How Economists Came to Accept Expected Utility Theory: The Case of Samuelson and Savage

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Abstract

Based on the correspondence between Paul Samuelson, Leonard Jimmie Savage, Milton Friedman and Jacob Marschak between May and September 1950, the article reconstructs the joint intellectual journey that led Samuelson to accept expected utility theory and Savage to revise his initial motivations for supporting it.

Keywords

Expected Utility Theory; Independence Axiom; Sure-Thing Principle; Paul Samuelson; Leonard Jimmie Savage; Milton Friedman; Jacob Marschak.

JEL Codes

B21: History of Economic Thought since 1925; Microeconomics.
B31: History of Economic Thought: Individuals.
D81: Criteria for Decision-Making under Risk and Uncertainty.

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Introduction

In the expected utility approach to decision-making under risk, the utility of a risky prospect is given by the sum of the utilities u of the alternative possible outcomes of the prospect, each weighted by the probability that the outcome will occur. Thus, for example, the utility of a lottery yielding a trip to London with probability p and \$2,000 with probability $(1-p)$ is given by $(u(\text{trip to London})\times p) + (u(\$2{,}000)\times(1-p))$. According to expected utility theory, the decision maker will choose the risky prospect with the highest expected utility. This theory dominated the economic analysis of individual decision-making under risk from the early 1950s to the 1990s. Beginning in the late 1970s, however, the accumulation of robust experimental evidence against the expected utility hypothesis prompted decision theorists to advance a number of alternative theories, such as prospect theory (Kahneman and Tversky 1979), Choquet expected utility (Schmeidler 1989), maxmin expected utility (Gilboa and Schmeidler 1989), rank-dependent utility (Quigging 1993) or the smooth model of ambiguity aversion (Klibanoff, Marinacci, and Mukerji 2005). As of the present moment, though, none of these alternative theories has reached the dominant status that expected utility theory once enjoyed. As Gilboa and Marinacci (2013, p. 232) have argued in a recent survey of the decision-theoretic literature, it is not clear that a single theory of decision making under uncertainty will replace expected utility theory, and "even if a single paradigm will eventually emerge, it is probably too soon to tell which one it will be." Because of the absence of a univocal alternative, and thanks also to its simplicity and adaptability, expected utility theory remains the primary model in numerous areas of economics dealing with risky decisions, such as finance, the theory of asymmetric information and game theory. Therefore, the question of why economists came to accept it in the first place remains pertinent.

Expected utility theory was originally advanced by Daniel Bernoulli in the eighteenth century, was adopted by some nineteenth-century economists, such as Alfred Marshall, but came under sustained criticism from the 1930s through to the early 1950s. In these decades, some economists argued that individuals evaluate risky alternatives by looking at the mean, the variance, and possibly other elements of the distribution of uncertain payoffs, rather than using expected utility (Hicks 1931). Others noted that an individual who places probabilities and utilities on a range of outcomes, and then calculates the

weighted average of utilities, is engaging in cardinal measurability of utility, which contrasts with the ordinal conception of utility that dominated utility analysis in the 1930s (Tintner 1942). Along with approaches based on the distribution of payoffs were models based on the minimax-criterion (Wald 1950), or on the idea that individuals focus only on the best-possible and the worst-possible outcomes of risky alternatives (Shackle 1949).

The fortunes of expected utility theory began to recover when John von Neumann and Oskar Morgenstern introduced a set of axioms of rational individual decision-making that implied expected utility theory in their *Theory of Games and Economic Behavior* (1944; second edition with an explicit proof of the expected utility theory theorem, 1947). Among the early supporters of the expected utility hypothesis in the von Neumann-Morgenstern version were Milton Friedman and Leonard Jimmie Savage (1948), both based at the University of Chicago, and Jacob Marschak (1948, 1950), a leading member of the Cowles Commission for Research in Economics.

Paul Samuelson of MIT was initially a severe critic of expected utility theory. Between mid-April and early May 1950, Samuelson (1950a, b, c) composed three papers in which he attacked von Neumann and Morgenstern's axiomatic system for its lack of transparency, contested the capacity of expected utility theory to explain empirical phenomena, identified and named what he said was a hidden axiom behind expected utility theory—the "Independence Axiom"—, and claimed this axiom was untenable. This axiom states that if an individual prefers a trip to London over a trip to Venice then, for any probability p and any amount of money \$K, he should also prefer the lottery yielding a trip to London with probability p and \$K with probability $(1-p)$, to the lottery yielding a trip to Venice with probability p and \$K with probability $(1-p)$. Samuelson argued that, rather satisfy than the Independence Axiom, the individual's ordinal preferences over risky alternatives should satisfy only one property besides completeness, transitivity and continuity—namely, what today we call monotonicity with respect to first-order stochastic dominance. In the case of choices over lotteries with monetary payoffs, monotonicity means that raising a payoff without changing the other payoffs, or increasing the probability of a larger payoff at the expense of the probability of a smaller payoff, raises preferability. For Samuelson (1950a, p. 169) any assumption beyond monotonicity would impose an arbitrary "straight-jacket" on individual preferences over risky alternatives.

By 1952, however, Samuelson had somewhat unexpectedly become a resolute supporter of the expected utility hypothesis. In a prominent conference on decision theory held in Paris in May 1952, he joined Friedman, Savage, and Marschak in advocating expected utility theory against the attacks of Maurice Allais and other opponents of the theory. In 1952 Samuelson also organized a symposium on expected utility theory that was published in the October 1952 issue of *Econometrica* and was instrumental in stabilizing expected utility theory as the dominant economic model of choice under risk.

Why did Samuelson change his mind? Accounts of Samuelson's conversion to expected utility theory based on published materials and personal recollections have been provided by Samuelson himself (for example, Samuelson 1947 [1983]), and by Fishburn and Wakker (1995) in their essay on the origin of the Independence Axiom.[1] The present article fills out these accounts by employing, for the first time, letters and other unpublished materials collected in the Samuelson Papers held at Duke University, the Savage Papers at Yale University, and the Friedman Papers at the Hoover Institution. These archives reveal that Samuelson's change of mind occurred mainly through an exchange of letters with Savage and, to a minor extent, Marschak and Friedman, between May and September 1950. This correspondence shows that Samuelson accepted expected utility theory only when Savage persuaded him of the normative force of the Independence Axiom. Samuelson frankly admitted his capitulation in a letter to Friedman dated August 25, 1950 (Samuelson Papers, Box 31):

> Dear Milton: … [L]et me make an important surrender. Savage's patient letters and the induced cogitation have convinced me that he is right on the only important difference between us. … I called the [Independence] assumption gratuitous, arbitrary, etc. … etc. (You know how I can lay it on when I get going.) But now I must eat my words. As you know I hate to change my mind, but I hate worse to hold wrong views, and so I have no choice.

[1] Other cases of prominent economists explicitly admitting to have changed their mind on some substantive issue are rare but not nonexistent. For instance, David Ricardo (1821) famously changed his mind about the effects of new machinery on the demand for labor, and John Maynard Keynes (1936) repudiated many of the views about employment, interest and money he had held before writing the *General Theory*. At least on one other occasion Samuelson also admitted to having been wrong, namely in the debate on the re-switching of production techniques (Samuelson 1966).

The correspondence also shows that, for Savage, his exchange with Samuelson modified his thinking about expected utility theory. Samuelson's arguments prompted Savage to streamline the normative defense of expected utility theory and formulate the Sure-Thing Principle, which is the central assumption of the subjective version of expected utility theory that Savage later advanced in *The Foundations of Statistics* (1954).

Based on the correspondence between Samuelson, Savage, Marschak and Friedman, the article reconstructs the joint intellectual journey that led Samuelson to accept expected utility theory and Savage to revise his motivations for supporting it. The article is organized around the main issues those four economists discussed in their correspondence: i) identifying the importance of the Independence Axiom as a key point under dispute; ii) the nature of the cardinal function featuring in expected utility theory, and the relationship between the Independence Axiom and the idea that the utilities of different commodities are independent; iii) the descriptive validity of expected utility theory; and iv) the normative appeal of the Independence Axiom. More detail on the history of expected utility theory between 1930 and 1950 is available in Moscati (forthcoming).

Identifying the Importance of the Independence Axiom

In *Theory of Games* (1944 [1947]), von Neumann and Morgenstern state a series of axioms about the individual's preferences over indifference classes of lotteries, and offer a proof that an individual obeying these axioms will then follow expected utility theory. Their axiomatization of the expected utility hypothesis theory includes the completeness, transitivity and continuity of preferences, but does not feature an assumption corresponding to what today we call the Independence Axiom. In the first of Samuelson's three 1950 papers, completed in April 1950 and called the "Japanese paper" because it was later published in a Japanese journal, Samuelson declared that he found von Neumann and Morgenstern's axioms opaque. On the one hand, he observed, they concern preference relations and seem therefore to be ordinal in nature; on the other hand, the axioms imply that the preference relations can be represented by the

expected-utility formula, which features a cardinal utility function.[2] Samuelson was puzzled by this apparent contradiction and could not understand how von Neumann and Morgenstern's ordinal axioms imply expected utility theory: "I am simply confused," he admitted (1950a, p. 172).

Samuelson guessed that von Neumann and Morgenstern had "implicitly added a hidden and unacceptable premise to their axioms" (p. 172), but in April 1950 he was unable to identify this hidden assumption. Nevertheless, Samuelson christened the hidden premise the "independence assumption" (p. 170, footnote 7) because he associated it with the assumption that the utilities of different commodities are independent or additively-separable.[3] Since the early twentieth century this assumption had been discredited in utility analysis because it rules out the substitutability and complementary of goods. In his *Foundations of Economic Analysis* (1947 [1983]) Samuelson had extensively criticized additive separability as farfetched, and shown that it implies the cardinal measurability of utility and further implausible features of the demand functions for commodities.

In Samuelson's second 1950 paper, completed by May 5 and published as a RAND Corporation memorandum on May 24, Samuelson made the hidden premise of von Neumann–Morgenstern axiomatics explicit, naming it the "Special Independence Assumption."[4] The adjective "special" was intended in a pejorative sense, emphasizing the dubiously restrictive character of the assumption. Samuelson (1950b, pp. 6-7) stated the assumption in terms of indifference:

> *Special Independence Assumption*: If two situations A and B are indifferent, so that V(A)=V(B) [the utility V(A) of A is equal to the utility V(B) of B], then … V(A,C)= V(B,C) [the utility of the probability mixture of situation A and situation C is equal to the utility of the probability mixture of situation B and situation C] *for all* C's.

In late April-early May 1950 Samuelson was unaware that he was not the first to have stated the Independence Axiom. Marschak (1948, 1950), RAND researcher

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[2] A utility function $U(x)$ is cardinal if is it unique up to positive linear transformations of the form $aU(x)+b$, where a>0.

[3] This means that the utility U of commodity bundle $(x_1,\ldots, x_n)$ can be expressed as $U(x_1,\ldots, x_n)=\sum_{i=1}^{n} U_i(x_i)$, where U_i is the utility function relative to commodity i.

[4] The RAND Corporation is a private think tank originally funded by the US Army Air Force in 1946 through the Douglas Aircraft Company with the goal of bringing together civil scientists from different backgrounds to work on interdisciplinary research projects with possible military applications. RAND became an independent nonprofit corporation in 1948, and Samuelson began collaborating with RAND in 1949.

Norman Dalkey (1949), and John Nash (1950), then still a Ph.D. student in mathematics at Princeton, had all put forward axiomatizations of expected utility theory including versions of the Independence Axiom (Fishburn and Wakker 1995; Bleichrodt, Li, Moscati, and Wakker forthcoming). Notably, in an article published in the April 1950 issue of *Econometrica*, Marschak (1950) called it Postulate IV_2.[5]

In Samuelson's third paper of 1950, completed in early May and published as a RAND memorandum on June 13, Samuelson referred to an oral communication in which Marschak had called his attention to Postulate IV_2. Accordingly, in a footnote Samuelson (1950c, p. 2) noted that what he had called the Special Independence Axiom "seems to be his [Marschak's] Postulate IV_2."

Samuelson circulated the Japanese paper to colleagues, requesting comments and criticism. On May 1, 1950, he sent a copy to Milton Friedman and asked him to forward a second copy to Leonard (Jimmie) Savage. Friedman did so in early May 1950, accompanying Samuelson's paper with a perplexed comment to Savage: "Dear Jimmie: … Can you figure out what it is about? I must confess I cannot" (Friedman Papers, Box 99). On May 19, 1950, Savage sent a long letter to Samuelson containing extensive comments on the Japanese paper; the letter was carbon copied to Friedman.

In the Japanese paper, Samuelson had also questioned the axiomatization of expected utility theory that Friedman and Savage (1948) had advanced. This axiomatization consisted of three assumptions that Friedman and Savage (p. 288) claimed implied expected utility theory and are logically equivalent to von Neumann and Morgenstern's axioms. Notably, Friedman and Savage's assumptions do not include the Independence Axiom. In the Japanese paper, Samuelson (1950a, pp. 121–23) argued that the von Neumann-Morgenstern and the Friedman-Savage axiomatic systems were not equivalent, and further doubted that the three Friedman-Savage assumptions actually implied expected utility theory. In his letter of May 19, 1950, to Samuelson, Savage acknowledged that Samuelson was right (Samuelson Papers, Box 67):

> Dear Professor Samuelson: … On reexamination I find that β [the Friedman-Savage axiomatic system] does not imply α [expected utility theory]. This is

[5] Formally, Marschak's Postulate IV_2 reads as follows: let A, B and C, be three different prospects, i.e., lotteries; if $A \sim B$, then $pA+(1-p)C \sim pB+(1-p)C$, where $0<p<1$ (1950, 120).

> because Milton and I slipped in leaving out of it something very like what … you call the basic hypothesis.

In the second part of his letter Savage provided a new set of axioms, collectively labeled as β′, and proved that they actually imply expected utility theory. In particular, Axiom 2″ of β′ is a version of the Independence Axiom, albeit expressed in terms of preference rather than indifference.[6]

After some discussion, which occupied their correspondence between May and July 1950 and which was hampered by terminological misunderstandings, by late July 1950 Samuelson and Savage came to agree that Samuelson's Special Independence Assumption, Savage's Axiom 2″, and Marschak's Postulate IV_2 are fundamentally equivalent. They also agreed that, if the preference relation over lotteries is complete, transitive and continuous, the Independence Axiom is a necessary and sufficient condition for expected utility theory. At that point, the only important difference between them concerned the plausibility of the Independence Axiom. Thus, in his letter to Savage of July 20, 1950, Samuelson re-focused the discussion on this latter point (Savage Papers, Box 29):

> Dear Dr. Savage: … I shall be interested in knowing what you think of the "plausibility" of the postulate V(A)=V(B) implies V(A,C)=V(B,C) for arbitrary C and arbitrary mixtures.

Samuelson himself maintained that the postulate was a "gratuitously-arbitrary-special-implausible hypothesis."

Independence Axiom, Independent Utilities, and Cardinal Utility

On April 20, 1950, Samuelson sent the Japanese paper to Jacob Marschak, who replied in a letter dated May 11, 1950 (Samuelson Papers, Box 66). Marschak accepted that his Postulate IV_2 and Samuelson's Special Independence Assumption are equivalent. However, he rejected Samuelson's association of the Independence Assumption, and thus Postulate IV_2, with the discredited hypothesis that the utilities of different commodities are additively separable.

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[6] Savage's Axiom 2″ reads as follows: let A, B and C be three different gambles; if $A \prec B$, then $pA+(1-p)C \preccurlyeq pB+(1-p)C$, where $0<p<1$.

Marschak stressed that while the latter has to do with the joint consumption of different goods—for example, beer *and* pretzels—Postulate IV_2 relates to the consumption of different goods in mutually-exclusive situations where a choice is being made between outcomes—that is, *either* beer *or* pretzels. Thus a man who is indifferent between beer and tea might well prefer the commodity bundle beer-and-pretzels to the commodity bundle tea-and-pretzels and, at the same time, be indifferent between a lottery consisting of "either beer or pretzels" and another lottery consisting of "either tea or pretzels." Marschak wrote:

> I should not expect … [the] man to tell me that the mere co-presence in the same lottery bag of tickets inscribed 'pretzels' with tickets inscribed 'tea' will contaminate (or enhance) the enjoyment of either the liquid or the solid that will be the subject's lot.

In his rejoinder of May 15, 1950 (Samuelson Papers, Box 66), Samuelson accepted that different lottery prizes are mutually exclusive in a probability sense. But he then returned to his earlier concern about the additive separability of utilities, and wrote that he could not understand why this "*ex ante* preference pattern" toward lotteries could generate, as is the case in expected utility theory, a utility indicator that "impute[s] an *independent* numerical score to each possible prize." The expected utility of a "beer or pretzels" lottery is in fact expressed by $(u(\text{beer})\times p) + (u(\text{pretzels})\times(1-p))$. But the expressions $u(\text{beer})$ and $u(\text{pretzels})$ seem to suggest that the utilities of beer and pretzels are independent of each other. For Samuelson, this was "a pun on words."

The solution came from Milton Friedman. In May 1950 William Baumol, then a young assistant professor at Princeton University, submitted to the *Journal of Political Economy* a paper arguing that expected utility theory involved a return to a cardinal conception of utility, which was eventually published as Baumol (1951). In a letter to Baumol dated June 3, 1950, and carbon copied to Savage and Samuelson, Friedman disclosed that he would serve as a referee for this paper, and commented on it (Samuelson Papers, Box 15). Friedman wrote the report on Baumol's paper between June and August 1950, and forwarded copies to Savage and Samuelson.

In oppositions to Baumol's claims, Friedman contended that expected utility theory "does not commit you in any way to 'cardinal utility' whatever that may mean" (Savage Papers, Box 29). Friedman labelled as g(A) the expected utility of

lottery A, and noticed that any monotonically increasing transformation G of g(A) continues to represent the preference order between lotteries:

> It is obvious that we can take as a utility function any member of the set G[g(A)], where the set is subject only to the restriction that G' be greater than zero.[7]

For Friedman, therefore, expected utility theory was not in contradiction with the ordinal approach to utility. Friedman argued that the cardinal function *u* featuring in the expected-utility formula and the utility function *U* expressing the individual's preferences over riskless outcomes are two different functions. According to Friedman, Baumol failed to see this difference. In order to avoid further confusion, Friedman suggested giving another name to the function *u* and proposed calling it "the choice generating function."

Friedman's interpretation of the nature of *u*, which was articulated by Friedman and Savage (1952) and quickly became the official view among utility theorists, has a number of important consequences. First, although expressions such as *u*(beer) and *u*(pretzels) in the expected-utility formula may suggest that the riskless utilities of beer and pretzels are independent of each other, this inference is unwarranted: the form of the function *u* carries no implications over the complementarity or substitutability of beer and pretzels in riskless situations. Second, even if the utilities of beer and pretzels were independent and could be represented by an additively-separable utility function *U*, this function would still differ from the choice-generating function *u*, in the specific sense that *u* need not be a positive linear transformation of *U*.[8]

Between May and August 1950, Samuelson apparently came to see expected utility theory as innocent of the charge that it involved a return to a cardinal conception of utility, and the Independence Axiom as innocent of any necessary relationship to the hypothesis that utilities are additively separable. This supposition is backed by the fact that in the two papers on expected utility theory that Samuelson completed in 1952 (1952 [1966]; 1952), he insisted on

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[7] More explicitly, if A is a lottery that yields payoff x_i with probability p_i, with i=1,…n, then g(A)=$\sum_{i=1}^{n} p_i u(x_i)$. I have slightly modified Friedman's notation to make it more consistent with that used in Samuelson's correspondence.

[8] A further consequence of Friedman's interpretation is that the possible concavity of the function *u*, which is associated with risk aversion, cannot be conceived of as an expression of the decreasing marginal utility of riskless outcomes, e.g., of money. This question, however, was not discussed in the Samuelson-Savage-Marschak-Friedman correspondence reconstructed in the present paper.

the ordinal nature of the Independence Axiom and the other assumptions underlying expected utility theory, stressed the fact that the expected utility hypothesis is about mutually-exclusive outcomes rather than joint consumption of different goods, and pointed out that "independence in probability situations puts *no* restriction whatsoever upon the dependence or independence that holds in the nonstochastic situation" (1952, p. 673). Friedman's proposed distinction seems to have eliminated one significant obstacle in Samuelson's accepting of the Independence Axiom and expected utility theory, but did not offer him any positive reasons to endorse them.

The Descriptive Validity of the Expected Utility Theory

Based on data from the U.S. Bureau of Labor Statistics, the National Bureau of Economic Research, books about the history of lotteries in different countries, and casual observation, Friedman and Savage (1948) identified three basic facts that a satisfactory theory of choice under risk should be able to explain: (i) individuals of all income levels buy insurance; (ii) individuals of all income levels purchase lottery tickets or engage in similar forms of gambling; (iii) most individuals both purchase insurance and gamble. Friedman and Savage (p. 297) claimed that, by assuming that the utility curve of money is first concave, then convex, and then concave again, expected utility theory can rationalize these three facts. In the Japanese paper, Samuelson (1950a, p. 168) contested this claim, noticing that, for instance, expected utility theory cannot explain "the perfectly possible case of a man who refuses fair small bets at all income levels and yet buys lottery tickets." More generally, Samuelson contended that the phenomena associated with gambling are "infinitely richer" than the expected utility hypothesis permits, and that there is as much to be learned about gambling "from Dostoyevsky as from Pascal."[9]

In his letter to Samuelson of May 19, 1950, Savage replied that the fact that expected utility theory is not consistent with every conceivable sort of behavior shows that the theory "is not simply tautological" (Samuelson Papers, Box 67). More forcefully, Savage advanced a simplicity defense, namely that expected utility theory should be accepted as a simple and acceptable approximation to

[9] Fyodor Dostoyevsky authored the autobiographical novel "The Gambler" (1867), whose protagonist was addicted to roulette. In his *Pensées* (1670), Blaise Pascal made a famous argument for believing in God based on a wager.

reality: "It is … the simplest theory of gambling behavior which has come to my attention and which seems at all consistent with the facts in any reasonably extensive range of contexts."

Savage also argued that the understanding of expected utility theory as a handy approximation of reality was also shared by Friedman, and also by von Neumann, whom he had known in the academic year 1941-1942 when studying in Princeton as a post-doctoral fellow: "I have repeatedly heard von Neumann express the idea in the most emphatic language. His interest, like Milton's and mine, in the theory stems from the belief that it is a skillfully chosen zero approximation to reality."

In his response to Savage of July 20, 1950, carbon-copied to Friedman and Marschak, Samuelson ironically rejoined that he found no particular merit in the fact that expected utility theory is non-tautological (Savage Papers, Box 29): "Both you and Milton express in separate letters pride that you have labored like lions and produced a *non-tautology*. I am sure there is a category of people who must be told that this is not necessarily a crime. … I do not see that I qualify for this category."

To refute Savage's simplicity argument, Samuelson contrasted the expected utility hypothesis with the theory of decision-making under risk that he had advocated in the Japanese paper, which theory is based on the hypothesis that preferences over risky alternatives are monotonic with respect to first-order stochastic dominance. Both theories make only one further assumption besides those concerning the completeness, transitivity and continuity of preferences, namely expected utility theory uses the Independence Axiom while Samuelson's theory uses stochastic-dominance monotonicity. Therefore, at the formal level both theories are equally simple.

Concerning the empirical implications of the two theories, Samuelson argued that where these implications differ "there is *no* factual evidence in favor of the special theory [that is, expected utility theory] and some against it." Samuelson also commented cursorily on the pioneering experiment to test the validity of the expected utility hypothesis conducted between 1948 and 1949 by Frederick Mosteller, a Harvard statistician associated with Friedman and Savage, and Philip Nogee, then a Harvard Ph.D. student in psychology.[10] Samuelson disparaged the design of the experiment – "[Nobody] could expect anything from this pitiful set-

[10] The findings of the experiment were published in Mosteller and Nogee (1951). On the Mosteller-Nogee experiment and the role Friedman and Savage played in its design, see Moscati (2016).

up" – and thus implicitly dismissed Mosteller and Nogee's claim that their experimental findings supported the empirical validity of the expected utility hypothesis. Thus, argued Samuelson, Savage's simplicity criterion backfires, supporting his own theory of decision-making under risk rather than expected utility theory. He concluded: "On the matter of simplicity, I know how I would wield Occam's Razor."

Arguably, Samuelson's argument had some effect on Savage who, in his subsequent letters to Samuelson, no longer insisted on the simplicity and descriptive power of expected utility theory. Friedman, by contrast, remained convinced that expected utility theory was empirically valid. In a letter to Samuelson dated September 13, 1950 (Samuelson Papers, Box 31), Friedman stressed that he accepted expected utility theory, not because he judged the axioms underlying it particularly plausible, but because he considered it a simple theory whose implications are not only far from obvious, but also consistent with much common experience:

> Dear Paul: … It has never seemed to me obviously true or necessary that individual's reactions to complicated gambles should be completely predictable from their reactions to two-side ones – which has always seemed to me the fundamental empirical content of the B[ernoulli]-M[arshall] hypothesis – and it still does not. At the same time, it has seemed … the simplest and most direct way to extend the usual utility analysis to choices involving risk, and not inconsistent with much common experience.

Notably, Friedman explicitly admitted that certain phenomena related to gambling cannot really be explained by expected utility theory, and predicted that "to handle some experience" the theory would "need complication."

The Normative Plausibility of the Independence Axiom

In his letter to Samuelson of May 11, 1950 (Samuelson Papers, Box 66), Marschak identified behavior satisfying the assumptions underlying expected utility theory, including the Independence Axiom (or, equivalently, Postulate IV_2), with rational behavior. He compared the argument for expected utility theory to Euclidean geometry, and behavior violating expected utility theory axioms with

non-Euclidean geometry.[11] Marschak admitted that among actual people the observation of "non-Euclidean habits" is likely, but argued that such conduct is not advisable: "It may be *usual* for village carpenters … to deviate from the advice of Euclidian geometers …. All the same, they would be better advised to behave rationally by following Euclid." As an example of non-advisable behavior, Marschak took what he called "love for danger"— that is, a violation of preference monotonicity with respect to stochastic dominance. Marschak rhetorically asked Samuelson whether, as a factory owner, he would hire a statistician "whose formula for quality control would be based on 'love for danger,' i.e., on rather liking the prospect of the factory being blown up, with 5% probability?"

Samuelson's rejoinder soon arrived. In his letter to Marschak of May 15, 1950 (Samuelson Papers, Box 66), Samuelson declared that, like Marschak, he was interested in "locating the 'natural' discontinuities which fence-out 'irrational' from 'rational' behavior." However, he contested the claim that the Independence Axiom or Postulate IV_2 should be included among the axioms defining rational, or Euclidean, behavior. For Samuelson, "IV_2 stands out like a sore-thumb as arbitrary and alien." Accordingly, he dismissed "any identification of 'non-Euclideanism' with non-IV_2-ism."

With respect to the love-for-danger example, Samuelson correctly rejected Marschak's identification of love for danger with a violation of the Independence Axiom:

> I would not hire Dostoyevsky to be my quality-control statistician; but until it can be shown that there is an iota of connection between "love of danger" and "*ex ante* lack of additive independence" [the Independence Axiom], the analogy is more confusing than clarifying.

Savage also advanced some normative arguments in defense of the Independence Axiom in letters from May to July 1950. However these arguments failed to impress Samuelson.[12] As already mentioned, in his letter to Savage of

[11] In economic discourse in the first half of the twentieth century the analogy with Euclidean and non-Euclidean geometries was far from infrequent. It was used by, among others, Pigou (1920), J.M. Clark (1921) and, most famously, Keynes (1936, p. 16) in a passage of the *General Theory* in which he compared the "the classical theorists" to "Euclidean geometers in a non-Euclidean world." Echoing Keynes, in a paper on fiscal policies Samuelson (1942, p. 593) opposed the Euclidean to the non-Euclidean world.

[12] The main argument concerned a hypothetical individual whom Samuelson named Ysidro, and whose preferences satisfy monotonicity with respect to stochastic dominance but not the

July, 20, 1950, Samuelson still dismissed the Independence Axiom as a "gratuitously-arbitrary-special-implausible hypothesis." It seemed that discussion had reached a deadlock, but then Savage came up with a new argument.

Savage's letter to Samuelson, dated August 12, 1950 (Samuelson Papers, Box 67), began by alluding to "a simple but important idea" not yet "set down in our correspondence to date." Savage considered three incomes A, B, C, two mutually exclusive events E and E', and two contracts I and II reading as follows:

I. In the event E Jimmie's income shall be C, and in the event E' it will be A.
II. In the event E Jimmie's income shall be C, and in the event E' it will be B.[13]

Savage argued that if he (as Jimmie) prefers income B to income A, he would certainly prefer contract II to I. The reason is that, by choosing contract II, "I guarantee that whichever of the [two] events occurs I will have nothing to reproach myself for." If one accepts the argument as compelling, continued Savage, one should also accept the Independence Axiom, because "if E and E' are disjoint random events of probabilities (1–*p*) and *p*" the Independence Axiom "is a special case."

The reader familiar with Savage's work will recognize that his "simple but important idea" is none other than the Sure-Thing Principle—that is, the central assumption of the subjective version of expected utility theory he later advanced in *The Foundations of Statistics* (1954). In this book (pp. 21–24), contracts I and II became acts *f* and *g*, and the Sure-Thing Principle was formalized as Postulate 2. The Postulate states that the preference between acts *f* and *g* should not depend on the situations in which the acts have the same consequence, such as event E in the above example, but only on the situations, like event E', in which two acts have different consequences. The letter to Samuelson of August 12, 1950, appears to be Savage's first statement of the Principle.

Savage's argument provoked Samuelson, who, in a letter dated August 16, 1950 (Savage Papers, Box 29), replied: "Dear Jimmie, I have read your letter hastily and translated its contents into the terminology defined in my earlier

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Independence Axiom. Savage argued that Ysidro's preferences are less rational than Samuelson believed, because they could lead Ysidro to accept a Dutch-book, i.e., a bet yielding a sure loss. However, the argument did not convince Samuelson. In a letter of August 16, 1950 (Savage Papers, Box 29), Samuelson wrote Savage that "as yet, I cannot tell that Ysidro has anything 'to reproach himself for.'" Only after accepting the Independence Axiom did Samuelson accept Savage's Dutch-book argument against Ysidro's preferences.

[13] I have slightly modified the symbols Savage used to make his notation more consistent with that used in the rest of this article.

letters." However, Samuelson's translation was misguided and, on August 18, 1950 (Samuelson Papers, Box 67), Savage wrote back to correct Samuelson's misunderstanding:

> Dear Paul, your points mistake the meaning of my last letter. … It therefore seems in order to give the argument … once more. If in every event which can possibly occur the consequence of action I is not preferred to that of action II, and if in some possible event the consequence of II is preferred to that of I, then any sane preferer would prefer II to I. Your [Special Independence Assumption] is a very special case of this.

Samuelson did not reply to Savage's letter, probably because he was going to meet Savage two weeks later at a meeting of the Econometric Society held at Harvard University. But we know that Savage's repetition of his argument hit home from the letter that Samuelson sent to Friedman on August 25, 1950 (Samuelson Papers, Box 31), in which, as quoted earlier, Samuelson admitted that Savage was right "on the only important difference between us." As Samuelson explained to Friedman, by this point the only important difference between him and Savage concerned the normative issue of whether the Independence Axiom should be included among the assumptions "defining 'rational behavior.'" Initially Samuelson believed that this was not the case, but Savage's idea had finally persuaded him:

> I have had to review my notion of what it is "reasonable" to postulate …. Is it reasonable to postulate that V(A)<V(B) implies V(A,C)<V(B,C) for all C? I must answer with a reluctant but firm Yes.

Samuelson as an Advocate of Expected Utility Theory

Samuelson and Savage both attended the Econometric Society meeting at Harvard in early September 1950. In the paper Samuelson presented at the meeting (a draft of which can be found in Samuelson Papers, Box 152), and for which Savage served as a discussant, he expanded on what he had already written to Friedman. In particular, he declared that thanks to Savage he had

come to see the Independence Axiom as a compelling requisite of rational behavior:

> What should be our definition of behavior by a "rational" man? … Dr. Savage has helped persuade me that the Independence Axiom *is not* so much a sore thumb as compared to weaker axioms as I had believed.

However, while Samuelson had come to accept expected utility theory because of the normative force of the Independence Axiom, he remained skeptical about the descriptive power of the theory. Thus, he maintained that expected utility theory does not provide "a very illuminating explanation" of the facts concerning gambling or investment behavior, not even "as a first approximation."

Between the Harvard meeting and the Paris conference of May 1952, Samuelson did not publish on expected utility theory. In Paris, he continued to downplay the descriptive power of the theory, arguing that "from the standpoint of explaining *actual behavior* of men on this planet, the Bernoulli utility hypothesis appears to me of rather trifling importance" (1952 [1966], p. 128). Samuelson also repeated that the decisive reason for his eventual endorsement of expected utility theory was that, thanks to Savage, he had come to see the Independence Axiom as "a natural if not inevitable concept" in the realm of choices between lotteries (p. 130). Samuelson even changed the label for the postulate from "Special Independence Assumption" to "Strong Independence Axiom" (p. 133). The term "special," which Samuelson had used to stress the restrictive and arbitrary character of the assumption, disappeared. It was replaced with the term "strong," by which Samuelson meant to point out that the Independence Axiom was now expressed in terms of preference rather than indifference.[14]

In 1952 Samuelson also organized a symposium on expected utility theory, which was published in the October issue of *Econometrica*. In this symposium, Malinvaud (1952) clarified how the Independence Axiom is hidden in von Neumann and Morgenstern's axiomatization of expected utility theory. Specifically, Malinvaud showed that the Independence Axiom is implied by the fact that von Neumann and Morgenstern's assumptions concern preferences over indifference classes of lotteries rather than preferences over single lotteries.

[14] Samuelson's Strong Independence Axiom reads as follows: for all lotteries A, B, and C, $A \succcurlyeq B$ if and only if $p\mathrm{A}+(1-p)\mathrm{C} \succcurlyeq p\mathrm{B}+(1-p)\mathrm{C}$, where $0<p<1$. More on the Paris conference in Mongin 2014.

Savage explained why an argument against the Independence Axiom put forward by Herman Wold was "a non sequitur" (Wold and others 1952, p. 663). Samuelson (1952, p. 672) restated the paper he had given at the Paris conference, and explicitly presented the Independence Axiom as "a version of what Dr. Savage calls the 'sure-thing principle.'"

The Evolution of Savage's Views

The discussions of summer 1950 between Samuelson and Savage changed not only Samuelson's thinking about expected utility theory but also Savage's. Samuelson's arguments induced Savage to stop advocating the theory by invoking its simplicity and descriptive power. More importantly, Samuelson's skepticism about the Independence Axiom pushed Savage to formulate the normative justification of it expressed by the Sure-Thing Principle.

Savage explicitly acknowledged the importance that the controversy with Samuelson had on the development of his own ideas. On July 3, 1951, he sent Samuelson the first draft of *The Foundations of Statistics* (Savage Papers, Box 29). In the letter accompanying the manuscript, Savage wrote:

> Dear Samuelson: Attached is some dittoed material which I hope soon to complete and redraft as a book. … This work owes much to the written and oral discussions you and I had last summer.

The evidence from the Samuelson-Savage correspondence contradicts an often-told "normative retreat story" about Savage (for example, in Jallais and Pradier 2005). According to this story, Savage retreated to the normative defense of expected utility theory presented in *The Foundations of Statistics* only after violating the theory himself at the Paris conference; this happened when, during a conference break, Allais presented Savage with the choice situations later associated with the expression "Allais paradox" (Allais and Hagen 1979). Savage affirmed that he preferred Lottery 1, which pays 100 million Francs with probability 1, to Lottery 2, which yields 500 million Francs with probability 0.10, 100 million Francs with probability 0.89, and 0 Francs with probability 0.01. He also stated that, between Lottery 3 yielding 100 million Francs with probability 0.11 and 0 Francs with probability 0.89, and Lottery 4 yielding 500 million Francs

with probability 0.10 and 0 Francs with probability 0.90, he preferred Lottery 4. However, this pair of preferences – Lottery 1 preferred to Lottery 2, and Lottery 4 preferred to Lottery 3 – violates expected utility theory.[15] In *The Foundations of Statistics*, Savage (1954, p. 103) argued that the preferences he had expressed in Paris were in conflict with the Sure-Thing Principle and were therefore erroneous. Accordingly, he corrected himself and argued that, upon reflection, he preferred Lottery 3 to Lottery 4.

Savage's letters to Samuelson show that for him the normative force of the Sure-Thing Principle, and therefore of the Independence Axiom, was a crucial motivation for endorsing expected utility theory well before the Paris conference, and independently of the Allais paradox. If there was one person responsible for Savage's normative turn, it was Samuelson, not Allais.

Summary and Conclusion

The Paris conference and the *Econometrica* symposium of 1952 marked the acceptance of expected utility theory as the mainstream model for risky choices in economics, and were instrumental in establishing the "Independence Axiom" as the standard name for the key underlying postulate. Indeed, most of the arguments in favor and against expected utility theory discussed in Paris and in the *Econometrica* symposium had been already addressed by Samuelson, Savage, Marschak and Friedman in their intense correspondence between May and September 1950.

But while these four major economists all came to accept expected utility theory, their reasons were not the same. Among them, only Friedman accepted expected utility theory because he judged it empirically valid. Marschak, in contrast, accepted the theory because he found the axioms underlying it normatively appealing (see also Marschak 1951). Samuelson remained skeptical about the descriptive power of expected utility theory, and only came to accept the theory when, through the lens of Savage's Sure-Thing Principle, he came to view the Independence Axiom as a requisite for rational behavior in conditions of

[15] To see why, notice that, according to expected utility theory, preferring Lottery 1 to Lottery 2 implies that $u(100) > 0.10u(500) + 0.89u(100) + 0.01u(0)$. On the other hand, preferring Lottery 4 to Lottery 3 implies that $0.10u(500) + 0.90u(0) > 0.11u(100) + 0.89u(0)$. It is easy to see that there exists no utility function u satisfying both inequalities, which implies that that pair of preferences cannot be rationalized by expected utility theory. More on Allais and his paradox in the essay in this journal by Munier (1991).

risk, and thus as normatively compelling. Savage initially advocated expected utility theory by appealing to its simplicity, empirical validity and normative plausibility, but his controversy with Samuelson induced him to focus on the normative defense of the theory, which he perfected by formulating the Sure-Thing Principle.

The correspondence between Samuelson and Savage of May-August 1950 enhanced the fortunes of expected utility theory in at least two important ways. First, it won over to the cause of expected utility theory a prominent economist, namely Samuelson, who after 1950 contributed to stabilizing the theory as the dominant economic model of choice under risk. Second, it induced Savage to articulate the Sure-Thing Principle, which later became the central normative argument in favor of the theory.

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