DR. MITHUN BHOWAL

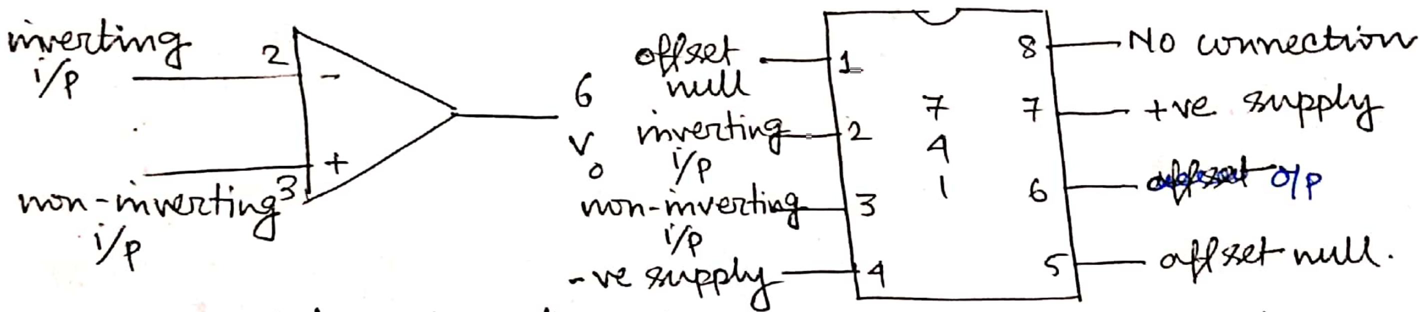
PHN NO: 9874577633

9038473239

Operational Amplifier (opamp)

Definition :- Operational amplifier abbreviated as opamp is a high gain direct couple amplifier whose response characteristics are controlled externally by negative feedback arrangement. The significance of the term 'operational' is that the opamp can perform mathematical operations such as summation, subtraction, integration, differentiation etc. They are also used as audio and vedio amplifiers.

circuit symbol :- The symbol of an opamp ~~is~~ and pin configuration is shown in fig below.



The amplifier has two i/ps ~~are~~ one is marked (+) called the inverting i/p. A signal applied to the +ve i/p will appear at the same phase at the o/p but when a signal is applied to the −ve i/p, it will be shifted by 180° at the o/p.

Characteristics of an ideal opamp :-

An ideal opamp exhibit the following characteristic

(i) Infinite voltage gain.

(ii) Infinite i/p resistance so that almost any signal source can drive it.

(iii) zero output resistance, so that output can drive an infinite number of other device.

(iv) zero output voltage when i/p voltage is zero.

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(v) Infinite bandwidth so that any frequency signal from 0 to ∞ HZ can be amplified without attenuation.

(vi) characteristic not drift with temperature.

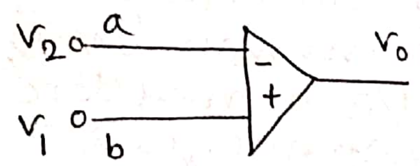
(vii) Infinite common mode rejection ratio (CMRR) so that the output common mode noise voltage is zero.

(viii) Infinite slew rate so that output voltage changes occur simultaneously with i/p voltage changes.

However a practical opamp differ from the ideal opamp.

For practicle opamp, the dc or the low frequency voltage gain is typcially 10^3 to 10^6. The bandwidth is finite, the voltage gain being constant upto several hundred kilohertz and then decreasing with increase in frequency. The i/p impedence is between 150 kΩ and a hundread MΩ. The o/p impedence lies in the range 0.75 to 1,00 Ω. The practical opamp donot have a perfect balance and their characteristics also change somewhat with temperature.

Common-mode rejection ratio:- (CMRR).



Let v_1 and v_2 be the voltage applied to the non inverting and inverting terminal of the opamp. In practicle the difference of the i/p signal i.e $v_d = v_1 - v_2$ —(1) and the average of the i/p signal $v_c = \frac{v_1 + v_2}{2}$ —(2) are amplifier to produce the o/p voltage v_0. Thus voltage v_c is called the common mode signal.

Then, $v_0 = A_1 v_1 + A_2 v_2$ —(3)

where A_1 = voltage gain when terminal a is grounded. A_2 = voltage gain when terminal b is grounded.

Solving eq. (1) and (2), we get

$$V_1 = V_c + \frac{V_d}{2}$$
$$V_2 = V_c - \frac{V_d}{2}$$

putting the value of V_1 and V_2 in eq. (5) we get

$$V_o = A_1\left(V_c + \frac{V_d}{2}\right) + A_2\left(V_c - \frac{V_d}{2}\right)$$
$$= \frac{1}{2}(A_1 - A_2)V_d + (A_1 + A_2)V_c$$
$$= A_d V_d + A_c V_c$$

where $A_d = \frac{1}{2}(A_1 - A_2)$ = voltage gain for difference signal.

and $A_c = A_1 + A_2$

= common mode signal

So, the common mode rejection ration is the ratio between the valtage gain for difference signal and common mode signal. So it is define by.

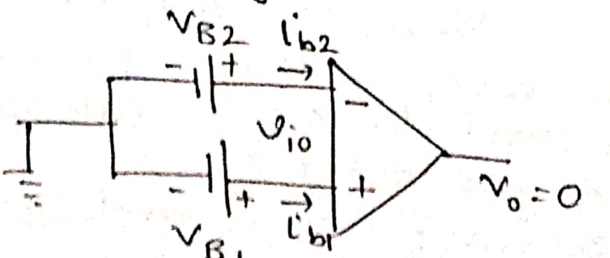
$$CMRR = \left| A_d / A_c \right|$$

It is also called the figure of merit of the opamp. Since A_d needs to be large and A_c very small, the amplifier must be so designed that the CMRR is much large than unity.

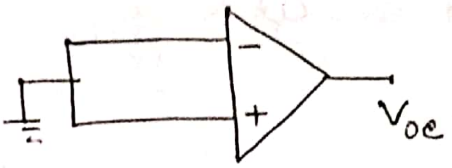
opamp offset voltage and current :—

An ideal OPAMP is perfectly balanced i.e the o/p $V_o = 0$ when two i/ps are equal ($v_1 = v_2$). In a practical opamp an unbalance is caused by a mismatch of i/p transistor. This mismatch causes the flow of unequal bias current through the i/p terminals and results in a non zero o/p voltage without any i/p signal. This ~~voltages (which should be applied between~~ offset voltage can be balanced by applying a small dc valtage between the i/p terminals.

The voltage which applied between the i/p terminals to balance the amplifier is called i/p offset voltage (v_{io}).



The voltage that appears at the output with the two i/p terminals grounded is called the output offset voltage (V_{oo})



The input bias current is defined as the average of two separate currents entering the input terminals of a balance amplifier. Thus,

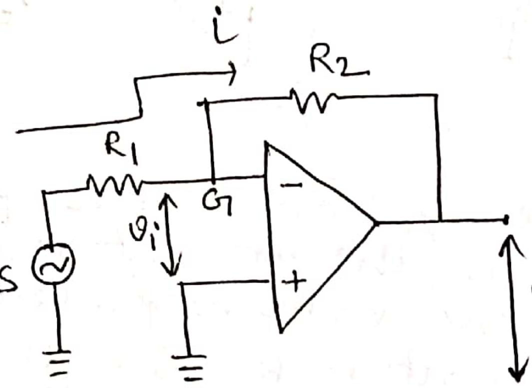
$$i_b = \frac{i_{b1} + i_{b2}}{2} \quad \text{when } v_o = 0$$

The input offset current is the difference between the two bias currents entering into the i/p terminals of balance amplifier.

$$i_{io} = i_{b1} - i_{b2} \quad \text{when } v_o = 0$$

Virtual ground :-

In fig shows the circuit of an inverting opamp with negative feedback. The negative feedback voltage through R_2 tends to cancel the i/p signal at point G.



and tries to keep the point G at ground potential. we know that $A = -\frac{v_o}{v_i}$

Since open loop gain $A \to \infty$, so for any finite v_o, $v_i \to 0$. Since i/p impedence $R_i \to \infty$, so practically no current enters into the opamp. input terminal. Thus there is a 'virtual ground' at point G.

The concept of virtual ground aris from the fact that the i/p voltage v_i at the inverting terminal of the opamp is force to such a small that for all practical purpose, it may be assumed to be zero. Hence it is referred to as virtual ground.

~~Slew Rate~~

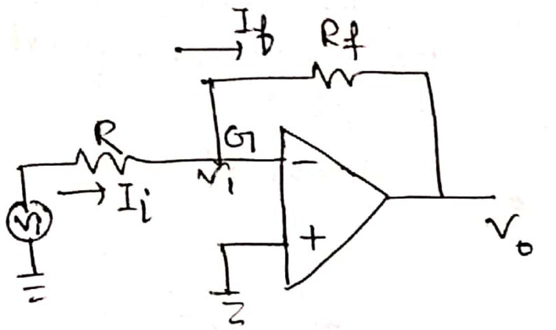
Slew Rate :-

Slew rate is defined as the maximum rate of change of output voltage per unit of time and is expressed in volt per microsecond.

So, $SR = \left.\frac{dv_o}{dt}\right|_{max}$ V/μs.

OPAMP application :-

(i) Inverting amplifier :-



~~To find~~

In the circuit a voltage V_i is applied to the inverting i/p and the o/p is feedback to the inverting i/p through a resistance R_f and let V_o be the o/p voltage.

Since i/p impedence of the opamp is infinitely large, the current through it is very small which can be neglected. This means the current in R is equal to the current in R_f. Let V_1 be the potential at point G, called the virtual ground.

Then current through R is, $\frac{V_i - V_1}{R}$

and current through R_f is, $\frac{V_o - V_1}{R_f}$

Due to virtual,

$$\frac{V_i - V_1}{R} = \frac{V_o - V_1}{R_f}$$

$$\Rightarrow V_i R_f - V_1 R_f = V_o R - V_1 R$$

$$\Rightarrow V_1 (R_f + R) = V_i R_f + V_o R$$

$\Rightarrow$ ~~$V_oR / V_iR_f =$~~

$$\Rightarrow V_o R + V_i R_f = 0 \qquad [\text{as } V_1 = 0]$$

$$\Rightarrow \frac{V_o}{V_i} = -R_f/R$$

$$\text{Gain } A_V = \frac{V_o}{V_i} = -\frac{R_f}{R}$$

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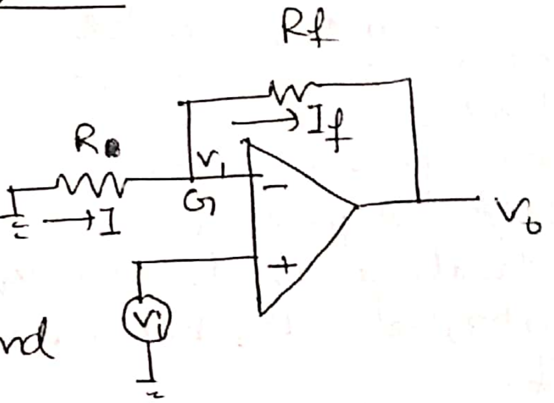
Thus the close loop gain is the ratio of the feedback resistance R_f to the i/p resistance R_1. The negative sign signifies that the o/p voltage is inverted with respect to the input voltage. So o/p voltage is 180° out of phase with the i/p.

(ii) Non-inverting amplifier:-



Here resistance R is applied to the inverting terminal while the i/p voltage V_i is applied to the non-inverting terminal. Let V_1 be the potential at virtual ground G.

Current flowing through R is $\frac{V_i}{R}$ and the current flowing through R_f is, $\frac{V_o - V_i}{R_f}$.

Due to virtual ground,

$$\frac{V_o - V_i}{R_f} = \frac{V_i}{R}$$

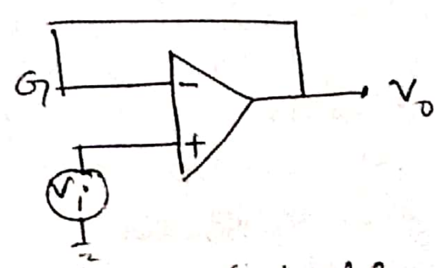
$$\Rightarrow \frac{V_o}{V_i} - 1 = \frac{R_f}{R}$$

$$\Rightarrow \frac{V_o}{V_i} = 1 + \frac{R_f}{R}$$

voltage gain, $A_V = \left(\frac{V_o}{V_i}\right) = 1 + \frac{R_f}{R}$

The voltage gain is greater than unity by a factor R_f/R. As the gain is positive, there is no phase difference between the i/p voltage V_i and the o/p voltage V_o.

(iii) unity gain buffer :-



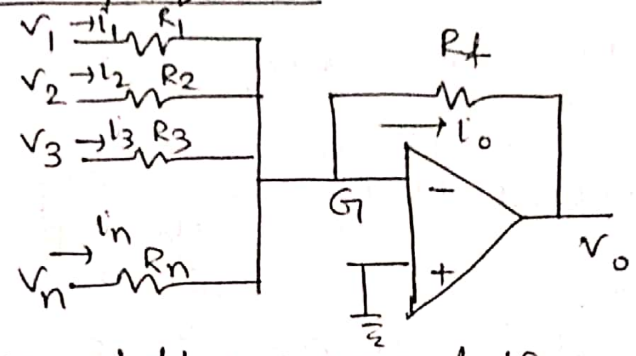
In fig. shows the circuit of unity gain buffer. It has practically infinite input resistance ($R = \infty$) and or zero output resistance ($R_f = 0$). So, the circuit can be used as a unity gain buffer.

$A_V = 1 + R_f/R = 1$. [as, $R = \infty$, $R_f = 0$]

Therefore it can be employed as an impedence matching device between a high impedence source and a low impedence load. It allows the i/p valtage to be transferred to the o/p without any change. ~~anal~~

(iv) Adder or summing amplifier :-

The circuit arrangement shown in fig can be used to obtained an output which is a sum of a number of i/p signal. Due to virtual ground, the sum of the current through $R_1, R_2, R_3 \ldots R_n$ becomes equal to the current in R_f.



So, $i_1 + i_2 + i_3 + \cdots + i_n = i_0$

$$\Rightarrow \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \cdots + \frac{v_n}{R_n} = -\frac{v_0}{R_f}$$

$$\Rightarrow v_0 = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 + \cdots + \frac{R_f}{R_n} v_n\right)$$

If ~~as~~ $R_1 = R_2 = R_3 = \cdots = R_n = R$, then

$$v_0 = -\frac{R_f}{R}(v_1 + v_2 + v_3 + \cdots + v_n)$$

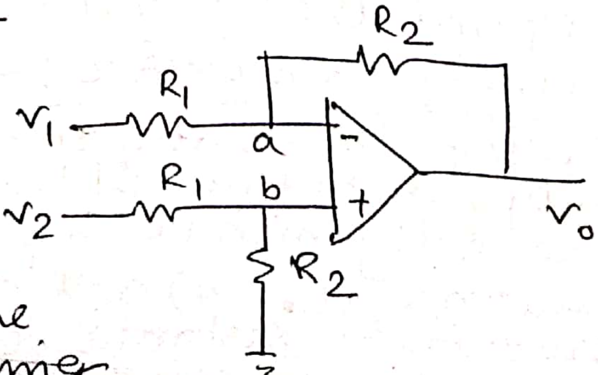
If $R_f = R$, then,

$$v_0 = -(v_1 + v_2 + v_3 + \cdots + v_n)$$

So, the o/p valtage is numerically equal to the algebrical sum of input valtage.

(v) Differential amplifier :-

In fig shows a differential amplifier to get an o/p valtage which is the difference of two i/p valtage.



Let v_1 ~~is the~~ and v_2 be the valtage applied at inverting and non-inverting terminal of opamp respectively

Since the open loop gain and input impedence of the opamp are infinite there exists a virtual short circuit at point on input terminal. So, the points a and b will have the same potential (consider v). So, applying KCL we get,

$$\frac{v_1 - v}{R_1} = \frac{v - v_o}{R_2} \quad \text{—— (1)}$$

and

$$\frac{v_2 - v}{R_1} = \frac{v}{R_2} \quad \text{—— (2)}$$

Subtracting eq. (1) from (2) we get,

$$\frac{v_2 - v - v_1 + v}{R_1} = \frac{v - v + v_o}{R_2}$$

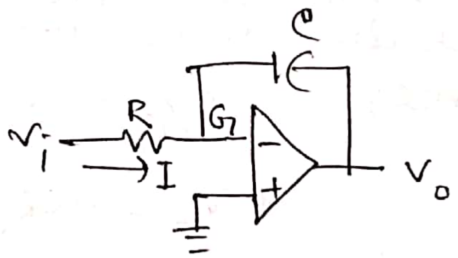
$$\Rightarrow \frac{v_2 - v_1}{R_1} = \frac{v_o}{R_2}$$

$$\Rightarrow v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

Thus the circuit amplifies the difference of two input signal.

(vi) Integrator:-

If the feedback resistance of a basic inverting amplifier is replaced by a capacitor c as shown in fig the circuit becomes an integrator that is it perform mathematical operation of integrator.



Now i/p valtage is given by $v_i = IR$

$$\Rightarrow I = v_i/R \quad \text{—— (1)}$$

and o/p valtage is,

$$v_o = -q/c$$

$$= -\frac{1}{c}\int_0^t I\,dt$$

$$= -\frac{1}{c}\int_0^t \frac{v_i}{R}\,dt \quad \text{[putting value from eq. (1)]}$$

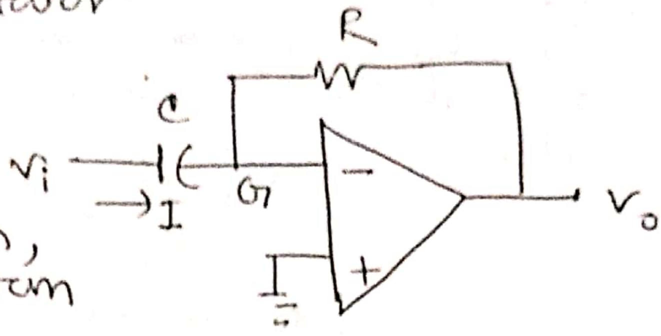
$$= -\frac{1}{CR}\int_0^t v_i\,dt$$

The o/p valtage is proportional to the $-1/RC$ times the integral of i/p signal and RC is the time constant of the integrator. Hence opamp used as an integrator.

(7)

(vii) Differentiator :-

In fig shows the differentiator or differentiation amplifier. As its name implies, the circuit perform mathematical operation of differentiation, i.e ~~is that~~ the o/p waveform is the derivative of the i/p waveform. The differentiator may be constructed from a basic inverting amplifier if an i/p resistance R is replace by a capacitor C.



Now the current flowing through the capacitor C is given by,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CV_i)$$

$$= C\frac{dV_i}{dt}$$

Current flowing through resistance R is given by, $I = -V_o/R$

Due to infinite input impedence of the opamp, these two current are equal.

Hence,
$$-V_o/R = C\frac{dV_i}{dt}$$

$$\Rightarrow V_o = -CR\frac{dV_i}{dt}$$

Thus the o/p voltage is proportional to the time derivative of the i/p voltage.

How does close loop operation change the performance of an OPAMP?

→ OPAMP are used with close loop operation ~~of~~ or feedback. This feedback basically negative, which reduces the gain of the operational amplifier but increases the stability, ~~d~~elemenate the noise. The feedback signal always effect the original input signal.

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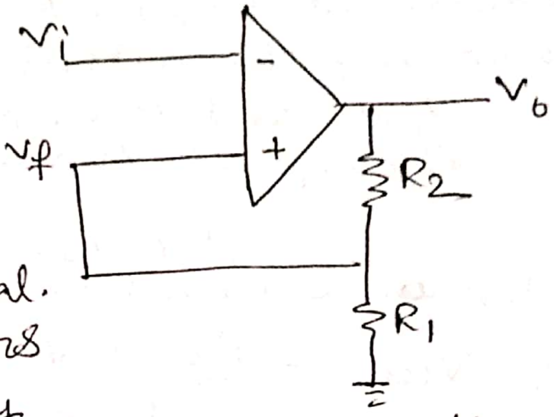
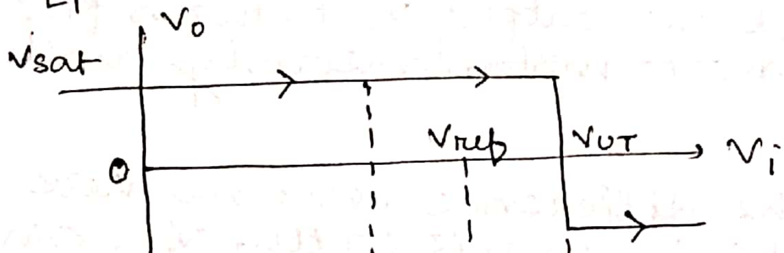
Schmitt trigger :-

With a positive feedback an opamp can be constructed to switch from one voltage level to another, showing the phenomenon of hysterisis such a circuit is called a regenerative comparator or more commonly a schmitt trigger.



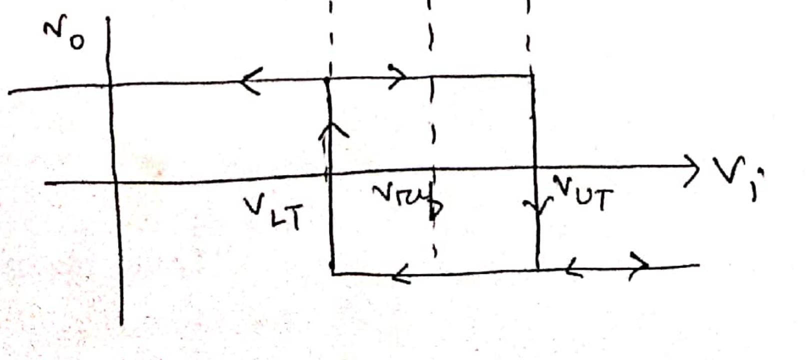
In fig shows a schmitt trigger. The i/p voltage is applied to the inverting terminal and feedback voltage to the non-inverting terminal. The i/p voltage V_i triggers the o/p voltage V_o every time ~~certain voltage~~ it exceeds certain voltage levels. These voltage levels are called upper threshold voltage (V_{UT}) and lower threshold voltage (V_{LT}). The hysterisis width is the difference between two threshold voltage i.e

$V_{UT} - V_{LT}$



fig(a)

fig(b)



fig(b)

Suppose the o/p voltage $V_o = +V_{sat}$ the voltage at +ve i/p terminal will be,

$$V_{ref} + \frac{R_2}{R_1+R_2}(V_{sat} - V_{ref}) = V_{UT}$$

This voltage is called upper threshold voltage V_{UT}. As long as, V_i is less than V_{UT} the output V_o remain at $+V_{sat}$. When V_i is just greater than V_{UT} the output regeneratively switches to $-V_{sat}$ and remain at this level as long as $V_i > V_{UT}$ as shown in fig (b)

For $V_o = -V_{sat}$, the voltage at the +ve i/p terminal is,

$$V_{ref} - \frac{R_2(V_{sat} + V_{ref})}{R_1+R_2} = V_{LT}$$

The voltage is referred to as lower threshold voltage V_{LT}. The i/p voltage V_i must become lesser than V_{LT} in order to cause V_o to switch from $-V_{sat}$ to $+V_{sat}$.

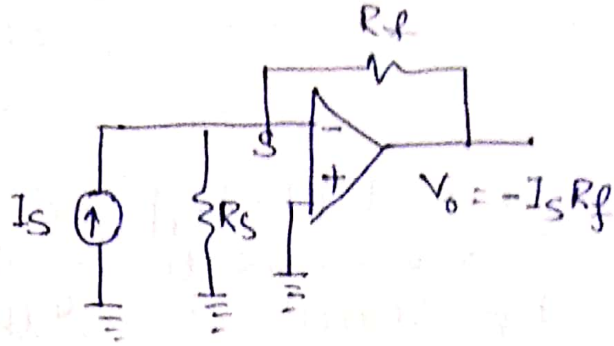
A regenerative transition takes place as shown in fig (b) and the output V_o returns from $-V_{sat}$ to $+V_{sat}$ almost instantaneously as shown in fig (c)

The difference between these two voltage is the hysteresis width V_H, can be written as,

$$V_H = V_{UT} - V_{LT} = \frac{2R_2 V_{sat}}{R_1+R_2}$$

Current to voltage converter :-

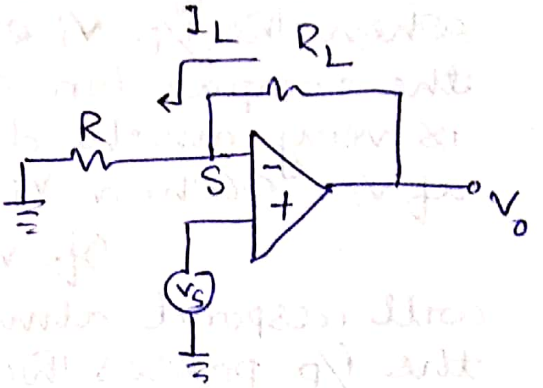
A device that generates a voltage proportional to a signal current applied to its input is called a current to voltage converter. A current to voltage converter is shown in fig.



Due to existance of the virtual ground at the point S. The current through RS is zero. Hence the entire current I_s will flow through the resistance Rf and will develop the o/p voltage given by $V_o = -I_s R_f$, i.e the o/p voltage is proportional to i/p current.

voltage to current converter :-

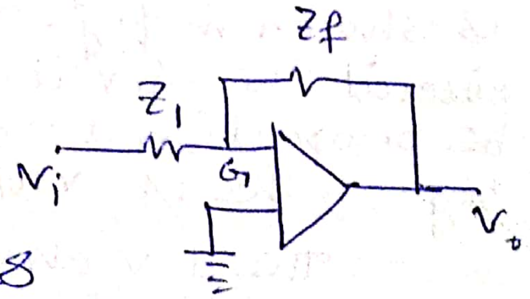
It is often required to convert a voltage signal to a proportional o/p current. This is required for example, when we drive a deflection coil in a television tube. A voltage to current converter is shown in fig.



For an i/p voltage Vs, the current in R_L is given by $I_L = \frac{V_s}{R}$, I_L is independent of R_L because of the virtual ground of the ~~an~~ opamp and proportional to i/p voltage. So the above arrangement can be used an voltage to current converter.

Phase shifter :-

If the feedback and i/p resistance of an inverting amplifier ~~can~~ be replaced by the impedence Z_1 and Z_f which have equal magnitudes but different phase angle.



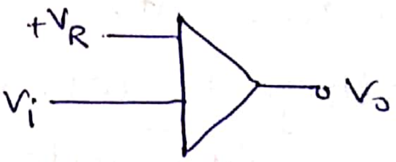
Hence, $\frac{v_o}{v_i} = -\frac{Z_f}{Z_1} = -\frac{|Z_f| \exp(j\theta_f)}{|Z_1| \exp(j\theta_1)}$

$= \exp[j(\pi + \theta_f - \theta_1)]$ — (1)

Since $|Z_f| = |Z_1|$ and $\exp(j\pi) = -1$. The angles θ_f and θ_1 are respectively the phase angle of Z_f and Z_1. Equation (1) shows that v_o leads v_1 by $(\pi + \theta_f - \theta_1)$ but $|v_o| = |v_1|$. Obviously, the current shifts the phase of a sinusoidal i/p voltage leaving its magnitude unaltered. The phase shift can be anything between 0° and 360°.

Comparator :-

Comparator is a circuit which compare i/p signal v_i with a reference voltage V_R when the i/p V_i exceeds V_R,

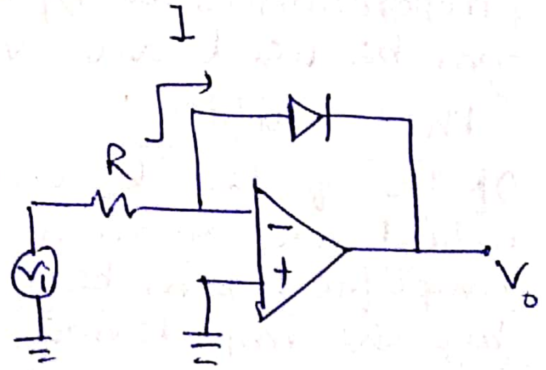


the comparator o/p V_o taken on a value which is very much difference from the magnitude of V_o. when V_i is smaller than V_R.

If V_R is set equal to zero, the o/p will respond almost discontinuously every time the i/p passes through zero. Such an arrangement is called zero crossing detector. If the i/p to an OPAMP comparator is a sine wave, the o/p is a ~~seqe~~ square wave. If zero crossing detector is used, a symmetrical square wave results.

OPAMP as a log amplifier :-

The arrangement to use OPAMP as a log amplifier is shown in fig. In this circuit the o/p voltage will be proportional to the log of the i/p voltage.



The I-V characteristics of the p-n junction diode is given by

$$I = I_s \left[\exp\left(\frac{eV}{\eta k_B T}\right) - 1 \right]$$

Here I is the diode current for the forward voltage V. Since $I \gg I_s$

$$I \simeq I_s \exp\left(\frac{eV}{\eta K_B T}\right)$$

$$\Rightarrow I/I_s = \exp\left(\frac{eV}{\eta K_B T}\right)$$

$$\Rightarrow \log\left(I/I_s\right) = eV/\eta K_B T$$

$$\Rightarrow V = \frac{\eta K_B T}{e} \log\left(I/I_s\right)$$

current flowing through R, $I = -V_i/R$,

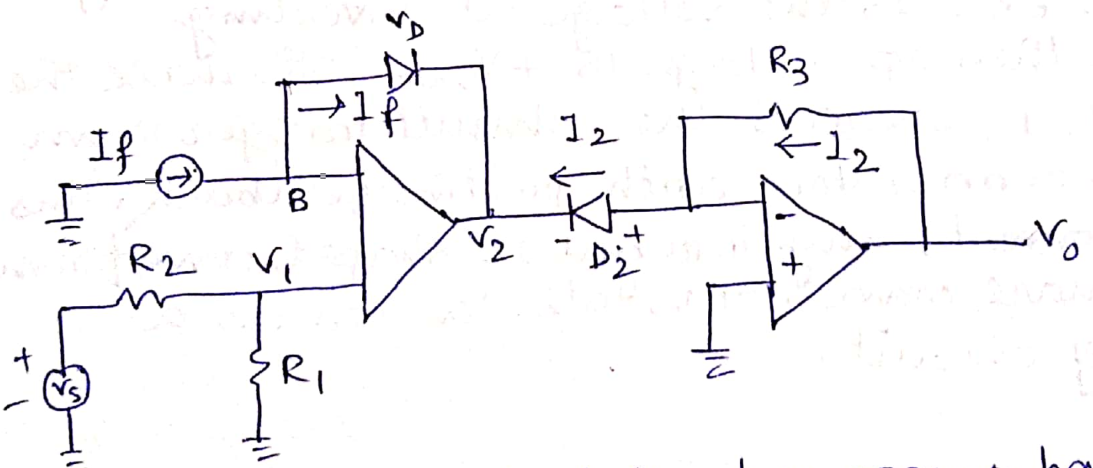
$$\therefore V = \frac{\eta K_B T}{e} \log\left(-\frac{V_i/R}{I_S}\right)$$

o/p voltage $V_o = V = -\frac{\eta K_B T}{e} \log\left(\frac{V_i}{I_S R}\right)$

Hence the o/p voltage is proportional to the log of the i/p voltage. Hence opamp can be used as a log amplifier.

Antilog amplifier :-

The arrangement to use opamp as a antilog amplifier is show in fig.



Here we assume that the two opamp have infinite input resistance and they behave as pure differential amplifier i.e.

$$V_2 = V_1 - V_D \qquad [V_1 = \text{voltage drop at node } B]$$

$$= \frac{R_1}{R_1+R_2} V_s - \frac{\eta K_B T}{e} \log\left(I_f/I_s\right) \quad \text{——} ①$$

Again $V_2 = -V_{D2} = -\frac{\eta K_B T}{e} \log \frac{I_2}{I_s}$ —— (2)

Also $V_o = I_2 R_3$ —— (3)

Equating (1) & (2)

$$\frac{R_1 V_S}{R_1+R_2} - \frac{\eta K_B T}{e}\left(\frac{I_f}{I_S}\right) = -\frac{\eta K_B T}{e} \log\left(\frac{I_2}{I_S}\right)$$

$$\Rightarrow \frac{R_1 V_S}{R_1+R_2} = -\frac{\eta K_B T}{e} \log_e\left(\frac{I_2}{I_f}\right)$$

$$\Rightarrow I_2 = I_f \text{ antilog}\left[\frac{R_1 V_S}{R_1+R_2}\left(-\frac{e}{\eta K_B T}\right)\right]$$

from eq. (3)

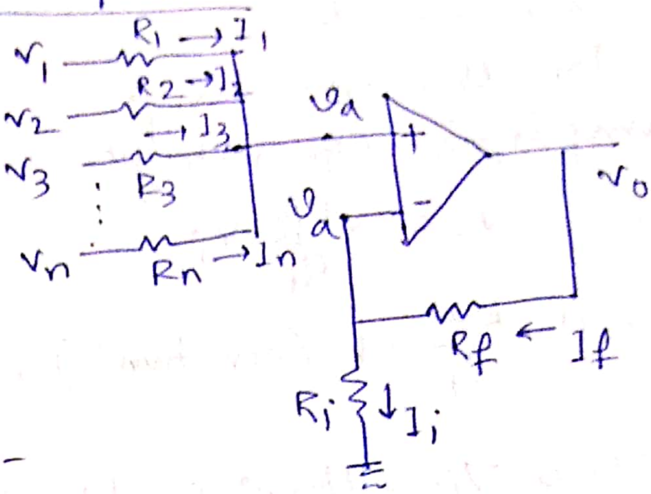
$$V_o = R_3 I_f \text{ antilog}\left[\frac{R_1}{R_1+R_2} \times \frac{(-V_S)e}{\eta K_B T}\right]$$

Hence o/p is proportional to antilog of the i/p.

What is the basic difference between a basic comparator and Schmitt trigger?

→ The basic comparator is a circuit with no feedback and it generates o/p based on the comparison of two i/ps. If voltage at the non-inverting terminal exceeds the voltage at inverting terminal then o/p voltage is $+V_{sat}$. Otherwise the o/p is $-V_{sat}$, whereas the schmitt trigger is an inverting comparator with positive feedback. This circuit converts any irregular shaped waveform to a square wave. It is also known as a squaring circuit.

Non-inverting summing amplifier



The circuit arrangement shown in fig can be used to obtained an o/p which is proportional to sum of number of i/p. For non-inverting summing amp. all i/ps ($v_1, v_2, v_3 \ldots v_n$) are connected to ~~inverse~~ non-inverting terminal through i/p resistance ($R_1, R_2, R_3 \ldots R_n$). Feedback resistance connected across inverting terminal. The voltage at inverting i/p and non-inverting i/p terminal is same V_a.

Due to virtual ground, current flowing through R_f = current flowing through R_i

$$\Rightarrow \frac{V_o - V_a}{R_f} = \frac{V_a - 0}{R_i}$$

$$\Rightarrow V_o = V_a\left(1 + \frac{R_f}{R_i}\right) \quad \text{—— (1)}$$

As, $I_1 + I_2 + I_3 + \cdots + I_n = 0$

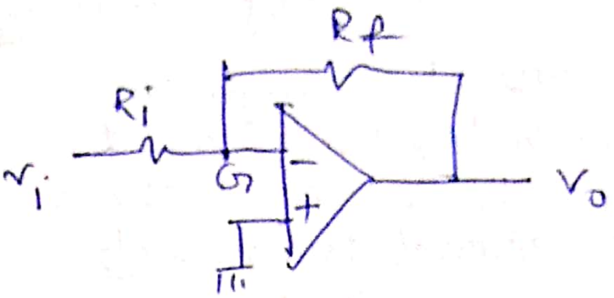
$$\Rightarrow \frac{V_1 - V_a}{R_1} + \frac{V_2 - V_a}{R_2} + \frac{V_3 - V_a}{R_3} + \cdots + \frac{V_n - V_a}{R_n} = 0$$

$$\Rightarrow V_a = \frac{v_1/R_1 + v_2/R_2 + \frac{v_3}{R_3} + \cdots + \frac{v_n}{R_n}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right)} \quad \text{—— (2)}$$

putting value of V_a in eq. (1)

$$v_o = \left(1 + \frac{R_f}{R_i}\right) \frac{v_1/R_1 + v_2/R_2 + v_3/R_3 + \cdots + v_n/R_n}{\left(1/R_1 + 1/R_2 + 1/R_3 + \cdots + 1/R_n\right)}$$

Scale changer :-



For the inverting amplifier o/p voltage

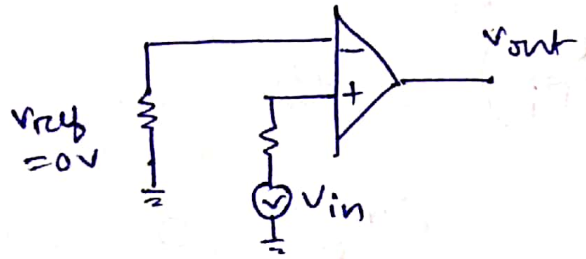
$$V_o = -\frac{R_f}{R_i} V_i$$

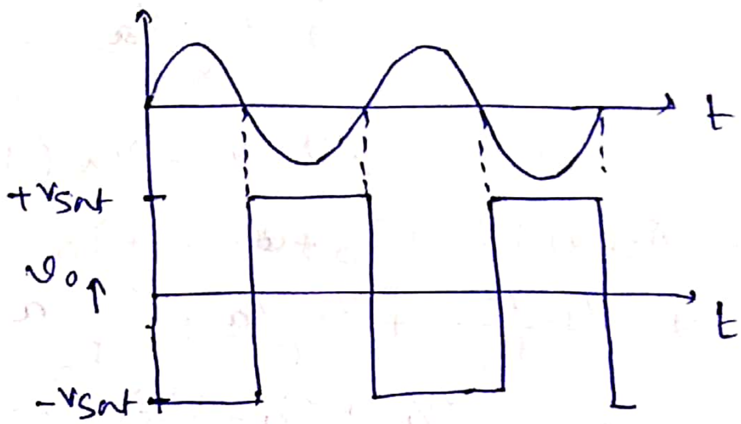
let $\frac{R_f}{R_i} = K$ (constant), the o/p can be written as

$$v_o = -K v_i$$

Thus o/p voltage scale is obtained by multiplying the i/p voltage scale by $-K$, called scale factor. Using precision resistors, accurate values of K can be achived. The inverting amplifier can serve as a scale changer. A low voltage can be accurately measured by amplifying the voltage by the scale changer.

zero crossing detector :-





The comparator can be used to generate a symmetric square wave from sine waves by just taking $V_{ref} = 0$ and choosing v_{in} as a sinusoidal voltage.

When v_{in} passes through zero towards positive values the output v_o is driven into negative saturation value $-V_{sat}$. On the other hand, when v_{in} passes through zero towards negative values the output v_o is driven into positive saturation $+V_{sat}$. Thus if $V_{ref} = 0$, the output changes almost discontinuously from one state to the other every time the input signal passes through zero. For this such a configuration is called zero crossing detector.

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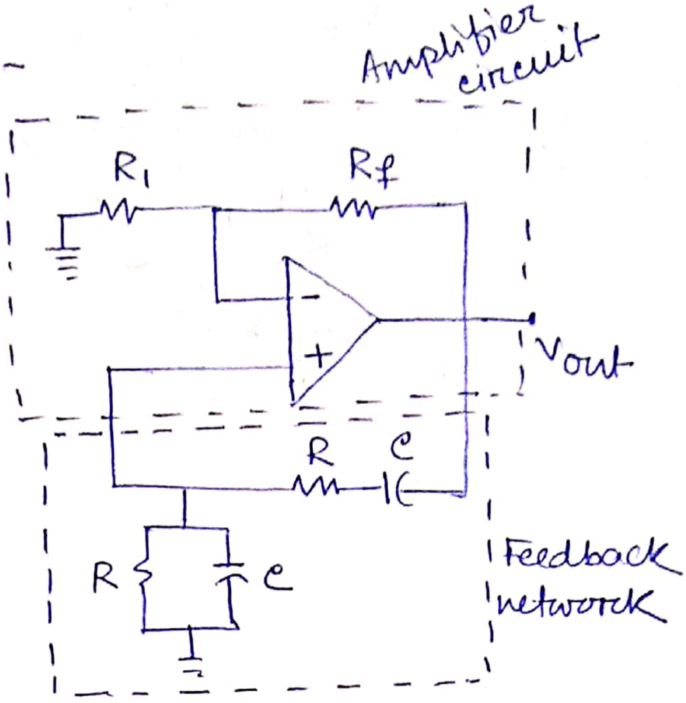
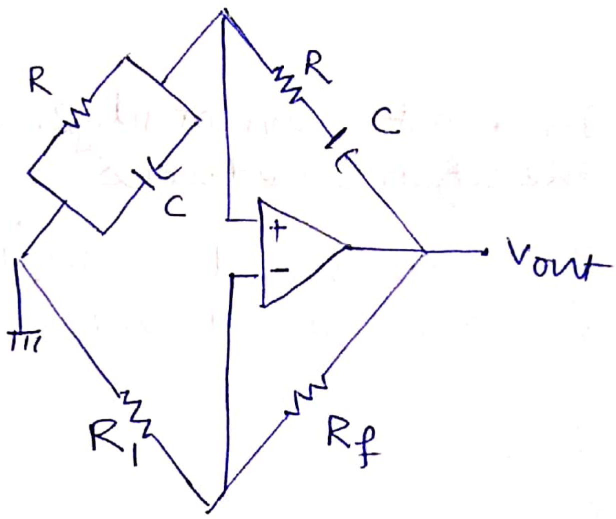
Wien Bridge Oscillator :-



The wien bridge is a lead-lag network of R-C, is introduced in between the amplifier input and output. The bridge has a R-C network in series connection and a parallel RC network in adjacent arm. The phase shift across the network lags with increasing frequency and leads with decreasing frequency. By introducing Wien bridge feedback network the oscillator becomes sensitive to a signal of only one particular frequency. This particular frequency is that at which Wien Bridge is balanced and for which the phase shift is 0°.



The impedence, $Z_3 = R \parallel \frac{1}{j\omega C}$

$$Y_3 = \frac{1}{R_3} + j\omega C$$

$Z_2 = R_f$

$Z_4 = R_1$

$$Z_1 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

using bridge balance equation,

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_1 Z_4 = Z_2 / Y_3$$

$$Z_2 = Z_1 Z_4 Y_3$$

$$R_f = R_1 \left(R - \frac{j}{\omega C}\right)\left(\frac{1}{R} + j\omega C\right)$$

$$R_f = \frac{RR_1}{R} + j\omega RR_1 C - \frac{jR_1}{\omega CR} + \frac{CR_1}{C}$$

$$R_f = \left(\frac{\cancel{R}R_1}{\cancel{R}} + \frac{\cancel{C}R_1}{\cancel{C}}\right) - j\left(\frac{R_1}{\omega C R_3} - \omega C_3 R R_1\right)$$

Equating imaginary and real part we have

$R_f = 2R_1$ and $\frac{R_1}{\omega C R} - \omega C R R_1 = 0$

$\Rightarrow \frac{R_f}{R_1} = 2$

$$\Rightarrow \frac{\cancel{R_1}}{\omega C R} = \omega C R \cancel{R_1}$$

$$\Rightarrow \omega^2 = \frac{1}{C C R R}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{C^2 R^2}}$$

$$\Rightarrow 2\pi f = \frac{1}{CR}$$

$$\Rightarrow f = \frac{1}{2\pi RC}$$

$\frac{R_f}{R_1} = 2$ and $f = \frac{1}{2\pi RC}$ for sustained oscillations this two conditions must be simultaneously satisfied. Thus