

AI5002: Assignment 13

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Download all Python codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/Assignment_13/simulation%20code.py

and latex-tikz codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/Assignment_13/latex_code.tex

1 PROBLEM

A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \quad (1.0.1)$$

If the expected value $E[X] = \frac{2}{3}$, then $Pr[X < 0.5]$ is.....

2 SOLUTION

First, we need to find out the values of a and b .
We know,

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad (2.0.1)$$

$$\Rightarrow \int_0^1 (a + bx) dx = 1 \quad (2.0.2)$$

$$\Rightarrow \left[ax + b \frac{x^2}{2} \right]_0^1 = 1 \quad (2.0.3)$$

$$\Rightarrow a + \frac{b}{2} = 1 \quad (2.0.4)$$

$$\Rightarrow 2a + b = 2 \quad (2.0.5)$$

$$E(X) = \frac{2}{3} \quad (2.0.6)$$

$$\Rightarrow \int_0^1 xf(x) dx = \frac{2}{3} \quad (2.0.7)$$

$$\Rightarrow \int_0^1 x(a + bx) dx = \frac{2}{3} \quad (2.0.8)$$

$$\Rightarrow \left[a \frac{x^2}{2} + b \frac{x^3}{3} \right]_0^1 = \frac{2}{3} \quad (2.0.9)$$

$$\Rightarrow \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \quad (2.0.10)$$

$$\Rightarrow 3a + 2b = 4 \quad (2.0.11)$$

Multiplying (2.0.5) with 2 and subtracting (2.0.11) from it, we get

$$a = 0 \quad (2.0.12)$$

Putting (2.0.12) in (2.0.5), we get

$$b = 2 \quad (2.0.13)$$

Using (2.0.12) and (2.0.13) in (1.0.1),

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \quad (2.0.14)$$

$$\therefore Pr(X < 0.5) = \int_0^{0.5} f(x) dx \quad (2.0.15)$$

$$= \int_0^{0.5} 2x dx \quad (2.0.16)$$

$$= 2 \left[\frac{x^2}{2} \right]_0^{0.5} = (0.5)^2 \quad (2.0.17)$$

$$= 0.25 \quad (2.0.18)$$

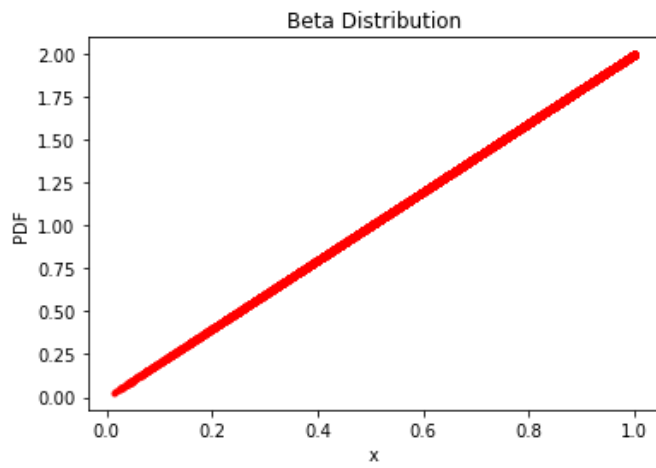


Fig. 0: Beta Distribution Plot