Final exam

April 24

Lecture question (recommended duration : 15 minutes)

What is an exponential distribution of parameter λ? Support, conditions on λ, density and cumulative functions, expectation and variance. Plot the graph of the density for $\lambda = 1$.

Exercice 1 (recommended duration : 20 minutes)

Soit la fonction définie sur $[0, 4]$:

$$\begin{cases} f(x) = kx & \text{pour } 0 \leq x \leq 2 \\ f(x) = 1 - kx & \text{pour } 2 \leq x \leq 4. \end{cases}$$

a. Compute k such that f is a density function over $[0, 4]$.

b. Represent the graph of f.

c. Let X a random variable whose density function is f. Determine $P\left(\frac{1}{2} \leq X \leq \frac{3}{2}\right)$.

d. Compute the probability of X being greater than 1 knowing that it is less than 3 .

Exercice 2 (recommended duration : 30 minutes)

The goal of this exercise is to solve the following problem : two counters are open at a bank. The service time $T1$ at the first counter (respectively, $T2$ at the second counter) follows an exponential distribution with mean 5 minutes (respectively, 8 minutes). The two counters are assumed to be independent. Two customers enter simultaneously, one choosing counter 1 and the other counter 2. We want to find the mean time δT after which the first customer exits and the mean time ΔT after which the last customer exits.

a. Introduce the variable $Z = min(T1, T2)$. Express the cumulative distribution function of Z in terms of those of $T1$ and $T2$.

b. Deduce the distribution of Z.

c. Find δT and ΔT .

Exercice 3 (recommended duration : 20 minutes)

A factory makes electronic components whose life duration is the random variable X of normal distribution with expectation 5 years and of standard deviation 6 months.

We randomly pick a component. Using the table, what is the probability that its life duration is :

a. less than 4 years and 3 months

b. greater than 5 years and 3 months

c. between 4 years and 6 months and 6 years

$1 - \frac{3}{4}$

$\frac{4 - 3}{4} = \frac{1}{4} = \frac{3.5}{2} = 1.75$

$6 \overline{)125}$ 2.8.
12
50