AI5002: Binomial Subtraction

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Download all Python codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/compulsory%20binomial%20assignment/binomial_simulation.py

and latex-tikz codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/compulsory%20binomial%20assignment/latex.tex

1 Problem

Let, $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, q)$, independently. Find the PMF of $X_1 - X_2$.

2 Solution

Given, $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, q)$, independently.
$\therefore n_2 - X_2 \sim Bin(n_2, p)$
By additive/ reproductive property of binomial,
$X_1 + n_2 - X_2 \sim Bin(n_1 + n_2, p)$
Let, $D = X_1 - X_2$.

$$P(D = d) = P(X_1 - X_2 = d) \tag{2.0.1}$$
$$= P(X_1 - X_2 + n_2 = d + n_2) \tag{2.0.2}$$
$$= \binom{n_1 + n_2}{n_2 + d} p^{n_2+d} q^{n_1-d}, d = -n_2 \text{ to } n_1 \tag{2.0.3}$$

3 Reproductive property

If $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, p)$, then
$Y = X_1 + X_2 \sim Bin(n_1 + n_2, p)$.

4 Proof

Let, $X_1 \sim Bin(n_1, p)$ and $X_2 \sim Bin(n_2, p)$
Then, $Y = X_1 + X_2$ takes values 0,1,2,....,$(n_1 + n_2)$

$$P(Y = y), y = 0, 1, 2, ..., (n_1 + n_2) \tag{4.0.1}$$
$$= P(X_1 + X_2 = y) \tag{4.0.2}$$
$$= \sum_{x_1=0}^{min(n_1,y)} P(X_1 = x_1, X_2 = y - x_1) \tag{4.0.3}$$
$$= \sum_{x_1=0}^{m} P(X_1 = x_1).P(X_2 = y - x_1), m = min(n_1, y) \tag{4.0.4}$$
$$= \sum_{x_1=0}^{m} \binom{n_1}{x_1} p^{x_1} q^{n_1-x_1}.\binom{n_2}{y - x_1} p^{y-x_1} q^{n_2-y+x_1} \tag{4.0.5}$$
$$= p^y.q^{n_1+n_2-y}.\sum_{x_1=0}^{m} \binom{n_1}{x_1}\binom{n_2}{y - x_1} \tag{4.0.6}$$
$$= \binom{n_1 + n_2}{y}.p^y.q^{n_1+n_2-y} \tag{4.0.7}$$

$\therefore \quad Y = X_1 + X_2 \sim Bin(n_1 + n_2, p)$

5 Problem

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

6 Solution

Let U_1 denotes the number of heads and U_2 denotes the number of tails that occur when a coin is tossed 6 times.
Clearly, $U_1 \sim Bin(n = 6, p)$
and $U_2 \sim Bin(n = 6, 1 - p = q)$.
$\therefore n - U_2 \sim Bin(6, p)$.
By reproductive property,

$$U_1 + n - U_2 \sim Bin(6 + 6, p) \tag{6.0.1}$$

$X = U_1 - U_2$. Using (2.0.3),

$$P(X = x) = \binom{6+6}{6+x} p^{6+x} q^{6-x}, x = -6 \text{ to } 6 \quad (6.0.2)$$

Suppose, the coin is unbiased. Then, $p = q = \frac{1}{2}$.

$$\therefore P(X = x) = \binom{12}{6+x} \left(\frac{1}{2}\right)^{12}, x = -6 \text{ to } 6 \quad (6.0.3)$$

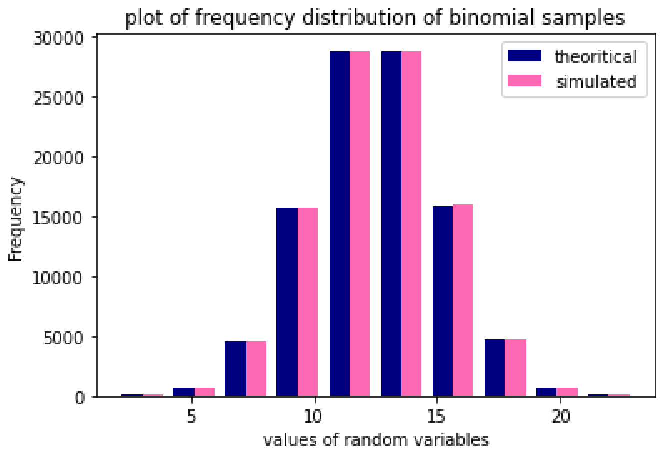

Fig. 0: Binomial Frequency plot