AI5002: Binomial Subtraction

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Download all Python codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_2/binomial_simulation.py

and latex-tikz codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/binomial%20subtraction/latex.tex

1 Problem

Let, $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, q)$, independently. Find the PMF of $X_1 - X_2$.

2 Solution

Given, $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, q)$, independently.
$\therefore n_2 - X_2 \sim Bin(n_2, p)$
By additive/ reproductive property of binomial,
$X_1 + n_2 - X_2 \sim Bin(n_1 + n_2, p)$
Let, $D = X_1 - X_2$.

$$P(D = d) = P(X_1 - X_2 = d) \tag{2.0.1}$$

$$= P(X_1 - X_2 + n_2 = d + n_2) \tag{2.0.2}$$

$$= \binom{n_1 + n_2}{n_2 + d} p^{n_2+d} q^{n_1-d}, d = -n_2 \text{ to } n_1 \tag{2.0.3}$$