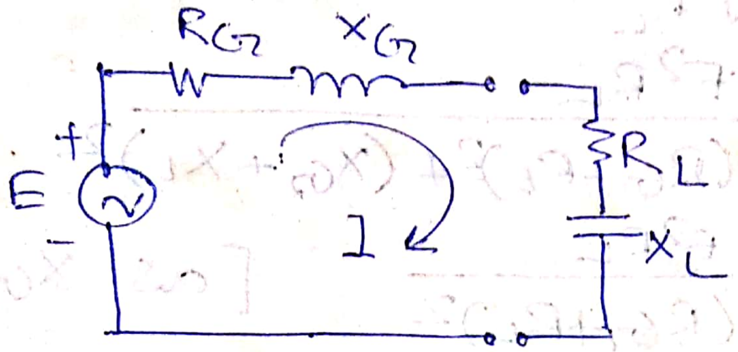
Maximum power transfer theorem:-

Any two terminal linear network will absorb maximum power from the generator if the load impedence is complex conjugate of the internal impedence of the generator.



Let us consider a circuit with Z_g and Z_L are connected in series with the voltage source E, Z_g is the impedence due to generator and Z_L is the ~~[illegible]~~ impedence due to load where $Z_G = R_G + jX_G$ and $Z_L = R_L + jX_L$

Let I be the instantaneous current flowing through the ckt, therefore,

$$I = \frac{E}{Z_G + Z_L}$$

$$= \frac{E}{(R_G + jX_G) + (R_L + jX_L)}$$

$$= \frac{E}{\sqrt{(R_G+R_L)^2+(X_G+X_L)^2}}$$

Now power absorb by the load,

$$P = I^2 R_L$$

$$= \frac{E^2 R_L}{(R_G+R_L)^2+(X_G+X_L)^2}$$

$$\Rightarrow \frac{\partial P}{\partial X_L} = \frac{-E^2 R_L \times 2(X_G+X_L)}{[(R_G+R_L)^2+(X_G+X_L)^2]^2}$$

For $P = P_{max}$, $\frac{\partial P}{\partial X_L} = 0$

$$\Rightarrow \frac{-E^2 R_L \cdot 2(X_G+X_L)}{[(R_G+R_L)^2+(X_G+X_L)^2]^2} = 0$$

$$\Rightarrow -2E^2 R_L (X_G+X_L) = 0$$

$$\Rightarrow X_G + X_L = 0$$

$$\Rightarrow X_G = -X_L$$

Again, $P = I^2 R_L$

$$= \frac{E^2 R_L}{(R_G+R_L)^2+(X_G+X_L)^2}$$

$$= \frac{E^2 R_L}{(R_G+R_L)^2} \quad [\text{as } X_G = -X_L]$$

$$\Rightarrow \frac{\partial P}{\partial R_L} = \frac{(R_G+R_L)^2 E^2 - E^2 R_L \times 2(R_G+R_L)}{(R_G+R_L)^4}$$

$$= \frac{[(R_G+R_L)^2 - 2R_L(R_G+R_L)]E^2}{(R_G+R_L)^4}$$

$$= \frac{E^2 (R_G+R_L)(R_G+R_L-2R_L)}{(R_G+R_L)^4}$$

$$= \frac{E^2 (R_G+R_L)(R_G-R_L)}{(R_G+R_L)^4} = \frac{E^2(R_G-R_L)}{(R_G+R_L)^3}$$

For $P = P_{max}$.

$$\frac{\partial P}{\partial R_L} = 0$$

$$\frac{E^2 (R_G - R_L)}{(R_G + R_L)^3} = 0$$

$$\Rightarrow E^2 (R_G - R_L) = 0$$

$$\Rightarrow R_G = R_L.$$

Hence maximum power Transfer Theorem is proved.

Superposition Theorem :-

In a network containing energy source and impedance, the current flowing at any point is the algebric sum of all current which would be seperately flow at that point by energy source.

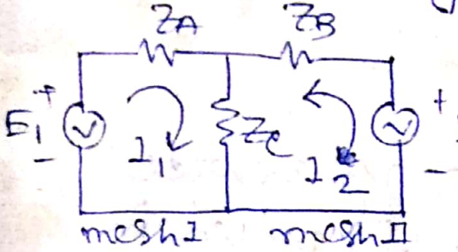


fig ①

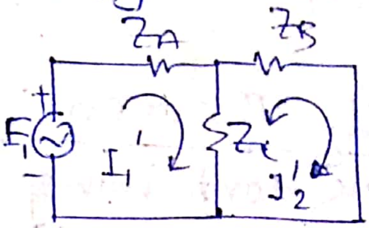


fig ②

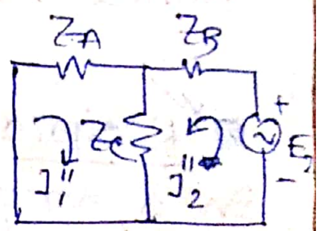


fig ③

Let us consider a ckt consisting of Z_A, Z_B, Z_C where Z_A, Z_B, Z_C are impedence ~~are~~ and voltage source E_1 and E_2.
I_1 and I_2 be the instantaneous current flowing through mesh I and II.
Now applying KVL we get,

$$E_1 = I_1 (Z_A + Z_C) + I_2 Z_C \quad \text{——} ①$$

$$E_2 = I_2 (Z_B + Z_C) + I_1 Z_C \quad \text{——} ②$$

when E_1 ON and E_2 OFF, then I_1' and I_2' are the current flowing through mesh I and II respectively.

So, $$E_1 = I_1'(Z_A + Z_C) + I_2' Z_C \quad \text{——} ③$$

$$0 = I_2'(Z_B + Z_C) + I_1' Z_C \quad \text{——} ④$$

When, E_1 'OFF' and E_2 'ON', then I_1'' and I_2'' ~~flo~~ are the current flowing through mesh I and II respectively.

$$0 = I_1''(Z_A + Z_C) + I_2'' Z_C \quad \text{—— (5)}$$

$$E_2 = I_2''(Z_B + Z_C) + I_1'' Z_C \quad \text{—— (6)}$$

Adding eq. (3) and (5), (4) and (6), we get,

$$E_1 = (I_1' + I_1'')(Z_A + Z_C) + (I_2' + I_2'') Z_C \quad \text{—— (7)}$$

$$E_2 = (I_2' + I_2'')(Z_B + Z_C) + (I_1' + I_1'') Z_C \quad \text{—— (8)}$$

comparing eq. (1) with (7) and (2) with (8) we get

$$I_1 = I_1' + I_1''$$

$$I_2 = I_2' + I_2''$$

Hence superposition theorem is proved.

■ **Reciprocity Theorem :-**

In a network containing energy source and impedence, if an e.m.f is applied in one mesh produces a certain current in the second mesh, then the same e.m.f acting in the second mesh will give an identical current in the first mesh.

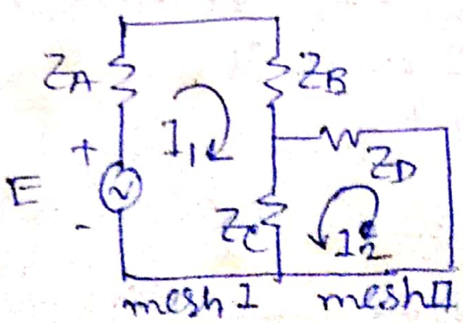


fig (1)

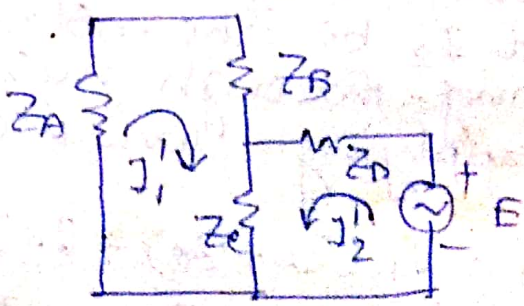


fig (2)

Let us consider a circuit with impedence Z_A, Z_B, Z_C and Z_D and a voltage source E.

$Z_A + Z_B + Z_C = Z_1$ [self impedance of mesh I]

$Z_C + Z_D = Z_2$ [self impedance of mesh II]

$Z_C = Z_{12}$ [mutual impedance of mesh I and II]

In fig ①, voltage source E is connected in mesh I, the current I_1 and I_2 flowing through mesh I and II respectively.

Now, applying KVL in fig ①,

$$E = Z_1 I_1 + Z_{12} I_2 \quad \text{——} ①$$

$$0 = Z_2 I_2 + Z_{12} I_1$$

$$= Z_{12} I_1 + Z_2 I_2 \quad \text{——} ②$$

using cramer's rule in eq. ① and ②,

$$I_2 = \frac{\begin{vmatrix} Z_1 & E \\ Z_{12} & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_{12} \\ Z_{12} & Z_2 \end{vmatrix}} = \frac{-EZ_{12}}{Z_1 Z_2 - (Z_{12})^2}$$

Now voltage source E is connected in mesh II and current I_1' and I_2' flowing through mesh I and II respectively.

Applying KVL in fig ②, we get,

$$E = Z_2 I_2' + Z_{12} I_1' \quad \text{——} ③$$

$$0 = Z_1 I_1' + Z_{12} I_2'$$

$$= Z_{12} I_2' + Z_1 I_1' \quad \text{——} ④$$

using cramer's rule in eq. ③ and ④,

$$I_1' = \frac{\begin{vmatrix} Z_2 & E \\ Z_{12} & 0 \end{vmatrix}}{\begin{vmatrix} Z_2 & Z_{12} \\ Z_{12} & Z_1 \end{vmatrix}} = \frac{-EZ_{12}}{Z_1 Z_2 - (Z_{12})^2}$$

$$\therefore I_2 = I_1'$$

Hence superposition theorem proved.