$$\rho(x,t) = \frac{1}{\sqrt{2\pi\sigma(t)^2}} exp(-\frac{x^2}{2\sigma(t)^2}) \tag{1}$$

$$\langle x(t)^2 \rangle = \int_{-\infty}^{+\infty} x^2 \rho(x,t) dx \tag{2}$$

$$\begin{aligned}
\langle x(t)^2 \rangle =& \frac{1}{\sqrt{2\pi\sigma(t)^2}} \int_{-\infty}^{+\infty} x^2 exp(-\frac{x^2}{2\sigma(t)^2}) dx \\
=& \frac{1}{\sqrt{2\pi\sigma(t)^2}} \times 2 \int_0^{+\infty} (\frac{x}{\sqrt{2\sigma(t)^2}})^2 exp(-(\frac{x}{\sqrt{2\sigma(t)^2}})^2) d(\frac{x}{\sqrt{2\sigma(t)^2}}) \times (2\sigma(t)^2)^{3/2} \\
=& \frac{2\sigma(t)^2}{\sqrt{\pi}} \times 2 \int_0^{+\infty} t^2 exp(-t^2) dt \\
=& \frac{2\sigma(t)^2}{\sqrt{\pi}} \Gamma(\frac{3}{2}) \\
=& \frac{2\sigma(t)^2}{\sqrt{\pi}} \times \frac{1}{2}\sqrt{\pi} \\
=& \sigma(t)^2
\end{aligned} \tag{3}$$