

AI5002: Binomial Assignment

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Download all Python codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/compulsory%20binomial%20assignment/binomial_simulation.py

and latex-tikz codes from

<https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/compulsory%20binomial%20assignment/latex.tex>

1 PROBLEM

Let, $X_1 \sim \text{Bin}(n_1, p)$ and $X_2 \sim \text{Bin}(n_2, q)$, independently. Find the PMF of $X_1 - X_2$.

2 SOLUTION

Given, $X_1 \sim \text{Bin}(n_1, p)$ and $X_2 \sim \text{Bin}(n_2, q)$, independently.

$\therefore n_2 - X_2 \sim \text{Bin}(n_2, p)$

By additive/ reproductive property of binomial,
 $X_1 + n_2 - X_2 \sim \text{Bin}(n_1 + n_2, p)$

Let, $D = X_1 - X_2$.

$$P(D = d) = P(X_1 - X_2 = d) \quad (2.0.1)$$

$$= P(X_1 - X_2 + n_2 = d + n_2) \quad (2.0.2)$$

$$= \binom{n_1 + n_2}{n_2 + d} p^{n_2 + d} q^{n_1 - d}, d = -n_2 \text{ to } n_1 \quad (2.0.3)$$

3 REPRODUCTIVE PROPERTY

If $X_1 \sim \text{Bin}(n_1, p)$ and $X_2 \sim \text{Bin}(n_2, p)$, then
 $Y = X_1 + X_2 \sim \text{Bin}(n_1 + n_2, p)$.

4 PROOF

Let, $X_1 \sim \text{Bin}(n_1, p)$ and $X_2 \sim \text{Bin}(n_2, p)$

Then, $Y = X_1 + X_2$ takes values $0, 1, 2, \dots, (n_1 + n_2)$

$$P(Y = y), y = 0, 1, 2, \dots, (n_1 + n_2) \quad (4.0.1)$$

$$= P(X_1 + X_2 = y) \quad (4.0.2)$$

$$= \sum_{x_1=0}^{\min(n_1, y)} P(X_1 = x_1, X_2 = y - x_1) \quad (4.0.3)$$

$$= \sum_{x_1=0}^m P(X_1 = x_1) \cdot P(X_2 = y - x_1), m = \min(n_1, y) \quad (4.0.4)$$

$$= \sum_{x_1=0}^m \binom{n_1}{x_1} p^{x_1} q^{n_1 - x_1} \cdot \binom{n_2}{y - x_1} p^{y - x_1} q^{n_2 - y + x_1} \quad (4.0.5)$$

$$= p^y \cdot q^{n_1 + n_2 - y} \cdot \sum_{x_1=0}^m \binom{n_1}{x_1} \binom{n_2}{y - x_1} \quad (4.0.6)$$

$$= \binom{n_1 + n_2}{y} \cdot p^y \cdot q^{n_1 + n_2 - y} \quad (4.0.7)$$

$\therefore Y = X_1 + X_2 \sim \text{Bin}(n_1 + n_2, p)$

5 PROBLEM

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

6 SOLUTION

Let U_1 denotes the number of heads and U_2 denotes the number of tails that occur when a coin is tossed 6 times.

Clearly, $U_1 \sim \text{Bin}(n = 6, p)$

and $U_2 \sim \text{Bin}(n = 6, 1 - p = q)$.

$\therefore n - U_2 \sim \text{Bin}(6, p)$.

By reproductive property,

$$U_1 + n - U_2 \sim \text{Bin}(6 + 6, p) \quad (6.0.1)$$

$X = U_1 - U_2$. Using (2.0.3),

$$P(X = x) = \binom{6+x}{6-x} p^{6+x} q^{6-x}, x = -6 \text{ to } 6 \quad (6.0.2)$$

Suppose, the coin is unbiased. Then, $p = q = \frac{1}{2}$.

$$\therefore P(X = x) = \binom{12}{6+x} \left(\frac{1}{2}\right)^{12}, x = -6 \text{ to } 6 \quad (6.0.3)$$

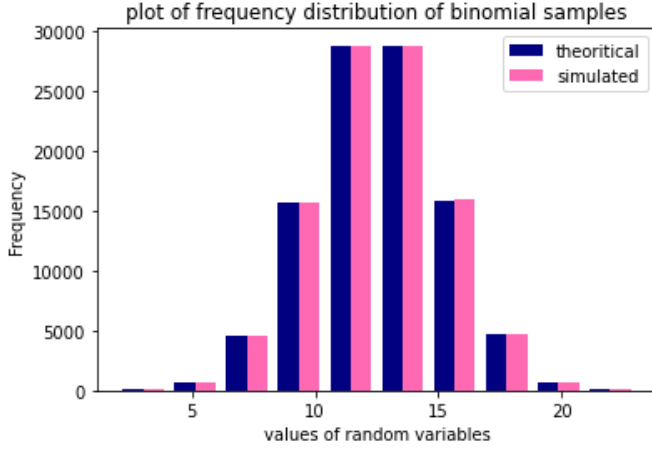


Fig. 0: Binomial Frequency plot