(X)$ and $V(X)$.

Exercise 3 [*]

A continuous random variable X admits for probability density function

$$\forall x \in \mathbb{R}, f(x) = \frac{c}{1+x^2}$$

a) Find the value of the constant c.
b) Compute the cumulative distribution function F_X.
c) Compute $P(X \geq 2)$ and $P(-3 \leq X \leq 4)$.
d) Compute $P\left(\frac{1}{3} \leq X^2 \leq 1\right)$.
e) Determine $E(X)$ and $V(X)$.

Exercise 4 [**]

A continuous random variable X admits for probability density function

$$f(x) = \begin{cases} kx^2 \text{ if } x \in [-3,6] \\ 0 \text{ otherwise} \end{cases}$$

a) Find the value of the constant k.

b) Compute the probability density functions of the variables $U = \frac{1}{3}(12 - X)$ and $V = X^2$.

c) Compute the expected value and the variance of X, U and V.

Exercise 5 [*]

A darts player finds that the distance R from the impact of the dart to the centre of the target (of

radius a) follows a law whose probability density function is $f(r) = c\left(1 - \frac{r^2}{a^2}\right)$ where c is a constant. Compute the probability of hitting the target within a radius b of the centre of the target (assuming that the target is always hit).

Exercise 6 [**] Temperature of a gas

Consider a box containing N particles of oxygen of mass $m = 2,7 \cdot 10^{-26}$ kg. Suppose that the speed of the particles is a random variable V whose probability density function is

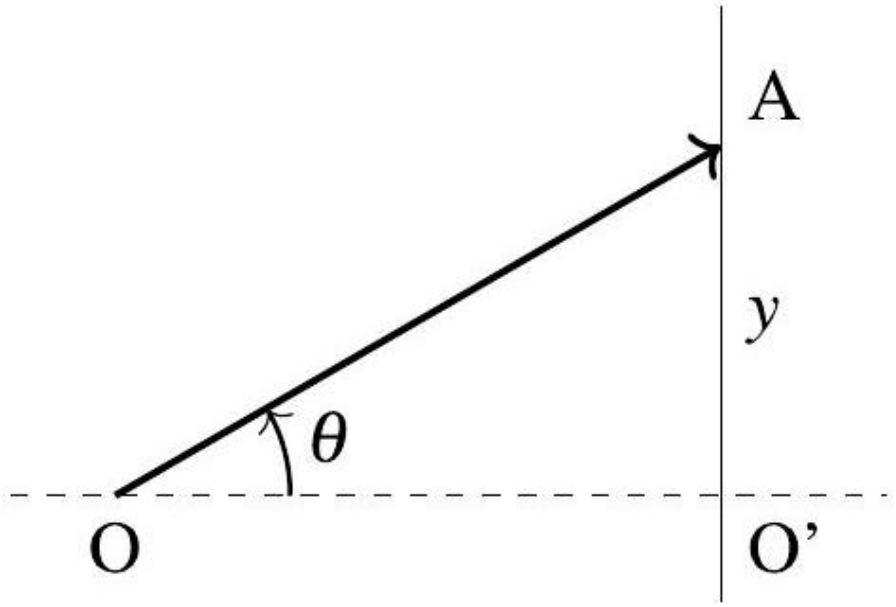
$$f(v) = \begin{cases} ce^{-\frac{|v-v_0|}{v_0}} \text{ if } v \geq 0 \\ 0 \text{ otherwise.} \end{cases} \qquad \text{where } c \in \mathbb{R} \text{ and } v_0 = 800 \text{ km/h}$$

a) Find the value of the constant c.

b) Let $K = \frac{1}{2}mV^2$ be the kinetic energy of a particle. Knowing that the gaz temperature T is defined by $kT \quad = (K) \quad$ where $k = 1,38 \cdot 10^{-23}$ J/K is Boltzmann's constant, compute the gaz temperature.

Exercise 7 [*]

A light source emits at point O uniformly in the plane $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Compute the probability density of the random variable Y equal to the algebraic value of O′A. Take the distance OO' equal to one metre.



Exercise 8 [*]

A cylindrical beam of particles of radius a is directed at a hemispherical target ABC of centre O. Let $f(r)$ be the uniform probability density function on $[0, a]$

of the continuous random variable $R =$ "distance of the particle from the axis OB ".