

AI5002: Binomial Addition

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AI20RESCH11003

Download all Python codes from

<https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/binomial%20subtraction/binomial%20.py>

and latex-tikz codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/binomial%20subtraction/binomial_addition.tex

1 PROBLEM

Find the PMF of the addition of two binomial random variables.

2 SOLUTION

Let, $X_1 \sim \text{Bin}(n_1, p)$ and $X_2 \sim \text{Bin}(n_2, p)$

Then, $Y = X_1 + X_2$ takes values $0, 1, 2, \dots, (n_1 + n_2)$

$$P(Y = y), y = 0, 1, 2, \dots, (n_1 + n_2) \quad (2.0.1)$$

$$= P(X_1 + X_2 = y) \quad (2.0.2)$$

$$= \sum_{x_1=0}^{\min(n_1, y)} P(X_1 = x_1, X_2 = y - x_1) \quad (2.0.3)$$

$$= \sum_{x_1=0}^m P(X_1 = x_1).P(X_2 = y - x_1), m = \min(n_1, y) \quad (2.0.4)$$

$$= \sum_{x_1=0}^m \binom{n_1}{x_1} p^{x_1} q^{n_1-x_1} \cdot \binom{n_2}{y-x_1} p^{y-x_1} q^{n_2-y+x_1} \quad (2.0.5)$$

$$= p^y \cdot q^{n_1+n_2-y} \cdot \sum_{x_1=0}^m \binom{n_1}{x_1} \binom{n_2}{y-x_1} \quad (2.0.6)$$

$$= \binom{n_1 + n_2}{y} \cdot p^y \cdot q^{n_1+n_2-y} \quad (2.0.7)$$

$$\therefore Y = X_1 + X_2 \sim \text{Bin}(n_1 + n_2, p)$$