MAT 2784 A Assignment # 6 Solutions

1. $\mathcal{L}\{3t^4 + 4t^3 - 5t^2 + 7t - 2\}$

$$= 3\left(\frac{4!}{s^5}\right) + 4\left(\frac{3!}{s^4}\right) - 5\left(\frac{2!}{s^3}\right) + 7\left(\frac{1}{s^2}\right) - 2\left(\frac{1}{s}\right)$$

$$= \boxed{\frac{72}{s^5} + \frac{24}{s^4} - \frac{10}{s^3} + \frac{7}{s^2} - \frac{2}{s}}$$

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2. $\mathcal{L}\{3e^{-\pi t} + 5\cos(3t) - \sin(2t)\}$

$$= 3\left(\frac{1}{s+\pi}\right) + 5\left(\frac{s}{s^2+9}\right) - \frac{2}{s^2+4} = \boxed{\frac{3}{s+\pi} + \frac{5s}{s^2+9} - \frac{2}{s^2+4}}$$

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3. $\mathcal{L}\{4\cosh(3t) + 7\sinh(2t)\}$

$$= 4\left(\frac{s}{s^2-9}\right) + 7\left(\frac{2}{s^2-4}\right) = \boxed{\frac{4s}{s^2-9} + \frac{14}{s^2-4}}$$

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4. $\mathcal{L}^{-1}\left\{\frac{3}{s^4} + \frac{6}{s^3} - \frac{8}{s^2} + \frac{5}{s}\right\}$

$$= 3\left(\frac{1}{3!}t^3\right) + 6\left(\frac{1}{2!}t^2\right) - 8(t) + 5(1) = \boxed{\frac{1}{2}t^3 + 3t^2 - 8t + 5}$$

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5. $\mathcal{L}^{-1}\left\{\frac{2s+14}{s^2-s-2}\right\}$

$$\left(\frac{2s+14}{s^2-s-2} = \frac{2s+14}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1}\qquad \left.\begin{matrix} A+B=2 \\ A-2B=14\end{matrix}\right\} \begin{matrix} A=6 \\ B=-4\end{matrix}\right)$$

$$= \mathcal{L}^{-1}\left\{\frac{6}{s-2} - \frac{4}{s+1}\right\}$$

$$= \boxed{6e^{2t} - 4e^{-t}}$$

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6. $\mathcal{L}^{-1}\left\{\frac{2s}{s^2+9} + \frac{7}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{2\left(\frac{s}{s^2+9}\right) + \frac{7}{2}\left(\frac{2}{s^2+4}\right)\right\}$

$$= \boxed{2\cos(3t) + \frac{7}{2}\sin(2t)}$$

7. $y'' - 5y' + 6y = 2e^t$, $y(0) = 6$, $y'(0) = 14$

let $Y(s) = \mathcal{L}\{y(t)\}$ and take the Laplace Transform of the DE to get $\mathcal{L}\{y''\} - 5\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = \mathcal{L}\{2e^t\}$

$$s^2 Y(s) - s y(0) - y'(0) - 5(sY(s) - y(0)) + 6Y(s) = 2/s-1$$

$$(s^2 - 5s + 6)Y(s) - 6s - 14 + 30 = 2/s-1$$

$$(s^2 - 5s + 6)Y(s) = 6s - 16 + 2/s-1$$

$$Y(s) = \frac{6s-16}{(s-2)(s-3)} + \frac{2}{(s-1)(s-2)(s-3)}$$

$$\frac{6s-16}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3} \qquad \left.\begin{array}{l} A+B=6 \\ -3A-2B=-16 \end{array}\right\} \begin{array}{l} A=4 \\ B=2 \end{array}$$

$$\frac{2}{(s-1)(s-2)(s-3)} = \frac{C}{s-1} + \frac{D}{s-2} + \frac{E}{s-3} \qquad C(s^2-5s+6) + D(s^2-4s+3) + E(s^2-3s+2) = 2$$

$C + D + E = 0$ ①

$-5C - 4D - 3E = 0$ ②

$6C + 3D + 2E = 2$ ③

$$\begin{array}{ll} 3\times① + ② & -2C - D = 0 \\ ③ - 2\times① & 4C + D = 2 \end{array} \Bigg\} \begin{array}{l} C = 1 \\ D = -2 \end{array} \Bigg\} \; E = 1$$

so $$Y(s) = \frac{4}{s-2} + \frac{2}{s-3} + \frac{1}{s-1} + \frac{-2}{s-2} + \frac{1}{s-3}$$

$$= \frac{1}{s-1} + \frac{2}{s-2} + \frac{3}{s-3}$$

$\therefore$ the solution to the IVP is $\boxed{y(t) = \mathcal{L}^{-1}\{Y(s)\} = e^t + 2e^{2t} + 3e^{3t}}$

8. $y' = 4x + 2y$; $y(0) = 0$ on $0 \le x \le 1$ with $h = 0.1$

so $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$, $\cdots$, $y_0 = 0$

Euler's Method: $$y_{n+1} = y_n + h f(x_n, y_n)$$
$$= y_n + (0.1)(4x_n + 2y_n)$$
$$= 0.4x_n + 1.2y_n$$

$y_1 = 0.4x_0 + 1.2y_0 = 0$
$y_2 = 0.4x_1 + 1.2y_1 = (0.4)(0.1) + 1.2(0) = 0.04$
$y_3 = 0.4x_2 + 1.2y_2 = (0.4)(0.2) + 1.2(0.04) = 0.128$
$y_4 = 0.4x_3 + 1.2y_3 = (0.4)(0.3) + 1.2(0.128) = 0.2736$
$y_5 = 0.4x_4 + 1.2y_4 = (0.4)(0.4) + 1.2(0.2736) = 0.4883$
$y_6 = 0.4x_5 + 1.2y_5 = (0.4)(0.5) + 1.2(0.4883) = 0.7860$
$y_7 = 0.4x_6 + 1.2y_6 = (0.4)(0.6) + 1.2(0.7860) = 1.1832$
$y_8 = 0.4x_7 + 1.2y_7 = (0.4)(0.7) + 1.2(1.1832) = 1.6998$
$y_9 = 0.4x_8 + 1.2y_8 = (0.4)(0.8) + 1.2(1.6998) = 2.3598$
$y_{10} = 0.4x_9 + 1.2y_9 = (0.4)(0.9) + 1.2(2.3598) = 3.1918$

true solution: $y' - 2y = 4x$, so $\mu(x) = e^{\int -2dx} = e^{-2x}$
$y(x) = e^{2x}\left(\int 4xe^{-2x}dx + C\right)$, $y(0)=0$, $\underline{y(x) = e^{2x} - 2x - 1}$

n	x_n	y_n	$y(x_n)$	error	relative error (%)
1	0.1	0	0.0214	0.0214	100
2	0.2	0.04	0.0918	0.0518	56
3	0.3	0.128	0.2221	0.0941	42
4	0.4	0.2736	0.4255	0.1519	36
5	0.5	0.4883	0.7183	0.2300	32
6	0.6	0.7860	1.1201	0.3341	30
7	0.7	1.1832	1.6552	0.4720	29
8	0.8	1.6998	2.3530	0.6532	28
9	0.9	2.3598	3.2496	0.8898	27
10	1.0	3.1918	4.3891	1.1973	27