Matrix Theory: Assignment 8

Debolena Basak
Roll No.: AI20RESCH11003
PhD Artificial Intelligence

***Abstract*—This document is based on orthonormal basis and orthonormal matrix.**

Download all latex-tikz codes from

https://github.com/Debolena/EE5609/tree/master/Assignment_8

1 Problem

Let $\mathbf{R}_n, n \geq 2$ be equipped with standard inner product. Let $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ be n column vectors forming an orthornormal basis of $\mathbf{R}_n$. Let $\mathbf{A}$ be a n x n matrix formed by the column vectors, $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$. Then,

1) $\mathbf{A} = \mathbf{A}^{-1}$
2) $\mathbf{A} = \mathbf{A}^T$
3) $\mathbf{A}^{-1} = \mathbf{A}^T$
4) $Det(\mathbf{A}) = 1$

2 solution

Given, $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ are orthonormal and form basis.
So, when they form column vectors of matrix $\mathbf{A}$, we can say that $\mathbf{A}$ is also orthonormal.

$$\therefore \mathbf{A^T A} = \mathbf{I} \tag{2.0.1}$$

$$\implies \mathbf{A^T A A^{-1}} = \mathbf{I A^{-1}} \tag{2.0.2}$$

$$\implies \mathbf{A^T} = \mathbf{A^{-1}} \tag{2.0.3}$$

Clearly, option 3 is the correct answer.