Assignment 2: Matrix Theory

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***Abstract*—This is a problem to balance a chemical equation using system of linear equations.**

Download all the latex-tikz codes from

https://github.com/Debolena/EE5609/blob/master/Assignment%202/latex%20code.tex

1 Problem

Balance the following chemical equation.

$$HNO_3 + Ca(OH)_2 \rightarrow Ca(NO_3)_2 + H_2O \quad (1.0.1)$$

2 Solution

Let the balanced version of (1.0.1) be

$$x_1HNO_3 + x_2Ca(OH)_2 \rightarrow x_3Ca(NO_3)_2 + x_4H_2O \quad (2.0.1)$$

which results in the following equations:

$$(x_1 + 2x_2 - 2x_4)H = 0 \quad (2.0.2)$$
$$(x_1 - 2x_3)N = 0 \quad (2.0.3)$$
$$(3x_1 + 2x_2 - 6x_3 - x_4)O = 0 \quad (2.0.4)$$
$$(x_2 - x_3)Ca = 0 \quad (2.0.5)$$

which can be expressed as

$$x_1 + 2x_2 + 0.x_3 - 2x_4 = 0 \quad (2.0.6)$$
$$x_1 + 0.x_2 - 2x_3 + 0.x_4 = 0 \quad (2.0.7)$$
$$3x_1 + 2x_2 - 6x_3 - x_4 = 0 \quad (2.0.8)$$
$$0.x_1 + x_2 - x_3 + 0.x_4 = 0 \quad (2.0.9)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 3 & 2 & -6 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \quad (2.0.10)$$

where,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad (2.0.11)$$

(2.0.10) can be reduced as follows:

$$\begin{pmatrix} 1 & 2 & 0 & -2 \\ 1 & 0 & -2 & 0 \\ 3 & 2 & -6 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xleftrightarrow[R_3 \leftarrow \frac{R_3}{3} - R_1]{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & -\frac{4}{3} & -2 & \frac{5}{3} \\ 0 & 1 & -1 & 0 \end{pmatrix} \quad (2.0.12)$$

$$\xleftrightarrow{R_2 \leftarrow -\frac{R_2}{2}} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & -\frac{4}{3} & -2 & \frac{5}{3} \\ 0 & 1 & -1 & 0 \end{pmatrix} \quad (2.0.13)$$

$$\xleftrightarrow[R_4 \leftarrow R_4 - R_2]{R_3 \leftarrow R_3 + \frac{4}{3}R_2} \begin{pmatrix} 1 & 2 & 0 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & -2 & 1 \end{pmatrix} \quad (2.0.14)$$

$$\xleftrightarrow[R_3 \leftarrow -\frac{3}{2}R_3]{R_1 \leftarrow R_1 - 2R_2} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -2 & 1 \end{pmatrix} \quad (2.0.15)$$

$$\xleftrightarrow{R_4 \leftarrow R_4 + 2R_3} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.16)$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_3]{R_1 \leftarrow R_1 + 2R_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.17)$$

Thus,

$$x_1 = x_4, x_2 = \frac{1}{2}x_4, x_3 = \frac{1}{2}x_4 \tag{2.0.18}$$

$$\implies \quad \mathbf{x} = x_4 \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.19}$$

by substituting $x_4 = 2$.

Hence, (2.0.1) finally becomes

$$2HNO_3 + Ca(OH)_2 \rightarrow Ca(NO_3)_2 + 2H_2O \tag{2.0.20}$$