AI5002: Assignment 14

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AI20RESCH11003

Download all Python codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_14/code.py

and latex-tikz codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_14/latex.tex

1 Problem

Gate Problem 20:
Let X be a random variable with probability density function

$$f(x) = \begin{cases} 0.2, & |x| \le 1 \\ 0.1, & 1 \le |x| \le 4 \\ 0, & otherwise \end{cases} \tag{1.0.1}$$

The probability $Pr(0.5 < X < 5)$ is.....

2 Solution

The CDF is:

$$F_X(x) = \begin{cases} 0, & x \le -1 \\ \int_{-1}^{x} 0.2dt, & |x| \le 1 \\ \int_{-1}^{1} 0.2dt + \int_{1}^{x} 0.1dt, & 1 \le |x| \le 4 \\ 1, & x > 4 \end{cases} \tag{2.0.1}$$

$$= \begin{cases} 0, & x \le -1 \\ 0.2(x+1), & |x| \le 1 \\ 0.4 + 0.1(x-1)dt, & 1 \le |x| \le 4 \\ 1, & x > 4 \end{cases} \tag{2.0.2}$$

$$Pr(0.5 < X < 5) \tag{2.0.3}$$
$$= Pr(0.5 < X < 4) \tag{2.0.4}$$
$$[\because f(x) = 0 \text{ for } x \in (4,5)] \tag{2.0.5}$$
$$= Pr(X < 4) - Pr(X \le 0.5) \tag{2.0.6}$$
$$= F_X(4) - F_X(0.5) \tag{2.0.7}$$
$$= [0.4 + 0.1(4-1)] - [0.2(0.5+1)] \tag{2.0.8}$$
$$= 0.7 - 0.3 \tag{2.0.9}$$
$$= 0.4 \tag{2.0.10}$$

Alternatively,

$$Pr(0.5 < X < 5) \tag{2.0.11}$$
$$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx + \int_{4}^{5} f(x)\,dx \tag{2.0.12}$$
$$= \int_{0.5}^{1} 0.2dx + \int_{1}^{4} 0.1dx + \int_{4}^{5} 0dx \tag{2.0.13}$$
$$= 0.2 \times 0.5 + 0.1 \times 3 + 0 \tag{2.0.14}$$
$$= 0.4 \tag{2.0.15}$$

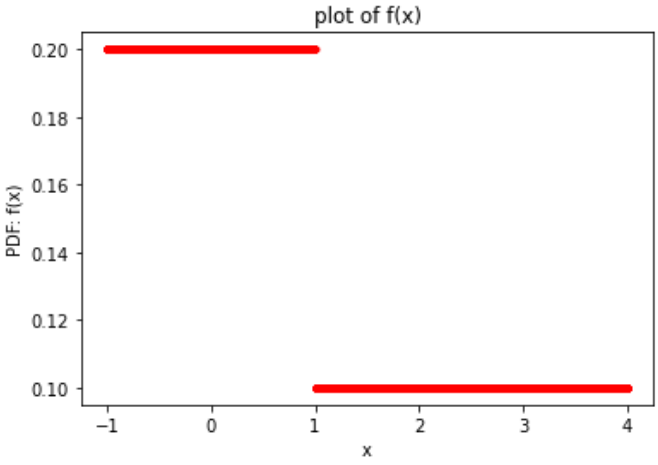

Fig. 0: PDF Plot