DSP End Sem Paper 2023

1. (a) $y(t) = x^2(t)$ → not a linear system because

It does not satisfy the principle of superposition because squaring the input makes it non-linear.

Example: $\left[x_1(t) + x_2(t) \neq x_1^2(t) + x_2^2(t)\right]$

(b) Unit Ramp function :→

The Unit Ramp function, denoted as $r(t)$, is defined as:

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

It increases linearly with time for $t \geq 0$.

(c) → Previously done. (Midsem 2024 → 1.(c))

(d) → " " (" " → 2.(a))

(e) → " " (" 2023 → 1.(b))

(f) Z transform of $\{-2, -1, 1, 2, 3, 4, 5\}$

The Z-transform of a discrete sequence $x[n]$ is given by:

$$X(z) = \sum_{n=0}^{N-1} x[n] z^{-n}$$

$$x[n] = \{-2, -1, 1, 2, 3, 4, 5\}$$

$$X(z) = -2z^0 - z^{-1} + z^{-2} + 2z^{-3} + 3z^{-4} + 4z^{-5} + 5z^{-6}$$

$$X(z) = -2 - \frac{1}{z} + \frac{1}{z^2} + \frac{2}{z^3} + \frac{3}{z^4} + \frac{4}{z^5} + \frac{5}{z^6}$$ [Ans]

2. (a) Briefly discuss about the classification of signals:

1. Continuous Time and Discrete-Time Signals:

a) Continuous Time Signal: Defined for every value of time t. Example: [$x(t) = \sin(t)$] ✓

b) Discrete-time Signal: Defined only at specific time instances n.

Example: [$x[n] = \{1, 2, 3, 4\}$]. ✓

2. Analog and digital Signals:

a) Analog signal: Amplitude can take any value in a continuous range. Example: Temperature Readings.

b) Digital Signal: Amplitude is quantized to specific discrete levels.

Example: Binary data (0s and 1s).

3. Deterministic and Random Signals:

a) Deterministic Signal: Completely predictable at any time.

Example: [$x(t) = \cos(t)$] ✓

b) Random Signal: Unpredictable and described statistically.

Example: Noise signals.

4. Periodic and Aperiodic Signals:

a) Periodic Signal: Repeat itself after a fixed interval 'T'.

Example: [$x(t) = \sin(2\pi t)$] ✓

b) Aperiodic Signal: Does not repeat over time.

Example: Exponential decay. [$x(t) = e^{-t}$] ✓

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5. Causal and Non-causal signals:

a) Causal Signal: Defined only for $t \geq 0$ or $n \geq 0$

Example: $[u(t)$ (Unit step).$]$

b) Non-causal signal: Exists for all time, including $t < 0$.

Example: $\left[x(t) = e^{|t|}\right]$

6. Energy and Power signal:

→ We will come to this question in the part → ⓒ

ⓑ Different operations on sequence:

1. Shifting:

a) Right shift: Delays the sequence by k units.

$$[y[n] = x[n-k]]$$

b) Left shift: Advances the sequence by k units.

$$[y[n] = x[n+k]]$$

2. Scaling:

a) Amplitude Scaling: Multiplying each element of the sequence by a constant a:

$$[y[n] = a \cdot x[n]]$$

b) Time Scaling: Changing the time index by a factor a. Applicable for discrete signals only if a is an integer:

$$[y[n] = x[a \cdot n]]$$

3. Reversal (Time Folding): →

Flips the sequence about $n=0$;

$$[y[n] = x[-n]]$$

4. **Addition:**

Adding two sequences point by point:

$$\boxed{y[n] = x_1[n] + x_2[n]}$$

5. **Multiplication:**

Pointwise multiplication of two sequence:

$$\boxed{y[n] = x_1[n] \cdot x_2[n]}$$

6. **Convolution:**

Combines two sequences to produce a third sequence:

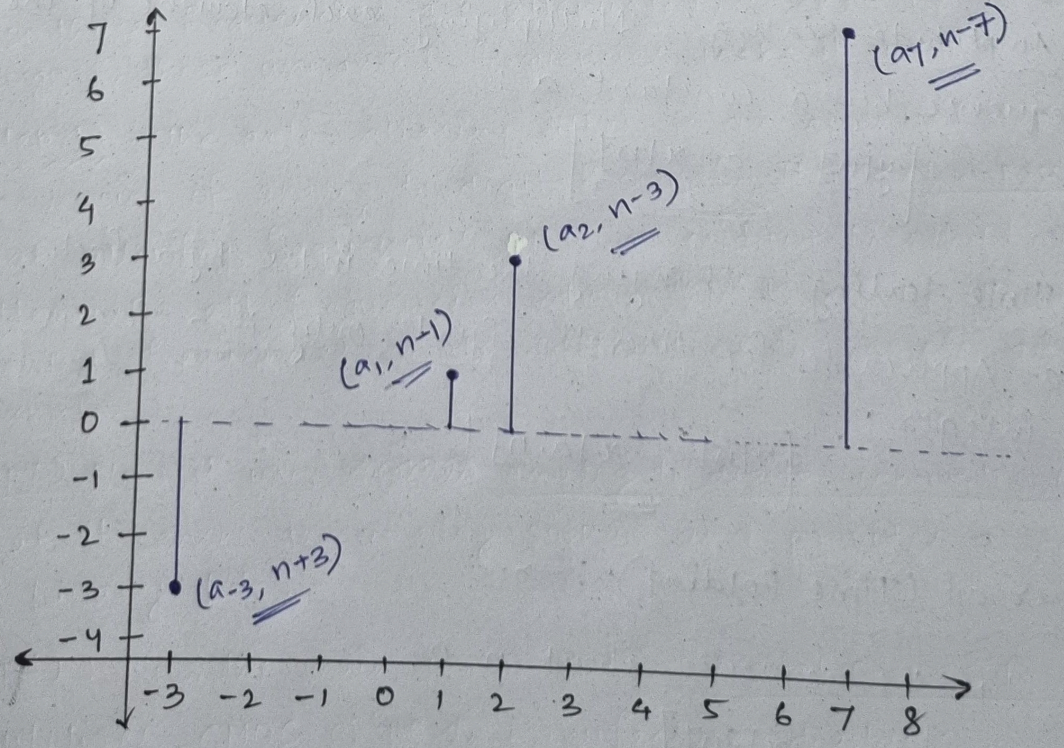
$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x_1[k] \cdot x_2[n-k]}$$

7. **Difference:**

Subtracting one sequence from another:

$$\boxed{y[n] = x_1[n] - x_2[n]}$$

⊛ $p[n] = a_{-3}\delta[n+3] + a_1\delta[n-1] + a_2\delta[n-3] + a_7\delta[n-7]$



(c) Energy and Power signals:

a) Energy signal:

* A signal is called an energy signal if it has finite energy and zero average power.

* Energy is calculated as: →

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \text{ (for continuous signals)}$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \text{ (for discrete signal)}$$

Examples: Exponentially decaying signals like $x(t) = e^{-|t|}$, rectangular pulses.

(b) Power signal:

* A signal is called a power signal if it has finite average power and infinite energy.

* Power is calculated as:

$$\left[P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \right] \text{ (for continuous signals)}$$

Examples: Sinusoidal signals like $x(t) = \sin(2\pi t)$, periodic signals.

3. a) Digital Signal Processing (DSP):

Digital Signal Processing is analysis and manipulation of signals using digital systems or algorithms. Signals like audio, video, and sensor data are first converted into digital form using analog-to-digital conversion. (ADC).

DSP is used to:

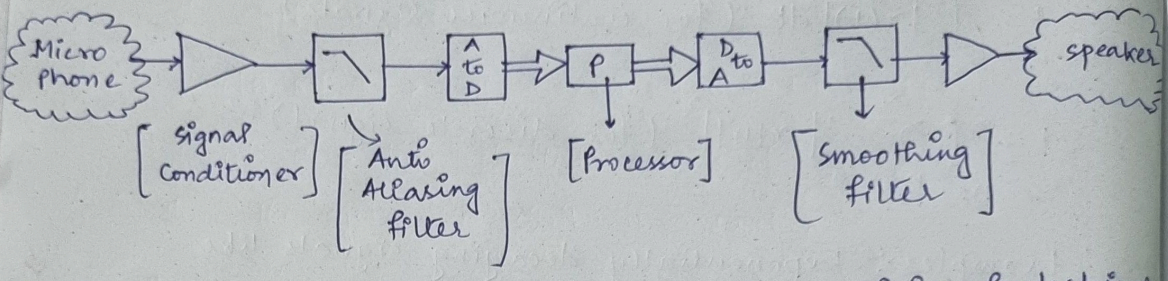
→ (i) Enhance signals by removing noise.

→ (ii) Extract useful information (e.g. speech recognition).

→ (iii) Perform operation like filtering, modulation, or compression.

Applications of DSP are found in communications, medical imaging, audio processing, radar systems, and multimedia. After processing, the digital signals can be converted back to analog using a digital-to-analog converter (DAC).

⊛ Components of Digital Signal Processing:



Step 1: In DSP block diagram, it starts from the receiving of electrical signals. It uses transducer at the input side such as microphone that transforms sound into an electrical signal.

Step 2: After getting an electrical signal, it gives to the input of operational amplifier to sense the analog signal so that it amplifies the signal.

Step 3: For transformation of analog to digital signal, we use anti-aliasing filter. It refers to anti-aliasing filter. It passes frequencies for a limited threshold value. Those frequencies which are higher than the limited threshold, so those frequencies are attenuated. To examine an analog signal, these unwanted frequencies make it complex.

Step 4): The anti-aliasing filter is an essential step in the conversion of analog to digital signal. It is a low-pass-filter that allowing frequencies up to a certain threshold. It attenuates all frequencies above this threshold. These unwanted frequencies create difficulties to sample an analog signal.

Step 5) Now it uses analog to digital converter (ADC) that it senses an analog signal and provides a sequence of binary bits.

Step 6): Now, the main component is digital signal Processor. It utilizes CMOS chips to manufacture digital signal processors.

step 7) Now it uses digital signal processor which is important to compare the acquisition rate of the ADC by slew rate of DAC.

step 8) Here, we uses a low pass filter i.e smoothing filter which removes high frequency components that are not necessary and refines the output.

step 9) At the last stage, we use op-amp as an amplifier that has output transducer (speaker).

ⓑ Already done [Midsem Paper 2023 1.ⓐ]

ⓒ **Linear Shift Invariant system:**

Linear Shift Invariant system must satisfies two key properties: $\longrightarrow$

1. **Linearity:**

   The system follows the principle of superposition. If:

   $x_1[n] \longrightarrow \text{System} \longrightarrow y_1[n]$ and

   $x_2[n] \longrightarrow \text{System} \longrightarrow y_2[n]$

   then for any constant a and b: $\longrightarrow$

   $$a x_1[n] + b x_2[n] \longrightarrow \text{system} \longrightarrow a y_1[n] + b y_2[n]$$

2. **Shift Invariance:**

   A time shift in the input results in the same time shift in the output. If: $\longrightarrow$

   $x[n] \longrightarrow \text{system} \longrightarrow y[n]$,

   then,

   $$x[n-k] \longrightarrow \text{system} \longrightarrow y[n-k].$$

Example:

Consider the system defined by:

$$y[n] = 3x[n] + 2x[n-1].$$

Linearity:

if $x_1[n] \longrightarrow y_1[n] \Rightarrow y_1[n] = 3x_1[n] + 2x_1[n-1]$

$x_2[n] \longrightarrow y_2[n] \Rightarrow y_2[n] = 3x_2[n] + 2x_2[n-1]$

now, for $ax_1[n] + bx_2[n]$

$\Rightarrow y[n] = 3(ax_1[n] + bx_2[n]) + 2(ax_1[n-1] + bx_2[n-1])$

$= ay_1[n] + by_2[n]$.

$\therefore$ The system is linear.

Shift - Invariance :

if $x[n] \longrightarrow y[n]$

$x[n-K] \longrightarrow y[n] = 3x[n-K] + 2x[n-K-1]$

which is a time-shifted version of $y[n-K]$.

Thus, the system is shift-invariant.

$\therefore$ The system is Linear Shift Invariant (LSI)

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, T(\delta(n-K))$$

(i) Linearity :

if $x_1(n) \longrightarrow y_1(n)$

then, $y_1(n) = \sum_{K=-\infty}^{\infty} x_1(K)\, T(\delta(n-K))$

if $x_2(n) \longrightarrow y_2(n)$

then, $y_2(n) = \sum_{K=-\infty}^{\infty} x_2(K)\, T(\delta(n-K))$

for $ax_1(n) + bx_2(n)$

$$\Rightarrow y(n) = \sum_{K=-\infty}^{\infty} [ax_1(K) + bx_2(K)]\, T(\delta(n-K))$$

$$= a\sum_{K=-\infty}^{\infty} x_1(K) + T(\delta(n-K)) + b\sum_{K=-\infty}^{\infty} x_2(K) + T(\delta(n-K))$$

$$= ay_1(n) + by_2(n)$$

$\longrightarrow$ proved

(ii) Shift Invariance:

For the system to be shift-invariant, shifting the input by n_0 should shift the output by the same amount.

$$x(n-n_0) \Rightarrow \quad y(n) = \sum_{k=-\infty}^{\infty} x(k-n_0)\, T(\delta(n-k))$$

$$\because m = k - n_0 \text{ (let)} \quad y(n) = \sum_{m=-\infty}^{\infty} x(m)\, T(\delta(n-m-n_0))$$

This output is not necessarily equal to $y(n-n_0)$, as $T(\delta(n-k))$ depends on the form of T.

$\therefore$ The system is not shift-variant. (Ans)

4. a) Characterization of LTI system:

(i) Impulse Response ($h(n)$):

a) The output of LTI system for an input $x(n)$ is determined by the convolution of the input $x(n)$ with the system's impulse response $h(n)$.

b) Mathematical Expression:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

c) Example: if $h(n) = \delta(n) + \delta(n-1)$ and $x(n) = \{1, 2\}$

then output is $y(n) = \{1, 3, 2\}$

(ii) Difference Equation:

a) LTI systems can be represented using linear constant-coefficient difference equations, describing the relationship between input and output.

b) Mathematical Expression:

$$\left[y(n) + a_1 y(n-1) = b_0 x(n) + b_1 x(n-1)\right]$$

c) Example: For $y(n) = 0.5y(n-1) + x(n)$, if $x(n) = \{0, 0, 1\}$, the output $y(n)$ is computed iteratively.

(iii) Frequency Response ($H(e^{j\omega})$):

a) The system's response in the frequency domain, obtained by taking the Fourier Transform of the impulse response $h(n)$.

b) Mathematical Representation:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

c) Example: if $h(n) = \{1, 1\}$, then

$$\boxed{H(e^{j\omega}) = 1 + e^{-j\omega}}$$

(iv) Transfer Function ($H(z)$):

a) The z-transform of the impulse response provides the system's transfer function, used for analyzing stability and designing filters.

b) Mathematical Representation:

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{n=-\infty}^{\infty} h(n)\, z^{-n}$$

c) Example: if $h(n) = \{1, -1\}$

$$\boxed{H(z) = 1 - z^{-1}}$$

(b) Already Done (1st Part) [Midsem 2024, 3·(a)]
2nd Part)

(*) Properties of Convolution:

a) Communitive:

$\rightarrow x[n] * h[n] = h[n] * x[n]$

b) Distributive over addition:

$\rightarrow x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$

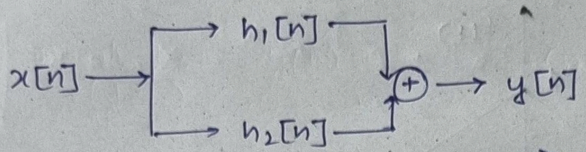
if, c) Cascade Connection:

$x[n] \rightarrow h_1[n] \rightarrow h_2[n] \rightarrow y[n]$ & $x[n] \rightarrow h_2[n] \rightarrow h_1[n] \rightarrow y[n]$

$\Rightarrow \quad x[n] \longrightarrow \boxed{h_1[n] * h_2[n]} \longrightarrow y[n]$

(convolution)

(iv) **Parallel Connection :**



$\Rightarrow \quad x[n] \longrightarrow \boxed{h_1[n] + h_2[n]} \longrightarrow y[n]$

(v) **Stability :** A LTI system is stable iff

$$\boxed{S = \sum_{K=-\infty}^{\infty} |h[K]| < \infty}$$

since $\quad |y[n]| = \left| \sum_{K=-\infty}^{\infty} h[K]\, x[n-K] \right| \leq \sum_{K=-\infty}^{\infty} \big| h[K] \big| x[n-K] < \infty$

When $\big(|x[n]| \leq Bx\big)$

$\therefore$ This is a sufficient condition proof.

(vi) **Causality :**

→ Those systems for which the output depends only on the input samples $y[n_0]$ depends only the input sequence values for $n \leq n_0$.

→ An LTI system is casual iff
$h[n] = 0 \quad \forall\, n < 0$.

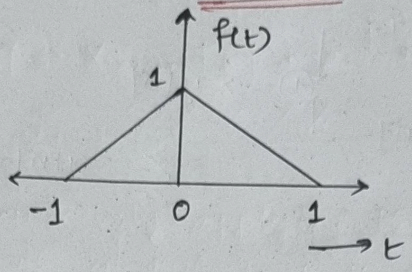
(c) (i) Refer to the steps [Midsem 2023 2.(i)]
(kichu to nije kore ne, sab e ki kore debo ?? 🙂)

(ii) Already done. [Midsem 2023, 2.(ii)]

5. b) (We will do it later).

What do you think ? I am going to avoid the Question ? ; No !!! Not at all buddy.

Come, let's see what's the solution !!!



From the picture:

a) f(t) is triangular and symmetric about $t=0$

b) It has a height of 1 at $t=0$ and linearly decreases 0 at $(t=\pm 1)$.

⊛ Mathematical Representation of f(t):

$$f(t) = \begin{cases} 1-|t| & \text{if } |t| \le 1 \\ 0, & \text{if } |t| > 1 \end{cases}$$

Formula:

$$\left[F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \right]$$

Since f(t) is non-zero only in the range $(-1 \le t \le 1)$ the integral becomes:

$$F(\omega) = \int_{-1}^{1} (1-|t|)\, e^{-j\omega t}\, dt$$

***

$$|t| = \begin{cases} t, & t \in [0,1] \\ -t, & t \in [-1,0) \end{cases}$$

$$= \int_{0}^{1} (1-t) e^{-j\omega t}\, dt + \int_{-1}^{0} (1+t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{1} e^{-j\omega t} dt - \int_{0}^{1} t e^{-j\omega t} dt + \int_{-1}^{0} e^{-j\omega t} dt + \int_{-1}^{0} t e^{-j\omega t} dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^1 + \left[\frac{te^{-j\omega t}}{-j\omega}\right]_0^1 - \int_0^1 \frac{e^{-j\omega t}}{-j\omega}\,dt + \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^0 + \int_{-1}^0 te^{-j\omega t}\,dt$$

$$=$$

$$\frac{1-e^{-j\omega}}{(j\omega)^2} - \frac{e^{-j\omega}}{j\omega} + \frac{1-e^{-j\omega}}{-j\omega} + \left[\frac{te^{-j\omega t}}{-j\omega}\right]_{-1}^0 - \int_{-1}^0 \frac{e^{-j\omega t}}{-j\omega}\,dt$$

$$\Rightarrow \frac{1-e^{-j\omega}}{(j\omega)^2} - \frac{e^{-j\omega}}{j\omega} + \frac{1-e^{-j\omega}}{-j\omega} + \left[\frac{te^{-j\omega t}}{-j\omega}\right]_{-1}^0 - \left[\frac{e^{-j\omega t}}{(-j\omega)^2}\right]_{-1}^0$$

$$= \frac{1-e^{-j\omega}}{(j\omega)^2} - \frac{e^{-j\omega}}{j\omega} + \frac{1-e^{-j\omega}}{-j\omega} + \frac{e^{j\omega}}{j\omega} + \frac{e^{j\omega}}{(j\omega)^2} - \frac{1}{(j\omega)^2}$$

→ The result is a sinc-squared function.

$$F(\omega) = \text{sinc}^2(\omega/2)$$

Where $\text{sinc}(\omega) = \frac{\sin(\omega)}{\omega}$ (Ans)

[It's very very confusing!!! sir?] No need to do this question!!

[FT of cts signals are not there in Exam]

ⓒ Fast Fourier Transform:

The Fast Fourier Transform (FFT) is an algorithm that efficiently computes the DFT. The DFT requires $O(N^2)$ complex multiplications, which becomes computationally expensive for large N. The FFT reduces the computational complexity to $O(N \log N)$ by exploiting symmetries in the DFT.

FFT algorithm Breakdown:

1) Divide and conquer:

a) The FFT splits the DFT into smaller DFTs, recursively breaking down the signal into smaller parts.

b) It takes advantage of the symmetry and periodicity of the complex exponential terms.

2) Radix-2 FFT:

a) The most common FFT algorithm is the Radix-2 FFT, which works when N is a power of 2.
It splits the signal into even and odd indexed parts:

$$\left[X[K] = X_{even}[K] + e^{-j\frac{2\pi}{N}K} \cdot X_{odd}[K] \right]$$

b) By recursively applying this process, the number of computations is drastically reduced.

Example:

Consider the sequence $x[n] = \{1, 0, -1, 0\}$ with $N = 4$

→ FFT Algorithm Breakdown:

1) Divide the sequence:

a) Even-indexed terms: $X_{even} = \{1, -1\}$

b) Odd-indexed terms: $X_{odd} = \{0, 0\}$

2) Compute the DFT of the Even and Odd Parts:

a) $X_{even}[K] = \{0, 2\}$

b) $X_{odd}[K] = \{0, 0\}$

3) Combine the Results:

The FFT algorithm then combines the results of the DFTs using the symmetry properties of the DFT. For $K = 0$ to $N/2 - 1$, we compute:

$$X[K] = X_{even}[K] + W_N^K \cdot X_{odd}[K]$$

$$X[K + N/2] = X_{even}[K] - W_N^K \cdot X_{odd}[K]$$

Where $W_N^K = e^{-j\frac{2\pi K}{N}}$ is the "twiddle factor".

* For $k=0$

$$X[0] = X_{even}[0] + W_4^0 \cdot X_{odd}[0] = 0 + 1 \cdot 0 = 0$$

$$X[2] = X_{even}[0] - W_4^0 \cdot X_{odd}[0] = 0 - 0 = 0$$

* For $k=1$

$$X[1] = X_{even}[1] + W_4^1 \cdot X_{odd}[1] = 2 + (-j) \cdot 0 = 2$$

$$X[3] = X_{even}[1] - W_4^1 \cdot X_{odd}[1] = 2 - 0 = 2$$

FFT result :

The FFT result for the sequence $x[n] = \{1, 0, -1, 0\}$ is :

$$X[k] = \{0, 2, 0, 2\} \quad \text{(Ans)}$$

6(a) **DFS (Discrete fourier series) :**

The discrete fourier series represents a periodic discrete-time signal as a finite sum of complex exponentials (sinusoids). It decomposes the signal into its frequency components, making it easier to analyze in the frequency domain.

Formula :

if $x[n]$ is periodic with period N, its DFS is : →

$$\left[X[k] = \sum_{n=0}^{N-1} \frac{1}{N} x[n] e^{-j\frac{2\pi kn}{N}} \right]$$

Where $[k = 0, 1, \cdots N-1]$

The original signal can be reconstructed as : →

$$\left[x[n] = \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N} kn} \right]$$

(b) Magnitude & Phase Spectrum:

(i) Magnitude spectrum:

The magnitude spectrum represents the amplitude of each frequency component in a signal. It is derived from the Fourier Transform and is defined as :→

$\cancel{|X(\omega)| = \sqrt{Re(X(\omega))^2}}$

$$\boxed{|X(\omega)| = \sqrt{Re(X(\omega))^2 + Im(X(\omega))^2}}$$

Where $X(\omega)$ is the Fourier Transform of the signal. It shows how the energy or strength of the signal is distributed across different frequencies.

Example : For a sinusoidal signal, the magnitude spectrum shows peaks at the frequencies present in the signal.

(ii) Phase spectrum :

The phase spectrum represents the phase angle of each frequency component in the signal. It indicates the shift or alingment of the sinusoidal components in time and is defined as :

$$\boxed{\angle X(\omega) = \tan^{-1}\left(\frac{Im(X(\omega))}{Re(X(\omega))}\right)}$$

It provides critical information about the signal's timing and structure.

Example : For a pure cosine wave, the phase spectrum would be zero, while for a sine wave, it would be $-\pi/2$.

Properties of DFS :

1. **Periodicity :** The DFS coefficients $X[k]$ are periodic with a period N :

$$X[k] = X[k+N].$$

2. **Linearity :** The DFS operation is linear. If two signals $x_1[n]$ and $x_2[n]$ have DFS coefficients $X_1[k]$ and $X_2[k]$, then ;

$a x_1[n] + b x_2[n]$ has DFS coefficients

↳ $a X_1[k] + b X_2[k]$, where a and b are constants

3. **Time Shifting :**

Shifting a signal $x[n]$ by n_0 in time introduces a phase shift in its DFS coefficients :

$$\text{DFS of } \left[x[n-n_0] = X[k] \cdot e^{-j\frac{2\pi}{N} k n_0} \right]$$

4. **Frequency Shifting :**

Multiplying a signal $x[n]$ by $e^{j\frac{2\pi}{N} k_0 n}$ shifts its DFS coefficients in frequency.

$$\text{DFS of } x[n] \cdot e^{j\frac{2\pi}{N} k_0 n} = X[k-k_0]$$

5. **Parseval's Theorem :**

The total energy of the signal in the time domain is equal to the total energy in the frequency domain.

$$\left(\sum_{n=0}^{N-1} |x[n]|^2 = N \sum_{k=0}^{N-1} |X[k]|^2 \right)$$

(a) Poles and zeroes of a z-transform:

1. Poles:
Poles are the values of z for which the denominator of X(z) becomes zero, making X(z) approach infinity. These determines the system's stability and behaviour.

2. Zeroes:
Zeroes are the values of z for which the numerator of the z-transform, X(z) becomes zero. These represent the frequencies where the system's response is nullified.

The poles and zeroes can be visualized on a z-plane which helps analyze the system's frequency response and stability.

(b) $y(n) = x(n+3)u(n) \longrightarrow$ (unit STEP function)

We know, $$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$$

substituting $y(n) = x(n+3)u(n)$

$[y(n) \neq 0 \text{ only for } n \geq 0]$

$$Y(z) = \sum_{n=0}^{\infty} x(n+3) z^{-n}$$

now, let $m = n+3$; $n = m-3$ and if $n=0$ then $m=3$

$$\Rightarrow Y(z) = \sum_{m=3}^{\infty} x(m) z^{-(m-3)}$$

$$\Rightarrow Y(z) = \sum_{m=3}^{\infty} x(m) z^{-m+3}$$

$$\Rightarrow Y(z) = z^3 \sum_{m=3}^{\infty} x(m) z^{-m}$$

$\therefore [Y(z) = z^3 X(z)]$ (Ans) here $X(z)$ is the z-transform of $x(n)$

7. (a) Butterworth Filters:

A Butterworth filters is a type of signal processing filter designed to have a flat frequency response in the passband. It avoids ripples in both the passband and stopband, providing a smooth transition.

Characteristics:

1. Flat Passband:

The Butterworth filter has a maximally flat magnitude response in the passband, meaning there are no ripples.

2. Order of the Filter:

The steepness of the transition from passband to stopband depends on the filter order. Higher order means sharper roll-off

3. Transfer Function:

The magnitude response of the ButterWorth filter is given by:

$$\left[\, |H(\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^{2N}} \,\right]$$

Where ω_c is the cutoff frequency and N is filter Order.

4. Phase Response:

The Butterworth filter does not have linear phase response, making it less ideal for applications requiring phase preservation.

Applications:

a) Used in audio processing to remove noise.

b) Commonly used in communication systems for signal conditioning.

c) Applied in image processing for smooth filtering.

Example:

A low-pass Butterworth filter with $N=2$ ensures a gradual roll-off, ideal for clean and smooth signal attenuation beyond the cutoff frequency.

(b) ROC of z-transform:

The region of Convergence (ROC) is the range of z values in the complex plane where the z-transform of a signal converges. It is crucial for determining the stability and causality of a system.

Key points:

1. **Definition:**

The ROC is the set of all values of z for which the z-transform series:

$$\left(X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \right)$$

converges (i.e → the series has a finite value).

2. **Dependence on the signal:**

The ROC depends on the nature of the signal $x[n]$.

a) For right-sided signals (nonzero for $n \geq 0$), the ROC is outside the outermost pole.

b) For left-sided signals (nonzero for $n < 0$), the ROC is inside the innermost pole.

c) For two-sided signals, the ROC lies between poles.

3. **Properties of ROC:**

a) The ROC does not include any poles of $X(z)$.

b) For finite-duration signals, the ROC is the entire z-plane except $z = 0$ or $z = \infty$.

c) For stable systems, the ROC must include the unit circle ($|z| = 1$).

4. **Applications:**

* Helps analyze the stability of systems (stable systems have ROC containing $|z| = 1$).

* Determines the causality of systems (Casual systems have ROC outside the outermost pole).

Example:

For $x[n] = a^n u[n]$ (a right-sided signal), the z-transform is:

$$X(z) = \frac{1}{1 - az^{-1}}, \quad |z| > |a| \text{ (ROC)}.$$

c) Already Done (Midsem Paper 2023 $\xrightarrow{1.a)}$ (Types of systems) point→ 3.

d) **Memory-less System:**

A memory less system is a system where the output at any time depends only on the input at that same time. It does not depend on past or future values of the input.

Characteristics of Memory-less System:

1. **No Memory Dependency:**

The output $y(t)$ is determined solely by the current input $x(t)$. For discrete systems, $y[n]$ depends only on $x[n]$.

2. **Mathematical Representation:**

For a memoryless system:

$y(t) = f(x(t))$ or $y[n] = f(x[n])$,

Where $f(\cdot)$ is any function.

3. **Examples:**

a) $y(t) = 3x(t)$: The output is three times the current input.

b) $y[n] = |x[n]|$: The output is the magnitude of the input at the same instant.

4. **Non-Examples:**

Systems like $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ or $y[n] = x[n-1]$ are not memory-less because they rely on past input values.

5. **Applications:**

Memory less systems are used in situations where the system's behaviour is independent of past inputs, such as amplifiers or simple scaling devices.

Conclusion:

A memory-less system is simple and instantenous, making it ideal for real-time processing where historical data is irrelevant.