

AI5002: Challenging Problem 1

Debolena Basak
AI20RESCH11003

Download all Python codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/Assignment_11/assignment11_code_drawing%20balls.py

and latex-tikz codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/Challenging%20Problem_1/latex.tex

1 PROBLEM

Let X be a random variable such that $E(X) = E(X^2) = 1$. Then, $E(X^{100}) = ?$

- 1) 0
- 2) 1
- 3) 2^{100}
- 4) $2^{100} + 1$

2 SOLUTION

Given,

$$E(X) = 1 \quad (2.0.1)$$

$$\text{and } E(X^2) = 1 \quad (2.0.2)$$

$$\therefore \text{Var}(X) = E(X^2) - E^2(X) = 1 - 1 = 0 \quad (2.0.3)$$

$\therefore X$ is a degenerate random variable degenerated at $X = 1$.

$$\therefore \text{Var}(X) = 0 \quad (2.0.4)$$

$$\implies E(X - E(X))^2 = 0 \quad (2.0.5)$$

$$\implies X = E(X) = 1, \text{ with probability } 1. \quad (2.0.6)$$

$$\implies P(X = 1) = 1 \quad (2.0.7)$$

$$\therefore E(X^{100}) = k^{100} \times P(X = k), k = 1 \quad (2.0.8)$$

$$= 1 \times P(X = 1) \quad (2.0.9)$$

$$= 1 \quad (2.0.10)$$

Hence, option (2) is the correct answer.