Challenge Question: Matrix Theory

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***Abstract*—This is a problem of a point, circle and tangent.**

Download all the latex-tikz codes from

https://github.com/Debolena/EE5609/tree/master/challenge%20problem%20on%208.10.20

1 Problem

Given a point $\mathbf{P}$ outside a circle whose equation is known. Find the equation(s) of the tangent(s) and the the distance from $\mathbf{P}$ to the point(s) of contact.

2 Solution

The equation of the circle can be expressed as

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

Let point

$$\mathbf{P} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \tag{2.0.2}$$

We know radius of circle and centre is given by

$$\sqrt{\|\mathbf{u}\|^2 - f} \tag{2.0.3}$$

$$and \quad \mathbf{c} = -\mathbf{u} \tag{2.0.4}$$

,respectively. Let the tangents from point $\mathbf{P}$ touch the circle at $\mathbf{q}$ and $\mathbf{r}$. At point $\mathbf{q}$, the normal vector is:

$$\mathbf{V}\mathbf{q} + \mathbf{u} \tag{2.0.5}$$

$$= \mathbf{q} - \mathbf{c}, \quad \because \mathbf{V} = \mathbf{I} \tag{2.0.6}$$

$$= \mathbf{n}_1, say. \tag{2.0.7}$$

Similarly, at point $\mathbf{r}$, the normal vector is:

$$\mathbf{r} - \mathbf{c} \tag{2.0.8}$$

$$= \mathbf{n}_2, say \tag{2.0.9}$$

Then, the equation of the tangents are:

$$\mathbf{n}_1^T(\mathbf{x} - \mathbf{q}) = 0 \tag{2.0.10}$$

$$\mathbf{n}_2^T(\mathbf{x} - \mathbf{r}) = 0 \tag{2.0.11}$$

We also have points of contact as,

$$\mathbf{q} = k_1\mathbf{n}_1 - \mathbf{u} \tag{2.0.12}$$

$$\implies \mathbf{q} = k_1\mathbf{n}_1 + \mathbf{c} \tag{2.0.13}$$

$$and \tag{2.0.14}$$

$$\mathbf{r} = k_2\mathbf{n}_2 - \mathbf{u} \tag{2.0.15}$$

$$\implies \mathbf{r} = k_2\mathbf{n}_2 + \mathbf{c} \tag{2.0.16}$$

where,

$$\kappa_i = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} \tag{2.0.17}$$

$$= \pm\sqrt{\frac{\|\mathbf{u}\|^2 - f}{\mathbf{n_i}^T\mathbf{n_i}}} \tag{2.0.18}$$

$$= \pm\frac{radius}{\|\mathbf{n_i}\|} \tag{2.0.19}$$

Using 2.0.10, 2.0.11, 2.0.13 and 2.0.16, we can solve for the points of contact $\mathbf{q}$ and $\mathbf{r}$. Therefore, distance from $\mathbf{P}$ to the points of contact are:

$$\|\mathbf{P} - \mathbf{q}\|, \tag{2.0.20}$$

$$\|\mathbf{P} - \mathbf{r}\| \tag{2.0.21}$$