DSP Midsem 2024) :→

1.
(a)

Continuous Time Signal	Discrete-time Signal
→ These signals are defined for all values of time and can take on any value at any point in time.	→ These signals are defined only at discrete intervals of time.
→ Represented mathematically as $x(t)$ where t is continuous.	→ Represented as $x[n]$, where n is an integer.
→ Typically represented as a smooth curve or waveform.	→ Represented as a sequence of samples.
→ Exists in the continuous domain (real time).	→ Exists in the discrete domain (sample points).
→ Requires analog storage and processing.	→ Can be stored and processed digitally using algorithms.

(b) Importance of sampling process in DSP :

Sampling is the process of converting a continuous-time signal into discrete-time signal by taking samples at regular intervals. It is a critical step in DSP for the following reasons:

1. Conversion to digital form:

DSP systems operate on digital signals, which require the original analog signals to be ~~comp~~ sampled and converted into a digital format.

2. Efficient Processing:

Digital signals can be processed using efficient algorithms and hardware, enabling advanced functionalities like filtering, compression, and modulation.

3. Preservation of Information:

The Nyquist-Shannon sampling Theorem states that a ~~continuous~~ continuous signal can be fully reconstructed from its samples if the sampling frequency is at least twice the maximum frequency of the signal. Proper sampling ensures no loss of critical information.

4. Storage and Transmission:

Digital signals, derived from sampling, are easier to store (as binary data) and transmit over digital channels without significant degradation.

5. Flexibility and Scalability:

Sampling allows for manipulation in a digital domain, such as resampling or changing the sampling rate to adapt to various system requirements.

ⓒ Unit STEP Sequence and Unit Impulse (delta) Sequence :→

The unit step sequence is a discrete-time signal that has a value of 0 for negative indices and a value of 1 for zero and positive indices.

* Mathematical Representation:

$$u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

* Example:

$$\begin{cases} \text{For } n = -2, -1, 0, 1, 2 \longrightarrow \\ u[n] = \{0, 0, 1, 1, 1\}. \end{cases}$$

The unit Impulse ~~function~~ sequence is a discrete time signal that has a value of 1 at $n=0$ and 0 for all other indices. It is also known as the Kronecker delta.

* Mathematical Representation:

$$\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$$

* Example:

$$\begin{cases} \text{For } n = -2, -1, 0, 1, 2. \\ \delta[n] = \{0, 0, 1, 0, 0\} \end{cases}$$

ⓓ discrete sequence: $x[n] = \{2, 4, 6, 8\}$ as a summation of scaled and shifted delta functions $\delta[n]$,

A sequence $x[n]$ can be expressed as:

$$\left\{ x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n-k] \right\}$$

This formula means that each element of $x[n]$ scales a delta function $\delta[n-k]$ at the appropriate index k.

Representation for $x[n] = \{2, 4, 6, 8\}$

$$\begin{cases} x[0] = 2 \\ x[1] = 4 \\ x[2] = 6 \\ x[3] = 8 \end{cases}$$

The summation will be:

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 8\delta[n-3]$$

(Answer)

2.
ⓐ Key Properties of an LTI (Linear Time-Invariant) System:

Linear Time-Invariant (LTI) systems have several important properties that make them fundamental in signal processing and system analysis:⟶

1. Linearity
2. Time Invariance
3. Causality
4. Stability
5. Impulse Response Representation (Convolution)

1. Linearity:

An LTI system is linear, meaning the principle of superposition applies. If the input to the system is a weighted sum of signals, the output will also be the same weighted sum of the outputs corresponding to each signal.

* Mathematical Expression:⟶

For an input $x_1[n]$ with output $y_1[n]$, and $x_2[n]$ with output $y_2[n]$;

if $T\{x_1[n]\} = y_1[n]$ and $T\{x_2[n]\} = y_2[n]$, then

$$T\{a_1 x_1[n] + a_2 x_2[n]\} = a_1 y_1[n] + a_2 y_2[n].$$

Where T represents the system, and a_1, a_2 are scalars.

2. Time Invariance :

An LTI system is time-invariant, meaning its behavior does not change over time. Shifting the input by a certain amount shifts the output by the same amount, without altering its form.

⊛ Mathematical Expression :→

For an input $x[n]$ with output $y[n]$;

If $T\{x[n]\} = y[n]$, then $T\{x[n-n_0]\} = y[n-n_0]$,

where n_0 is the time shift.

ⓑ $h_A[n] = \{1, 2\}$.

$h_B[n] = \{2, 1\}$.

ⓘ Systems connected in series :

In series connection, the equivalent impulse response is the convolution of the individual impulse responses of the systems:

$$\left[h_{series}[n] = h_A[n] * h_B[n] \right]$$

convolution operation

→ **Calculation :**

$h_A[n] = \{1, 2\}$

$h_B[n] = \{2, 1\}$

$$h_{series}[n] = \sum_{K=-\infty}^{\infty} h_A[K] \cdot h_B[n-K]$$

$$h_{series}[n] = \{1 \cdot 2, (1 \cdot 1 + 2 \cdot 2), 2 \cdot 1\}$$

$$\{h_A[0] \cdot h_B[0], (h_A[0] \cdot h_B[1] + h_B[0] + h_A[1]), h_A[1] \cdot h_B[1]\}$$

$$= \{2, 5, 2\}$$

(Ans)

ⓘⓘ Systems connected in Parallel :

In parallel connection, the equivalent impulse response is the sum of the individual impulse responses of the systems.

$$h_{parallel}[n] = h_A[n] + h_B[n]$$

→ **Calculation :**

$h_A[n] = \{1, 2\}$

$h_B[n] = \{2, 1\}$

$$h_{parallel}[n] = \{1+2, 2+1\}$$

$$\{h_A[0] + h_B[0], h_A[1] + h_B[1]\}$$

$$= \{3, 3\}.$$

(Answer)

③ ⓐ Convolution in the context of Discrete Time Signal:

Convolution is a mathematical operation used to determine the output of a linear time-invariant (LTI) system when its input and impulse response are known.
For discrete-time signals, the convolution of an input $x[n]$ with the system's impulse response $h[n]$ is defined as: →

$$\boxed{y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]}$$

Where: $y[n]$ = the output sequence.
$x[k]$ = the input sequence.
$h[n-k]$ = the time reversed and shifted version of the impulse response.

⊛ Steps to solve Discrete Convolution (Just a Revision !!!)

1. Flip the impulse Response: Reverse $h[n]$ to get $h[-n]$
2. Shift: $h[-n]$: Move $h[-n]$ to $h[n-k]$, where k represents the time index.
3. Multiply: Multiply $x[k]$ and $h[n-k]$ element wise for all k.
4. Sum: Sum the results of the multiplication to get $y[n]$

Follow these 4 steps when you are solving convolution sum problem [A friendly Advice !!!]

⊛ Importance of Convolution in LTI systems:

1. Output Determination: Convolution calculates the output $y[n]$ of an LTI system by combining the input $x[n]$ and the system's impulse response $h[n]$, capturing the system's behaviour for any input.
2. System Characterization: The impulse response $h[n]$ fully defines the system. Convolution uses $h[n]$ to model how to the system processes signal.

3. <u>Linear Superposition</u> :

Convolution leverages the superposition principle, breaking complex inputs into simple impulses and summing their responses.

4. <u>Signal Processing</u> :

Essential in filtering, smoothing, and detecting signals, convolution is the backbone of operations like noise reduction and feature extraction.

(b) $x[n] = \{1, 2, 1\}$

$h[n] = \{1, 1\}$

$$y[n] = \sum_{k=0}^{Lx-1} x[k] \cdot h[n-k]$$

Range of k = $(-\text{len } h[n] + 1)$ to $(\text{len } x[n] + \text{Len } h[n] - 1)$

$= -1$ to 4

k	-1	0	1	2	3	4
$x[k]$		1	2	1		
$h[-k]$	1	1				
$h[1-k]$		1	1			
$h[2-k]$			1	1		
$h[3-k]$				1	1	
$h[4-k]$					1	1

$y[0] = 1 \times 0 + 1 \times 1 = 1$

$y[1] = 1 \times 1 + 1 \times 2 = 3$

$y[2] = 1 \times 2 + 1 \times 1 = 3$

$y[3] = 1 \times 1 + 1 \times 0 = 1$

$y[n] = \{1, 3, 3, 1\}$ (Ans)