AI5002: Assignment 8

Debolena Basak
AI20RESCH11003

Download all Python codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_8/python_assignment_8.py

and latex-tikz codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_8/latex.tex

1 Problem

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X=x)=\begin{cases}0.1, & \text{if } x=0\\ kx, & \text{if } x=1 \text{ or } 2\\ k(5-x) & \text{if } x=3 \text{ or } 4\\ 0, & \text{otherwise}\end{cases} \tag{1.0.1}$$

1) Find the value of k.
2) What is the probability that you study at least two hours ? Exactly two hours? At most two hours?

2 Solution

1) We know, sum of the all probabilities = 1

$$\sum_{x=0}^{4} P(X=x)=1 \tag{2.0.1}$$
$$\implies 0.1+k+2k+2k+k=1 \tag{2.0.2}$$
$$\implies 0.1+6k=1 \tag{2.0.3}$$
$$\implies k=0.15 \tag{2.0.4}$$

2) Probability of studying atleast 2 hours

$$=\sum_{x=2}^{4} P(X=x) \tag{2.0.5}$$
$$=P(X=2)+P(X=3)+P(X=4) \tag{2.0.6}$$
$$=2k+2k+k \tag{2.0.7}$$
$$=5*0.15 \tag{2.0.8}$$
$$=0.75 \tag{2.0.9}$$

Probability of studying exactly 2 hours

$$=P(X=2) \tag{2.0.10}$$
$$=0.15*2 \tag{2.0.11}$$
$$=0.30 \tag{2.0.12}$$

Probability of studying atmost 2 hours

$$=\sum_{x=0}^{2} P(X=x) \tag{2.0.13}$$
$$=P(X=0)+P(X=1)+P(X=2) \tag{2.0.14}$$
$$=0.1+0.15+0.30 \tag{2.0.15}$$
$$=0.55 \tag{2.0.16}$$

NOTE:
Suppose A= event of studying atleast 2 hours
B= event of studying atmost 2 hours
Then clearly, $A \cap B$ is the event of studing exactly two hours.
We can verify here that

$$P(A)+P(B)-P(A\cap B) \tag{2.0.17}$$
$$=0.75+0.55-0.30 \tag{2.0.18}$$
$$=1 \tag{2.0.19}$$