

AI5002: Assignment 8

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Download all Python codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/Assignment_8/python_assignment_8.py

and latex-tikz codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/Assignment_8/latex.tex

1 PROBLEM

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x , has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5 - x) & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases} \quad (1.0.1)$$

- 1) Find the value of k .
- 2) What is the probability that you study at least two hours ? Exactly two hours? At most two hours?

2 SOLUTION

- 1) We know, sum of the all probabilities = 1

$$\begin{aligned} \sum_{x=0}^4 P(X = x) &= 1 & (2.0.1) \\ \Rightarrow 0.1 + k + 2k + 2k + k &= 1 & (2.0.2) \\ \Rightarrow 0.1 + 6k &= 1 & (2.0.3) \\ \Rightarrow k &= 0.15 & (2.0.4) \end{aligned}$$

- 2) Probability of studying atleast 2 hours

$$= P(X \geq 2) \quad (2.0.5)$$

$$= \sum_{x=2}^4 P(X = x) \quad (2.0.6)$$

$$= P(X = 2) + P(X = 3) + P(X = 4) \quad (2.0.7)$$

$$= 2k + 2k + k \quad (2.0.8)$$

$$= 5 * 0.15 \quad (2.0.9)$$

$$= 0.75 \quad (2.0.10)$$

- Probability of studying exactly 2 hours

$$= P(X = 2) \quad (2.0.11)$$

$$= 0.15 * 2 \quad (2.0.12)$$

$$= 0.30 \quad (2.0.13)$$

- Probability of studying atmost 2 hours

$$= P(X \leq 2) \quad (2.0.14)$$

$$= \sum_{x=0}^2 P(X = x) \quad (2.0.15)$$

$$= P(X = 0) + P(X = 1) + P(X = 2) \quad (2.0.16)$$

$$= 0.1 + 0.15 + 0.30 \quad (2.0.17)$$

$$= 0.55 \quad (2.0.18)$$

NOTE:

Suppose A = event of studying atleast 2 hours

B = event of studying atmost 2 hours

Then clearly, $A \cap B$ is the event of studing exactly two hours.

We can verify here that

$$P(A) + P(B) - P(A \cap B) \quad (2.0.19)$$

$$= 0.75 + 0.55 - 0.30 \quad (2.0.20)$$

$$= 1 \quad (2.0.21)$$

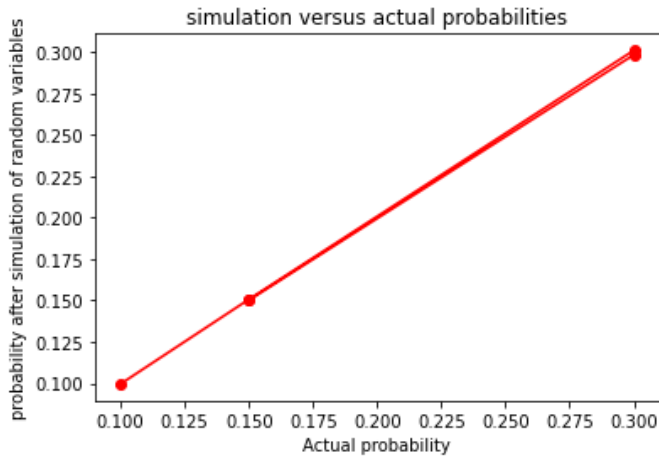


Fig. 2: Actual Vs Simulated Plot

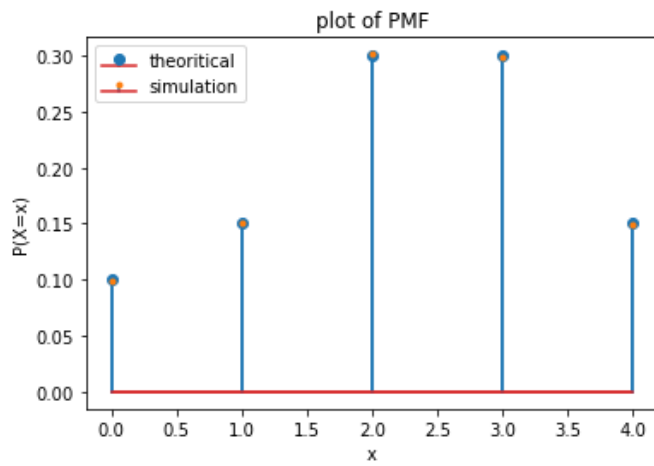


Fig. 2: PMF Plot

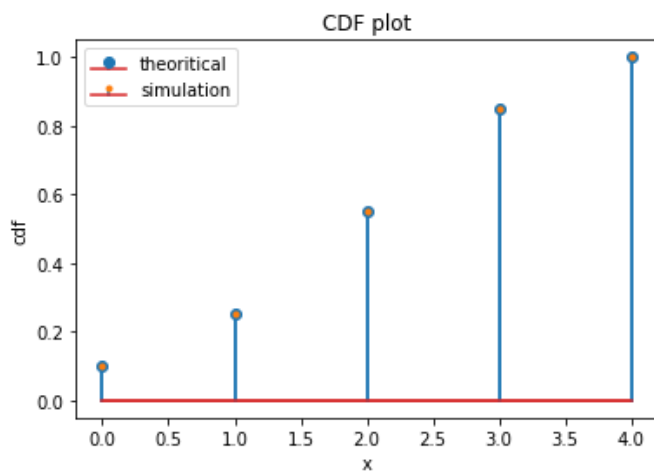


Fig. 2: CDF Plot