Midsem Paper 2023

1. ⓐ What is system? Discuss different types of systems

Part-1)

System: In the context of Digital Signal Processing (DSP) a system is a set of rules or processes that takes an input signal, performs operations on it, and produces an output signal. It acts as a transformation entity between the input and output signals. Mathematically, a system is represented as ⟶

$$y(t) = T\{x(t)\}$$

Where $x(t)$ is the input, $y(t)$ is the output, and T denotes the system.

Types of systems:

1. Linear & Non-linear:

ⓐ Linear system: Follows the principles of superposition and scaling. Example: $[y(t) = 2x(t)]$

ⓑ Non-linear system:
Does not follow superposition and scaling.
Example: $[y(t) = x^2(t)]$

2. Time-Invariant and Time-variant:

ⓐ Time-Invariant: System properties does not change with time. Example: $[y(t) = x(t) + 3]$

ⓑ Time-Variant: System properties can change with time. Example: $[y(t) = x(t) - t]$

3. Casual and Non-Casual:

ⓐ Casual system: The output depends only on the present and past inputs, not future inputs.
Example: $[y[n] = x[n] + x[n-1]]$

(b) **Non-Casual system:** The output depends on future inputs.

Example: $[y[n] = x[n] + x[n+1]]$

4. **Stable and Unstable systems:**

(a) **Stable system:** Produces bounded output for any bounded input.

Example: $[y[n] = 0.5x[n]]$

(b) **Unstable system:** Produces unbounded output for any bounded input.

Example: $[y[n] = x[n] + nx[n-1]]$

5. **Dynamic and Static systems:**

(a) **Dynamic system:** Output depends upon past or future values of the input.

Example: $[y[n] = x[n] + x[n-1]]$

(b) **Static system:** Output depends upon only on the current input.

Example: $[y[n] = 2x[n]]$

b) $x(t) = \cos^2(2t - \pi/3)$ is periodic or not?

We know,

$$[x(t+T) = x(t)] \longrightarrow \text{periodic w.r.t } T.$$

We know,

$$\left[\cos^2\theta = \frac{1+\cos 2\theta}{2}\right]$$

$$x(t) = \frac{1}{2} + \frac{1}{2}\cos\left(4t - \frac{2\pi}{3}\right)$$

↓ *** comparing with $\cos(\omega t \pm \theta)$

$\omega = 4$, $\left(T = \frac{2\pi}{\omega}\right)$

$T = \frac{2\pi}{4} = \frac{\pi}{2}$

$\therefore$ Periodicity $(T) = \frac{\pi}{2}$

$\therefore$ $x(t) = \cos^2(2t - \pi/3)$ is periodic. (Answer)

2. (i) You can solve the question by your own.
Follow the steps from Midsem 2024 Paper
Q3 (a)

(ii) $h[n] = a^n u[n]$; $|a|<1$ and $x[n] = u[n]$

$$y[n] = x[n] * h[n] = \sum_{K=-\infty}^{\infty} x[K]\, h[n-K]$$

$$= \sum_{K=0}^{\infty} (1)\, a^n u[n-K] = \sum_{K=0}^{\infty} a^n u[n-K]$$

$$= \sum_{K=0}^{\infty} a^n = a^0 + a^1 + a^2 + a^3 + \cdots$$

$$\left[y[n] = \frac{1}{1-a} \right] \text{ (Answer)}$$

3. Discrete Fourier Transform (DFT):

The DFT is applied to discrete signals or sequences. It transforms a finite sequence of data points in the time domain into another sequence of the same length in the frequency domain. The DFT is particularly useful for processing periodic and finite-duration signals.

Given a sequence $x[n]$ of length N, the DFT is defined as:

$$X[K] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{N}kn}, \quad K = 0, 1, \cdots N-1$$

Where:
1. $x[n]$: discrete time-domain signal.
2. $X[K]$: corresponding frequency-domain components (DFT coefficient)
3. N : Number of samples in the sequence.
4. $e^{-j\frac{2\pi}{N}kn}$: Complex exponential term (basis function for the transformation)
5. k : Frequency index.

* The DFT takes a sequence of values (signal) and maps them to their corresponding frequency components.

* Each frequency component $X[K]$ contains information about the amplitude and phase of a sinusoid of frequency K/N that forms part of the original signal.

* Properties of DFT :

1. Periodicity :

   (a) The DFT assumes that the input signal is periodic with period N.

   (b) $X[K+n] = X[K]$, meaning the frequency components repeat every n samples.

2. Symmetry :

   * If $x[n]$ is real-valued, The DFT exhibits symmetry, with $X[K] = X[N-K]^*$, where * denotes the complex conjugate.

3. Linearity :

   * The DFT is a linear transformation, meaning that the DFT of a sum of signals is the sum of their DFTs.

4. Convolution Theorem :

   The DFT of a convolution of two signals in the time domain is the pointwise product of their DFTs.

Reference : slide 3, 4, 11 from

" Discrete Fourier Transform_New.pptx.pdf "

(What a extension!!!)