Study of the nominal exchange rate between the Brazilian real and the US dollar 2000-2020

Econometrics

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I. Introduction

1. Context and motivation

The study of the nominal exchange rate between the between the Brazilian real and the US dollar is a major issue in understanding Brazil's macroeconomic dynamics, particularly in a context of high exposure to global monetary fluctuations. We live in a globalized world, and in such a world the value of a national currency directly affects the competitiveness of exports, the attractiveness of foreign investments and the macroeconomic stability.

Our study aims to identify the main economic factors that influence the evolution of the nominal exchange rate of the real against the US dollar over the period 2000 to 2020.

We picked this topic due to the volatility observed over the past two decades of the Brazilian real. A period marked by time of crisis, accelerated growth, significant monetary shifts and fluctuating international capital flows. As a major emerging economic, Brazil offers an interesting case to analyze how internal and external factors influence the nominal exchange rate.

2. Theorical framework and determinants of the nominal exchange rate

The nominal exchange rate reflects the price of one currency in terms of another, in this case, the number of Brazilian reals required to purchase one US dollar. It results from the supply and demand for currencies in the foreign exchange market, influenced by a range of macroeconomic factors. According to economic theory, the main explanations for exchange rate fluctuations include:

- *Purchasing Power Parity (PPP) (Ricardo, 1817),* which states that exchange rates should be adjusted to equalize price levels between countries.
- *Interest Rate Parity (Keynes*), suggesting that interest rate differentials between two countries are offset by expected exchange rate movements.
- *Current Account Models (Mundell-Fleming),* linking trade imbalances to currency flows.

Based on these theoretical insights, we have selected a set of relevant explanatory variables, considering Brazil's specific economic context and its relationship with the United States.

Our econometric model includes the following variables:

- **BRL/USD Nominal Exchange Rate (dependent variable):** (*TxdechanBra)* Measures the value of the real relative to the US dollar. Changes in this variable are the core focus of our study.

- **Brazil's GDP**: (PibBra) Reflects overall economic health. A growing GDP suggests increased productivity and can strengthen the domestic currency. Expected relationship: negative.

- **Brazilian Exports**: (expBra) As exports increase, demand for the real also rises (since payments are received in local currency), leading to potential appreciation. Expected relationship: negative.

- **Brazilian Imports**: (ImpBra) Higher imports increase demand for foreign currency, potentially depreciating the real. Expected relationship: positive.

- **Brazilian Real Interest Rate**: (TxdinteretBra) A higher rate attracts foreign capital inflows, supporting currency appreciation. Expected relationship: positive.

- **US Real Interest Rate**: (TxdinteretUSA) A rise in US rates tends to attract capital to the US, weakening emerging market currencies like the real. Expected relationship: negative.

- **Foreign Direct Investment (FDI) in Brazil**:(IdeBra) Reflects the country's attractiveness to international investors. Increased FDI is generally associated with currency appreciation. Expected relationship: negative or ambivalent, depending on whether FDI is productive or speculative.

For the econometric analysis, we will use multiple regression models to examine the relationships between the exchange rate and the explanatory variables. The following model specification will be used to analyze the data:

3. <u>Study period</u>

This period was chosen based on historical and economic considerations. Since 1999, Brazil has adopted a floating exchange rate regime, abandoning the previously managed exchange rate bands. This shift has allowed market forces to determine the exchange rate more freely, making econometric analysis more meaningful.

The period from 2000 to 2020 includes:

- *A phase of strong economic growth (2000–2010),*

- *Increasing instability marked by political and economic crises (2014–2016),*

- *A deep recession followed by a slow recovery,*

- *The global impact of the COVID-19 pandemic in 2020.*

This range of economic conditions provides a rich context to observe how the nominal exchange rate behaves under different macroeconomic scenarios.

4. Objective of the study

Through a rigorous econometric approach, our goal is to quantify the impact of the explanatory variables on the nominal exchange rate of the Brazilian real against the US dollar. This work will allow us to empirically test the validity of major economic theories on exchange rate behavior, while offering a concrete analysis of Brazil's monetary dynamics over the past two decades.

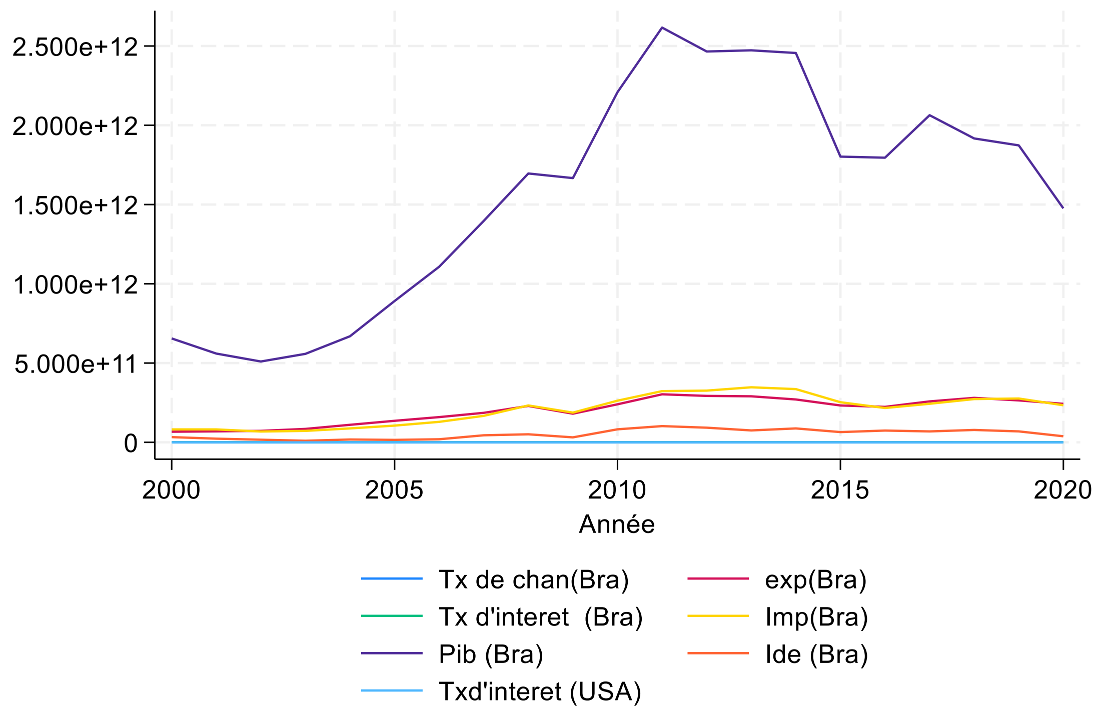
II. The model

1. Methodology

The data used in this analysis spans from 2000 to 2020 and includes key macroeconomic variables for Brazil and the United States. All the data used in our study comes from the Perspective Monde database and the World Bank, so we can confirm the reliability of this information. It contains annual data for exchange rates, exports, imports, GDP, interest rates, and foreign direct investments.

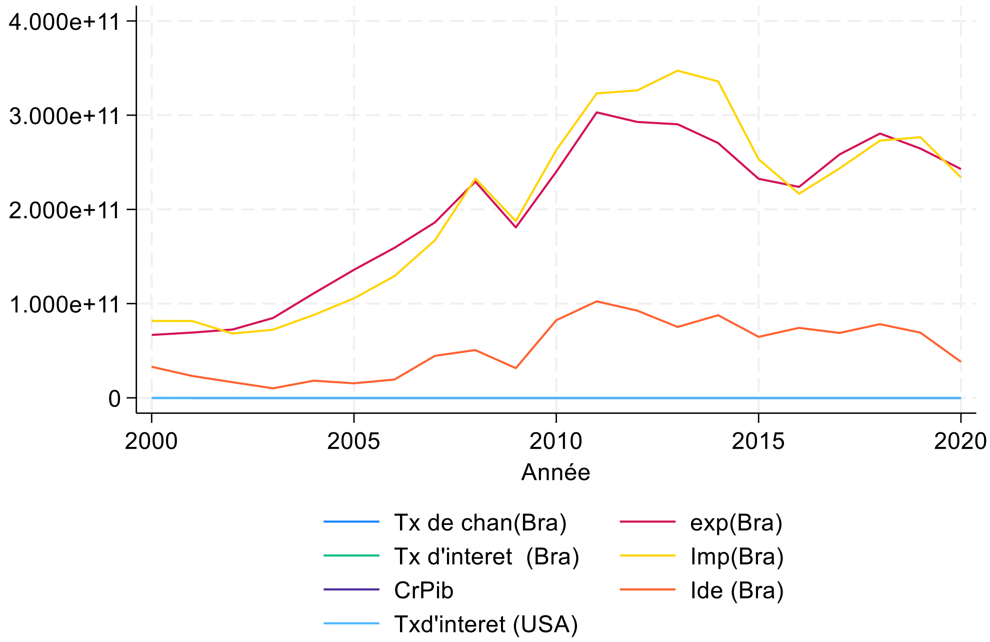
For the econometric analysis, we will use multiple regression models [2] to examine the relationships between the exchange rate and the explanatory variables.

But first we'll do some descriptive statistics and graphics:



We get from the previous graphic that the GDP appears to have no relationship with the dependent variable. It would be better to consider GDP growth, which more effectively captures fluctuations or economic cycles. So we'll add a new variable CrPib = 100*(D.PibBra/L.PibBra)

On the other hand, exports and imports have a strong relationship, they fluctuate in the same way, with a potential correlation risk to verify.

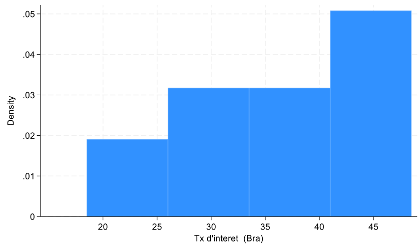


After adding this new variable, exports and imports remain strongly correlated with each other, but the FDI does not have a strong relationship with the dependent variable, which is the exchange rate. All other variables have a strong correlation with the exchange rate.

We get:

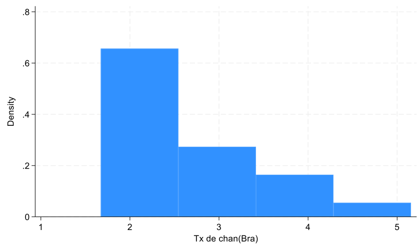
```
.* summarize

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
       Année |         21        2010    6.204837       2000       2020
  TxdechanBra |         21    2.673773    .8922786   1.672829   5.155179
      expBra |         21    2.00e+11    8.09e+10   6.68e+10   3.03e+11
Txdinteret~a |         21     35.9832     8.99176   18.49884   48.50473
      ImpBra |         21    2.05e+11    9.55e+10   6.83e+10   3.47e+11
-------------+---------------------------------------------------------
      PibBra |         21    1.56e+12    7.08e+11   5.10e+11   2.62e+12
      IdeBra |         21    5.22e+10    2.94e+10   1.01e+10   1.02e+11
Txdinteret~A |         21    2.844892    1.434602   1.162899   6.813792
       CrPib |         20    5.564223    17.56163  -26.62133   33.22097
```

And the following histograms:



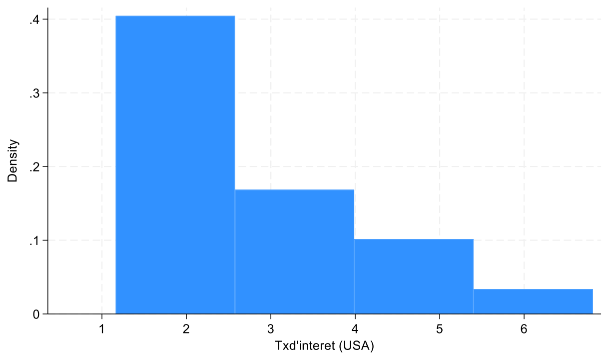
Histogram of the interest rate in Brazil

We observe a right-skewed distribution, with higher density concentrated in the upper bins (around 40-50 bps). This suggests that in the sample, higher interest rates occur more frequently compared to lower rates. The distribution appears to be positively skewed, which may reflect macroeconomic conditions such as inflationary pressures or central bank policy shifts during the observed period.From an econometric standpoint, if this variable is used as a regressor, transformation or standardization might be considered depending on the model assumptions (e.g., normality in OLS). If interest rates are the dependent variable, further investigation into the factors causing this skewness would be warranted.



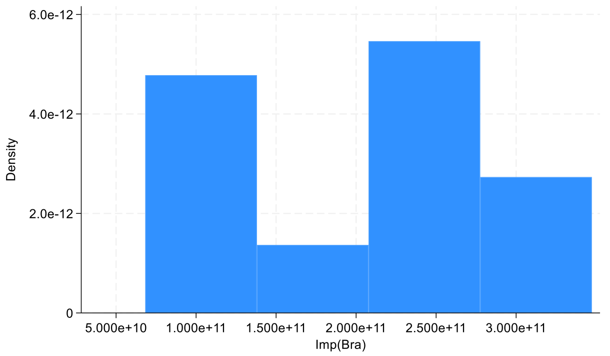
Histogram of the exchange rate

The density is heavily concentrated around lower values, particularly around 2 bps, which indicates a left-skewed distribution (or negatively skewed, depending on the interpretation of the rate). Econometrically, this implies that low exchange rate levels are more common in the dataset, whereas higher values are relatively rare. This kind of distribution could signal a stable currency environment with occasional volatility spikes. If exchange rate volatility is being modeled, this distribution would suggest potential non-normality and heteroskedasticity, which may require corrective modeling techniques (e.g., GARCH models).

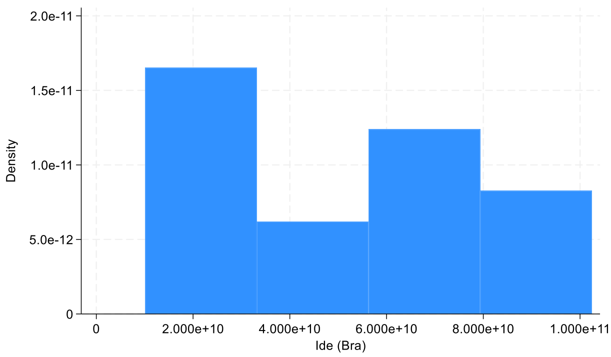


Histogram of the interest rate in the US

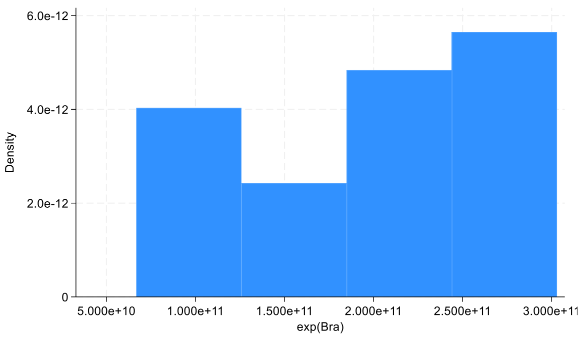
The shape is strongly right-skewed (positively skewed), with most observations clustered around 2%, and a long tail extending toward higher interest rate levels. Such a distribution is typical of macroeconomic interest rate data over time, reflecting periods of low policy rates with occasional hikes. From an econometric modeling perspective, this distribution implies that stationarity and transformations may be necessary before applying models that assume normality or homoscedasticity. If the interest rate is the dependent variable, models accounting for structural breaks or regime changes may also be relevant.



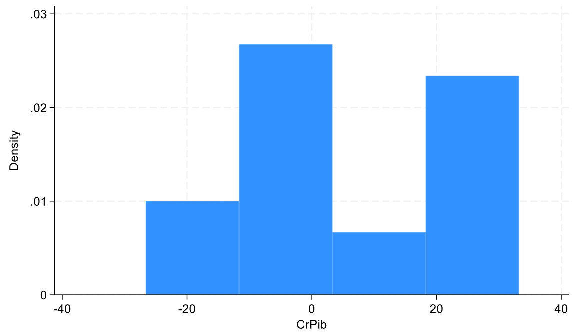
Histogram of importation in Brazil



Histogram of the IDE of Brazil



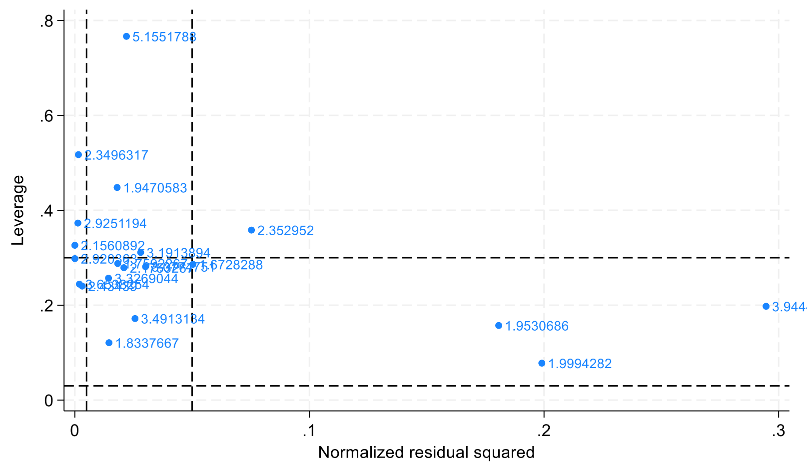
Histogram of exportation in Brazil



Histogram of the growth rate of IDE in Brazil

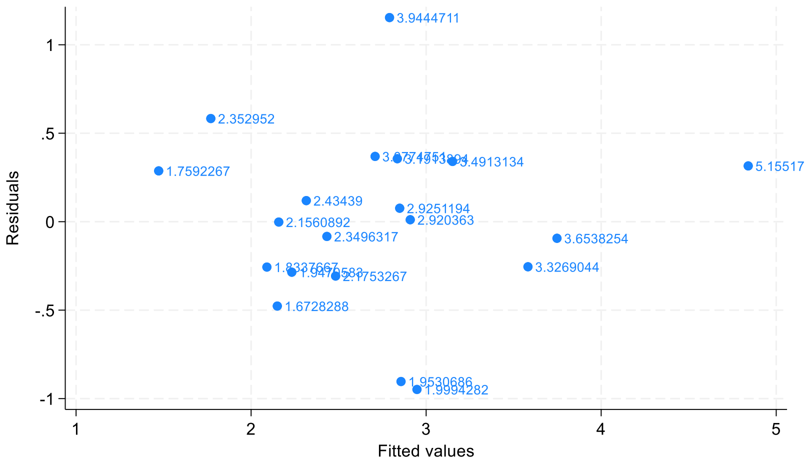
Shows the distribution of the variable CrPib. The X-axis: Value of CrPib and the Y-axis: Density (relative frequency). The variable appears to be bimodal or at least not normally distributed, with clusters around negative, near-zero, and positive values. This could indicate heterogeneity in the data — perhaps different country groups or time periods with different economic conditions. If CrPib is a dependent variable in the regression, non-normality could affect the validity of inference (especially in small samples like ours).

We now must detect outliers, and we'll detect them thanks to rvfplot:



The X-axis: Normalized residual squared, indicates how far an observation's residual is from zero. The Y-axis is the Leverage, indicates how far an observation's predictor values are from the mean. The Dashed lines represent thresholds for high leverage or influence.

Points in the top-right quadrant are potentially influential observations — they have both high leverage and high residuals, meaning they could disproportionately affect the regression results. The observation around (0.02, 0.8) with label 5.1551788 is of particular concern: high leverage and high residual. These points should be investigated further. They might indicate outliers, data entry errors, or important subgroups in the data.



The X-axis represents fitted values from the regression model. The Y-axis is residuals (difference between observed and predicted values). The points are labeled with observation indices or IDs. Ideally, residuals should be randomly scattered around the horizontal line at zero, indicating homoscedasticity (constant variance of errors). In this case, there seems to be a slight curvature, which may suggest model misspecification — e.g., omitted variables or non-linear relationships. Some points lie farther from the rest (e.g., 5.1551788 and 3.9444711), again highlighting potential outliers.

The results provide insight into the relationship between the exchange rate and the explanatory variables, highlighting statistically significant relationships and their economic implications

2. <u>Regression and tests</u>

After defining the model and selecting the relevant variables, we proceed with estimating the regression coefficients.

```
. reg TxdechanBra CrPib expBra ImpBra IdeBra TxdinteretUSA TxdinteretBra
|
      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(6, 13)        =      5.13
       Model |  10.6688674         6  1.77814457   Prob > F        =    0.0066
    Residual |  4.50578242        13  .346598648   R-squared       =    0.7031
-------------+----------------------------------   Adj R-squared   =    0.5660
       Total |  15.1746498        19   .79866578   Root MSE        =    .58873

------------------------------------------------------------------------------
  TxdechanBra | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        CrPib |  -.0387203   .0083756    -4.62   0.000    -.0568148   -.0206258
       expBra |   3.28e-11   8.32e-12     3.94   0.002     1.49e-11    5.08e-11
       ImpBra |  -3.35e-11   1.17e-11    -2.87   0.013    -5.87e-11   -8.32e-12
       IdeBra |   3.08e-12   1.69e-11     0.18   0.858    -3.35e-11    3.96e-11
TxdinteretUSA |  -.1779665   .1419636    -1.25   0.232    -.4846603    .1287273
TxdinteretBra |  -.0439383   .0419104    -1.05   0.314    -.1344802    .0466036
        _cons |   5.093416   2.342845     2.17   0.049     .0320077    10.15482
------------------------------------------------------------------------------
```

The regression model explains **70.31%** of the variation in the exchange rate of the Brazilian real (TxdechanBra), with an adjusted R-squared of **56.60%.** The overall model is statistically significant (F = 5.13, p = 0.0066). Among the explanatory variables, CrPib, expBra, and ImpBra are significant, meaning they have a meaningful impact on the exchange rate. However, IdeBra, TxdinteretUSA, and TxdinteretBra are not statistically significant and do not appear to influence the exchange rate in this model.

By using the backward elimination (remove the variable with the highest p-value from a regression model), We remove the variable "IdeBra" and make another regression.

```
. reg TxdechanBra CrPib expBra ImpBra  TxdinteretUSA TxdinteretBra

      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(5, 14)        =      6.61
       Model |  10.6573865         5  2.13147731   Prob > F        =    0.0023
    Residual |   4.5172633        14  .322661664   R-squared       =    0.7023
-------------+----------------------------------   Adj R-squared   =    0.5960
       Total |  15.1746498        19   .79866578   Root MSE        =    .56803

------------------------------------------------------------------------------
  TxdechanBra | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
--------------+----------------------------------------------------------------
        CrPib |  -.0385574    .008035    -4.80   0.000    -.0557907   -.0213241
       expBra |   3.25e-11   7.88e-12     4.13   0.001     1.56e-11    4.94e-11
       ImpBra |  -3.19e-11   7.60e-12    -4.20   0.001    -4.82e-11   -1.56e-11
TxdinteretUSA |  -.1777113   .1369671    -1.30   0.215    -.4714765    .1160538
TxdinteretBra |  -.0387587   .0296849    -1.31   0.213    -.1024265    .0249091
        _cons |   4.802959   1.654941     2.90   0.012     1.253464    8.352454
------------------------------------------------------------------------------
```

The overall model is still statistically significant (F = 5.13, p = 0.0023) with a good R-squared (70.23 %) We've decided to keep "TxdinteretUSA" and "TxdinteretBra," although insignificant at the 5% threshold. Since the number of observations is small (20), we might omit important variables by removing them. We will decide if it is worthy after the various tests.

3. Diagnostics and model validation

To ensure the validity and robustness of our model, we perform several diagnostic tests, including tests for multicollinearity, heteroskedasticity, and autocorrelation. These tests help to verify the assumptions of the regression model and ensure that the results are reliable.

We start with the multicollinearity test with VIF. The Variance Inflation Factor (VIF) test is commonly used to detect multicollinearity by measuring how much the variance of an estimated coefficient increases due to multicollinearity. We suppose from the first graph that could be a possibility of multicollinearity. The hypothesis are :

H0 (null hypothesis): There is no multicollinearity – the variable is not highly correlated with the others.

HA (alternative hypothesis): There is multicollinearity – the variable is highly correlated with one or more other independent variables

Multicollinearity test

```
***Multicolinearity test
* ***Multicolinearity test */
 vif

    Variable |       VIF       1/VIF
-------------+----------------------
      ImpBra |     29.82    0.033538
      expBra |     21.61    0.046272
xdinteret~a |      3.97    0.252109
xdinteret~A |      1.43    0.698543
       CrPib |      1.17    0.852897
-------------+----------------------
    Mean VIF |     11.60
```

The VIF results indicate a high level of multicollinearity, particularly for ImpBra (VIF = 29.82) and expBra (VIF = 21.61), which are well above the common threshold of 10. This suggests that these two variables are highly correlated with other predictors in the model and may distort the reliability of the coefficient estimates.

Correction of multicollinearity

We want to correct multicollinearity by transforming the two correlated variables into a trade balance. Economically, replacing expBra and ImpBra with a trade balance variable (exports minus imports) captures the net trade effect on the exchange rate, reducing redundancy while preserving the relevant economic information.

```
. gen solde_com= expBra-ImpBra

. reg TxdechanBra CrPib solde_com  TxdinteretUSA TxdinteretBra

      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(4, 15)        =      8.81
       Model |  10.6442237         4  2.66105593   Prob > F        =    0.0007
    Residual |  4.53042611        15  .302028408   R-squared       =    0.7014
-------------+----------------------------------   Adj R-squared   =    0.6218
       Total |  15.1746498        19   .79866578   Root MSE        =    .54957

------------------------------------------------------------------------------
 TxdechanBra | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       CrPib |  -.0382991   .0076747    -4.99   0.000    -.0546573   -.0219408
   solde_com |   3.21e-11   7.32e-12     4.39   0.001     1.65e-11    4.77e-11
TxdinteretUSA |  -.1847874    .128107    -1.44   0.170    -.4578411    .0882662
TxdinteretBra |  -.0429792   .0203991    -2.11   0.052    -.0864588    .0005004
       _cons |   5.092798   .7975564     6.39   0.000     3.392847    6.792749
------------------------------------------------------------------------------

. vif

    Variable |       VIF       1/VIF
-------------+----------------------
   solde_com |      2.33    0.428269
Txdinteret~a |      2.00    0.499734
Txdinteret~A |      1.34    0.747447
       CrPib |      1.14    0.875069
-------------+----------------------
    Mean VIF |      1.70
```

The VIF results show low multicollinearity, with all values well below the critical threshold of 10, indicating stable and reliable coefficient estimates. We can move on to test the heteroskedasticity with a new model TxdechanBra= B1* CrPib + B2* solde_com + B3*TxdinteretUSA + B4* TxdinteretBra + cons

Heteroskedasticity test

The Breusch-Pagan and White tests are two commonly used methods to detect heteroskedasticity in a regression model. They help determine whether the variance of the residuals is constant (homoskedasticity) or not, which is an important assumption for the validity of statistical inferences.

```
.
. hettest

Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: Fitted values of TxdechanBra

H0: Constant variance

    chi2(1) =   0.00
Prob > chi2 = 0.9492

.
. estat imtest, white

White's test
H0: Homoskedasticity
Ha: Unrestricted heteroskedasticity

   chi2(14) =  14.47
Prob > chi2 = 0.4150
```

```
Cameron & Trivedi's decomposition of IM-test

--------------------------------------------------
              Source |       chi2     df         p
---------------------+----------------------------
  Heteroskedasticity |      14.47     14    0.4150
            Skewness |       4.44      4    0.3503
            Kurtosis |       0.97      1    0.3248
---------------------+----------------------------
               Total |      19.88     19    0.4019
--------------------------------------------------

. *Aucune preuve d'heterodestacité
```

Both the Breusch-Pagan and White tests for heteroskedasticity show no evidence of heteroskedasticity in the model. The Breusch-Pagan test yields a p-value of 0.9492 (>0.05), indicating that the null hypothesis of constant variance cannot be rejected. Similarly, White's test shows a p-value of 0.4150(0.005), further supporting the absence of heteroskedasticity in the data indicating constant variance of the residuals. Lets continue with another test

Auto-Correlation test

Autocorrelation can lead to inefficient estimates and biased statistical tests, making the Durbin-Watson statistic an important diagnostic tool for time series data.It is used to detect autocorrelation in the residuals of a regression model, which occurs when residuals are correlated across observations.

Durbin–Watson d-statistic(5, 20) = 1.733364

. scalar rho = 1-(1.733364/2)

0.133318

The Durbin-Watson statistic of 1.733364 suggests a positive autocorrelation, as it is closer to 2, indicating no strong autocorrelation in the residuals. The calculated value of rho (0.1333) further confirms this, as it is a small positive value, indicating weak positive autocorrelation.

So the model TxdechanBra= B1* CrPib + B2* solde_com + B3*TxdinteretUSA + B4* TxdinteretBra + cons has no multicollinearity ,no sign of herocesdaticity and no sign of auto-correlation.A quasi-perfect model.

Show Test

In 2010, Brazil surpassed the United Kingdom to become the 6th largest global economy, marking a peak in its economic growth. However, after 2011, the country faced a decline due to various factors, including decreasing demand for raw materials and a shift in China's economic strategy. This period was followed by a significant economic and political crisis, which worsened in 2014 with corruption scandals. By 2015, Brazil entered a deep recession, experiencing sharp declines in exports and GDP. Given this dramatic shift in the country's economic trajectory, 2010 is a logical choice as a breakpoint for the Chow test, reflecting the onset of a period of both growth and subsequent decline. We'll now perform a Chow test considering the year 2010 as a break, with a regression after 2010 and before to test these arguments.

The Chow test is used to determine whether there is a structural break in a time series dataset, typically by comparing two different time periods or groups to see if the relationship between variables changes.

We first define a break point (2010) by creating a binary variable

We Run regressions for the two periods (before and after 2010):

We Store the regression results In A and B

Compare the two regression models with suest

Test for structural break

The null hypothesis (H_0) is that the coefficients from the two periods are the same (no structural break).

The alternative hypothesis (H_1) is that at least one coefficient is different between the two periods.

The Prob > chi2 = 0.0000 show that the test statistic is significant at the 0.05 level. That confirms the break

```
***Chow test considering the year 2010 as a break
/* **Chow test considering the year 2010 as a break */


. gen break = Année > 2010
variable break already defined
r(110);

. gen break1 = Année> 2010

. gen avant_break = Année <= 2010


**Regression for the period after 2010
/* *Regression for the period after 2010 */

. regress TxdechanBra solde_com CrPib TxdinteretUSA TxdinteretBra if break1 == 1

      Source |       SS           df       MS      Number of obs   =        10
-------------+----------------------------------   F(4, 5)         =     13.45
       Model |  9.38440784         4  2.34610196   Prob > F        =    0.0069
    Residual |  .872447659         5  .174489532   R-squared       =    0.9149
-------------+----------------------------------   Adj R-squared   =    0.8469
       Total |  10.2568555         9  1.13965061   Root MSE        =    .41772

------------------------------------------------------------------------------
 TxdechanBra | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
   solde_com |   2.82e-11   7.46e-12     3.78   0.013     9.03e-12    4.74e-11
       CrPib |  -.0190147   .0122661    -1.55   0.182    -.0505458    .0125164
TxdinteretUSA |   .6759994   .2508877     2.69   0.043     .0310719    1.320927
TxdinteretBra |  -.0693651   .0275133    -2.52   0.053    -.1400903    .0013602
       _cons |   4.195997   .9816559     4.27   0.008      1.67257    6.719424
------------------------------------------------------------------------------
```

```
'Store the estimated coefficients
'* Store the estimated coefficients

*/
 estimates store A



 ***Regression before 2010
 regress TxdechanBra solde_com TxdinteretUSA TxdinteretBra if avant_break == 1

      Source |       SS           df       MS      Number of obs   =        11
-------------+----------------------------------   F(3, 7)         =     18.81
       Model |  2.10148763         3  .700495875   Prob > F        =    0.0010
    Residual |   .26061866         7  .037231237   R-squared       =    0.8897
-------------+----------------------------------   Adj R-squared   =    0.8424
       Total |  2.36210629        10  .236210629   Root MSE        =    .19295

------------------------------------------------------------------------------
 TxdechanBra | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
   solde_com |   5.23e-12   3.43e-12     1.52   0.172    -2.89e-12    1.33e-11
xdinteretUSA |  -.1945838   .0401645    -4.84   0.002    -.2895578   -.0996099
xdinteretBra |   .0567754   .0101161     5.61   0.001     .0328546    .0806962
       _cons |   .6161933   .3976589     1.55   0.165    -.3241205    1.556507
------------------------------------------------------------------------------


 estimates store B
```

We need to compare the two regressions models to make a conclusion, and we'll make two hypotheses H0, the coefficients are identical in both regression processes, and H1, at least one coefficient is different.

```
**Compare the 2 regression models
/* *Compare the 2 regression models */
. suest A B

Simultaneous results for A, B                            Number of obs = 20

------------------------------------------------------------------------------
             |               Robust
             | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
A_mean       |
   solde_com |   2.82e-11   6.02e-12     4.69   0.000     1.64e-11    4.00e-11
       CrPib |  -.0190147   .0041989    -4.53   0.000    -.0272444    -.010785
TxdinteretUSA |   .6759994   .1324585     5.10   0.000     .4163855    .9356133
TxdinteretBra |  -.0693651   .0143074    -4.85   0.000     -.097407   -.0413232
       _cons |   4.195997   .5895533     7.12   0.000     3.040494      5.3515
-------------+----------------------------------------------------------------
A_lnvar      |
       _cons |  -1.745891   .2382229    -7.33   0.000    -2.212799   -1.278982
-------------+----------------------------------------------------------------
B_mean       |
   solde_com |   6.25e-12   3.92e-12     1.60   0.110    -1.43e-12    1.39e-11
       CrPib |  -.0047093   .0054793    -0.86   0.390    -.0154485    .0060299
TxdinteretUSA |  -.1677973   .0373483    -4.49   0.000    -.2409987    -.094596
TxdinteretBra |   .0523494   .0152167     3.44   0.001     .0225253    .0821735
       _cons |   .7823852   .6798076     1.15   0.250    -.5500132    2.114784
-------------+----------------------------------------------------------------
B_lnvar      |
       _cons |  -3.278898   .2185157   -15.01   0.000    -3.707181   -2.850615
------------------------------------------------------------------------------

.**H₀ (first hypothesis) : the coefficient are identical in both regression process.

**H₁ (alternative) : At least one coefficient is different.
/* *H₁ (alternative) : At least one coefficient is different.  */
```

```
. test [A_mean= B_mean]

 ( 1)  [A_mean]solde_com - [B_mean]solde_com = 0
 ( 2)  [A_mean]CrPib - [B_mean]CrPib = 0
 ( 3)  [A_mean]TxdinteretUSA - [B_mean]TxdinteretUSA = 0
 ( 4)  [A_mean]TxdinteretBra - [B_mean]TxdinteretBra = 0
       Constraint 1 dropped

           chi2(  3) =   75.88
         Prob > chi2 =    0.0000

. * There is a break observable in 2010 we reject H0
```

A break is observable in 2010 we reject H0, so we'll consider this break in 2010 by introducing a dummy variable that takes the value of 1 for observations after 2010 and 0 for those before, and the interactions to test if the effect of the explanatory variables changes before and after the break.

```
. gen ruptur2010 = (Année >= 2010)

*** Take into account the interactions to test if the effect of the explanatory
variables changes before and after the break.
/* **Take into account the interactions to test if the effect of the explanatory
variables changes before and after the break. */
. gen interaction2010 = solde_com * ruptur2010

. gen interaction1=ruptur2010*CrPib
(1 missing value generated)

.  gen interaction2=ruptur2010*TxdinteretUSA

.  gen interaction3=ruptur2010*TxdinteretBra

.


. reg TxdechanBra CrPib solde_com  TxdinteretUSA TxdinteretBra interaction2010
interaction1 interact
> ion2 interaction3 ruptur2010

      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(9, 10)        =     13.67
       Model |  14.0341057         9  1.55934508   Prob > F        =    0.0002
    Residual |   1.1405441        10   .11405441   R-squared       =    0.9248
-------------+----------------------------------   Adj R-squared   =    0.8572
       Total |  15.1746498        19   .79866578   Root MSE        =    .33772

------------------------------------------------------------------------------
    TxdechanBra | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
----------------+-------------------------------------------------------------
          CrPib |  -.0121956   .0181691    -0.67   0.517    -.0526789    .0282877
      solde_com |   1.43e-11   1.83e-11     0.78   0.453    -2.65e-11    5.51e-11
  TxdinteretUSA |  -.1633692   .1038165    -1.57   0.147    -.3946867    .0679483
  TxdinteretBra |   .0458789   .0368502     1.25   0.242    -.0362284    .1279862
interaction2010 |   1.33e-11   1.93e-11     0.69   0.506    -2.96e-11    5.62e-11
   interaction1 |  -.0139319   .0194862    -0.71   0.491    -.0573498     .029486
   interaction2 |   .7832375   .2211002     3.54   0.005     .2905955     1.27588
   interaction3 |  -.1071759   .0423085    -2.53   0.030    -.2014451   -.0129067
     ruptur2010 |   2.981618   1.887132     1.58   0.145    -1.223175     7.18641
          _cons |   1.032245   1.721438     0.60   0.562    -2.803358    4.867848
------------------------------------------------------------------------------
```

We remove the insignificant variables and then perform a multicollinearity test for the new model.

```
. reg TxdechanBra CrPib solde_com  TxdinteretUSA   interaction2 interaction3

      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(5, 14)        =     25.79
       Model |  13.6887506         5  2.73775013   Prob > F        =    0.0000
    Residual |  1.48589919        14  .106135656   R-squared       =    0.9021
-------------+----------------------------------   Adj R-squared   =    0.8671
       Total |  15.1746498        19  .79866578    Root MSE        =    .32578

------------------------------------------------------------------------------
 TxdechanBra | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       CrPib |    -.0262   .0049255    -5.32   0.000    -.0367642   -.0156358
   solde_com |  2.42e-11   3.47e-12     6.99   0.000     1.68e-11    3.17e-11
TxdinteretUSA | -.2581868   .0849947    -3.04   0.009    -.4404824   -.0758912
 interaction2 |  .9612298   .1740229     5.52   0.000     .5879879    1.334472
 interaction3 | -.0472991   .0128969    -3.67   0.003    -.0749602    -.019638
       _cons |  3.343123    .286783    11.66   0.000     2.728035    3.958211
------------------------------------------------------------------------------
```

This regression shows a strong overall model fit with an R-squared of 0.9021, meaning about 90% of the variation in the exchange rate (TxdechanBra) is explained by the independent variables. All coefficients are statistically significant at the 1% level, indicating a robust relationship between the exchange rate and factors such as trade balance (solde_com), interest rates, and the interactions representing structural shifts. Notably, the positive coefficient of interaction2 and negative of interaction3 suggest a structural change in how U.S. and Brazilian interest rates impact the exchange rate after the 2010 break.In addition,the model is statiscally significant (0.0000)

Multicolinearity Test

```
. vif

    Variable |       VIF       1/VIF  
-------------+----------------------
interaction3 |      7.95    0.125780
interaction2 |      7.52    0.132902
Txdinteret~A |      1.68    0.596699
   solde_com |      1.49    0.671147
       CrPib |      1.34    0.746569
-------------+----------------------
    Mean VIF |      4.00
```

Two variables are the cause of the multicollinearity: a result that we were expecting since the new variables that we added to the model are deduced from previous ones (they're the product of former variables by the indicator variable) so we have a problem of imperfect multicollinearity.

Let's perform a heteroskedasticity test.

Heteroskedasticity Test

```
. estat hettest

Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: Fitted values of TxdechanBra

H0: Constant variance

    chi2(1) =   0.25
Prob > chi2 = 0.6202

. estat imtest, white

White's test
H0: Homoskedasticity
Ha: Unrestricted heteroskedasticity

   chi2(18) =  19.88
Prob > chi2 = 0.3398

Cameron & Trivedi's decomposition of IM-test

--------------------------------------------------
              Source |       chi2     df         p
---------------------+----------------------------
  Heteroskedasticity |      19.88     18    0.3398
            Skewness |       8.33      5    0.1392
            Kurtosis |       0.00      1    0.9499
---------------------+----------------------------
               Total |      28.21     24    0.2514
--------------------------------------------------
```

Both the Breusch–Pagan and White tests show high p-values (0.6202 and 0.3398, respectively), indicating we fail to reject the null hypothesis of homoskedasticity. This means there is **no evidence of heteroskedasticity** in the model, and the assumption of constant variance of residuals is upheld.

Auto-Correlation Test

```
. estat dwatson

Durbin-Watson d-statistic(  6,    20) =  1.997006

. scalar rho = 1-(1.997006/2)

. display rho
.001497
```

The Durbin–Watson statistic is approximately 2 (1.997), which is very close to the ideal value indicating no first-order autocorrelation in the residuals. The calculated ρ (rho) is near zero (0.0015), confirming **the absence of serial correlation in the model.**

The Final model is : TxchangeBra=B1*CrPibBra +B2* interaction2 + B3*interaction3 + B4*solde_com + B5txdinteretUSA + cons

III. Conclusion

The final model identifies key macroeconomic variables—such as Brazil's GDP growth, interest rate differentials, and trade balance—as significant drivers of the Brazilian exchange rate. The inclusion of interaction terms highlights structural changes in relationships after 2010, capturing the evolving economic context. Overall, the model is statistically robust and consistent, with no signs of heteroskedasticity or autocorrelation, confirming the reliability of the estimations. These policy measures could help mitigate the exchange rate volatility and contribute to economic stability [4].

1. Limits

Despite the insights gained from the econometric analysis, there are several limitations that should be considered when interpreting the results. These limitations primarily stem from the dataset, the time period under consideration, and the factors not included in the model. Firstly, the sample size in this analysis is relatively small, and using quarterly data instead of annual data would likely provide a more accurate reflection of exchange rate fluctuations. Unfortunately, quarterly data was not readily available for the entire period of interest. Moreover, while the model includes economic variables such as interest rates, exports, and GDP, it excludes financial and psychological factors that also influence exchange rates. Market liquidity, volatility, and investor sentiment are not captured in this analysis, yet these factors significantly impact exchange rate behavior. Additionally, the period under study covers a time of significant political and economic changes in Brazil. In 2010, Brazil became the 6th largest economy in the world, but by 2015, it was downgraded to the 9th largest due to a series of economic crises, including the global decline in commodity demand, falling export prices, and internal political instability. The Brazilian economy faced a significant downturn in the years that followed, characterized by recession, rising inflation, and increased unemployment. The political crisis, marked by corruption scandals, further destabilized the economy. These factors have had profound impacts on the Brazilian Real, making it difficult to fully explain the currency's volatility with just the economic variables included in this analysis. While the econometric model provides valuable insights, it cannot capture the full complexity of the Brazilian economic environment during this period [3].

IV. References

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