Matrix Theory: Assignment 7

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Abstract—**This document is to find the trace of a matrix.**

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1 PROBLEM

Problem 30 of UGC NET JUNE 2018:
The trace of a matrix

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}^{20} \quad (1.0.1)$$

is

1) 7^{20}
2) $2^{20} + 3^{20}$
3) $2.2^{20} + 3^{20}$
4) $2^{20} + 3^{20} + 1$

2 SOLUTION

Let,

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (2.0.1)$$

To find the eigen values of $\mathbf{A}$:

$$\left| \mathbf{A} - \lambda \mathbf{I} \right| = 0 \quad (2.0.2)$$

$$\implies \begin{vmatrix} 2 - \lambda & 1 & 0 \\ 0 & 2 - \lambda & 0 \\ 0 & 0 & 3 - \lambda \end{vmatrix} = 0 \quad (2.0.3)$$

$$\implies (2 - \lambda)(2 - \lambda)(3 - \lambda) = 0 \quad (2.0.4)$$

$$\implies \lambda = 2, 2, 3 \quad (2.0.5)$$

Eigen values of $\mathbf{A}$ are 2,2,3.
Hence, the eigen values of $\mathbf{A}^{20}$ are: $2^{20}, 2^{20}$ and 3^{20}

respectively.
So, the trace of $\mathbf{A}^{20}$ is:

$$tr = 2^{20} + 2^{20} + 3^{20} \quad (2.0.6)$$

$$= 2.2^{20} + 3^{20} \quad (2.0.7)$$

Therefore, option 3 is the required answer.