Bonus Problem

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***Abstract*—This is a problem to prove that a set of orthogonal vectors are linearly independent.**

Download all the latex-tikz codes from

https://github.com/Debolena/EE5609/blob/master/challenge%20problems/latex

1 Problem

Suppose that a non-zero set of vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ are mutually orthogonal, i.e,

$$\mathbf{v_i}'\mathbf{v_j} = 0 \tag{1.0.1}$$

for $i \neq j$. Prove that these vectors are also linearly independent.

2 Solution

Here, $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ are a set of mutually orthogonal vectors.

We are to show that they are linearly independent.

Let us consider scalars $a_1, a_2, ..., a_n$.

$$a_1\mathbf{v_1} + a_2\mathbf{v_2} + ... + a_j\mathbf{v_j} + ... + a_n\mathbf{v_n} = 0 \tag{2.0.1}$$

$$\implies \mathbf{v_j}'(a_1\mathbf{v_1} + a_2\mathbf{v_2} + ... + a_j\mathbf{v_j} + ... + a_n\mathbf{v_n}) = \mathbf{v_j}'.\mathbf{0} = 0 \tag{2.0.2}$$

$$\implies a_j.\mathbf{v_j}'\mathbf{v_j} = 0 \quad \because \mathbf{v_j}'.\mathbf{v_i} = 0, \forall i \neq j \tag{2.0.3}$$

$$\implies a_j = 0, \quad \forall j \quad \because \mathbf{v_j}'.\mathbf{v_j} > 0 \tag{2.0.4}$$

Hence, $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ are a set of linearly independent vectors.