AI5002: Assignment 13

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Download all Python codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_13/simulation%20code.py

and latex-tikz codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_13/latex_code.tex

1 Problem

A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx, & 0 < x < 1 \\ 0, & otherwise \end{cases} \tag{1.0.1}$$

If the expected value $E[X] = \frac{2}{3}$, then $Pr[X < 0.5]$ is..........

2 Solution

First, we need to find out the values of a and b. We know,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1 \tag{2.0.1}$$

$$\implies \int_0^1 (a + bx)\ dx = 1 \tag{2.0.2}$$

$$\implies \left[ax + b\frac{x^2}{2}\right]_0^1 = 1 \tag{2.0.3}$$

$$\implies a + \frac{b}{2} = 1 \tag{2.0.4}$$

$$\implies 2a + b = 2 \tag{2.0.5}$$

$$E(X) = \frac{2}{3} \tag{2.0.6}$$

$$\implies \int_0^1 xf(x)\, dx = \frac{2}{3} \tag{2.0.7}$$

$$\implies \int_0^1 x(a + bx)\, dx = \frac{2}{3} \tag{2.0.8}$$

$$\implies \left[a\frac{x^2}{2} + b\frac{x^3}{3}\right]_0^1 = \frac{2}{3} \tag{2.0.9}$$

$$\implies \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \tag{2.0.10}$$

$$\implies 3a + 2b = 4 \tag{2.0.11}$$

Multiplying (2.0.5) with 2 and subtracting (2.0.11) from it, we get

$$a = 0 \tag{2.0.12}$$

Putting (2.0.12) in (2.0.5), we get

$$b = 2 \tag{2.0.13}$$

Using (2.0.12) and (2.0.13) in (1.0.1),

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise \end{cases} \tag{2.0.14}$$

$$F_X(x) = Pr(X < x) \tag{2.0.15}$$

$$= \int_{-\infty}^{x} f(t)\, dt \tag{2.0.16}$$

$$= \int_0^x 2t\ dt \tag{2.0.17}$$

$$= 2\left[\frac{t^2}{2}\right]_0^x \tag{2.0.18}$$

$$= x^2, 0 < x < 1 \tag{2.0.19}$$

Thus, the CDF is:

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ x^2, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases} \tag{2.0.20}$$

$$\therefore Pr(X < 0.5) = F_X(0.5) \tag{2.0.21}$$
$$= (0.5)^2 \tag{2.0.22}$$
$$= 0.25 \tag{2.0.23}$$

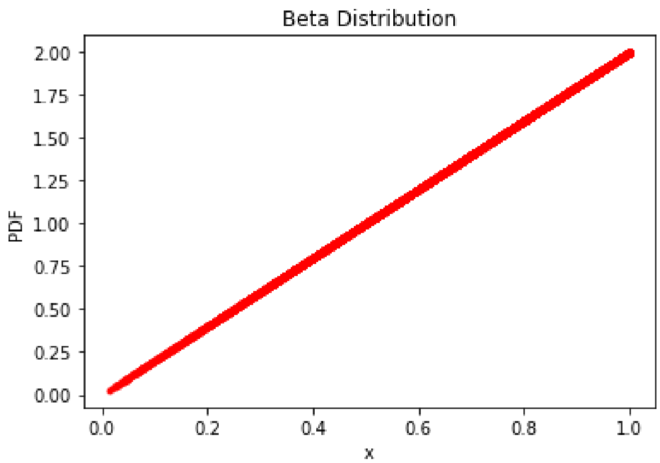

Fig. 0: PDF Plot