Defuzzyfication

Defuzzification Methods

- Fuzzy rule based systems evaluate linguistic if-then rules using fuzzification, inference and composition procedures. They produce fuzzy results which usually have to be converted into crisp output.
- To transform the fuzzy results in to crisp, defuzzification is performed.
- Defuzzification is the process of converting a fuzzified output into a single crisp value with respect to a fuzzy set. The defuzzified value in FLC (Fuzzy Logic Controller) represents the action to be taken in controlling the process.
- **Different Defuzzification Methods**
  - Center of Sums Method (COS)
  - Center of gravity (COG) / Centroid of Area (COA) Method
  - Center of Area / Bisector of Area Method (BOA)
  - Weighted Average Method
  - Maxima Methods
    - First of Maxima Method (FOM)
    - Last of Maxima Method (LOM)
    - Mean of Maxima Method (MOM)

Center of gravity (COG) / Centroid of Area (COA) Method

This method provides a crisp value based on the center of gravity of the fuzzy set. The total area of the membership function distribution used to represent the combined control action is divided into a number of sub-areas. The area and the center of gravity or centroid of each sub-area is calculated and then the summation of all these sub-areas is taken to find the defuzzified value for a discrete fuzzy set.

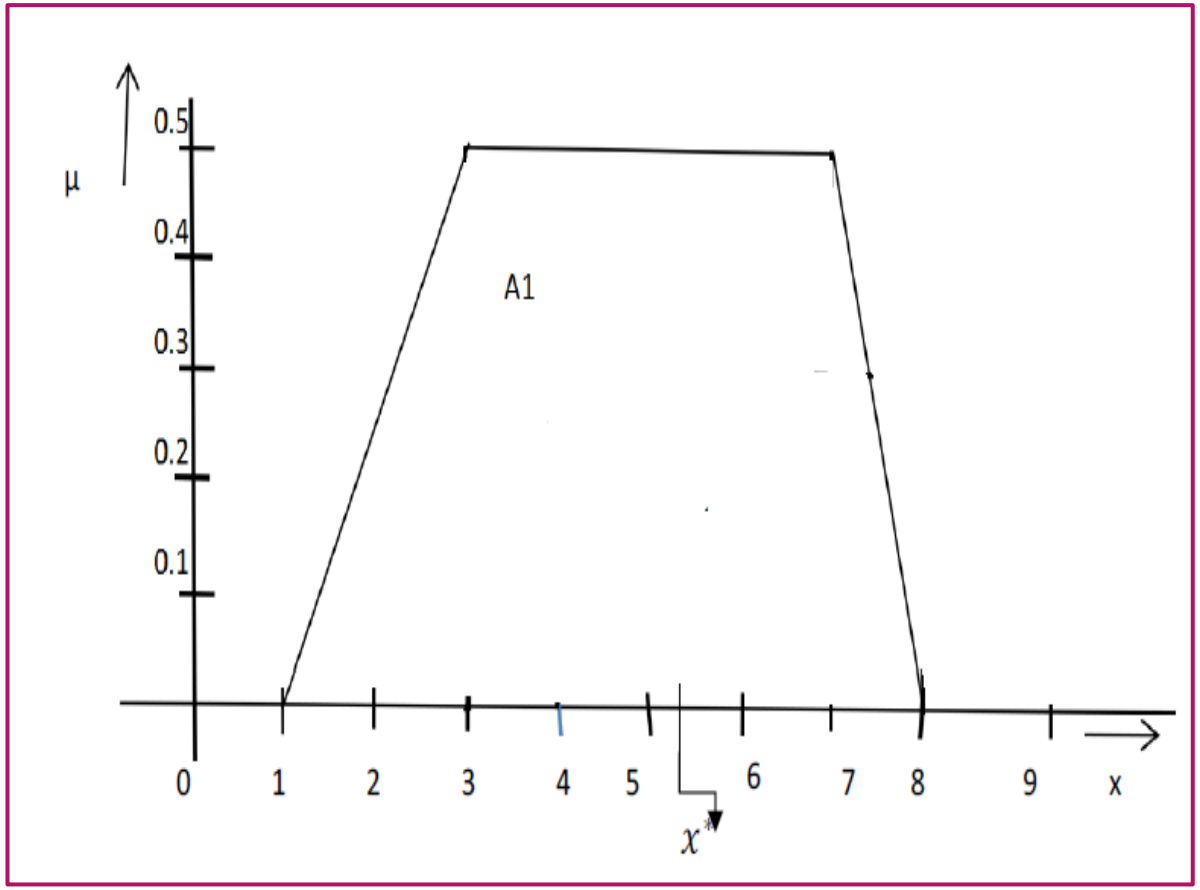
For discrete membership function, the defuzzified value denoted as x^* using COG is defined as:

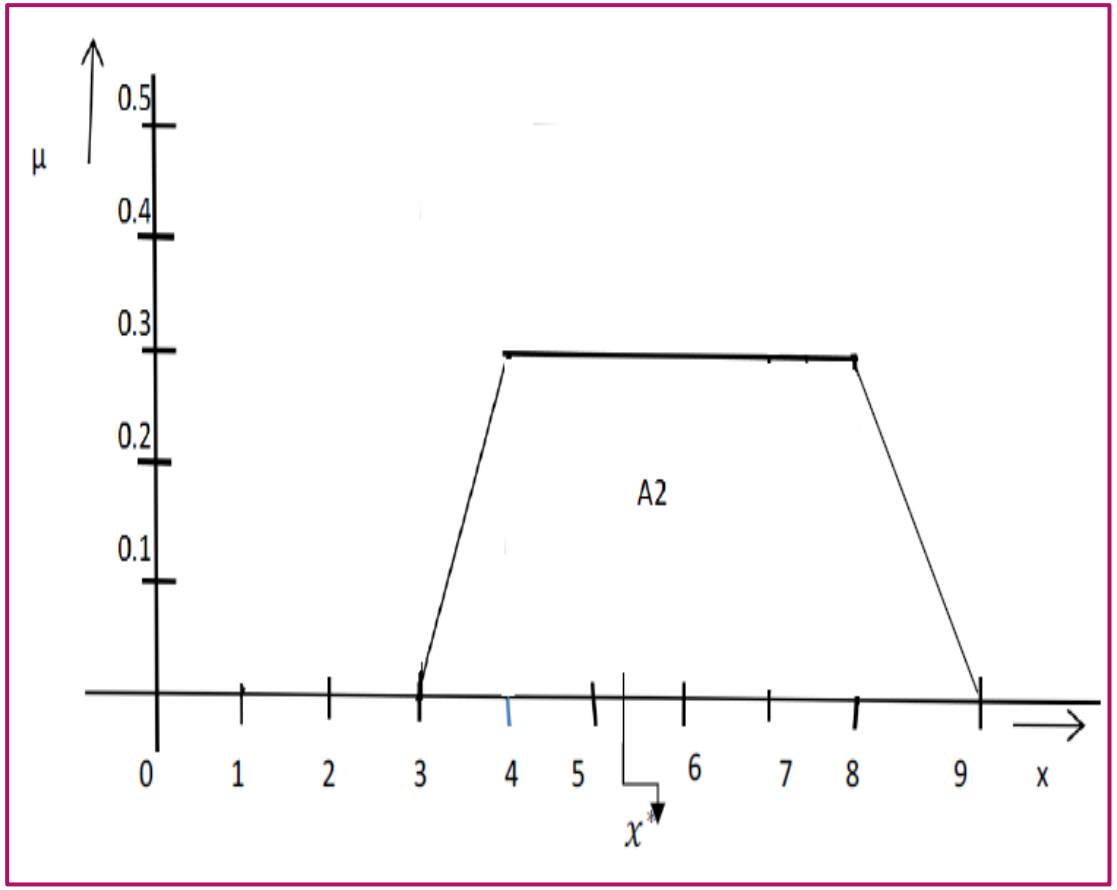
$$x^* = \frac{\sum_{i=1}^{n} x_i \cdot \mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)}$$, Here x_i indicates the sample element, $\mu(x_i)$ is the membership function, and n represents the number of elements in the sample.

For continuous membership function, x^* is defined as :

$$x^* = \frac{\int x\, \mu_A(x)\, dx}{\int \mu_A(x)\, dx}$$

Example: Fuzzy Set C1 and C2





Combined Fuzzy Set

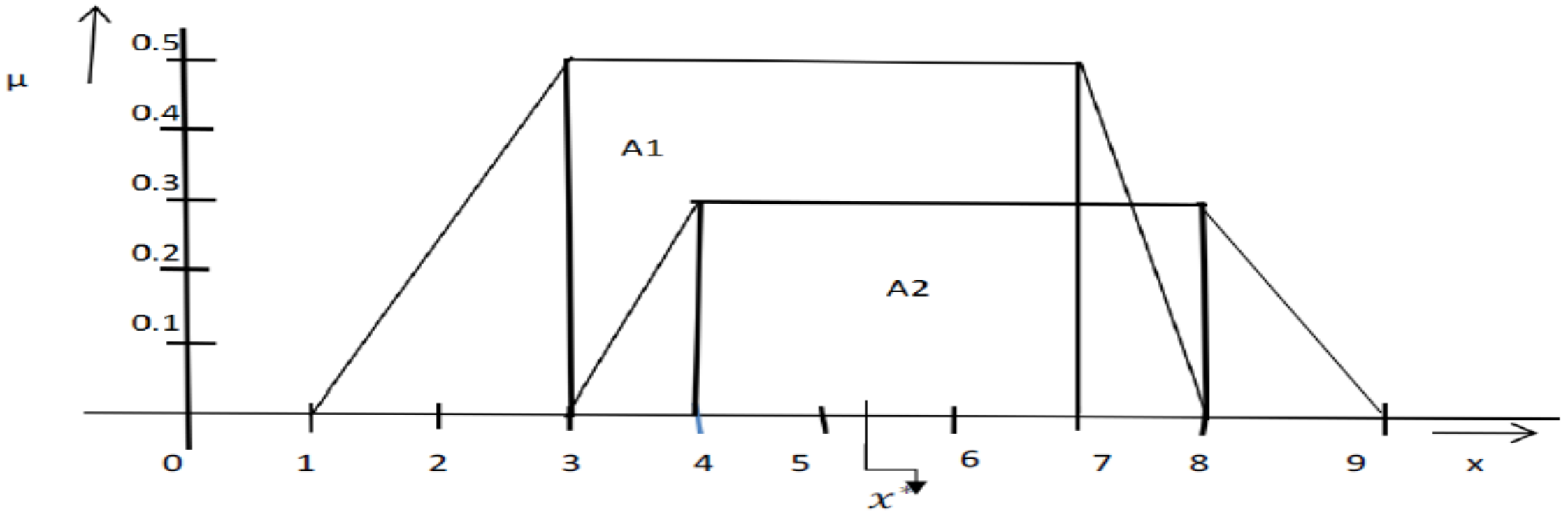


Figure 1 : Fuzzy sets C_1 and C_2

Calculation of Area

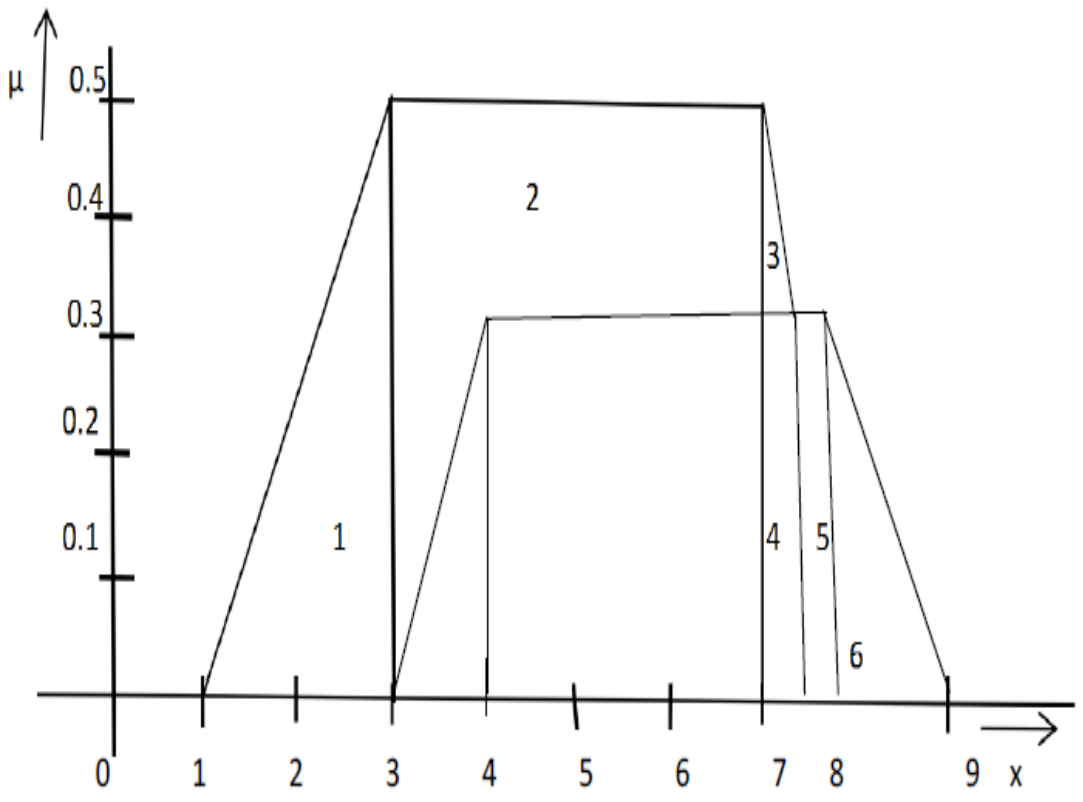


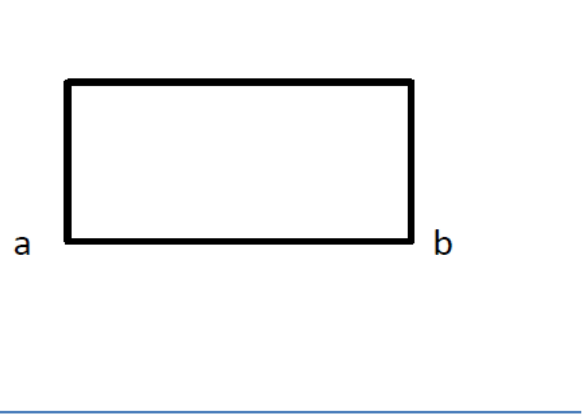
Figure 2 : Fuzzy sets C1 and C2

The defuzzified value x^* using COG is defined as:

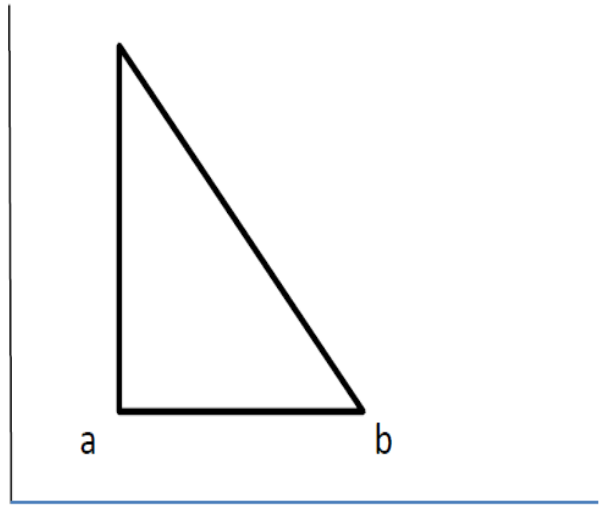
$$x^* = \frac{\sum_{i=1}^{N} A_i \times \bar{x}_i}{\sum_{i=1}^{N} A_i}$$, Here N indicates the number of sub-areas, A_i and $\bar{x}_i$ represents the area and centroid of area, respectively, of i^{th} sub-area.

In the aggregated fuzzy set as shown in figure 2. , the total area is divided into six sub-areas.
For COG method, we have to calculate the area and centroid of area of each sub-area.
These can be calculated as below.
The total area of the sub-area 1 is ½ * 2 * 0.5 = 0.5
The total area of the sub-area 2 is (7-3) * 0.5 = 4 * 0.5 = 2
The total area of the sub-area 3 is ½ * (7.5-7) * 0.2 = 0.5 * 0.5 *0.2 =.05
The total area of the sub-area 4 is 0.5* 0.3 = .15
The total area of the sub-area 5 is 0.5* 0.3 = .15
The total area of the sub-area 6 is ½ *1* 0.3 = .15

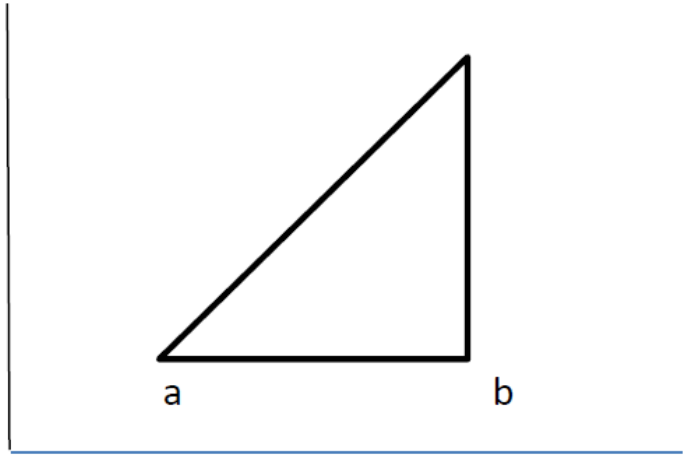
Calculation of Centroids of primary shapes



Centroid = (a+b)/2



Centroid = a + (b-a)/3



Centroid = a + (a+b)/3

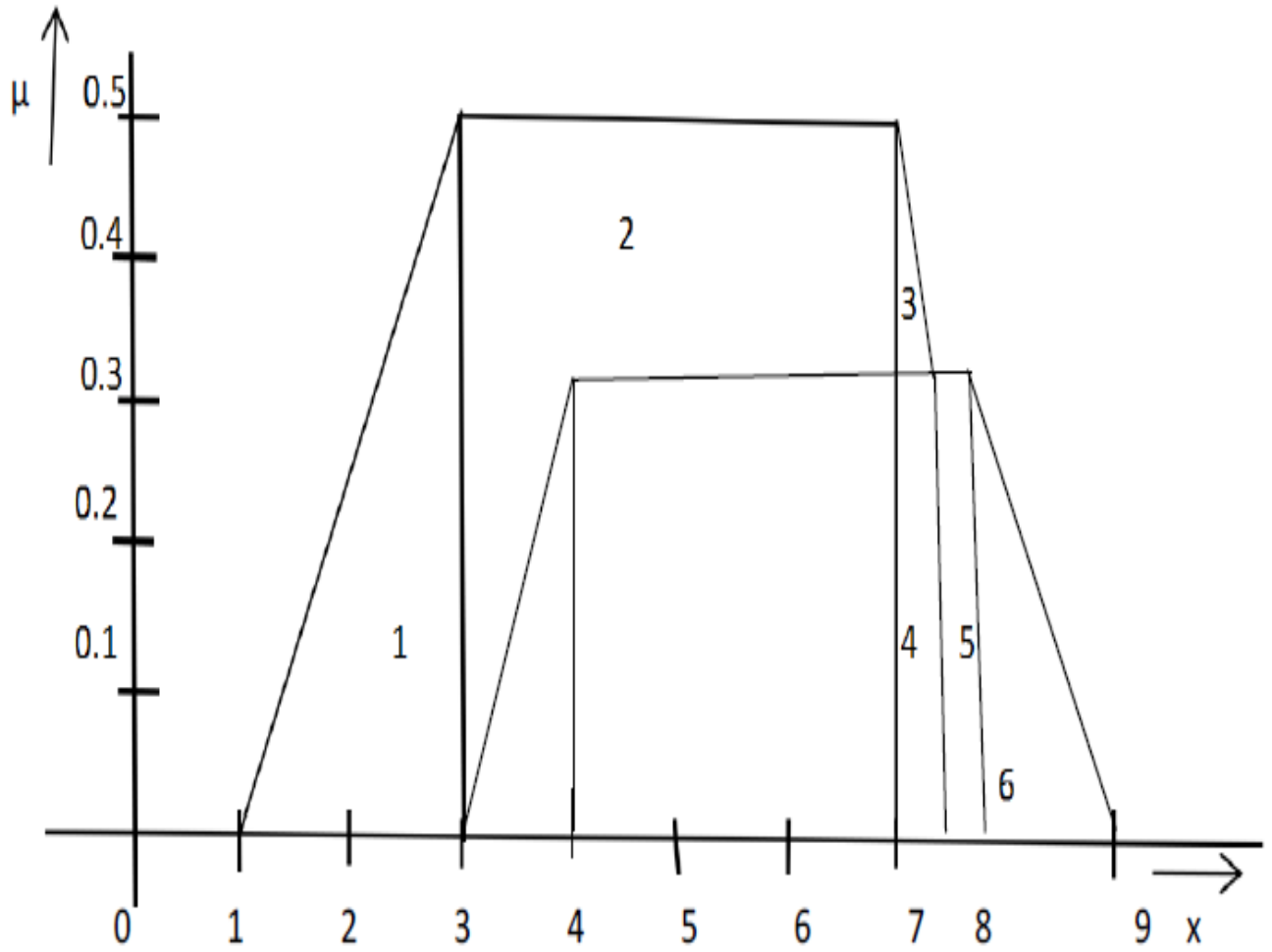


Figure 2 : Fuzzy sets C1 and C2

Calculation of Centroids:

Centroid of 1 = 1+ (1+3)/3 = 2.333
Centroid of 2 = (3+7)/2 = 5.00
Centroid of 3 = 7+(7.5-7)/3 = 7.166
Centroid of 4 = (7.5+7)/2 = 7.25
Centroid of 5 = (8+7.5)/2 = 7.75
Centroid of 6 = 8+(9-8)/3 = 8.333

Table 1

Sub-area number	Area(A_i)	Centroid of area($\overline{x_i}$)	$A_i \cdot \overline{x_i}$
1	0.5	2.333	1.1665
2	02	5	10
3	.05	7.166	0.3583
4	.15	7.25	1.0875
5	.15	7.75	1.1625
6	.15	8.333	1.2499

The defuzzified value x^* will be $\frac{\sum_{i=1}^{N} A_i \times \overline{x_i}}{\sum_{i=1}^{N} A_i}$

$$= \frac{(1.1665+10+0.3583+1.0875+1.1625+1.2499)}{(0.5+2+.05+.15+.15+.15)}$$

= (15.0247)/3 =5.008

$x^* = 5.008$

It will be Continued….