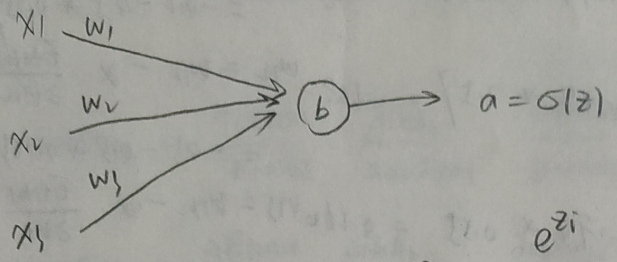
推导

softmax $S_i = \frac{e^{z_i}}{\sum_k e^{z_k}}$



$z_i = \sum_j w_{ij} x_{ij} + b$ $\quad a_i = \frac{e^{z_i}}{\sum_k e^{z_k}}$

cross entropy

$C = -\sum_i y_i \ln a_i$

$\frac{\partial C}{\partial z} = \sum_j \left(\frac{\partial C_j}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_i} \right)$

① $\frac{\partial C_j}{\partial a_j} = \frac{\partial(-y_j \ln a_j)}{\partial a_j} = -y_j \frac{1}{a_j}$

② i=j 时

$\frac{\partial a_j}{\partial z_i} = \frac{\partial \left(\frac{e^{z_i}}{\sum_k e^{z_k}} \right)}{\partial z_i} = \frac{e^{z_i} \cdot \sum_k e^{z_k} - e^{z_i} \cdot e^{z_i}}{\left(\sum_k e^{z_k} \right)^2} = \frac{e^{z_i}}{\sum_k e^{z_k}} \cdot \left(1 - \frac{e^{z_i}}{\sum_k e^{z_k}} \right)$

$= a_i (1 - a_i)$

i≠j 时

$\frac{\partial a_j}{\partial z_i} = \frac{\partial \left(\frac{e^{z_j}}{\sum_k e^{z_k}} \right)}{\partial z_i} = \frac{-e^{z_i} \cdot e^{z_j}}{\left(\sum_k e^{z_k} \right)^2} = -a_i a_j$

$\frac{\partial C}{\partial z_i} = \sum_j \left(\frac{\partial C_j}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_i} \right) = \sum_{j=i} \left(-y_j \frac{1}{a_j} \cdot a_i (1 - a_i) \right) + \sum_{j \neq i} \left(-y_j \frac{1}{a_j} \cdot -a_i a_j \right)$

$= -y_i (1 - a_i) + a_i \sum_{j \neq i} y_j = a_i \sum_j y_i - y_i = a_i - y_i$

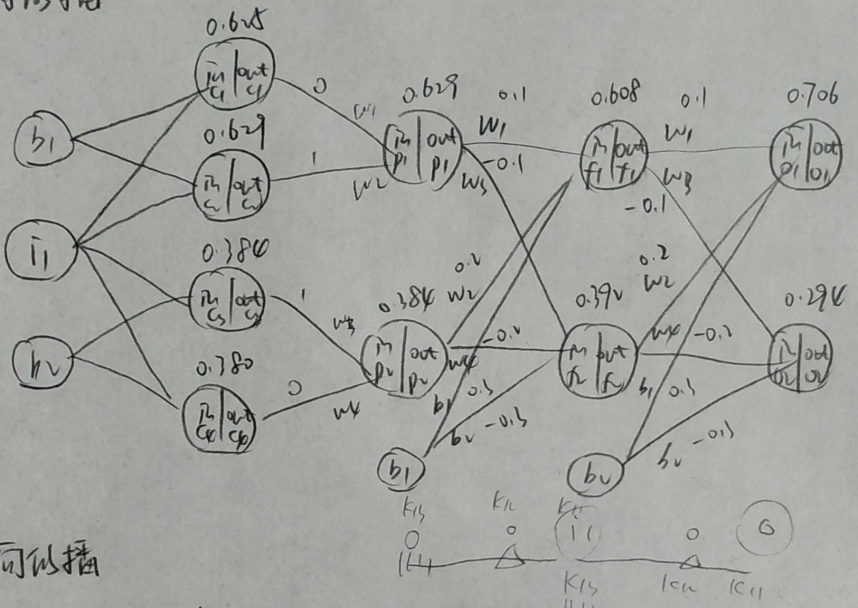
装……订……线

fc2层

$in_{o1} = w_{11} * outf_1 + w_{12} * outf_2 + w_{13} * outf_3 + w_{14} * outf_4 + b_1$

$in_{o2} = w_{21} * outf_1 + w_{22} * outf_2 + w_{23} * outf_3 + w_{24} * outf_4 + b_2$

反向传播



正向传播

conv

$in_{c1} = i_1 * k_{11} + b_1$

$in_{c2} = i_1 * k_{12} + b_1$

$in_{c3} = i_1 * k_{21} + b_2$

$in_{c4} = i_1 * k_{22} + b_2$

$0 * k_{11} + 0 * k_{12} + i_1 * k_{13}$

$0 * k_{11} + i_1 * k_{12} + 0 * k_{13}$

$i_1 * k_{11} + 0 * k_{12} + 0 * k_{13}$

k_{11}	k_{12}	k_{13}	b_1
0.1	0.2	0.3	0.5

k_{21}	k_{22}	k_{23}	b_2
0.3	0.2	0.1	-0.5

pool

$in_{p1} = max(outc_1, outc_2)$

$in_{p2} = max(outc_3, outc_4)$

fc1

$in_{f1} = outp_1 * w_1 + outp_2 * w_2 + b_1$

$in_{f2} = outp_3 * w_3 + outp_4 * w_4 + b_2$

fc2

$in_{o1} = outf_1 * w_1 + outf_2 * w_2 + b_1$

$in_{o2} = outf_3 * w_3 + outf_4 * w_4 + b_2$

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反向传播

$$coef_{o1} = \frac{\partial E_{total}}{\partial net_{o1}} = out_{o1} - o_1 = 0.706 - 1 = -0.294$$

$$coef_{o2} = \frac{\partial E_{total}}{\partial net_{o2}} = out_{o2} - o_2 = 0.294 - 0 = 0.294$$

$$w_1 = w_1 - \alpha \cdot \frac{\partial E_{total}}{\partial w_1} = w_1 - \alpha \cdot \frac{\partial E_{total}}{\partial net_{o1}} \cdot \frac{\partial net_{o1}}{\partial w_1} = 0.1 - 0.5 \cdot coef_{o1} \cdot out_{f1} = 0.1 - 0.5 * (-0.294) * 0.608$$
$$= 0.189$$

$$w_2 = w_2 - \alpha \cdot \frac{\partial E_{total}}{\partial w_2} = w_2 - \alpha \cdot \frac{\partial E_{total}}{\partial net_{o1}} \cdot \frac{\partial net_{o1}}{\partial w_2} = 0.2 - 0.5 * coef_{o1} \cdot out_{f1} = 0.2 - 0.5 * (-0.294) * 0.608$$
$$= 0.258$$

$$w_3 = w_3 - \alpha \frac{\partial E_{total}}{\partial w_3} = w_3 - \alpha \frac{\partial E_{total}}{\partial net_{o2}} \cdot \frac{\partial net_{o2}}{\partial w_3} = -0.1 - 0.5 \cdot coef_{o2} \cdot out_{f1} = -0.1 - 0.5 * 0.294 * 0.608$$
$$= -0.189$$

$$w_4 = w_4 - \alpha \frac{\partial E_{total}}{\partial w_4} = w_4 - \alpha \frac{\partial E_{total}}{\partial net_{o2}} \cdot \frac{\partial net_{o2}}{\partial w_4} = -0.2 - 0.5 coef_{o2} \cdot out_{f1} = -0.2 - 0.5 * 0.294 * 0.608$$
$$= -0.258$$

$$b_1 = b_1 - \alpha \frac{\partial E_{total}}{\partial b_1} = b_1 - \alpha \frac{\partial E_{total}}{\partial net_{o1}} \cdot \frac{\partial net_{o1}}{\partial b_1} = 0.3 - 0.5 \cdot coef_{o1} \cdot 1 = 0.3 - 0.5 * (-0.294) * 1 = 0.447$$

$$b_2 = b_2 - \alpha \frac{\partial E_{total}}{\partial b_2} = b_2 - \alpha \frac{\partial E_{total}}{\partial net_{o2}} \cdot \frac{\partial net_{o2}}{\partial b_2} = -0.3 - 0.5 \cdot coef_{o2} \cdot 1 = -0.3 - 0.5 * 0.294 * 1 = -0.447$$

$$coef_{f1} = \frac{\partial E_{total}}{\partial net_{f1}} = \left(\frac{\partial E_{total}}{\partial out_{o1}} \cdot \frac{\partial out_{o1}}{\partial net_{o1}} + \frac{\partial E_{total}}{\partial out_{o2}} \cdot \frac{\partial out_{o2}}{\partial net_{o1}} \right) \frac{\partial out_{f1}}{\partial net_{f1}}$$

$$= \left(coef_{o1} \cdot w_1 + coef_{o2} \cdot w_3 \right) out_{f1} (1 - out_{f1})$$

$$= \left(-0.294 * 0.189 + 0.294 * (-0.189) \right) * 0.608 * (1 - 0.608)$$

$$= -0.026$$

$$coef_{f2} = \frac{\partial E_{total}}{\partial net_{f2}} = \left(\frac{\partial E_{total}}{\partial out_{o1}} \cdot \frac{\partial out_{o1}}{\partial net_{o1}} + \frac{\partial E_{total}}{\partial out_{o2}} \cdot \frac{\partial out_{o2}}{\partial net_{o1}} \right) \frac{\partial out_{f2}}{\partial net_{f2}}$$

$$= \left(coef_{o1} \cdot w_2 + coef_{o2} \cdot w_4 \right) out_{f2} (1 - out_{f2})$$

$$= \left(-0.294 * 0.258 + 0.294 * (-0.258) \right) * 0.649 * (1 - 0.649)$$

$$= -0.036$$

$$w_1 = w_1 - \alpha \frac{\partial E_{total}}{\partial w_1} = w_1 - \alpha \frac{\partial E_{total}}{\partial net_{f1}} \cdot \frac{\partial net_{f1}}{\partial w_1} = 0.1 - 0.5 coef_{f1} \cdot in_{o1} = 0.1 - 0.5 * (-0.026) * 0.5$$
$$= 0.101$$

$$w_2 = w_2 - \alpha \frac{\partial E_{total}}{\partial w_2} = w_2 - \alpha \frac{\partial E_{total}}{\partial net_{f1}} \cdot \frac{\partial net_{f1}}{\partial w_2} = 0.2 - 0.5 * coef_{f1} \cdot in_{o1} = 0.2 - 0.5 * (-0.026) * 0.5$$
$$= 0.205$$

$$w_3 = w_3 - \alpha \frac{\partial E_{total}}{\partial w_3} = w_3 - \alpha \frac{\partial E_{total}}{\partial net_{f2}} \cdot \frac{\partial net_{f2}}{\partial w_3} = -0.1 - 0.5 coef_{f2} \cdot in_{o1} = -0.1 - 0.5 * (-0.036) * 0.5$$
$$= -0.089$$

$$w_4 = w_4 - \alpha \frac{\partial E_{total}}{\partial w_4} = w_4 - \alpha \frac{\partial E_{total}}{\partial net_{f2}} \cdot \frac{\partial net_{f2}}{\partial w_4} = -0.2 - 0.5 coef_{f2} \cdot in_{o1} = -0.2 - 0.5 * (-0.036) * 0.5$$
$$= -0.182$$

$$b_1 = b_1 - \alpha \frac{\partial E_{total}}{\partial b_1} = b_1 - \alpha \frac{\partial E_{total}}{\partial net_{f1}} \cdot \frac{\partial net_{f1}}{\partial b_1} = 0.3 - 0.5 \cdot coef_{f1} \cdot 1 = 1 * (0.026) * 0.5 - 0.3 = -0.287$$

$$b_2 = b_2 - \alpha \frac{\partial E_{total}}{\partial b_2} = b_2 - \alpha \frac{\partial E_{total}}{\partial net_{f2}} \cdot \frac{\partial net_{f2}}{\partial b_2} = -0.3 - 0.5 \cdot coef_{f2} \cdot 1 = 1 * (0.036) * 0.5 - 0.3 = -0.282$$

$$coef_{p1} = \frac{\partial E_{total}}{\partial in_{p1}} = \left(\frac{\partial E_{total}}{\partial in_{f1}} \cdot \frac{\partial in_{f1}}{\partial out_{p1}} + \frac{\partial E_{total}}{\partial in_{f2}} \cdot \frac{\partial in_{f2}}{\partial out_{p1}}\right) \cdot \frac{\partial out_{p1}}{\partial in_{p1}}$$

$$= (coef_{f1} \cdot w_1 + coef_{f2} \cdot w_3) \cdot 1$$

$$= (-0.026 * 0.108 + (-0.036) * (-0.089)) = 0.0004$$

$$coef_{p2} = \frac{\partial E_{total}}{\partial in_{p2}} = \left(\frac{\partial E_{total}}{\partial in_{f1}} \cdot \frac{\partial in_{f1}}{\partial out_{p2}} + \frac{\partial E_{total}}{\partial in_{f2}} \cdot \frac{\partial in_{f2}}{\partial out_{p2}}\right) \cdot \frac{\partial out_{p2}}{\partial in_{p2}}$$

$$= (coef_{f1} \cdot w_2 + coef_{f2} \cdot w_4) \cdot 1$$

$$= (-0.026 * 0.205 + (-0.036) * (-0.193)) * 1 = 0.002$$

$$coef_{c1} = \frac{\partial E_{total}}{\partial in_{c1}} = \frac{\partial E_{total}}{\partial in_{p1}} \cdot \frac{\partial in_{p1}}{\partial out_{c1}} \cdot \frac{\partial out_{c1}}{\partial in_{c1}}$$

$$= coef_{p1} \cdot w_1 \cdot out_{c1}(1 - out_{c1})$$

$$= 0.0004 * 0 * 0.625(1 - 0.625) = 0$$

$$coef_{c2} = \frac{\partial E_{total}}{\partial in_{c2}} = \frac{\partial E_{total}}{\partial in_{p1}} \cdot \frac{\partial in_{p1}}{\partial out_{c2}} \cdot \frac{\partial out_{c2}}{\partial in_{c2}}$$

$$= coef_{p1} \cdot w_2 \cdot out_{c2}(1 - out_{c2})$$

$$= 0.004 * 1 * 0.629(1 - 0.629) = 0.0001$$

$$coef_{c3} = \frac{\partial E_{total}}{\partial in_{c3}} = \frac{\partial E_{total}}{\partial in_{p2}} \cdot \frac{\partial in_{p2}}{\partial out_{c3}} \cdot \frac{\partial out_{c3}}{\partial in_{c3}}$$

$$= coef_{p2} \cdot w_3 \cdot out_{c3}(1 - out_{c3})$$

$$= 0.002 * 1 * 0.384(1 - 0.384) = 0.0005$$

$$coef_{c4} = \frac{\partial E_{total}}{\partial in_{c4}} = \frac{\partial E_{total}}{\partial in_{p2}} \cdot \frac{\partial in_{p2}}{\partial out_{c4}} \cdot \frac{\partial out_{c4}}{\partial in_{c4}}$$

$$= coef_{p2} \cdot w_4 \cdot out_{c4}(1 - out_{c4})$$

$$= 0.002 * 0 * 0.380(1 - 0.380) = 0$$

$$k_{11} = k_{11} - \alpha \frac{\partial E_{total}}{\partial k_{11}} = k_{11} - \left(\frac{\partial E_{total}}{\partial in_{c1}} \cdot \frac{\partial in_{c1}}{\partial k_{11}} + \frac{\partial E_{total}}{\partial in_{c2}} \cdot \frac{\partial in_{c2}}{\partial k_{11}}\right) * \alpha$$

$$= k_{11} - (coef_{c1} \cdot i_1 + coef_{c2} \cdot 0)\alpha = 0.1 - (0 * 0.1 + 0.0001 * 0)0.5 = 0.1$$

$$k_{12} = k_{12} - \frac{\partial E_{total}}{\partial k_{12}} = k_{12} - \left(\frac{\partial E_{total}}{\partial in_{c1}} \cdot \frac{\partial in_{c1}}{\partial k_{12}} + \frac{\partial E_{total}}{\partial in_{c2}} \cdot \frac{\partial in_{c2}}{\partial k_{12}}\right) * \alpha$$

$$= k_{12} - (coef_{c1} \cdot 0 + coef_{c2} \cdot 0)\alpha = 0.2 - (0 * 0 + 0.0001 * 0).05 = 0.2$$

$$k_{13} = k_{13} - \alpha \frac{\partial E_{total}}{\partial k_{13}} = k_{13} - \left(\frac{\partial E_{total}}{\partial in_{c1}} \cdot \frac{\partial in_{c1}}{\partial k_{13}} + \frac{\partial E_{total}}{\partial in_{c2}} \cdot \frac{\partial in_{c2}}{\partial k_{13}}\right) * \alpha$$

$$= k_{13} - (coef_{c1} \cdot 0 + coef_{c2} \cdot i_1)\alpha = 0.3 - (0 * 0 + 0.0001 * 1).05 = 0.3$$

$$b_1 = b_1 - \alpha \frac{\partial E_{total}}{\partial b_1} = b_1 - \left(\frac{\partial E_{total}}{\partial \hat{n}_{c1}} \cdot \frac{\partial \hat{n}_{c1}}{\partial b_1} + \frac{\partial E_{total}}{\partial \hat{n}_{c4}} \cdot \frac{\partial \hat{n}_{c4}}{\partial b_1} \right) * \alpha$$

$$= b_1 - (coef_{c1} \cdot 1 + coef_{c4} \cdot 1) * \alpha = 0.5 - (0.0 * 1 + 0.0001 * 1) * 0.5 \downarrow$$

$$k_{11} = k_{11} - \alpha \frac{\partial E_{total}}{\partial k_{11}} = k_{11} - \left(\frac{\partial E_{total}}{\partial \hat{n}_{c3}} \cdot \frac{\partial \hat{n}_{c3}}{\partial k_{11}} + \frac{\partial E_{total}}{\partial \hat{n}_{c4}} \cdot \frac{\partial \hat{n}_{c4}}{\partial k_{11}} \right) * \alpha$$

$$= k_{11} - (coef_{c3} \cdot i_1 + coef_{c4} \cdot 0) * \alpha = 0.3 - (0.0005 * 0.1 + 0 * 0) * 0.5 = 0.3 \downarrow$$

$$k_{12} = k_{12} - \alpha \frac{\partial E_{total}}{\partial k_{12}} = k_{12} - \left(\frac{\partial E_{total}}{\partial \hat{n}_{c3}} \cdot \frac{\partial \hat{n}_{c3}}{\partial k_{12}} + \frac{\partial E_{total}}{\partial \hat{n}_{c4}} \cdot \frac{\partial \hat{n}_{c4}}{\partial k_{12}} \right) * \alpha$$

$$= k_{12} - (coef_{c3} \cdot 0 + coef_{c4} \cdot 0) * \alpha = 0.2 - (0.0005 * 0 + 0 * 0) * 0.5 = 0.2$$

$$k_{21} = k_{21} - \alpha \frac{\partial E_{total}}{\partial k_{21}} = k_{21} - \left(\frac{\partial E_{total}}{\partial \hat{n}_{c3}} \cdot \frac{\partial \hat{n}_{c3}}{\partial k_{21}} + \frac{\partial E_{total}}{\partial \hat{n}_{c4}} \cdot \frac{\partial \hat{n}_{c4}}{\partial k_{21}} \right) * \alpha$$

$$= k_{21} - (coef_{c3} \cdot 0 + coef_{c4} \cdot i_2) * \alpha = 0.1 - (0.0005 * 0 + 0 * 0.1) * 0.5 = 0.1$$

$$b_2 = b_2 - \alpha \frac{\partial E_{total}}{\partial b_2} = b_2 - \left(\frac{\partial E_{total}}{\partial \hat{n}_{c3}} \cdot \frac{\partial \hat{n}_{c3}}{\partial b_2} + \frac{\partial E_{total}}{\partial \hat{n}_{c4}} \cdot \frac{\partial \hat{n}_{c4}}{\partial b_2} \right) * \alpha$$

$$= b_2 - (coef_{c3} \cdot 1 + coef_{c4} \cdot 1) * \alpha = -0.5 - (0.0005 * 1 + 0 * 1) * 0.5 = 0.5$$

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