

Network theorem

Network definitions :-

Branch : Each individual circuit component such as resistor, inductor, capacitor etc, is called a circuit element. A group of such element, usually in series and having two terminal is called a branch of the circuit.

Network : A network is a combination of circuit elements or branches interconnected in some way.

Linear circuit :- It is the circuit whose parameters remain constant with change in applied voltage or current.

Non linear circuit :- It is a circuit whose parameters change with voltage or current.

Active network :- It is a network which contain one or more than one source of e.m.f. An active network consist of an active element like a battery or transistor.

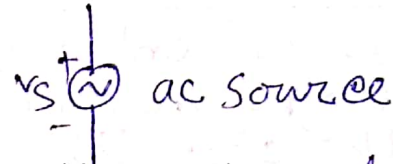
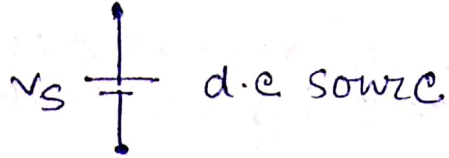
passive network :- when a network does not contain any source of e.m.f, it is called passive network. A passive network consist of resistance, inductance or capacitance as passive element.

Mesh or loop :- A loop is any close path form by a number of branches in a network. A mesh is the simplest possible close path.

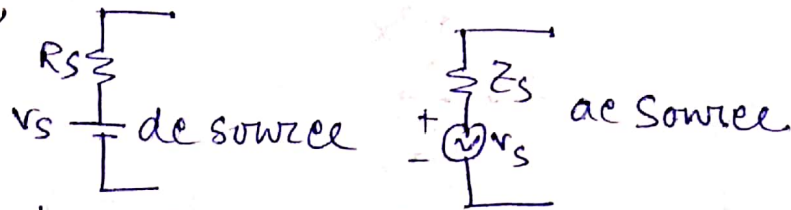
Node or junction :- A node or junction is simply a common point where two or more circuit component meet.

voltage source :-

An ideal voltage source is a voltage generator whose output is independent of the current delivered by the generator. An ideal voltage source must have zero internal impedance. So, the load

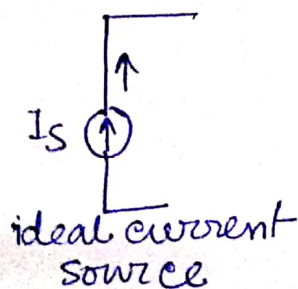


current approaches infinity when load resistance connected across it, approaches zero. Thus no practical source can be an ideal voltage source. A practical voltage source always has some internal impedance. So, a practical voltage source can be represented by an ideal voltage source in series with a resistance as shown in fig,

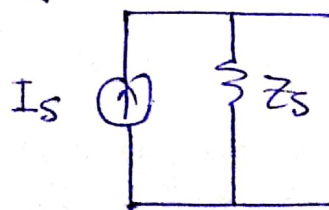


current source :-

An ideal current source is a current generator which supplies a current independent of the voltage across the terminals of the current generator. An ideal current source must have



infinite internal impedance. But practically it is impossible. Certain practical current source can be represented by an ideal current source in parallel with an internal impedance Z_s as shown in fig below:



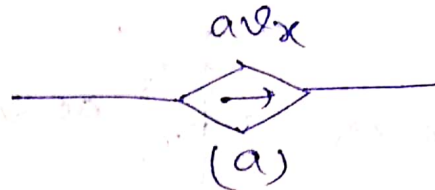
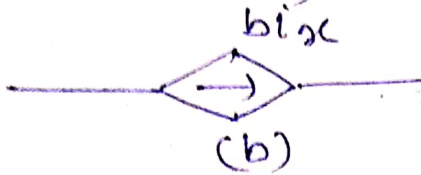
Independent source :-

A voltage or current source independent of any other voltage or current existing in the circuit to which these sources are connected.

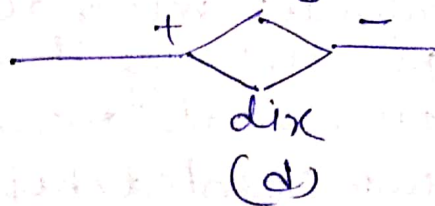
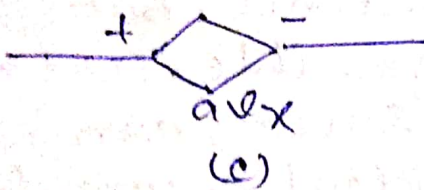
Dependent source :-

voltage or current source may also be dependent on a voltage or current existing somewhere else in the circuit.

The current supplied by a dependent current source is determined by either a voltage or a current existing somewhere else in the circuit. We can accordingly have a voltage controlled current source (a) or a current controlled current source (b)



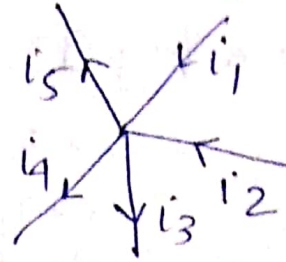
The voltage supplied by a dependent voltage source is determined by either a voltage or a current existing somewhere else in the circuit is called voltage controlled voltage source (c) or a current controlled voltage source (d)



Kirchhoff's current law:-

The algebraic sum of currents at any node of a circuit is zero.

As KCL, the algebraic sum of currents entering a node must be equal to the algebraic sum of the currents leaving a node.



$$i_1 + i_2 - i_3 - i_4 - i_5 = 0$$

$$\Rightarrow i_1 + i_2 = i_3 + i_4 + i_5$$

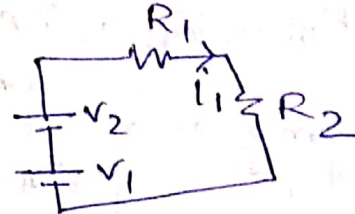
Kirchhoff's voltage law:-

The algebraic sum of voltages in any closed path of network that is traversed in a single direction is zero.

As per KVL,

$$-v_1 + (-v_2) + iR_1 + iR_2 = 0$$

$$\Rightarrow -v_1 - v_2 + iR_1 + iR_2 = 0$$



Thevenin's Theorem:-

Any two terminal linear network containing energy source and impedance may be replaced by an equivalent circuit consisting of a voltage source (E') in series with the impedance (Z') where E' is the open circuit voltage measured between the terminal and Z' is the impedance between the terminal when all energy source have been replaced by their internal impedance.

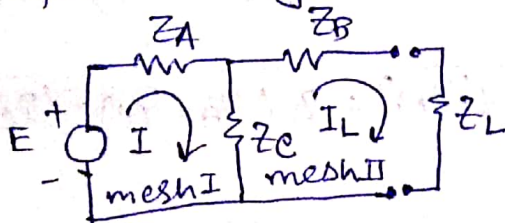


fig ①

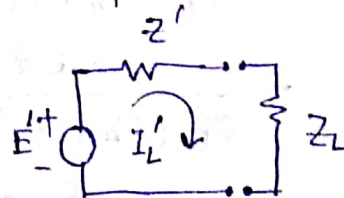


fig ②

Let us consider Z_A , Z_B , Z_C as impedance and Z_L is the load impedance. I and I_L be the current flowing through mesh I and mesh II respectively due to energy source E . A thevenin equivalent circuit as shown in fig ②.

From fig ② we get,

$$I_L' = \frac{E'}{Z' + Z_L} \quad \text{--- (I)}$$

Now from fig ①, we get

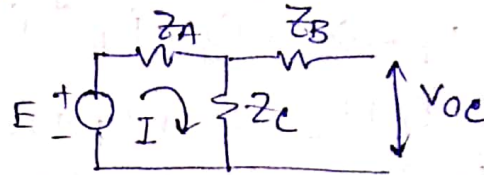
$$E = I(Z_A + Z_C) - I_L Z_C \quad \text{--- (II)}$$

$$0 = -I Z_C + I_L (Z_B + Z_C + Z_L) \quad \text{--- (III)}$$

To find equivalent voltage source, Z_L is removed and we get,

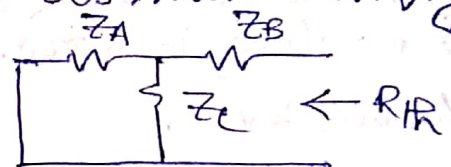
$$E' = V_{OC} = I Z_C$$

$$= \frac{E Z_C}{Z_A + Z_C} \quad \text{--- (IV)}$$



To find the internal resistance, voltage source is removed by a short circuit as shown in fig.

$$Z' = R_{TH} = Z_B + \frac{Z_A Z_C}{Z_A + Z_C}$$



Now using Cramer's rules, we get,

$$I_L = \frac{\begin{vmatrix} (Z_A + Z_C) & E \\ -Z_C & 0 \end{vmatrix}}{\begin{vmatrix} (Z_A + Z_C) & -Z_C \\ -Z_C & (Z_B + Z_C + Z_L) \end{vmatrix}}$$

$$= \frac{E Z_C}{(Z_A + Z_C)(Z_B + Z_C + Z_L) - (Z_C)^2}$$

$$= \frac{E Z_C}{Z_B (Z_A + Z_C) + Z_C (Z_A + Z_C) + Z_L (Z_A + Z_C) - (Z_C)^2}$$

$$= \frac{E Z_C}{Z_B (Z_A + Z_C) + Z_A Z_C + (Z_C)^2 + Z_L (Z_A + Z_C) - (Z_C)^2}$$

$$= \frac{E Z_C}{Z_B (Z_A + Z_C) + Z_A Z_C + Z_L (Z_A + Z_C)}$$

$$= \frac{E Z_C}{(Z_A + Z_C)}$$

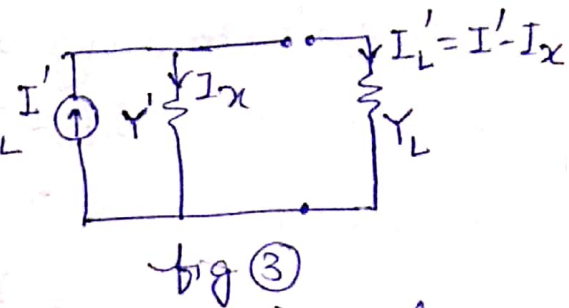
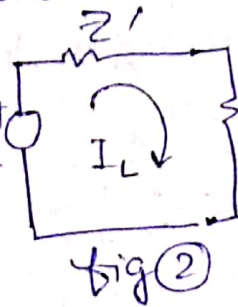
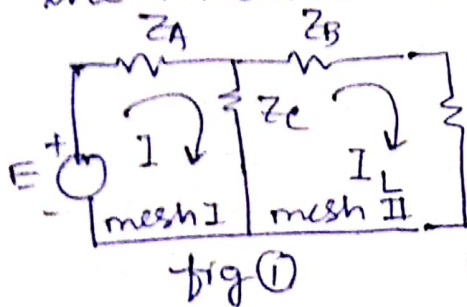
$$= \frac{Z_B + \frac{Z_A Z_C}{(Z_A + Z_C)} + Z_L}{1}$$

$$= \frac{E'}{Z' + Z_L} = I_L'$$

Hence Thevenin's Theorem proved.

Norton's Theorem :-

Any two terminal linear network containing energy source and impedance may be replaced by an equivalent circuit consisting of a current source (I') in parallel with the admittance (Y') where I' is the short circuit current measured between the terminals and (Y') is the admittance measured between the terminal when all energy source have been replaced by the internal admittance.



Let us consider Z_A, Z_B, Z_C as impedance and Z_L is the load impedance, I and I_L be the current flowing through mesh I and II respectively due to energy source E . Now from Khenen's theorem, we get,

$$I_L = \frac{E'}{Z' + Z_L} \quad \text{--- (1)}$$

Since the reciprocal of the impedance is called admittance.

$$Z' = \frac{1}{Y'}, \text{ and } Z_L = \frac{1}{Y_L}$$

$$\begin{aligned} Z' + Z_L &= \frac{1}{Y'} + \frac{1}{Y_L} \\ &= \frac{Y' + Y_L}{Y' Y_L} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} \text{from eq. (1), } I_L &= \frac{E'}{\frac{Y' + Y_L}{Y' Y_L}} = \frac{E' Y' Y_L}{Y' + Y_L} \\ &= \frac{I' Y_L}{Y' + Y_L} \quad \text{--- (3) } [\because I' = E' Y'] \end{aligned}$$

Let I_X be the current flowing through Y' and the current flowing through Y_L is $I_L' = I' - I_X$
 potential drop across $Y' = I_X / Y'$
 " " " $Y_L = (I' - I_X) / Y_L$

potential drop across Y' = potential drop across Y_L

$$\Rightarrow \frac{I_x}{Y'} = \frac{I' - I_x}{Y_L}$$

$$\Rightarrow I_x Y_L = I' Y' - I_x Y'$$

$$\Rightarrow I_x (Y_L + Y') = I' Y'$$

$$\Rightarrow I_x = \frac{I' Y'}{Y_L + Y'} \quad \text{--- (4)}$$

current flowing through Y_L is,

$$I_L' = I' - I_x$$

$$= I' - \frac{I' Y'}{Y_L + Y'}$$

$$= \frac{I' Y_L + I' Y' - I' Y'}{Y_L + Y'}$$

$$= \frac{I' Y_L}{Y_L + Y'} \quad \text{--- (5)}$$

comparing eq. (3) & (5) we get,

$$I_L = I_L'$$

Hence Norton's Theorem proved,