

Quiz 4

Quelques formules:

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t} d\omega$$

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$x(t-t_0) \xleftarrow{\text{CTFT}} e^{-j\omega t_0} X(j\omega)$$

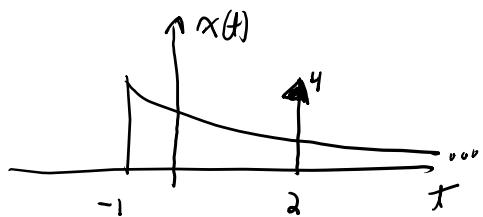
$$\frac{dx(t)}{dt} \xleftarrow{\text{CTFT}} j\omega X(j\omega)$$

$$x(at) \xleftarrow{\text{CTFT}} \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$e^{-at} u(t) \quad (\text{Re}\{a\} > 0) \xleftarrow{\text{CTFT}} \frac{1}{j\omega + a}$$

$$\delta(t) \xleftarrow{\text{CTFT}} 1$$

Question 1 Quelle est la transformée de Fourier du signal $x(t) = e^{-5t}u(t+1) + 4\delta(t-2)$?



$$\begin{aligned}
 X(j\omega) &= \int_{-\infty}^{+\infty} (e^{-5t}u(t+1) + 4\delta(t-2)) e^{-j\omega t} dt \\
 &= \int_{-1}^{\infty} e^{-5t} e^{-j\omega t} dt + \int_{2}^{+\infty} 4\delta(t-2) e^{-j\omega t} dt \\
 &= \frac{1}{-5-j\omega} [e^{-5t} e^{-j\omega t}]_{-1}^{\infty} + 4 \int_{2}^{\infty} \delta(t-2) e^{-j\omega t} dt \\
 &= \frac{1}{-5-j\omega} \cdot (0 - e^{-5} e^{j\omega}) + 4 e^{-j2\omega} \cdot 1 = \frac{e^{-5} e^{j\omega}}{j\omega + 5} + 4 e^{-j2\omega}
 \end{aligned}$$

Ou, avec propriétés:

$$e^{-st} u(t) \xrightarrow{\text{TF}} \frac{1}{j\omega + s}$$

$$e^{-s(t+1)} u(t+1) \xrightarrow{\text{TF}} \frac{e^{j\omega}}{j\omega + s}$$

$$e^s e^{-s(t+1)} u(t+1) \xrightarrow{\text{TF}} \frac{e^s e^{j\omega}}{j\omega + s}$$

$$\delta(t) \xrightarrow{\text{TF}} 1$$

$$4\delta(t-2) \xrightarrow{\text{TF}} 4e^{-j2\omega}$$

$$e^{-st} u(t+1) + 4\delta(t-2) \xrightarrow{\text{TF}} \frac{e^s e^{j\omega}}{j\omega + s} + 4e^{-j2\omega}$$

Question 2 Quel est le signal $x(t)$ dont la transformée de Fourier est $X(j\omega) = \frac{1}{(j\omega+1)(j\omega+4)}$?

$$X(j\omega) = \frac{A}{j\omega+1} + \frac{B}{j\omega+4}$$

$$A = X(j\omega) \Big|_{j\omega=-1} = \frac{1}{j\omega+4} \Big|_{j\omega=-1} = \frac{1}{-1+4} = \frac{1}{3}$$

$$= \frac{1}{3} \frac{1}{j\omega+1} - \frac{1}{3} \frac{1}{j\omega+4}$$

$$B = X(j\omega) \Big|_{j\omega=-4} = \frac{1}{j\omega+1} \Big|_{j\omega=-4} = \frac{1}{-4+1} = -\frac{1}{3}$$

$$e^{-at} \xrightarrow[\text{RÉEL}]{a>0} \frac{1}{j\omega+a}$$

$$\left(\text{VÉRIF: } \frac{1}{3} \frac{1}{j\omega+1} - \frac{1}{3} \frac{1}{j\omega+4} = \frac{\frac{1}{3}(j\omega+4) - \frac{1}{3}(j\omega+1)}{(j\omega+1)(j\omega+4)} = \frac{1}{(j\omega+1)(j\omega+4)} \right)$$

$$x(t) = \frac{1}{3} e^{-t} u(t) - \frac{1}{3} e^{-4t} u(t)$$