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ANALISYS OF HOSPITAL BED CAPACITY VIA QUEUING THEORY AND SIMULATION

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ABSTRACT

Eighty percent of Brazilian population is assisted by the public health system (SUS), while the rest uses private health insurance. The existing number of beds do not meet the entire demand and Brazilian government has made efforts to improve the system. The Health Ministry has analyzed a new proposal for planning the bed capacity. This proposal uses a queuing model and a new categorization of hospitalization and beds, taking into account specialties, age of the patients, and appropriate occupancy rates. In this paper, we use the proposed technique to estimate the required beds for Belo Horizonte, a mid-sized city in Brazil. We compare number of beds required by centralized and non-centralized administration. A simulation model was developed to analyze the dynamic behavior of the system and searching for the best configuration. This model was used to evaluate the results obtained by queuing model and to check its usability.

1 INTRODUCTION

Belo Horizonte has a population around 2.5 million inhabitants . The city has several public hospitals that admit patients from the metropolitan area and from other cities of the state of Minas Gerais, as a regional referral health.

SUS has a Central Admissions that manages part of the demand. The Central manages the hospitalization of approximately 39% of patients. The Central Admissions works as a single-server queuing system. All hospitals are gathered and can be treated as a single bed pool. Centralization tends to improve the service performance, i.e., low refusal and high utilization rates. However, the centralization management is not comprehensive, and about 61% of patients by-pass the central because there is an agreement between Central Admissions and the major hospitals of the city. This fact decrease the performance of the system, causing increased queues, large waiting times, high rates of refusal, and hospitalizations in poor condition, i.e., patients in the corridor of the wards and patients hospitalized in rooms of Emergency Departments.

The number of beds required is estimated based on governmental guidelines. These guidelines are driven by simplistic assumptions, such as average length of stay (LoS), unique arrival rate of patients, and occupancy rate from 80% to 85%. Other empirical ratio are used, such as a ratio from 2.5 to 3.0 beds per 1,000 inhabitants. Guidelines do not consider differences in LoS for each hospitalization type or by age group. Specific arrival rates for each type of hospitalization or seasonal effects are not taking into account. These practices leads to overcrowding of the system.

The Ministry of Health is currently revising a new proposal for estimating the bed capacity in order to improve the system performance. This proposal is a simple technique that uses queuing model to estimate the bed capacity for each specialty. The technique taking into account the size of the hospitals, the multiple arrival rates of patients and LoS for each specialty. If the Health Ministry approve these guidelines, they will be used to estimate the bed capacity for all Brazilian cities .

In this work, a simulation model was developed to analyze the SUS system of Belo Horizonte, a mid-sized city located in southeastern Brazil. A simulation-optimization was implemented to search for scenarios with low refusal rate and minimum level of quality. The simulation model is also used to evaluate the results obtained by queuing model and to check its usability in the system of this city. This can help the government to further investigate the new proposal.

The paper is organized as follows: section 2 presents a brief overview of the use of simulation and queuing theory for solving problems of bed capacity planning, section 3 details the current system of the city, section 4 presents the proposed method and the simulation model, section 5 discusses the results and, section 6 presents a short conclusion.

2 QUEUING MODELS AND SIMULATION APPLIED TO HOSPITAL PLANNING

The required number of hospital beds to meet demand is a recurrent problem in health management. Therefore, concerns on hospital capacity planning is an old problem in healthcare (Newshome 1933, Hardie 1959) as well as the use of Operations Research (OR) techniques to solve that problem. Surveys and reviews of OR applications in healthcare often address the subject of queuing theory and simulation in order to estimate the number of hospital beds (Rais and Viana 2010, Fomundam and Herrmann 2007, Gunal and Pidd 2010, Lakshmi and Appa Iyer 2013). In a quick literature review, we can find several works that have used queuing theory and simulation models in planning and management of bed capacities.

Gorunescu, McClean and Millard (2002) proposed a queuing model for bed-occupancy management and optimization. That work shows how a queuing model may be used to improve the hospital management. The work analyze bed occupancy and the probability of refusal admission. They illustrated the methodology using data from St. George’s Hospital, London, UK.

Jones (2009) discussed a variety of factors that affect the hospital bed planning and emergency admissions. In other work, Jones (2011) explored the relation between occupancy rates and size of the hospital. That work shows that smaller hospitals or bed pools should operate at lower average occupancy to yield similar service quality of the larger hospitals. The author uses queuing theory to explain this direct connection. The level of patients “turn-away” related to occupancies is also explored in the paper.

Harper and Shahani (2002) developed a simulation model for the planning and management of hospital beds. They built the model for use by The Royal Berkshire and Battle Hospitals NHS Trust, Reading, UK. The work analyzes both bed occupancies and refusal admission rates. Authors state that the model was also used for several needs of both hospitals.

Holm, Lurås, and Dahl (2011) developed a Discrete Event Simulation (DES) model and an optimization algorithm to allocate beds among hospital wards in order to minimize crowding. The DES model describe the patient flow through wards and the output of DES model is an utilization bed matrix, which is the input of optimization algorithm that allocates the available beds among the wards. The model was used in a Norwegian hospital.

Kokangul (2008) also developed a combination of deterministic and stochastic approach to optimize the bed capacity of a Pediatric Intensive Care Unit of a teaching hospital.

Kumar (2011) proposes a simulation modeling of patient flow to optimize the number of hospital beds. In this work, he proposes six scenarios involving emergency and routine admissions in a county hospital in USA.

Zhu, Hen, and Teow (2010) state that typical queuing model is not able to describe complex workflows such as patient flows of the Intensive Care Unit (ICU) systems. Then, they proposed a DES model to estimate ICU bed capacity. The authors use the DES model to analyze the ICU system of a Singapore Government Hospital.

3 THE PUBLIC HOSPITALIZATION SYSTEM OF BELO HORIZONTE

The public hospitalization system of Belo Horizonte does not work fully centralized, i.e., only 39% of the patients pass through the Central Admissions and the rest demand hospitals directly. The goal is that all patients pass through the central, but currently it is not possible due to: i) logistical problems; ii) lack of a centralized information system and; iii) lack of a quantitative analysis of the gains of centralization.

SUS in this city had in 2012, 38 hospitals and 6021 beds. This number of beds was not sufficient to meet the entire demand and many patients have to wait in queue or they were admitted precariously.

The system officially registered 224,455 admissions in 2012. According to the Central Admissions, 65,259 patients could not be admitted or they were admitted in precarious situations. Thus, we conclude that the annual demand for admissions was 289,704 patients, of which 29.07% can be considered refusal, as there was no official admission registration.

Managers know that there is a need for increased number of beds and changes in system management, but there is a lack of quantitative analysis. This work aims to provide a deeper quantitative analysis of the system.

4 THE PROPOSED METHOD

The authors of this paper have participated in a multidisciplinary group of experts, who proposed to the Brazilian Ministry of Health, in 2013, a new approach to planning and analysis of public hospital bed capacity (Campos et al. 2013). This proposal change the current rules to calculate bed capacity for Brazilian public hospitals. The proposed technique includes new categories of hospital beds and hospitalization, which are based on various characteristics related to age, illness, hospital procedures, arrival rates, LoS, and seasonality.

The number of beds is estimated using queuing theory and occupancies can vary according to hospital sizes (Jones 2011). The choice for queuing models lies in the fact that this modeling is simple and can easily be applied by managers of several Brazilian cities. The occupancy rate also varies with the specialty. The description of the categories of hospital beds and occupancy rates is described in the subsection 4.1.

4.1 Types of beds and hospitalization

Hospital beds and hospitalization were divided into the following types:

Standard beds:

- Neonatal beds: beds for neonatal admissions (0 – 28 days old);
- Pediatric beds: beds for pediatric admissions (28 days – 14 years old);
- Adults beds: beds for adults admission (≥ 15 years old);
- Pediatric surgical beds: beds for pediatric surgical admissions (0 – 14 years old); .
- Adults surgical beds: beds for adults surgical admissions (≥ 15 years old);
- Obstetric beds: beds for obstetrical procedures;

- Psychiatric beds: beds for patients admitted in psychiatric hospitals;
- Other beds: beds for patients with chronic diseases, patients in rehabilitation and others.

ICU beds:

- Neonatal ICU beds: beds for neonatal ICU admissions (0 – 28 days old);
- Pediatric ICU beds : beds for pediatric ICU admissions (28 days – 14 years old);
- Adults ICU beds: beds for adults ICU admissions (≥ 15 years old).

4.2 Appropriate occupancy rates

According to Jones (2011) the average occupancy rate depends on the size of hospital beds or pool of beds. However, occupancy rate must not exceed 90% to avoid high refusal rates. Moreover, maximum occupancy rates between 82% – 85% are required in order to maintain a safe acquired infection level. Figure 1 shows occupancy rates for insignificant refusal rates, i.e., less than 0.1%.

For hospitals up to 100 beds, occupancy rate, for refusal rate less than 0.1%, can be calculated by equation (1). For hospitals over 100 and less than 1,000 beds we must use equation (2). Both equations were obtained by regression analysis on the data showed in Figure 1.

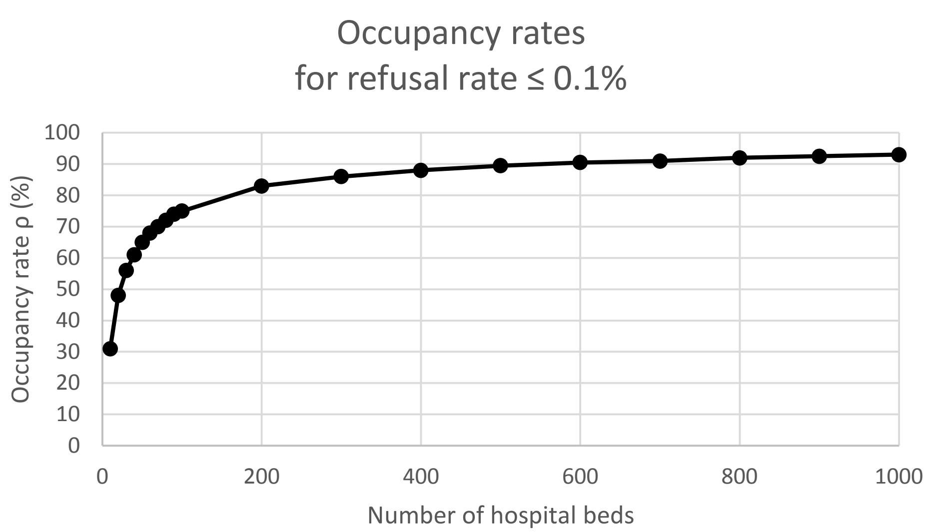

Figure 1 – Occupancy rates for refusal rate ≤ 0.1% (Jones 2011).

$$\rho = 18.726 \ln(NB) - 9.4039 \tag{1}$$

$$\rho = 7.4508 \ln(NB) + 42.484 \tag{2}$$

In equations (1) and (2) **ρ** is occupancy rate (%) for refusal rate less than 0.1% and ***NB*** is the number of hospital beds. Jones (2011) also presents curves for refusal rates between 0.1% and 20%.

4.3 Required number of hospital beds

If we suppose that the process of hospitalization is a Poisson process, we can determine the number of beds required for each specialty ***s*** using equation (3).

$$NB_s = \frac{\lambda_s . T_s}{365.\rho_s} \tag{3}$$

In equation (3), all parameters are related to the specialty $\boldsymbol{s}$, where $\boldsymbol{NB_s}$ is the number of required beds, $\boldsymbol{\lambda_s}$ is the annual demand, $\boldsymbol{T_s}$ is the average LoS (days) and $\boldsymbol{\rho_s}$ is the desired occupancy rate for each hospital or bed pool.

It is important note that physical availability of beds must be taken into account, because, in Brazil, some beds are not available all the time for public admissions. Beds may be unavailable during a period of the time and we must compute the utilization only during the time in which the beds are available for SUS. Thus, the occupancy rate $\boldsymbol{\rho_s = F.\rho_r}$, where $\boldsymbol{F}$ is the physical availability of beds and $\boldsymbol{\rho_r}$ is the effective utilization.

Using equation (3) we can estimate, for different occupancy rates, the required number of beds for each specialty. Hospitals can be divided into pools of beds for each specialty. Consequently, the same hospital may have different sizes for each specialty and therefore different values of $\boldsymbol{\rho_s}$. On the other hand, if there is a centralized administration to manage multiple hospitals, pools of beds for each specialty can include beds of several hospitals. Section 5 shows the results of applying equations (1) , (2) and (3) assuming a centralized and non-centralized admission.

4.4 The simulation model

In this work, a simulation model was developed in order to analyze the dynamic performance of the system. Moreover, a simulation-optimization model was also implemented to search for good feasible solutions, i.e., good configuration for the system. The purpose of simulation model is analyzing scenarios under centralized admission management and comparing the results against the current scenario and those obtained by the queuing model.

The model has two modules: i) the arrival of patients and their distribution by hospitalization category; ii) the hospitalization of patients.

Input data cover all months of 2012. All data were obtained from the databases of the Ministry of Health and the Central Admissions of the city.

4.4.1 Patients arrival and screening

As there are no records relating to arrival times of patients to hospitals or to the Central Admissions, we assume that the arrival is a Poisson process, i.e., the inter-arrival time follows an exponential distribution. The hourly arrival rate of patients of each specialty is obtained by dividing the annual number of hospitalizations by 8,760h. The number of admissions for each specialty is shown in Table 1.

The total demand is the total number of admissions increased by a factor of 1.2907 in order to include refusals, which are 29.07% of official admissions, according to data collected in the Central Admissions. Therefore, the total demand in 2012 was 289,704. This demand was used to calculate the daily arrival rate of patients, $\boldsymbol{\lambda}$ = 793.71.

Table 1 – Admissions in public hospitals (SUS) – Belo Horizonte (2012).

Type of hospitalization	**Number of admission (H_s)**	**%**	**Patients distribution (%)**		
			non-ICU Hosp	**ICU Hosp**	**Direct ICU Hosp**
Neonatal	7,078	3.15	59.38	21.72	18.90
Pediatrics (≤ 14 yo)	18,921	8.43	91.52	4.75	3.73
Adults (15 – 59 yo)	35,422	15.78	85.88	6.78	7.33
Adults (≥ 60 yo)	26,245	11.69	82.85	8.07	9.08
Pediatric Surg. (≤ 14 yo)	8,857	3.95	86.60	7.25	6.15
Adult Surg. (15 – 59 yo)	58,514	26.07	87.71	7.09	5.20
Adult Surg. (≥ 60 yo)	24,550	10.94	74.35	15.83	9.82
Obstetrics	32,567	14.51	98.94	0.70	0.36
Psychiatry	5,233	2.33	100.00	0.00	0.00
Other (chronic, rehab.)	7,068	3.15	99.75	0.08	0.17
Total	**224,455**	**100.00**	**---**	**---**	**---**

4.4.2 Hospitalization

Admissions were separated by specialties as we can see in Table 1. We have divided the admissions into three main groups:

- Regular admissions that not require later admission in ICU;
- Admissions that require a later admission in ICU;
- Admissions directly in ICU.

The distribution of patients in these three groups is shown in last three columns of Table 1. Patients are entities that arrive in the system according to the rate $\boldsymbol{\lambda}$ and receive attributes that indicate the main group and the specialty of the admission. After arrival in the system, the patients are admitted or refused if there is no free bed. The LoS for each specialty is shown in Table 2.

The simulation model has considered all hospital beds as a pool, i.e., the model simulate a centralized admissions system. The number of each type of bed is shown in Table 3. This table also show the annual demand for each type of bed. This demand includes refusals. In fact, the total demand for hospitalizations shown in Table 3 (310,182) corresponds to 289,704 patients because some of them use both standard and ICU beds.

Table 2 – Length of Stay (LoS).

Type of Hospitalization	Length of Stay (days)		Type of Hospitalization	Length of stay (days)	
	Avg	Expression		Avg	Expression
Neonatal	6.16	LogN(6.19, 5.34)	ICU Neonatal	12.50	0.5 + Exp(12)
Pediatrics (≤ 14 yo)	4.90	Exp(4.9)	ICU Pediatrics (≤ 14 yo)	11.40	0.5+97*Beta(0.611, 4.55)
Adults (15 – 59 yo)	8.63	Exp(8.63)	ICU Adults (15 – 59 yo)	7.70	Exp(7.7)
Adults (≥ 60 yo)	10.30	Gamma(7.88, 1.3)	ICU Adults (≥ 60 yo)	8.80	Exp(8.8)
Pediatric Surg. (≤ 14 yo)	2.86	Exp(2.86)	ICU Ped. Surg (<= 14 yo)	6.56	0.5+Weib(5.24, 0.787)
Adult Surg. (15 – 59 yo)	3.63	Exp(3.63)	ICU Adult Surg (15 – 59 yo)	5.30	Exp(5.3)
Adult Surg. (≥ 60 yo)	4.43	Exp(4.43)	ICU Adult Surg (≥ 60 yo)	4.67	97*Beta(0.411, 8.13)
Obstetrics	2.49	Gamma(1.08, 2.31)	ICU Obstetrics	2.45	0.5+Weib(2.14, 1.33)
Psychiatry	15.60	0.5+45*Beta(0.834, 1.65)	Other ICU (chronic,rehab.)	10.55	0.5+LogN(11.2, 18.7)
Other (chronic, rehab.)	12.95	45*Beta(0.359, 0.889)	**ICU = Intensive Care Unit**		

Table 3 – Current number of beds and demand (2012).

Type of bed	Number of beds	Annual Demand (λ_s)
Neonatal	166	7,409
Pediatrics	556	23,511
Adults	1,788	73,166
Pediatrics Surgery	130	10,728
Adults Surgery	1,628	100,171
Obstetrics	282	41,883
Psychiatry	419	6,754
Other	306	9,107
ICU Neonatal	154	3,711
ICU Pediatrics	99	3,602
ICU Adults	493	30,139
Total	**6,021**	**310,182**

4.4.3 Simulation-optimization

To find the best configuration for the system, we use a classical simulation-optimization strategy (Fu 2002). The decision variables are the number of beds for each specialty ($\boldsymbol{NB_s}$), which are resources of the system. The objective is finding a set of input variables ($\boldsymbol{NB_s}$) that minimize the expected value of the objective function. As the analysis of the objective function as well as the constraints can only be made via simulation, this model is not a model of deterministic optimization, i.e, is not an optimization model in the formal sense. Thus, the best solution means good feasible solution, because we do not have guarantee of optimality.

The simulation-optimization model can be written as follows:

$$\text{Min } N = \sum_{s=1}^{n} NB_s \tag{4}$$

Subject to constraints:

$$R_s \leq 0.001 \quad \forall\, s \tag{5}$$

$$P_S \geq 1.2907 H_s \qquad \forall\, s \tag{6}$$

$$NB_s \in Z^+ \quad \forall\, s \tag{7}$$

Equation (4) represents the objective, which is to minimize the total number of beds $\boldsymbol{N}$. Constraint (5) limits the rate of refusal by specialty (R_s) up to 0.1%. Constraint (6) requires that the annual number of patients ($\boldsymbol{P_s}$) admitted in each specialty be at least the value shown in Table 1 ($\boldsymbol{H_s}$) plus refusals (29.07%). Constraint (7) requires that the number of beds must be non-negative integer. Actually, after a few rounds of trials we can also establish upper and lower bounds for each $\boldsymbol{NB_s}$ to reduce the time to solve the problem.

5 ANALYSIS AND RESULTS

5.1 Number of beds estimated by queuing model

If we consider that the process of admission is a Poisson process, we can use equations (1) and (2) to determine the appropriate occupancy rate $\boldsymbol{\rho_s}$ for each specialty. For this purpose, we consider the average number of beds in each specialty in the city hospitals. We considered only hospitals over 10 beds. Hospitals up to 10 beds were ignored because they are not representative in this context. The results are shown in Table 4.

Table 4 – Average number of beds and appropriate occupancy rate for current system.

Type of bed	Number of beds (2012)	Number of Hospitals (≥ 10 beds)	Average number of beds	ρ_s (%)
Neonatology	166	5	33	56.18
Pediatrics	556	10	56	65.84
Adults	1,788	17	105	77.17
Pediatrics Surgery	130	4	33	55.79
Adults Surgery	1,628	21	78	74.90
Obstetrics	282	7	40	59.81
Psychiatry	419	4	105	77.70
Other	306	4	77	71.82
ICU Neonatology	154	6	26	51.37
ICU Pediatrics	99	7	14	40.21
ICU Adults	493	13	38	58.68

If we use the average LoS $\boldsymbol{T_s}$, demands $\boldsymbol{\lambda_s}$, occupancy rates $\boldsymbol{\rho_s}$, respectively shown in Table 2, 3 and 4, we can calculate the appropriate number of beds using equation (3). The results are shown in Table 5.

Table 5 – Number of beds for current system (non-centralized admissions).

Type of bed	λ_s	T_s (days)	Ideal ρ_s (%)	NB_s	**Number of beds (2012)**	ρs (%) (2012)
Neonatology	7,409	6.16	56.18	223	166	58.36
Pediatrics	23,511	4.90	65.84	479	556	43.98
Adults	73,166	9.33	77.78	2,404	1,788	81.01
Pediatrics Surgery	10,728	2.86	55.79	151	130	50.10
Adults Surgery	100,171	3.84	72.07	1,462	1,628	50.15
Obstetrics	41,883	2.49	59.81	478	282	78.50
Psychiatry	6,754	15.60	77.70	372	419	53.38
Other	9,107	12.95	71.82	450	306	81.81
ICU Neonatology	3,711	12.50	51.37	247	154	63.93
ICU Pediatrics	3,602	9.34	40.21	229	99	72.15
ICU Adults	30,139	6.28	58.68	884	493	81.50
Total or Avg	**310,182**	**5.98**	**68.54**	**7,379**	**6,021**	**65.45**

As we can see in Table 5, the number of required beds to meet entire demand efficiently is 7,379 beds. In this calculation, we consider a non-centralized administration, i.e., the current scenario. Thus, the system require more 1,358 beds to meet the entire demand and finish the current refusal.

If we suppose a centralized administration, the beds of all hospitals could be treated as a pool. This change tends to increase the occupancy rate, without compromising the quality of service. Therefore, centralization could achieve similar results by using a smaller number of beds. In centralized administration, each specialty is a pool of beds, which have at least 100 beds in the system of Belo Horizonte. Thus, we can consider a minimum occupancy rate ρ_s = 76.80% for all specialties. The results are shown in Table 6.

Table 6 – Number of beds for centralized admission.

Type of bed	λ_s	T_s (days)	Ideal ρ_s (%)	NB_s	**Number of beds (2012)**	ρ_s (%) (2012)
Neonatology	7,409	6.16	76.80	163	166	58.36
Pediatrics	23,511	4.90	76.80	411	556	43.98
Adults	73,166	9.33	76.80	2,434	1,788	81.01
Pediatrics Surgery	10,728	2.86	76.80	109	130	50.10
Adults Surgery	100,171	3.84	76.80	1,372	1,628	50.15
Obstetrics	41,883	2.49	76.80	372	282	78.50
Psychiatry	6,754	15.60	76.80	376	419	53.38
Other	9,107	12.95	76.80	421	306	81.81
ICU Neonatology	3,711	12.50	76.80	165	154	63.93
ICU Pediatrics	3,602	9.34	76.80	120	99	72.15
ICU Adults	30,139	6.28	76.80	675	493	81.50
Total or Avg	**310,182**	**5.98**	76.80	**6,618**	**6,021**	**65.45**

The results in Table 6 show that system needs 597 more beds to operate the system efficiently. If the government choose this type of hospital administration the increased number of beds will be roughly the half number required by non-centralized administration.

5.2 Number of bed estimated by simulation model

The simulation model developed for SUS of Belo Horizonte considered only centralized administration, as non-centralized administration is already used by the current system. The model was used to analyze system performance and estimating the optimal number of beds to meet entire demand. The results are compared with results obtained by queuing theory.

The simulation model was implemented in the Arena Rockwell Software and simulation-optimization was implemented in the same software using OptQuest for Arena .

Firstly, the OptQuest ran 1,500 simulations to find a good feasible solution. Secondly, we executed simulation rounds with 40 replications (length = 1 year and warm-up = 3 months) to refine the candidate solutions. The best solution is shown in Table 7. As we can see, it is possible to operate the system using only 6,000 beds (approximately the current number), but the values of occupancies rates are very high for some specialties.

Table 7 – Number of beds for refusal rates $\leq 0.1\%$.

Type of bed	**Number of beds (2012)**	$\boldsymbol{NB_s}$	$\boldsymbol{\rho_s}$ **(%)**	***CI*** **(α = 0.05)**
Neonatology	166	170	75.07	±0.4137
Pediatrics	556	390	82.26	±0.2323
Adults	1,788	2,000	93.93	±0.1265
Pediatrics Surgery	130	390	71.74	±0.2989
Adults Surgery	1,628	1,160	93.20	±0.1263
Obstetrics	282	380	77.78	±0.1450
Psychiatry	419	320	92.34	±0.3144
Other	306	380	89.82	±0.4303
ICU Neonatology	154	200	64.11	±0.4255
ICU Pediatrics	99	180	51.32	±0.4357
ICU Adults	493	700	74.57	±0.2496
Total	**6,021**	**6,000**	**---**	**---**

Thus, we ran the system again including the constraint (8), which limits the utilization rates at 77%, which is approximately the rate of Europe and USA.

$$\rho_s \leq 0.77 \qquad \forall\, s \tag{8}$$

This occupancy rate tends to result in a refusal less than 0.1%, as the pools of beds for each specialty have at least 100 beds in Belo Horizonte. At this time, OptQuest ran 2,850 simulations and the best solutions were also refined. A best solution is shown in Table 8.

The simulation model found a configuration for centralized administration that require more 783 beds than the current system. The difference between results from queuing model and the simulation is 186 beds. This difference can be attributed to the fact that the simulation model represents most faithfully the system, considering its dynamic behavior and making a deeper analysis of it. The difference between the total number of beds required by each model (6,618 and 6,804), shows that the simulation model requires about 2.81% more beds.

Table 8 – Number of beds for refusal rate ≤ 0.1% and occupancy rates ≤ 77%.

Type of bed	**Number of beds (2012)**	NB_s	ρ_s **(%)**	***CI*** **(α = 0.05)**
Neonatology	166	170	0.7498	±0.3085
Pediatrics	556	420	0.7663	±0.2590
Adults	1,788	2,445	0.7686	±0.1042
Pediatrics Surgery	130	117	0.7381	±0.3343
Adults Surgery	1,628	1,418	0.7632	±0.0979
Obstetrics	282	389	0.7605	±0.1441
Psychiatry	419	400	0.7483	±0.3140
Other	306	452	0.7581	±0.3961
ICU Neonatology	154	170	0.7558	±0.5802
ICU Pediatrics	99	123	0.7590	±0.4790
ICU Adults	493	700	0.7451	±0.2027
Total	**6,021**	**6,804**	**---**	**---**

6 CONCLUSION

The queuing model proposed to the Brazilian Ministry of Health to estimate bed capacity for each type of hospitalization seems to be realistic for use in Belo Horizonte. Comparison of the results with those obtained by simulation shows a good fitting, although we have note a short underestimate. If we consider the fact that it is almost impossible to build a simulation model for every 5,564 cities in Brazil, the queuing model proposed seems to be an alternative better than the empirical method currently used to estimate the number of beds for Brazilian public hospitals.

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