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# Fair referee assignments for professional football leagues

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**Abstract**

Assignment of referees to football games is an important problem faced in professional football leagues. Despite its importance, the problem has received limited academic attention. This paper presents a model and analysis of the problem for fair referee assignments, and develops a constructive heuristic and a local search procedure for its solution. Results from an extensive computational study show that the methods are effective in solving the problem in a second of computation time and yielding an excellent solution quality.


*Keywords:* Referee assignment; Sports; Heuristics; Local search

## 1. Introduction

Professional sports have received increasing interest all over the world, and, consequently, now represent significant economic value [1]. Among all the professional sports, football (U.S.: soccer) is one of the most popular and economically significant, evidenced by the huge economic impact of the FIFA World Cup [2]. Broadcasting rights and merchandize sales bring professional football clubs tremendous revenues. According to Jones' report [3], Real Madrid enjoyed €275.7M of revenues and was the champion of the *money league* in 2006. In the same report, we see that the revenues of the top 10 clubs averaged €213.3M.

With its increasing economic value football is no longer simply a sport, but is in fact an industry. Consequently, it has received increasing academic interest from operations researchers. However, the literature on football constitutes a small portion of the literature on all professional sports [4,5].

A line of research in optimization in football is concerned with scheduling games in different leagues (Austrian [6], Brazilian [7,8], Chilean [9], Danish [10], Dutch [11], German [6] and Italian [12]) that require considerably different objective functions and constraints. In all these papers, the model is specific to the league under consideration. An alternative approach adopted in other leagues ignores the league-specific requirements, but generates a schedule framework of games for the entire season and then (possibly) randomly assigns teams to the positions in the framework. In football a season consists of a number of *rounds*, in which every pair of teams meet exactly once, and each round consists of a number of *stages* in which each team plays exactly once. In other words a season is a *round-robin tournament*. The seasonal framework for the leagues adopting the second approach can be obtained by any method developed for scheduling round-robin tournaments [13].

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Silva et al. [14] develop a Monte-Carlo simulation approach to predicting the number of points necessary to qualify for play-offs or avoid elimination in the Brazilian league. Ribeiro and Urrutia [15] propose an optimization approach to this problem, which solves a number of integer programs sequentially and predicts which teams will qualify for the play-offs. The authors note that the Brazilian league is followed by a large population and the television and radio stations air shows in which experts discuss the likelihood of each team to qualify for the play-offs. The authors also report that their method is more accurate than the experts.

An important component of a football league is the referees. Football games require four referees: one center, two assistants and a fourth referee. Two assistant referees are also known as the linesmen. They watch the lines bordering the field noting when the ball goes out of bounds, watch off-side players and sometimes call fouls that are not noticed by the center referee. The fourth referee keeps record of bookings (yellow and red cards) and substitutions and is available as a substitute for the center referee in case he or she cannot continue officiating. The center referee is the chief authority in a football game, and will be referred to as "*the referee*" in this paper. It is very common to see players and managers argue with the referee when they do not agree with his or her calls. Furthermore, it is common to see players or club representatives blame the referee for the outcome after a lost game, even stating that they do not want that referee in their future games. This, in turn, creates a tension between the referees and the clubs, including players, managers and fans. Therefore, the assignment of referees is extremely important.

Duarte et al. [16] are concerned with assigning referees to tournament games. The authors state that amateur leagues are more like tournaments in that, on a given day, more than one game is played at a location and a referee can officiate multiple games. The authors consider the center and assistant referees. They model three referee positions for each game and aim to fill all the positions for a given tournament. They consider constraints to avoid assigning a referee to (i) two different games played at the same time or two games played at different times and locations on the same day, (ii) a game on the days the referee is not available, (iii) more than a predetermined number of games, and finally (iv) a game for which the referee does not meet the minimum skill level. The skill level is assessed for each referee and game, and is the only performance-based component of the model.

Gil-Lafuente [17] addresses the Spanish football league (La-Liga), noting that currently the assignments are made by computer (referred to as *the machine*). The author discusses the trade-off between the fairness of computer-based random assignments and the accuracy of human-based performance-oriented assignments. At one extreme, completely random assignments are fair and they cannot be questioned for favoritism; whereas, at the other extreme, human experts can assign referees to games and ensure that overall performance of the assignments is maximized. Gil-Lafuente develops a new system that combines the strengths of computer- and human-based approaches. His system assesses referees and games at a certain stage for several performance criteria and generates a coefficient of performance for each referee-game pair. He proposes to solve the emerging assignment problem (AP) in each stage and thus optimize the referee assignments for the entire season. Moreover, referees are re-assessed in each stage, thereby allowing for the evolution of their skills and performance to be considered.

Turkish football association (TFF) has an organization called central referee commission (MHK) that assigns referees to games on a weekly (stage-by-stage) basis. A key principle stated in the instructions of MHK is that frequent assignment of the same referee to a team's games is to be avoided [18]. In this paper, we focus on this principle and name it the *fair assignment* principle. In Section 2 we build a mathematical formulation for fair assignments throughout a complete season, and investigate its structural properties. In Sections 3 and 4 we develop both a constructive heuristic and a local search procedure for approximate solution of the model. In Section 5 we present our computational study and in Section 6 we discuss the results obtained from the computational study as well as how our model and solution methods can be extended to cover different application scenarios. Finally, in Section 7 we make our concluding remarks and provide possible future research directions.

## 2. A mathematical formulation

### *2.1. Motivation*

This research has been motivated by the Turkish Premier League's (TPL) 2005–2006 season. The referee assignments for this particular season (see [19]) were closely followed by the authors. Thirty-four (34) different referees were assigned to 306 games throughout the season, which consists of 18 teams and 34 stages. We have observed that the distribution of referees to teams over the stages is far from uniform. We will give several examples of unfairness here.

To avoid using real names, we refer to the teams with letters A, B,…, R and referees with numbers 1, 2,…, 34. Team F played its games with 18 different referees, however, referee 11 officiated seven of those games, which is far above the average ($\frac{34}{18}$). Furthermore, since each team plays 34 games and there are 34 referees, theoretically each referee can officiate exactly one of each team's games. In other words, a different referee can be assigned to each game of a certain team. Other examples of un-evenness are that referee 6 is assigned to team O's two consecutive games (in the 18th and 19th stages) and that referee 9 is assigned to both games played between teams C and E (at the first and 18th stages). In this paper, our goal is to develop an optimization model and efficient solution methods to obtain a fair assignment of referees to the games.

### 2.2. Preliminaries

Let $Z$ be the number of teams in the league and $z = 1, \ldots, Z$ be the team index. The number of stages is denoted by $W$ and $w = 1, \ldots, W$ is the stage index. Note that $W$ is a function of $Z$ and the number of rounds in the season. If $Z$ is even (odd), then a round consists of $Z - 1$ ($Z$) stages. If $\rho$ is the number of rounds, then the season consists of $W = \rho(Z-1)$ ($W = \rho Z$) stages for a league with an even (odd) number of teams. $G$ denotes the number of games in each stage, and the games are indexed by $g$. Note that $G$ is also a function of $Z$. That is, $G = \lfloor Z/2 \rfloor$, where $\lfloor x \rfloor$ is the largest integer that is less than or equal to $x$. Let $R$ be the number of referees and $r$ be the index of the referees. Note that, since all the games at a stage can be played at the same time, the number of referees must be greater than or equal to the number of games ($R \geqslant G$). We refer to the $z$th team as $T_z$, $g$th game of the $w$th stage as $M_{w,g}$, and the $r$th referee as $H_r$.

We are given the schedule of games throughout the season, i.e., we know which teams play in which games. This information is stored in binary parameters $h_{w,g,z}$ and $a_{w,g,z}$. If $M_{w,g}$ is a home (away) game for $T_z$, then $h_{w,g,z} = 1$ ($a_{w,g,z}$), otherwise it is 0. We define a binary decision variable $x_{w,g,r}$ denoting whether $H_r$ is assigned to $M_{w,g}$. Since our ultimate goal is to fairly assign referees to each team's games, we define a binary state variable $e_{w,r,z} = \sum_{g=1}^{G} (h_{w,g,z} + a_{w,g,z}) x_{w,g,r}$ that denotes whether $H_r$ is assigned to $T_z$'s game in stage $w$.

In this section, we have defined our notation flexibly, i.e., allowing various (even or odd) number of teams and rounds in the season. Generally, professional football leagues contain an even number of teams and are played over two rounds. In the case of the TPL, there are $Z = 18$ teams playing two rounds, where the second round is the same as the first except the home and away teams are swapped. This yields $W = 34$ and $G = 9$. Also note that $h_{w,g,z} = a_{w+17,g,z}$ and $a_{w,g,z} = h_{w+17,g,z}$ for each $w = 1, \ldots, 17$, $g = 1, \ldots, 9$ and $z = 1, \ldots, 18$.

### 2.3. The model

Our model can be considered as a constraint satisfaction model with two types of constraints, namely hard and soft constraints. Hard constraints must be satisfied in any feasible assignment. Soft constraints, lead to a more desirable assignment, however, their violation does not make the assignment infeasible. We first define two hard and five soft constraints, then incorporate the extent of the violations of the soft constraints into an objective function, and finally present the entire optimization model.

Our first hard constraint is based on the availability of the referees. As a referee cannot officiate more than one game in a stage, we have

$$\sum_{g=1}^{G} x_{w,g,r} \leqslant 1, \quad \forall w, r.$$

The second hard constraint assures that exactly one referee is assigned to each game:

$$\sum_{r=1}^{R} x_{w,g,r} = 1, \quad \forall w, g.$$

Our first soft constraint aims to disperse the assignment of a referee to a team's games over the season. A related constraint is commonly used in the theory of assembly line scheduling in just-in-time (JIT) manufacturing systems [20]. In JIT assembly lines one produces several different models of the same product. Ideally the assembly line is balanced so that each model takes exactly the same amount of time (cycle time) in each station on the line, which is very rare in practice. Especially in automobile assembly, certain options such as sun-roof appear in a subset of the models, creating an uneven workload in the stations assembling those options. To prevent line stoppages one aims to evenly distribute models that require a certain option as uniformly as possible over the planning horizon, which is generally handled by a constraint that mandates scheduling a number of cars without a certain option between two cars with that option in any subsequence. We refer to this constraint as the *spacing constraint*. Similarly, in order to fairly assign referees to games, we aim to space the assignment of a certain referee to a team's consecutive games. We desire $w_1 - w_2 \geqslant s$ for each $1 \leqslant w_2 < w_1 \leqslant W$ with $e_{w_1,r,z} = e_{w_2,r,z} = 1$, and for each $r$ and $z$, where $s$ is the pre-determined spacing parameter. We measure the spacing constraint's violation with

$$V_1 = \sum_{\substack{1 \leqslant w_2 < w_1 \leqslant W \\ e_{w_1,r,z} = e_{w_2,r,z} = 1}} \sum_{r=1}^{R} \sum_{z=1}^{Z} \max\{\mathrm{sign}\{s + w_2 - w_1\}, 0\}.$$

The second and third soft constraints address the number of times a referee is assigned to a team's games throughout the season. We demonstrate their difference from each other and from the spacing constraint on an example. In a league with 18 teams, if $s = 3$, then a certain referee can be assigned to 12 of a particular team's games (at stages $1, 4, 7, \ldots, 31, 34$) without violating the spacing constraint. However, if we want to limit such assignments to six, then clearly there should be a violation. We define two pre-determined parameters $o_\mathrm{a}$ and $o_\mathrm{h}$, denoting the maximum number of all (home and away) games and only home games of a team desired to be assigned to a referee, respectively. Accordingly, we define two constraints, namely, *referee-team all assignments constraint* and *referee-team homegame assignments constraint*. We measure the violations of these two constraints by

$$V_2 = \sum_{r=1}^{R} \sum_{z=1}^{Z} \max\left\{ \sum_{w=1}^{W} e_{w,r,z} - o_\mathrm{a}, 0 \right\},$$

and

$$V_3 = \sum_{r=1}^{R} \sum_{z=1}^{Z} \max\left\{ \sum_{w=1}^{W} \sum_{g=1}^{G} h_{w,g,z} x_{w,g,r} - o_\mathrm{h}, 0 \right\}.$$

The fourth soft constraint is related to the fairness of assignments among the referees. The number of games assigned to a referee over a season should be greater for the referees with higher skill levels. We define a pre-determined parameter $l_r$ for each $H_r$, denoting the minimum number of games desired to be assigned to $H_r$. Note that $\sum_{r=1}^{R} l_r \leqslant WG$ is a necessary condition for feasibility, as there are $WG$ games in the season. We refer to this constraint as the *referee minimum assignment constraint* and measure its violation by

$$V_4 = \sum_{r=1}^{R} \max\left\{ \sum_{w=1}^{W} \sum_{g=1}^{G} x_{w,g,r} - l_r, 0 \right\}.$$

The fifth and last soft constraints are concerned with games between the same two opponents. Recall that $M_{w,g}$ and $M_{w+W/2,g}$ are played by the same pair of teams, once at one's home and once at the other's. We refer to this constraint as the *same game constraint* and it requires that those two games are not officiated by the same referee. The violation in the same game constraint is measured by

$$V_5 = \sum_{w=1}^{W/2} \sum_{g=1}^{G} \sum_{r=1}^{R} \max\{x_{w,g,r} + x_{w+W/2,g,r} - 1, 0\}.$$

The five soft constraints defined above are mutually independent. No one of the constraints can be expressed as a combination of the others. In fact, each of the soft constraints reflects an important desired characteristic of fair referee assignments, and, hence, its elimination would change the assignment significantly. For example, if we eliminate the same game constraint, then assignment of the same referee to the two games between two teams can be frequently observed. Alternatively, if we eliminate the spacing constraint, then a referee can be assigned to $o_\mathrm{h}$ consecutive home games and $o_\mathrm{a} - o_\mathrm{h}$ consecutive away games of a particular team starting at a particular stage, thereby eliminating the possibility of violating the same game constraint. Clearly, in this example, referees seem to be assigned to teams, which is far from fair. Putting all the constraints together, we present our model below.

Minimize

$$\begin{aligned}\sum_{i=1}^{5} V_i = & \sum_{\substack{1 \leqslant w_2 < w_1 \leqslant W \\ e_{w_1,r,z}=e_{w_2,r,z}=1}} \sum_{r=1}^{R} \sum_{z=1}^{Z} \max\{\mathrm{sign}\{s + w_2 - w_1\}, 0\} \\ & + \sum_{r=1}^{R} \sum_{z=1}^{Z} \max\left\{\sum_{w=1}^{W} e_{w,r,z} - o_\mathrm{a}, 0\right\} \\ & + \sum_{r=1}^{R} \sum_{z=1}^{Z} \max\left\{\sum_{w=1}^{W} \sum_{g=1}^{G} h_{w,g,z} x_{w,g,r} - o_\mathrm{h}, 0\right\} \\ & + \sum_{r=1}^{R} \max\left\{\sum_{w=1}^{W} \sum_{g=1}^{G} x_{w,g,r} - l_r, 0\right\} \\ & + \sum_{w=1}^{W/2} \sum_{g=1}^{G} \sum_{r=1}^{R} \max\{x_{w,g,r} + x_{w+W/2,g,r} - 1, 0\}. \end{aligned} \tag{1}$$

Subject to

$$\sum_{g=1}^{G} x_{w,g,r} \leqslant 1, \quad w = 1, \ldots, W, \quad r = 1, \ldots, R, \tag{2}$$

$$\sum_{r=1}^{R} x_{w,g,r} = 1, \quad w = 1, \ldots, W, \quad g = 1, \ldots, G, \tag{3}$$

$$e_{w,r,z} - \sum_{g=1}^{G} (h_{w,g,z} + a_{w,g,z}) x_{w,g,r} = 0, \quad w = 1, \ldots, W, \quad r = 1, \ldots, R, \quad z = 1, \ldots, Z, \tag{4}$$

$$x_{w,g,r} \in \{0, 1\}, \quad w = 1, \ldots, W, \quad g = 1, \ldots, G, \quad r = 1, \ldots, R. \tag{5}$$

In model (1) is the objective function consisting of five violation measures, constraints (2) and (3) are the hard constraints, (4) ties the state variable $e_{w,r,z}$ to the decision variable $x_{w,g,r}$, and finally, (5) defines the decision variable as a binary variable. Note that, although all the functions are linear in the decision variable, the objective function includes non-smooth functions such as sign and max, and thus is non-linear.

### 2.4. Complexity and lower bounds

The model is built using binary variables and the objective function is a combination of five soft constraints. Finding a solution to an instance of the problem such that the objective function value is zero is equivalent to finding a feasible solution to a constraint satisfaction problem. In fact that constraint satisfaction problem is an instance of the satisfiability problem (SAT) where the number of variables in a clause is a function of $W$, $G$, $R$ and $Z$. It is well-known that SAT is NP-complete [21]. We also note that our problem is similar in nature to the problem introduced by McAloon et al. [22], where an assignment of games to periods is sought such that no team plays in the same period more than twice throughout the season. Considering periods as referees, our problem is a general case of their problem, even with only one soft constraint (referee-team all assignment constraint). As no one to date has proposed a polynomial-time solution method for McAloon et al.'s [22] problem, we are unlikely to find one for our problem. As we have already stated, the problem is a combinatorial optimization problem for which a polynomial time solution method may not exist. In fact, in our problem, we can assign $G$ referees out of $R$ to the games in $R!/G!$ different ways in each stage. Since we have $W$ independent stages, there exist $(R!/G!)^W$ distinct assignments. Even small values of these three parameters render enumerative approaches impractical. Therefore, in this paper, we first investigate the nature of the problem and obtain some lower bounds on (soft) constraint violations, and then develop two approximate solution methods.

We first focus on the spacing constraint, and consider a certain referee-team pair. The referee can be assigned to the team's games every $s$ stages. If the first assignment occurs in the first stage, then the referee can be assigned to at most $\lfloor (W-1)/s \rfloor + 1$ games of the team. For example, if $W = 34$ and $s = 5$, a referee can be assigned to a team's games at stages 1, 6, 11, 16, 21, 26, 31, that is seven games throughout the season. Note that, the last assignment is at stage 31, leaving a slack of 3. In case of positive slack, the assignments can be arranged more flexibly. Since there are $Z$ referees, at most $\min\{W, Z(\lfloor (W-1)/s \rfloor + 1)\}$ games can be officiated by a referee without violating the spacing constraint. Considering all the $R$ referees, we see that at most $R\min\{W, Z(\lfloor (W-1)/s \rfloor + 1)\}$ games can be assigned a referee without violation. Since there are $WG$ games in the season, the following is a lower bound for the violation in the spacing constraint:

$$LB_1 = \max\{0, WG - R\min\{W, Z(\lfloor (W-1)/s \rfloor + 1)\}\}.$$

As for the referee-team all assignments constraint, a referee can officiate at most $o_{\mathrm{a}}$ games for one team. Since each game involves two teams, a referee can officiate at most $o_{\mathrm{a}}Z/2$ games throughout the season. Thus, considering all the referees, we state that at most $o_{\mathrm{a}}ZR/2$ games can be assigned a referee, without violating the referee-team all assignments constraint. Recalling that we have $WG$ games in the season, the following is a lower bound for the referee-team all assignments constraint:

$$LB_2 = \max\{0, WG - o_{\mathrm{a}}ZR/2\}.$$

Using the similarity between the referee-team homegame assignments and the referee-team all assignments constraints, we note that a referee can officiate at most $o_{\mathrm{h}}Z$ games throughout the season, yielding a total of $o_{\mathrm{h}}ZR$ assignable games in the season. The derivation of the following lower bound is straightforward:

$$LB_3 = \max\{0, WG - o_{\mathrm{h}}ZR\}.$$

The referee minimum assignment constraint is concerned with the number of games officiated by a referee and is based on a user-defined parameter $l_r$ for each referee. Thus, the total number of games that should be assigned to all the referees is $\sum_{r=1}^{R} l_r$. Once again recalling that there are $WG$ games in the season, we obtain the following lower bound for the referee minimum assignment constraint:

$$LB_4 = \max\left\{0, WG - \sum_{r=1}^{R} l_r\right\}.$$

The last soft constraint is the easiest to analyze for a lower bound. Recall that $R \geqslant G = Z/2$ and the number of stages in a round is $Z - 1$, assuming an even number of teams. Thus, for each $Z > 2$, the games between the same pair of teams in the first and second rounds can be assigned a different referee, thus the violation in the same game constraint can be totally eliminated. If $Z = 2$ and $R = 1$, on the other hand, the same (and only) referee must be assigned to the only two games, causing one unit of violation in the same game constraint. Since this is a trivial case, we simply disregard it and state the following lower bound for the same game constraint:

$$LB_5 = 0.$$

## 3. A constructive heuristic procedure

In this section, we develop a constructive heuristic method. Our method considers stages one-at-a-time and in increasing order. In each stage($w$), $RG$ possible assignments are evaluated (each referee can be assigned to each game). For each possible assignment ($r - g$ pair) the violations in the soft constraints are calculated and the total $C_{r,g}$ is noted as the cost of assigning $H_r$ to $M_{w,g}$. Note that all the soft constraints are referee-specific; i.e., the objective function

```
Algorithm Constructive_Assignment_Heuristic
  1 Initialize all variables.
  2 For each stage
    2.1 Calculate the cost of assignment for each referee and game.
    2.2 Find an optimal assignment for the current stage.
    2.3 Update all variables regarding the current stage.
  3 Return the complete solution.
```

Fig. 1. Pseudo-code for algorithm Constructive_Assignment_Heuristic.

value can be obtained by summing up $R$ smaller values obtained separately for each referee. Having calculated the $C_{r,g}$ values, we build an assignment model for the stage being considered as follows.

Minimize

$$\sum_{r=1}^{R}\sum_{g=1}^{G} C_{r,g}\, y_{r,g}. \tag{6}$$

Subject to

$$\sum_{g=1}^{G} y_{r,g} \leqslant 1, \quad r = 1, \ldots, R, \tag{7}$$

$$\sum_{r=1}^{R} y_{r,g} = 1, \quad g = 1, \ldots, G, \tag{8}$$

$$y_{r,g} \in \{0, 1\}, \quad r = 1, \ldots, R, \quad g = 1, \ldots, G. \tag{9}$$

In this AP formulation, we define $y_{r,g}$ as our binary decision variable denoting whether $H_r$ is assigned to $M_{w,g}$. Note that constraints (7) and (8) are the hard constraints from our original model. The optimal solution of this AP can be obtained in $O(R^3)$ time using the well-known Hungarian Method [23]. After solving the AP, the solution is translated into our original notation and variable values are updated. The constructive heuristic terminates when all the stages are considered, and a complete solution is obtained. We call this method the *constructive assignment heuristic* and provide its pseudo-code in Fig. 1.

## 4. A local search procedure

Local search is a widely used technique to solve combinatorial optimization problems. The key to success with a local search method is the ability to find improving neighbor solutions and to calculate the change in the objective value quickly, which requires one to study the problem structure to define a good neighborhood function. For our problem we define two neighborhood functions as follows.

In any solution there are exactly $WG$ assignments and in each stage exactly $G$ of the $R$ referees are assigned. Recall that hard constraints are defined for each stage separately, i.e., a feasible solution can be constructed by considering stages independently as we do in the constructive heuristic in the previous section. Therefore, we define our neighborhood function to reflect this property, i.e., modify assignments in exactly one stage. In a certain stage swapping the assignments of two distinct referees yields a new assignment, only if at least one of the referees selected is assigned to a game in that stage. Thus, we select our first referee among the ones that are already assigned to a game in that stage. The selection of the second referee is more flexible as the only constraint to consider is to select a different referee. With this approach, we can select the first referee in $G$ different ways and the second one in $R-1$ different ways. Since a selection considers one stage only, we can select the stage in $W$ different ways. Therefore, any given assignment has exactly $WG(R-1)$ neighbor assignments. For a league with 18 teams and two rounds, $W = 34$ and $G = 9$, the number of neighbors is $306(R-1)$, which can be a large number depending on $R$. We refer to this neighborhood as the *general neighborhood* and develop an alternative neighborhood of a smaller size.

Swapping the assignment of two referees does not necessarily reduce constraint violations. For example, if no constraint violations are observed in the last stage of a certain assignment, no improvement can be obtained by searching in that stage. Thus, we restrict the neighborhood to sections where a constraint violation is observed, and, for a given solution, we first extract information regarding constraint violations. For the spacing and same game constraints, we note the stage and game indices (in pairs) for which a violation is observed. Therefore, it is much easier to locate the point at which to search for improvements: we go to the stage noted and find out which referee is assigned to the game noted, thereby obtaining the first referee instantly. The second referee can be selected in $R-1$ different ways. As to the referee-team all and homegame assignments constraints we store the indices of referees and teams (in pairs) for which a violation is observed. For such a pair, one can either store the indices of stages where the referee is assigned to the team's game in a linked list or go over all stages to find such a stage. When that stage is located, we already have the first referee and again selecting the second referee can be done in $R-1$ different ways. Similarly, for the referee minimum assignment constraint, we store the indices of the referees for which a violation is observed. For such a referee we need to locate a stage at which he or she is assigned to a game and select the second referee in $R-1$ different ways. The number of solutions in this *special neighborhood* is a function of the number of constraint violations. Even in the worst case the number of constraint violations is less than *WG*, thus the size of the special neighborhood is smaller than that of the general neighborhood ($WG(R-1)$). With efficient use of data structures, the time to reach locations where a constraint violation is observed can be kept at negligible levels. Thus, we expect that the special neighborhood can be explored in less time than the general neighborhood, and address this in our computational study.

The general and special neighborhoods can be used separately or jointly in an implementation of local search. In case of utilizing both neighborhoods, we first explore the special and then the general neighborhood. The reason is that using the special neighborhood can yield improvements in a shorter time, but it may not explore all possible improvements. Therefore, using the general neighborhood on an intermediate solution that contains fewer constraint violations should take less time. We define a parameter *Neighborhood* for our local search procedure that can take the following values: *general*, *special* and *both*.

Another important policy in a local search procedure is the number of improving neighbor solutions to evaluate before making a move. At one extreme, one makes a move as soon as an improving neighbor is found, and at the opposite extreme one evaluates all the neighbor solutions, selects the neighbor that improves the objective function most and then makes the move. These two policies are known as *first improving* and *best improving*, respectively. A compromise policy is to evaluate a pre-determined number of neighbors or to search until a pre-determined number of improving neighbors are identified and then move to the best improving among the evaluated neighbor solutions. In our preliminary experiments, we tried both extremes and several compromise policies and observed that the first improving policy works best for our problem.

Since local search terminates at a local optimum, the quality of the solution obtained depends on the initial solution used as a starting point. Generally there is a trade-off involved in the generation of initial solutions. Solutions can be

```
Algorithm Local_Search(Neighborhood,Initial Solution)
  Initialization
    1.1 If Initial Solution = constructive
        Find initial solution using Algorithm
        Constructive_Assignment_Heuristic.
    1.2 If Initial Solution = random
        Find initial solution randomly.
  2 Set current solution as the initial solution.
  Improvement
    3.1 If Neighborhood ∈ {special, both}
        3.1.1 Identify soft constraint violation points.
        3.1.2 Search for an improving neighbor in the special neighborhood.
        3.1.3 If an improving neighbor exists
            3.1.3.1 Update current solution.
            3.1.3.2 Update soft constraint violation points.
            3.1.3.3 Go to Step 3.1.2
    3.2 If Neighborhood ∈ {general, both}
        3.2.1 Search for an improving neighbor in the general neighborhood.
        3.2.2 If an improving neighbor exists
            3.2.2.1 Update current solution.
            3.2.2.2 Go to Step 3.2.1
  4 Return current solution.
```

Fig. 2. Pseudo-code for algorithm `Local_Search`(*neighborhood*,*Initial Solution*).

randomly generated almost instantly but their objective function values may be very large, causing the search to take too long or terminate at a poor local optimum. Advanced solution mechanisms such as constructive heuristics provide much better initial solutions, however, they may take a considerable amount of time in the initialization phase. We address this trade-off in our computational study via parameter *Initial Solution* which can take either *constructive* or *random* values. The pseudo-code for our local search procedure is given in Fig. 2.

## 5. Computational study

### 5.1. Experiment plan

This study is primarily concerned with the referee assignments in the TPL. We use seasonal schedules of the TPL (see [24]) in our computational study. Our first experimental factor is the number of referees. Analyzing the historical data, we observed that for the TPL, 28–30 referees are named at the beginning of the season and about five referees can be added as the season proceeds. As discussed earlier, in the 2005–2006 season, 34 different referees were assigned. This seems a sufficient number for a league with 18 teams. On the other hand, the number of referees can be as small as $R = G = 9$. However, the referees are generally assigned as fourth referees to games in the stages when they are not assigned as the center referee. Thus, in each stage, we need nine center and nine fourth referees who are selected from the same pool. Accordingly, we experiment with small ($R = 18$), medium ($R = 26$) and large ($R = 34$) numbers of referees.

We consider the distribution of the targeted number of games assigned to each referee as another experimental factor with two levels, *equal* and *not equal*. In the case of equal distribution, each referee is targeted to officiate the same number of games. In the other case, we split the referees in two equal-sized categories based on their skill-levels. The referees in the first category are targeted to officiate twice as many games as those in the second category. For instance, the 306 games can be distributed to 18 referees equally by targeting 17 for each, or in two categories such that the referees in the first category officiate 22 games each and the others 11 games each. Note that in this example only 297 of the 306 games are allocated to the referees. This is due to rounding fractional numbers down to avoid an infeasible targeted allocation. Regarding the allocation policy, we define a parameter *Flexibility* $\in \{0.7, 1\}$ that defines the number of games to be allocated by *Flexibility WG*.

Note that the parameters mentioned so far define the $l_r$ values for each referee without violating the referee minimum assignment constraint. In other words, the selection of levels for the above parameters assure that $LB_4 = 0$. We follow the same principle in defining all parameters. Recall that $LB_5 = 0$ by definition and, thus, we need to consider only three other constraints.

The spacing constraint requires parameter $s$. The assignment $s = 1$ cannot be considered as it allows assigning the same referee to a team's games in two consecutive stages. Large values of $s$ are not desired either, since they over constrain the problem. Practically, $s \in \{2, 3, 4\}$ is reasonable. This set of values also yields $LB_1 = 0$.

Focusing on the homegame assignments constraint we note that since the number of referees $R \in \{18, 26, 34\}$ is always greater than the number of home games for each team (17), $o_h = 1$ satisfies $LB_3 = 0$. Furthermore, it guarantees that no referee is assigned to more than one home game of any team, serving to reduce the influence of a team's fans on the referees. To introduce flexibility we let $o_h \in \{1, 2\}$. Similarly, for the all assignments constraint, we let $o_a \in \{3, 4\}$ which satisfies $LB_2 = 0$ and introduces some flexibility to the assignments.

In summary, we have six parameters with two or three values each (summarized in Table 1) yielding a total of 144 possible combinations. In our computational study we consider all 144 of those combinations.

### 5.2. Methods

In this study, we first generate 1000 random solutions and summarize the number of violations observed. Then, we evaluate the actual referee assignments of the 2005–2006 season of the TPL and compare them with the random solutions. Next, we run our constructive heuristic and compare its solutions to both random and actual TPL solutions. Finally, we run our local search procedure. Recall that the local search procedure takes two parameters with three and two values, respectively. Therefore, we run the local search procedure for six different settings. We compare the results obtained for each setting and compare them with each other. We also compare the results from our local search procedure to those of the random and constructive methods, as well as the actual assignments. The methods used in the

Table 1
Experimental parameters

| Parameter | Values |
|---|---|
| $R$ | 18, 26, 34 |
| *Equal* | True, false |
| *Flexibility* | 0.7, 1 |
| $s$ | 2, 3, 4 |
| $o_a$ | 3, 4 |
| $o_h$ | 1, 2 |

Table 2
Methods run in the computational experiments

| Method | $R$ values |
|---|---|
| `Random`(1000) | 18, 26, 34 |
| `Actual_TPL_2005-06` | 34 |
| `Constructive_Assignment_Heuristic` | 18, 26, 34 |
| `Local_Search`(*general*,*constructive*) | 18, 26, 34 |
| `Local_Search`(*special*,*constructive*) | 18, 26, 34 |
| `Local_Search`(*both*,*constructive*) | 18, 26, 34 |
| `Local_Search`(*general*,*random*) | 18, 26, 34 |
| `Local_Search`(*special*,*random*) | 18, 26, 34 |
| `Local_Search`(*both*,*random*) | 18, 26, 34 |

computational study are summarized in Table 2. All the methods are coded in Microsoft C#.NET and run on a desktop computer with a P4-3.4 GHz CPU and 2 GB of memory. Emerging APs in the constructive heuristic are solved using CPlex 9.1, a commercial optimization package.

### 5.3. Research questions

Having developed an optimization model for the problem and two heuristic methods to solve it, we seek answers for the following six research questions in our computational study:

(1) What are typical values for $V_1, \ldots, V_5$?
(2) Is the actual assignment of the 2005–2006 season better than a random solution?
(3) How does the constructive assignment heuristic compare to the random and actual assignments?
(4) What is the best parameter setting for the local search procedure?
(5) How does the local search procedure compare to the random, actual and constructive heuristic assignments?
(6) Do any of the methods generate ideal assignments, that is, satisfying all the constraints?

### 5.4. Results

To address the first research question we randomly generate 1000 (full season) assignments and evaluate them for all possible settings of the experimental factors. The results are summarized in Table 3. In each row, the results are presented in four lines for four different statistics: average, standard deviation, minimum and maximum. The last column reports computational time required to obtain the results (in seconds). Table 3 shows that $R$ significantly affects all five (soft) constraints, i.e., the number of violations in the spacing, referee-team all and homegame assignments and same game assignment constraints decreases as $R$ increases, whereas the number of violations for the referee minimum assignment constraint increases as $R$ increases. Overall, the total number of constraints violated decreases as $R$ increases. From the detailed results not reported here, we determine that each parameter affects only one constraint. That is, *Equal* and

Table 3
Evaluation of random assignments

| $R$ | Stat. | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $\sum_{i=1}^{5} V_i$ | Time (s) |
|---|---|---|---|---|---|---|---|---|
| 18 | Avg. | 62.35 | 35.67 | 65.17 | 17.67 | 8.52 | 189.38 | 3.1 |
| | St. dev. | 6.98 | 5.82 | 5.14 | 4.45 | 2.86 | 14.25 | 0.0 |
| | Min. | 15 | 3 | 9 | 0 | 0 | 50 | 3.0 |
| | Max. | 119 | 84 | 125 | 72 | 20 | 375 | 3.3 |
| 26 | Avg. | 43.67 | 14.71 | 45.98 | 18.28 | 5.90 | 128.54 | 3.9 |
| | St. dev. | 6.01 | 4.01 | 4.81 | 4.57 | 2.41 | 11.96 | 0.1 |
| | Min. | 6 | 0 | 3 | 0 | 0 | 22 | 3.8 |
| | Max. | 92 | 45 | 100 | 73 | 16 | 275 | 4.1 |
| 34 | Avg. | 33.53 | 7.37 | 35.47 | 26.24 | 4.50 | 107.11 | 4.7 |
| | St. dev. | 5.39 | 2.97 | 4.61 | 4.86 | 2.12 | 10.93 | 0.2 |
| | Min. | 4 | 0 | 1 | 0 | 0 | 14 | 4.5 |
| | Max. | 74 | 30 | 83 | 87 | 16 | 239 | 5.4 |

Table 4
Evaluation of the actual assignment of the 2005–2006 season of the TPL

| $R$ | Stat. | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $\sum_{i=1}^{5} V_i$ |
|---|---|---|---|---|---|---|---|
| 34 | Avg. | 10.67 | 14.50 | 30.00 | 65.00 | 3.00 | 123.17 |
| | St. dev. | 8.27 | 10.61 | 24.25 | 24.81 | 0.00 | 37.21 |
| | Min. | 1 | 4 | 6 | 39 | 3 | 53 |
| | Max. | 21 | 25 | 54 | 91 | 3 | 194 |

*Flexibility* affect $V_4$, $s$ affects $V_1$, $o_{\mathrm{a}}$ affects $V_2$, and $o_{\mathrm{h}}$ affects $V_3$ as expected. Time, on the other hand is affected by $R$ only, with $R = 34$ taking the most computational time.

The evaluation of the actual referee assignments for the 2005–2006 season of the TPL is summarized in Table 4. Comparing the random and actual assignments, we see that average total violation is lower for the random assignments. Considering constraints one-at-a-time we see that for the spacing, referee-team homegame assignments and same game assignment constraints, the actual assignment is better than random, whereas for the other two constraints it is worse. Thus, we cannot conclude which of the assignments is strictly better than the other. In other words, answering the second research question, the actual assignment for the 2005–2006 season is not better than a random assignment on average.

The total number of violated constraints is greater than 100 for both random and actual assignments, leaving room for improvement by heuristic methods, since we know that the lower bound for each constraint is 0. Whether we can reach an ideal assignment (with zero total violation) can be extracted from Tables 5 and 6.

As shown in Table 5, the constructive heuristic yields a total violation smaller than 100 for all $R$ values and 42 for $R = 34$. This is a significant improvement over the random and actual methods. Moreover, the spacing and same game constraints are never violated. Among all constraints, the referee-team homegame assignments constraint is the only one for which the constructive heuristic is outperformed by the actual method. As to the computation time consumed by the constructive heuristic, we see that it runs in a second, for the $R = 34$ case, which is only a fraction of the time required to randomly generate 1000 assignments. Consequently, we state that the constructive heuristic developed in this paper can be used to generate good assignments in real-time, which answers the third research question.

The answers to the last three research questions are found in Table 6. In the table, we see a trade-off between obtaining the initial solution using the constructive heuristic or randomly. That is, a random initial solution leads to a faster convergence, while, the constructive heuristic's solution leads to a better local optimum. Similarly, there is a trade-off between using the general or special neighborhood alone. However, using both sequentially outperforms using either of them individually. Answering the fourth research question, we state that using the constructive

Table 5
Performance of the constructive assignment heuristic

| $R$ | Stat. | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $\sum_{i=1}^{5} V_i$ | Time (s) |
|---|---|---|---|---|---|---|---|---|
| 18 | Avg. | 0.00 | 23.79 | 62.29 | 5.42 | 0.00 | 91.50 | 0.8 |
| | St. dev. | 0.00 | 17.46 | 40.29 | 6.22 | 0.00 | 46.27 | 0.0 |
| | Min. | 0 | 1 | 15 | 0 | 0 | 19 | 0.8 |
| | Max. | 0 | 60 | 111 | 15 | 0 | 180 | 1.1 |
| 26 | Avg. | 0.00 | 9.75 | 44.38 | 2.67 | 0.00 | 56.79 | 0.9 |
| | St. dev. | 0.00 | 9.71 | 31.69 | 4.37 | 0.00 | 34.36 | 0.1 |
| | Min. | 0 | 0 | 5 | 0 | 0 | 7 | 0.8 |
| | Max. | 0 | 35 | 86 | 13 | 0 | 123 | 1.2 |
| 34 | Avg. | 0.00 | 5.58 | 33.42 | 3.00 | 0.00 | 42.00 | 1.0 |
| | St. dev. | 0.00 | 7.10 | 26.20 | 3.67 | 0.00 | 27.88 | 0.0 |
| | Min. | 0 | 0 | 3 | 0 | 0 | 3 | 0.9 |
| | Max. | 0 | 28 | 75 | 11 | 0 | 103 | 1.1 |

Table 6
Performance of the local search procedure

| Neighborhood | Initial solution | $R$ | Stat. | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $\sum_{i=1}^{5} V_i$ | Time (s) |
|---|---|---|---|---|---|---|---|---|---|---|
| General | Constructive | 34 | Avg. | 0.00 | 0.00 | 0.38 | 0.13 | 0.00 | 0.50 | 1.4 |
| | | | St. dev. | 0.00 | 0.00 | 1.00 | 0.33 | 0.00 | 1.11 | 0.2 |
| | | | Min. | 0 | 0 | 0 | 0 | 0 | 0 | 1.1 |
| | | | Max. | 0 | 0 | 6 | 1 | 0 | 6 | 2.0 |
| Special | Constructive | 34 | Avg. | 0.00 | 0.00 | 0.00 | 2.88 | 0.00 | 2.88 | 1.1 |
| | | | St. dev. | 0.00 | 0.00 | 0.00 | 3.38 | 0.00 | 3.38 | 0.0 |
| | | | Min. | 0 | 0 | 0 | 0 | 0 | 0 | 1.0 |
| | | | Max. | 0 | 0 | 0 | 11 | 0 | 11 | 1.2 |
| Both | Constructive | 34 | Avg. | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 | 0.15 | 1.1 |
| | | | St. dev. | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 | 0.36 | 0.0 |
| | | | Min. | 0 | 0 | 0 | 0 | 0 | 0 | 1.0 |
| | | | Max. | 0 | 0 | 0 | 1 | 0 | 1 | 1.2 |
| General | Random | 34 | Avg. | 3.38 | 0.00 | 0.06 | 0.10 | 0.00 | 3.54 | 0.57 |
| | | | St. dev. | 1.00 | 0.00 | 0.24 | 0.31 | 0.00 | 1.13 | 0.22 |
| | | | Min. | 2 | 0 | 0 | 0 | 0 | 2 | 0.19 |
| | | | Max. | 5 | 0 | 1 | 1 | 0 | 6 | 1.11 |
| Special | Random | 34 | Avg. | 3.33 | 0.00 | 0.00 | 17.04 | 0.00 | 20.38 | 0.01 |
| | | | St. dev. | 0.95 | 0.00 | 0.00 | 17.40 | 0.00 | 17.39 | 0.02 |
| | | | Min. | 2 | 0 | 0 | 0 | 0 | 2 | 0.00 |
| | | | Max. | 4 | 0 | 0 | 50 | 0 | 52 | 0.06 |
| Both | Random | 34 | Avg. | 3.33 | 0.00 | 0.00 | 0.08 | 0.00 | 3.42 | 0.08 |
| | | | St. dev. | 0.95 | 0.00 | 0.00 | 0.28 | 0.00 | 1.01 | 0.06 |
| | | | Min. | 2 | 0 | 0 | 0 | 0 | 2 | 0.02 |
| | | | Max. | 4 | 0 | 0 | 1 | 0 | 5 | 0.22 |

heuristic to obtain an initial solution and the searching in both neighborhoods sequentially generates excellent assignments in about a second of computer time, thus it is the best setting for the local search procedure. The answer to the fifth research question is that the local search procedure outperforms the constructive heuristic, and, hence, random and actual methods, as well (in order to reduce the size of the table, we do not show the results for $R = 18$ and 26).

The answer to the final research question is positive for the local search procedure with the constructive heuristic used to obtain the initial assignment. Moreover, only one constraint is violated in the worst case. This result clearly shows that the research goal in designing the heuristic method has been achieved.

## 6. Discussion

In this section we discuss how one can enhance the model and solution methods developed in this paper for practical use. The first scenario we address is the consideration of all four referees. This means that the second hard constraint should be repeated for all four referee positions. The first hard constraint, on the other hand, should be duplicated as the center and the fourth referees are chosen from the same pool, and the linesmen are chosen from a separate pool. As to the spacing constraint, one first has to simply repeat the constraint four times. However, a trivial solution under this new structure is to team-up four referees (center, two assistants and the fourth) and assign them as a team to the same games at all times. However, in order to provide learning and improvement opportunities as well as ensuring fairness, the assignment of the same set of referees to the games should be avoided. Thus, a spacing constraint to space every pair of referees should be defined and incorporated into the model. The rest of the soft constraints are straightforward, one needs to repeat each constraint once for each referee type. The solution methods also should be modified, and one should expect a slower convergence and a larger number of constraint violations.

In the current model, it is assumed that every referee is available in each stage. Accordingly, in the first hard constraint, we limit the assignment of a referee at a stage to at most one. However, in practice, there might be some stages when a referee may not be available. In that case, one has to modify the constraint associated with that referee and stage and set the right-hand side to zero. Note that this will affect the solution methods. One has to modify the methods to first check which referees are available at each stage and to consider only the available ones.

With the model and solution methods developed here, one can either assign referees for the entire season at once or proceed stage-by-stage. To do it stage-by-stage, one generates the entire season's assignment at the beginning. These assignments are used as recommendations made by the computer. Then one can assign the first stage's referees either exactly as recommended by the computer or with some modifications. Actual assignments should then be entered into the model by means of the decision and state variables. Then one re-runs the solution method, this time with the first stage fixed. The method will then generate the best assignment it can find for the rest of the league. This process continues iteratively until all stages are evaluated and, actually, played.

In football, there are generally two national competitions: the league and the cup. In Turkey, these are the TPL and the Turkish Cup (TC). The cup games are played in a knock-out fashion. That is, in each stage one of the opponents is eliminated while the other one proceeds to the next stage. The first stage of the cup games include teams from the lower divisions, and such teams have a chance to proceed to higher stages, and even win the cup. However, in reality, especially in the higher stages of cup games, a majority of the teams are from the top division. Moreover, the referees who officiate the cup games are the same referees who officiate the league games. Therefore, spacing a referee's assignment to a team's games should include the cup games, as well. In the model, one can insert a special stage for each stage of the cup competition. The important aspects are that the special stages will not have exactly $G$ games and $Z$ teams, and one does not know which teams will play the cup games in a certain stage until the previous stage is played. Clearly, this requires that one cannot plan the entire season at once: planning must proceed stage-by-stage.

Finally, we note that in our formulation the soft constraints are treated equally important. As the results of our computational study show, assignments with zero violation can be obtained. However, in practice, if one faces an over-constrained scenario, then satisfying all the constraints might not be possible. In that case, associating weights to the soft constraints can be considered. The solution methods proposed in this paper can easily be tailored to work on the weighted model.

## 7. Conclusions and future research

In this paper, we have addressed fair referee assignments for professional football leagues, in particular, with the Turkish Premier League in mind. This problem is new as the literature considering referee assignments is limited to performance-based assignments only. Therefore, the mathematical formulation of the problem is the first significant contribution of this paper. The paper develops a constructive heuristic and a local search procedure for approximate solutions of the problem and analyzes their performance through an extensive computational study. The results of the computational study show that using the constructive heuristic to find an initial solution and the local search procedure to improve that initial solution yields excellent quality solutions in a second of computing time. Hence, the development of these solution methods is the second significant contribution of the paper.

Referee assignments require considering multiple criteria, including fairness as defined in this paper. Another important criterion is the performance obtained from the assignment. Performance-based assignment (studied by Gil-Lafuente [17]) assures that the referees that are not capable of officiating a certain game are not assigned to that game and games a referee is assigned to reflect the referee's skill level. That is, the referees with higher skill levels officiate more critical games. Fairness and performance are two conflicting criteria. Their incorporation in an optimization model and combined solution is a future research direction for the authors of this paper.

Another possible future research direction is the consideration of all four referees that are assigned to a football game. Both the modeling and the solution are challenging tasks due to the increase in the number of variables and constraints as discussed in the previous section. Similarly, referee assignments in other sports with different referee characteristics is another possible research direction.

In formulating our model, we have used the existing body of knowledge on different operations research problems. In particular, we have formulated the spacing constraint using the JIT scheduling problem. In a broader sense, the referee AP can be seen as a periodic scheduling problem where the assignment of a referee to the games of a particular team has a periodic nature. While one desires to avoid short periods, it can be inevitable unless the number of referees is greater than the number of games in the season. Studying the referee assignment problem from a periodic scheduling perspective appears interesting, and it may lead to new formulations of the problem. Consequently, the existing solution methods for problems such as periodic maintenance scheduling problem or real-time scheduling problem with periodic and sporadic tasks may be adapted to solve the referee assignment problem.

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