AI5002: Assignment 5

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1 Problem

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

2 Solution

Let $X \in \{0, 1\}$ be the random variable denoting an accident or not an accident, where 1 represents an accident and 0 represents not an accident.

Let $Y \in \{0, 1, 2\}$ be the random variable denoting whether the person is a scooter driver, car driver or a truck driver. Let 0, 1 and 2 represent scooter, car and truck drivers respectively

Given are:

$$P(Y = 0) = \frac{2000}{12000} = \frac{2}{12} = \frac{1}{6} \quad (2.0.1)$$

$$P(Y = 1) = \frac{4000}{12000} = \frac{4}{12} = \frac{1}{3} \quad (2.0.2)$$

$$P(Y = 2) = \frac{6000}{12000} = \frac{6}{12} = \frac{1}{2} \quad (2.0.3)$$

$$P(X = 1|Y = 0) = 0.01 \quad (2.0.4)$$

$$P(X = 1|Y = 1) = 0.03 \quad (2.0.5)$$

$$P(X = 1|Y = 2) = 0.15 \quad (2.0.6)$$

We have to find $P(Y = 0|X = 1)$
Using Bayes Theorem,

$$P(Y = 0|X = 1) = \frac{P(X = 1|Y = 0)}{\sum_{k=0}^{2} P(X = 1|Y = k) P(Y = k)} \quad (2.0.7)$$

$$= \frac{0.01 \times \frac{2}{12}}{0.01 \times \frac{2}{12} + 0.03 \times \frac{4}{12} + 0.15 \times \frac{6}{12}} \quad (2.0.8)$$

$$= \frac{1}{52} \quad (2.0.9)$$

$$= 0.01923 \quad (2.0.10)$$