

*ITI1100A*

# Group Discussion 3

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*UNIVERSITY OF OTTAWA*

# Discussion Objective

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Some practice problems on Gate Level Minimization of Boolean Expression

# Basic K-Map Representation

Two Variable K-Map

$m_0$	$m_1$
$m_2$	$m_3$

(a)

		$y$	
		0	1
$x$	0	$m_0$ $x'y'$	$m_1$ $x'y$
	1	$m_2$ $xy'$	$m_3$ $xy$

(b)

Three Variable K Map

$m_0$	$m_1$	$m_3$	$m_2$
$m_4$	$m_5$	$m_7$	$m_6$

(a)

		$y$			
		00	01	11	10
$x$	0	$m_0$ $x'y'z'$	$m_1$ $x'y'z$	$m_3$ $x'yz$	$m_2$ $x'yz'$
	1	$m_4$ $xy'z'$	$m_5$ $xy'z$	$m_7$ $xyz$	$m_6$ $xyz'$

$z$

(b)

# Question 3.1

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Simplify the following Boolean functions, using three-variable maps:

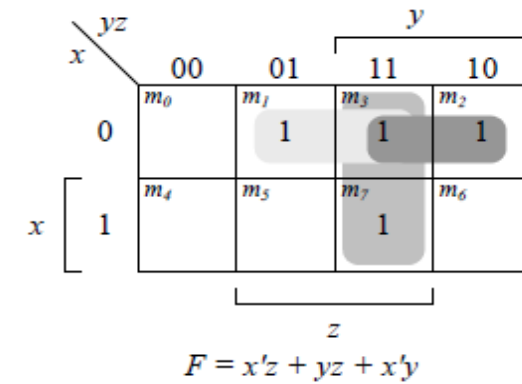
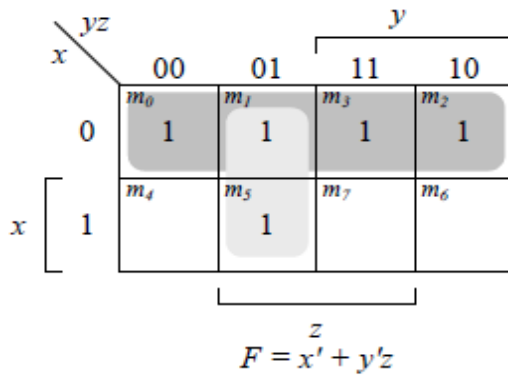
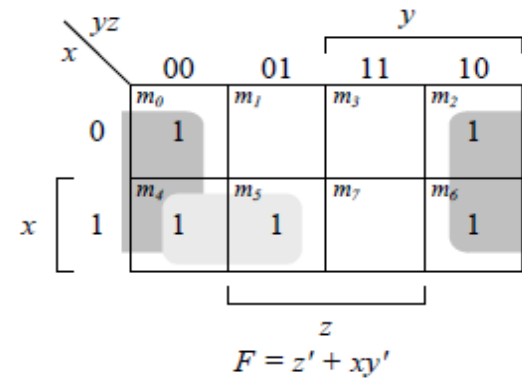
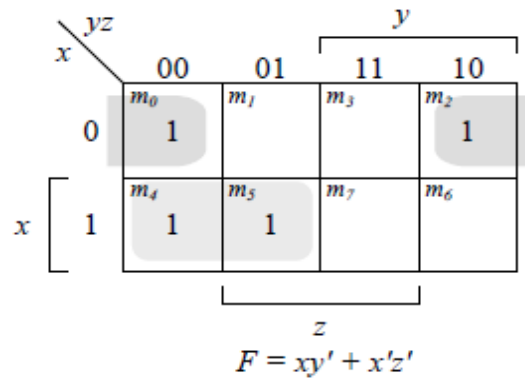
(a)  $F(x, y, z) = (0, 2, 4, 5)$

(b)  $F(x, y, z) = (0, 2, 4, 5, 6)$

(c)  $F(x, y, z) = (0, 1, 2, 3, 5)$

(d)  $F(x, y, z) = (1, 2, 3, 7)$

# Solution 3.1



# Question 3.3

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Simplify the following Boolean expressions, using three-variable maps:

(a)  $F(x, y, z) = xy + x' y' z' + x' yz'$

(b)  $F(x, y, z) = x' y' + yz + x' yz'$

(c)  $F(x, y, z) = x'y + yz' + y'z'$

(d)  $F(x, y, z) = x'yz + xy'z' + xy'z$

# Solution 3.3

		y			
		00	01	11	10
x	0	$m_0$ 1	$m_1$	$m_3$	$m_2$ 1
	1	$m_4$	$m_5$	$m_7$ 1	$m_6$ 1

(a)  $F = xy + x'y'z' + x'yz'$   
 $F = xy + x'z'$

		y			
		00	01	11	10
x	0	$m_0$ 1	$m_1$ 1	$m_3$ 1	$m_2$ 1
	1	$m_4$	$m_5$	$m_7$ 1	$m_6$

(b)  $F = x'y' + yz + x'yz'$   
 $F = x' + yz$

		y			
		00	01	11	10
x	0	$m_0$ 1	$m_1$	$m_3$ 1	$m_2$ 1
	1	$m_4$ 1	$m_5$	$m_7$	$m_6$ 1

(c)  $F = x'y + yz' + y'z'$   
 $F = x'y + z'$

		y			
		00	01	11	10
x	0	$m_0$	$m_1$	$m_3$ 1	$m_2$
	1	$m_4$ 1	$m_5$ 1	$m_7$	$m_6$

(d)  $F = x'yz + xy'z' + xy'z$   
 $F = x'yz + xy'$

# Question 3.6

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Simplify the following Boolean expressions, using four-variable maps:

(a)  $A'B'C'D' + AC'D' + B'CD' + A'BCD + BC'D$

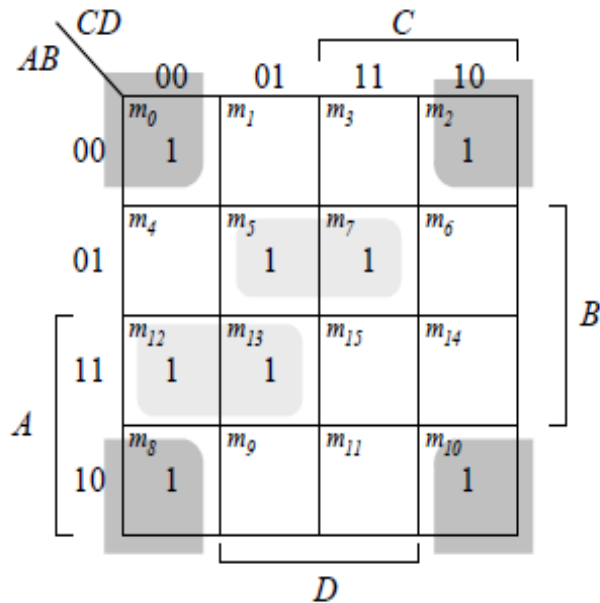
(b)  $x'z + w'xy' + w(x'y + xy')$

(c)  $A'B'C'D + AB'D + A'BC' + ABCD + AB'C$

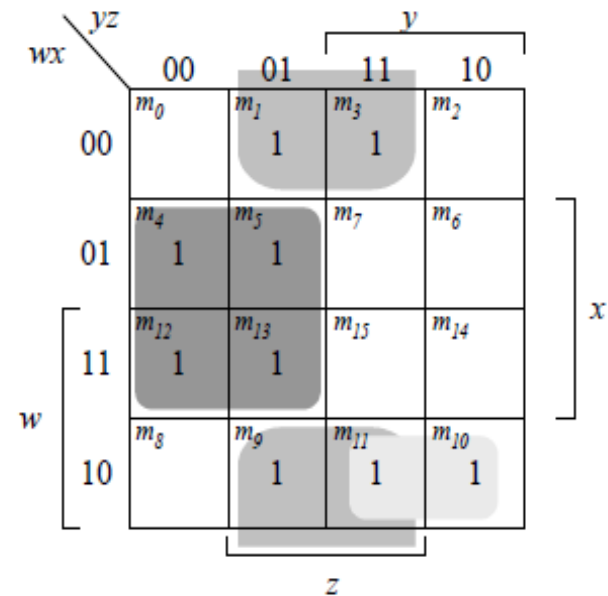
(d)  $A'B'C'D' + BC'D + A'C'D + A'BCD + ACD'$



# Solution 3.6



(a)  $F = B'D' + A'BD + ABC'$



(b)  $F = xy' + x'z + wx'y$

# Solution 3.6 (Continuation)

AB \ CD		C			
		00	01	11	10
A	00	$m_0$	$m_1$ 1	$m_3$	$m_2$
	01	$m_4$ 1	$m_5$ 1	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_{15}$ 1	$m_{14}$
	10	$m_8$	$m_9$ 1	$m_{11}$ 1	$m_{10}$ 1

D

B

(c)  $F = A'BC' + B'C'D + ACD + AB'C$

AB \ CD		C			
		00	01	11	10
A	00	$m_0$ 1	$m_1$ 1	$m_3$	$m_2$
	01	$m_4$	$m_5$ 1	$m_7$ 1	$m_6$
	11	$m_{12}$	$m_{13}$ 1	$m_{15}$	$m_{14}$
	10	$m_8$	$m_9$ 1	$m_{11}$	$m_{10}$

D

B

(d)  $F = C'D + A'BD + A'B'C'$

# Question 3.9

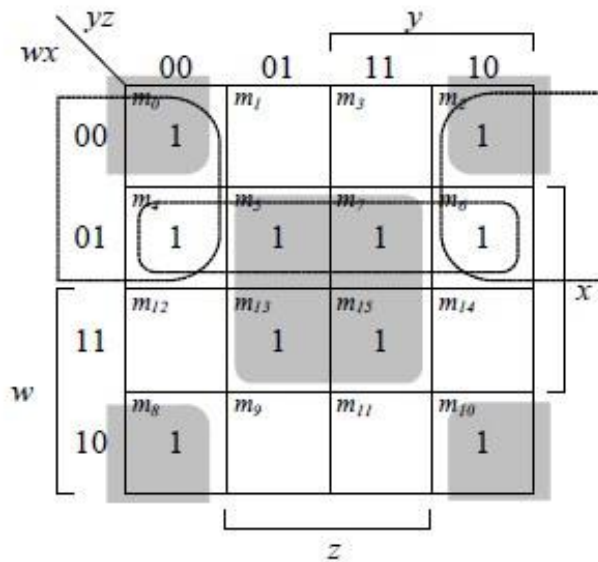
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Find all the prime implicants for the following Boolean functions, and determine which are essential:

(a)  $F(w, x, y, z) = \Sigma(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

(b)  $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

# Solution 3.9

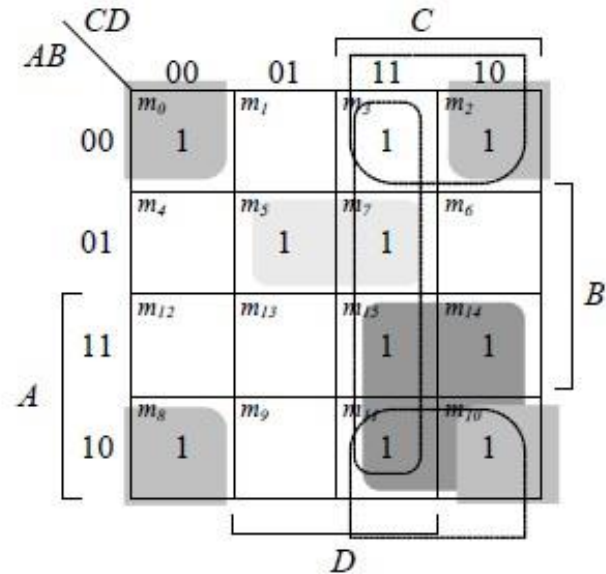


(a)

**Essential:**  $xz, x'z'$

**Non-essential:**  $w'x, w'z'$

$$F = xz + x'z' + (w'x \text{ or } w'z')$$



(b)

**Essential:**  $B'D', AC, A'BD$

**Non-essential:**  $CD, B'C$

$$F = B'D' + AC + A'BD + (CD \text{ OR } B'C)$$

# Question 3.13

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Simplify the following expressions to (1) sum-of-products and (2) products-of-sums:

(a)  $x'z' + y'z' + yz' + xy$

(b)  $ACD' + C'D + AB' + ABCD$

# Solution 3.13

(a)  $F = x'z' + y'z' + yz' + xy = x'z' + z' + xy = z' + xy$

		yz			
		00	01	11	10
x	0	m <sub>0</sub> 1	m <sub>1</sub>	m <sub>3</sub>	m <sub>2</sub> 1
	1	m <sub>4</sub> 1	m <sub>5</sub>	m <sub>7</sub> 1	m <sub>6</sub> 1

$y$ 
 $z$

$$F' = x'z + y'z$$

$$F = (x + z')(y + z')$$

(b)  $F = ACD' + C'D + AB' + ABCD$

		CD			
		00	01	11	10
AB	00	m <sub>0</sub>	m <sub>1</sub> 1	m <sub>3</sub>	m <sub>2</sub>
	01	m <sub>4</sub>	m <sub>5</sub> 1	m <sub>7</sub>	m <sub>6</sub>
	11	m <sub>12</sub>	m <sub>13</sub> 1	m <sub>15</sub> 1	m <sub>14</sub> 1
	10	m <sub>8</sub> 1	m <sub>9</sub> 1	m <sub>11</sub> 1	m <sub>10</sub> 1

$C$ 
 $D$

$$F = AC + AB' + C'D$$

$$F' = A'C + A'D' + BC'D'$$

$$F = (A + C')(A + D)(B' + C + D)$$

# Question 3.17

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Draw a NAND logic diagram that implements the complement of the following function:

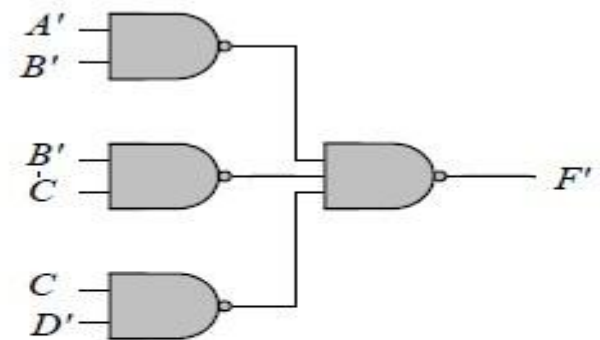
$$F(A, B, C, D) = \Sigma(0, 1, 2, 3, 6, 10, 11, 14)$$

# Solution 3.17

AB \ CD		C			
		00	01	11	10
A	00	$m_0$ 1	$m_1$ 1	$m_3$ 1	$m_2$ 1
	01	$m_4$	$m_5$	$m_7$	$m_6$ 1
	11	$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$ 1
	10	$m_8$	$m_9$	$m_{11}$ 1	$m_{10}$ 1

D

B



$$\begin{aligned}
 F &= A'B' + B'C + CD' \\
 F &= ((A + B)(B + C')(C' + D))' \\
 F &= ((A'B')'(B'C)'(CD')')' \\
 F' &= (A'B')(B'C)(CD')
 \end{aligned}$$



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# THANK YOU