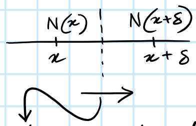
# Notes on PDEs

Monday, June 3, 2019 1:24 PM

Deriving Fick's Equations:




Particles moving into right chamber

$$\frac{1}{2}N(x) - \frac{1}{2}N(x+\delta)$$

flux (Divide by area · time)

$$J_x = -\frac{1}{2A\tau}\left(N(x+\delta) - N(x)\right)$$

Recall, that $\langle x^2(n)\rangle = n\delta^2 = \frac{t}{\tau}\delta^2 = 2Dt$

we took $D = \frac{\delta^2}{2\tau}$

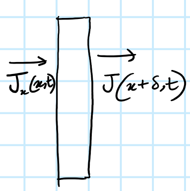
$$J_x = \frac{-\delta^2}{2\tau\delta}\left(\frac{N(x+\delta)}{A\delta} - \frac{N(x)}{A\delta}\right)$$

Volume!

$$J_x = -\frac{D}{\delta}\left(\underline{C(x+\delta) - C(x)}\right)$$

concentration difference

$$J_x = -D\frac{\partial C}{\partial x} \quad \text{(Fick's 1st law)}$$




Change in concentration over time:

$$\frac{1}{\tau}\left(C(t+\tau) - C(t)\right)$$

$$= -\frac{1}{\tau}\left(J_x(x+\delta) - J_x(x)\right)\frac{A\tau}{A\delta}$$

As $\tau \to 0$ and $\delta \to 0$

$$\frac{\partial C}{\partial t} = -\frac{\partial J_x}{\partial x}$$

or $$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$