Université d'Ottawa
Faculté de génie

Bureau des études supérieures

University of Ottawa
Faculty of Engineering

Graduate Office

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University of Ottawa

Winter – February, 06, 2016

ELG2136 - MidTerm Exam

90 minutes

Student Name: _____

Student Number: _____

Course Section (Choose A or B):

Question:	**QUESTION I**	**QUESTION II**	**QUESTION III**	Total
Points:	24	50	35	109
Score:				

PLEADE READ the instuctions on the back of this page.

General Instructions

- *Answer **all** questions in this section using the answer booklet(s) provided. Answers are expected to be succinct but complete.*

- *Your answers should be very clear and concise. Unintelligible answers will not be marked*

- *You must adhere to the specific instructions given next to the questions, where applicable.*

- *In the situations where your solution requires concrete steps to calculate some numerical answer, you must show the steps you used indicating clearly how you proceeded to calculate the answer to the question.*

- *The examination booklet includes lined paper sheets. The lines on those sheets are intended to guide you in writing and developing your answers in an organized manner.*

- *The space provided next to each question is generally more than sufficient for you to write the correct answer. However, if you needed more space, you can use the extra pages at the end of the exam.*

- *Questions during the exam are generally not allowed. However, you may ask questions only if you have any of the following specific concerns,*

  - *perceived error or missing information in a question*
  - *a missing paper from the booklet,*

  *If you must ask a question arising from the above concerns, then you must formulate your question so that an answer with **Yes** or **No** is sufficient to address your concern. Do not expect any further detailed answer.*

  *If the answer to your question is **Yes**, it would mean that you have legitimate point in your concern, in which case the professor will have to address, for your particular case, and perhaps for the rest of the students. Any other questions during the exams are strictly prohibited.*

- *No answers on the formula sheet will be marked.*

- *Crossed out answer will not be marked*

QUESTION I. Consider the circuit shown in Figure 1.
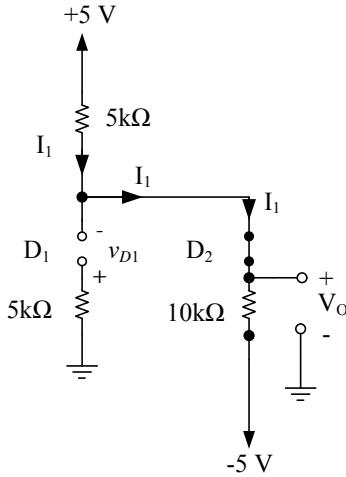
Figure 1:

(a) (12 points) Calculate the the value of the Voltage V_O if the diodes are ideal.

Solution:
Assumption. Assuming D_1 to be OFF and D_2 to be ON as shown in Figure 2.
Analysis.

$$I_1 = \frac{5 - (-5)}{15^{k\Omega}} = \frac{2}{3} \text{ mA} \tag{1}$$

$$V_o = 5 - I_1^{\text{mA}} \times 5^{k\Omega} = \frac{5}{3} \text{ V} \tag{2}$$

Validation.

$$v_{D_1} = 0 - V_o = -\frac{5}{3} \text{ V} = -1.633 \text{ V} < 0 \text{ V} \tag{3}$$

Since $v_{D_1} < 0$ then the diode D_1 is OFF as assumed. Since $I_1 > 0$ then the current in D_2 is flowing from the Anode to the Cathode. This means that D_2 is ON as assumed.

Figure 2:

You can continue the analysis steps on the back of this page ...
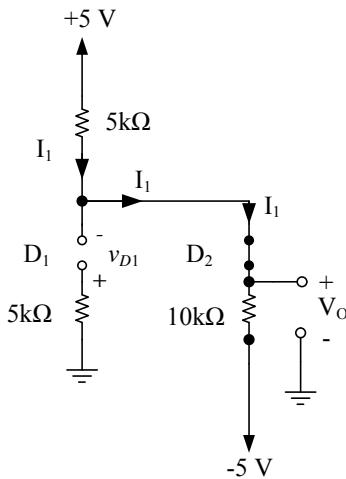
(b) (12 points) Repeat the previous problem if the diodes are modelled using the battery + ideal diode + resistance model, with a battery voltage $V_{D,\text{ON}} = 0.8$ V, and $r_D = 0\Omega$.

Solution:

Assumption. Assuming D_1 to be OFF and D_2 to be ON as shown in Figure 3.

Analysis.

$$I_1 = \frac{5 - (-5) - 0.8}{15^{k\Omega}} = \frac{9.2}{15} \text{ mA} \tag{4}$$

$$V_o = 5 - I_1^{\text{mA}} \times 5^{k\Omega} - 0.8 = 1.133 \text{ V} \tag{5}$$

Validation.

$$v_{D_1} = -0.8 - (V_o + 0.8) = -1.133 \text{ V} \tag{6}$$

Since $v_{D_1} < 0$ then the diode D_1 is OFF as assumed. Since $I_1 > 0$ then the current in D_2 is flowing from the Anode to the Cathode. This means that D_2 is ON as assumed.

Figure 3:

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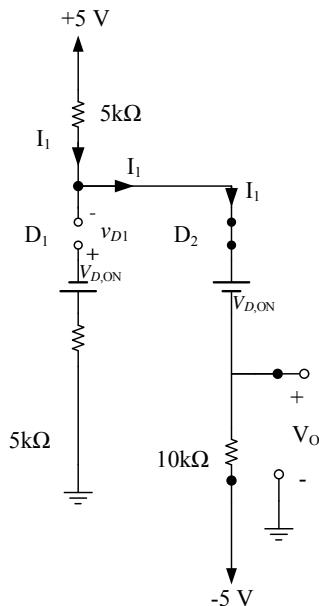
QUESTION II. (50 points) The diodes of the circuit shown in Figure 4 are **non-ideal** ones that can be modelled with a battery source of voltage $V_{D,ON}$, ideal diodes, and zero forward resistance, $r_D = 0$.

Plot the relation between I_{R_1} versus v_{in}.

Assume that $V_B = 2 \times V_{D,ON}$, $R_1 = 3k\Omega$ and $R_2 = 2k\Omega$.

IMPORTANT Instructions

You **MUST** show clearly on your plot the exact coordinates of all the break points and the exact slopes of each linear segment in your graph. A graph lacking this information will not be considered for marking.

You **MUST** also use the graph area provided on Figure 7 to sketch the relation you find.
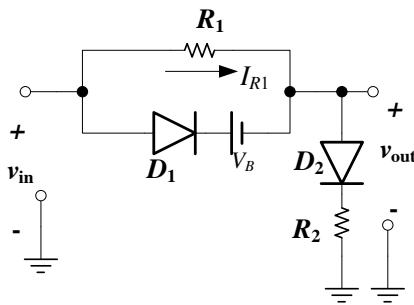

Figure 4:
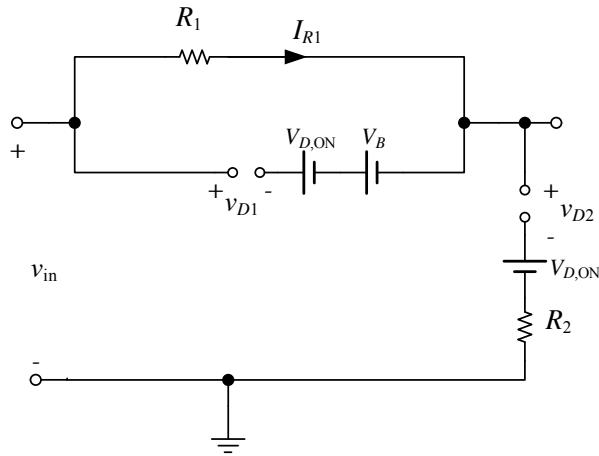
Figure 5:

Solution:

Phase 1. Initial State.

Assume v_{in} is extremely negative value ($v_{in} \ll 0$). It then follows that both D_1 and D_2 will be OFF as shown in Figure 5.

Phase 2. First Analysis: (D_1 and D_2 are OFF)

$$I_{R_1} = 0 \qquad (7)$$

and in this case we will have

You can continue the analysis steps on the back of this page . . .

$$v_{\text{out}} = v_{\text{in}} \tag{8}$$

Phase 3. Monitoring the state of the Diodes to predict the first Breakpoint (BP).

For D_1

$$
\begin{aligned}
v_{D_1} &= v_{\text{in}} - (v_{\text{out}} + 3 \times V_{D,\text{ON}}) \\
&= -3 \times V_{D,\text{ON}}
\end{aligned}
\tag{9}
\tag{10}
$$

v_{D_1} is thus independent from v_{in}, and is given by a negative value. It, therefore, appears as though D_1 will remain OFFregardless of the value of v_{in}.

For D_2

$$
\begin{aligned}
v_{D_2} &= v_{\text{out}} - V_{D,\text{ON}} \\
&= v_{\text{in}} - V_{D,\text{ON}}
\end{aligned}
\tag{11}
\tag{12}
$$

Hence, D_2 will switch if v_{D_2} at the point at which it would turn from negative to positive. In other words, D_2 remains OFF for as long as $v_{D_2} < 0$, that is, using (12)

$$
\begin{aligned}
v_{\text{in}} - V_{D,\text{ON}} &< 0 \\
v_{\text{in}} &< V_{D,\text{ON}}
\end{aligned}
\tag{13}
\tag{14}
$$

Thus, D_2 is the diode that will switch first and the first Breakpoint in this case is given at $v_{\text{in}} = V_{D,\text{ON}}$.
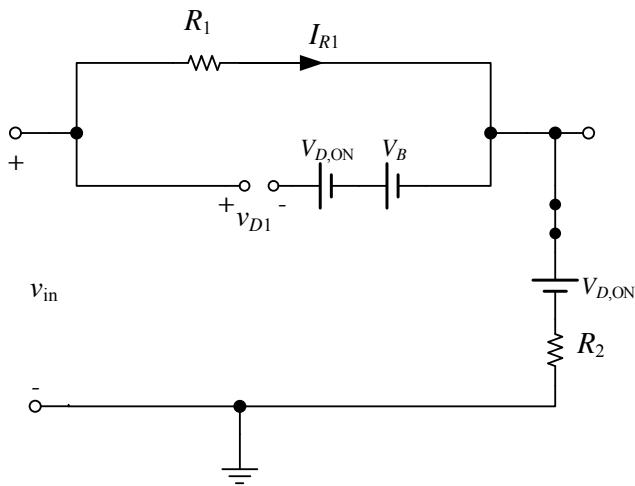
Figure 6:

Solution: Phase 4. Analysis after the first Breakpoint. (D_1 **OFF and** D_2 **ON**)

Here D_2 is ON and D_1 is OFF. Here the circuit will appear as shown in Figure 6. A KVL around
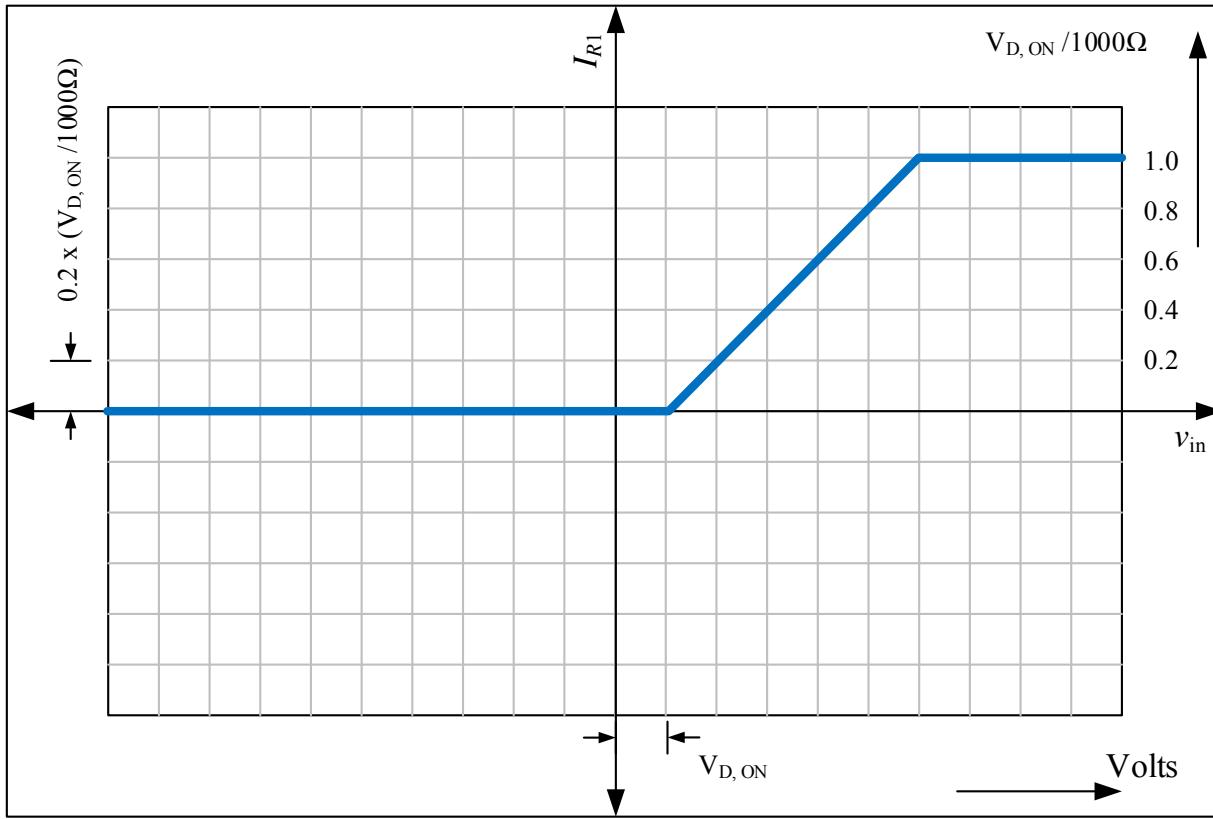
Figure 7:

the loop gives

$$I_{R_1} = \frac{v_{\text{in}} - V_{D,\text{ON}}}{R_1 + R_2} \tag{15}$$

$$= \frac{1}{R_1 + R_2} v_{\text{in}} - \frac{1}{R_1 + R_2} V_{D,\text{ON}} \tag{16}$$

$$= \frac{V_{D,\text{ON}}}{5000} \times \frac{v_{\text{in}}}{V_{D,\text{ON}}} - \frac{1}{5000} V_{D,\text{ON}} \tag{17}$$

$$= 0.2 \times \frac{V_{D,\text{ON}}}{1000} \times \frac{v_{\text{in}}}{V_{D,\text{ON}}} - 0.2 \times \frac{1}{1000} V_{D,\text{ON}} \tag{18}$$

Phase 5. Monitoring to find the next Breakpoint.

We need to find v_{D_1} as a function of v_{in}.

$$v_{D_1} = v_{\text{in}} - (v_{\text{out}} + 3 \times V_{D,\text{ON}}) \tag{19}$$

$$= v_{\text{in}} - (I_{R_1} \times R_2 + V_{D,\text{ON}} + 3 \times V_{D,\text{ON}}) \tag{20}$$

$$= v_{\text{in}} - \left(\frac{2}{5} v_{\text{in}} - \frac{2}{5} V_{D,\text{ON}} + 4 \times V_{D,\text{ON}} \right) \tag{21}$$

$$= 0.6 \times v_{\text{in}} - 3.6 \times V_{D,\text{ON}} \tag{22}$$

D_1 remains OFF for as long as v_{D_1} is less than 0. Hence, using (22), D_1 will remain OFF for as long as

$$0.6 \times v_{\text{in}} - 3.6 \times V_{D,\text{ON}} < 0 \tag{23}$$

You can continue the analysis steps on the back of this page . . .

or as long as

$$v_{\text{in}} < 6 \times V_{D,\text{ON}} \tag{24}$$

Hence the switching voltage for D_1 occurs at $v_{\text{in}} = 6 \times V_{D,\text{ON}}$.

Stage 6. Analysis post the second Breakpoint. (both diodes are ON)

Here we will have both Diodes are ON as shown in Figure 8. Using KVL around the upper loop we get

$$-3V_{D,\text{ON}} + I_{R_1} \times R_1 = 0 \tag{25}$$

which gives

$$I_{R_1} = 3V_{D,\text{ON}} \times \frac{1}{3000} = V_{D,\text{ON}} \text{ mA} \tag{26}$$
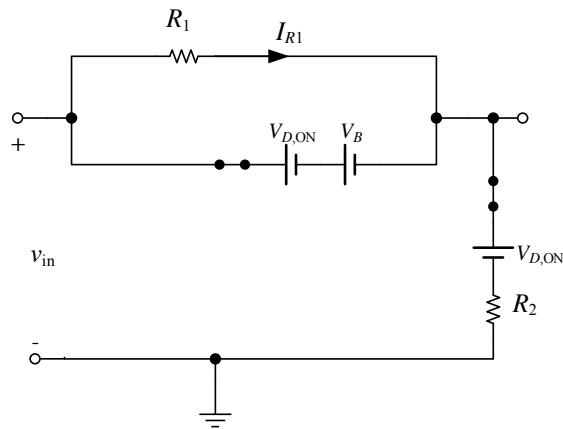
Figure 8:

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QUESTION III. Consider the circuit shown in Figure 9, where

- The supply v_S is a sinusoidal source with frequency 50 Hz.
- The PN junction (real) diodes can be modelled using 0.75 constant source and zero resistance, $(r_D = 0)$, when conducting in the forward region.
- The resistance $R = 10k\Omega$.

For convenience, the waveform of the supply voltage has been sketched as the solid black curve on Figure 10, on the next page. Other dashed grey lines and curves on that figure are merely provided for guidance in answering the following questions.
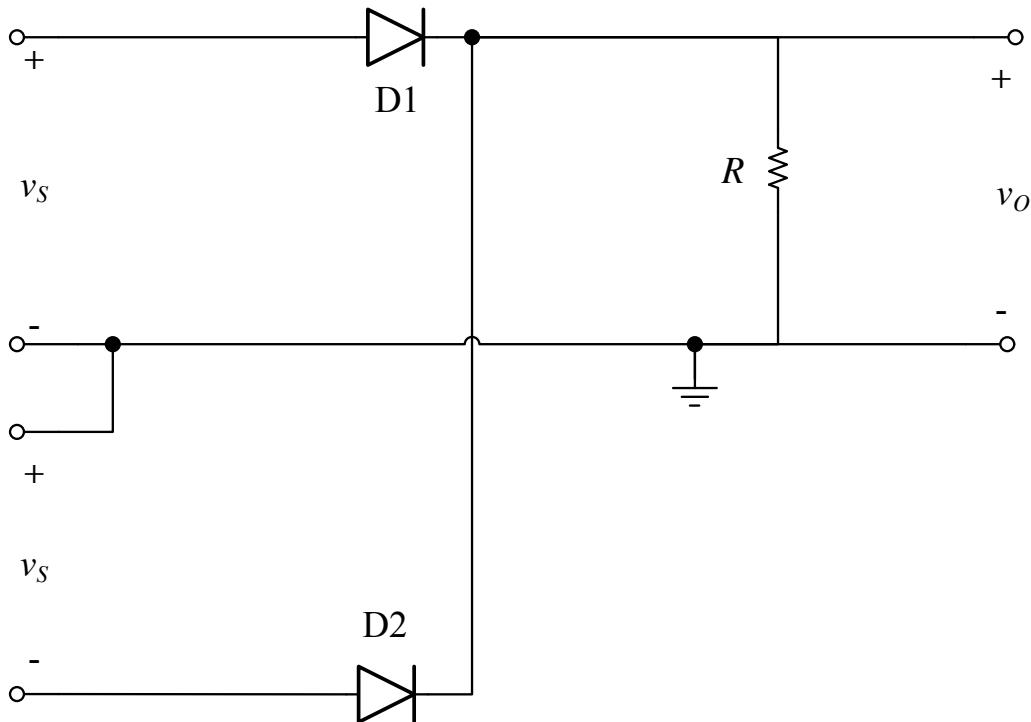
Figure 9:

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(a) (3 points) Complete the entries in Table below, referring to the waveform sketched on Figure 10.

Peak Value of the Supply Voltage v_S (Volts)	RMS Value of the Supply Voltage v_S (Volts)	Time instant t_a (in milliseconds)	Time instant t_b (in milliseconds)
6	**Solution:** $\frac{6}{\sqrt{2}} = 4.24$	**Solution:** 10	**Solution:** 20

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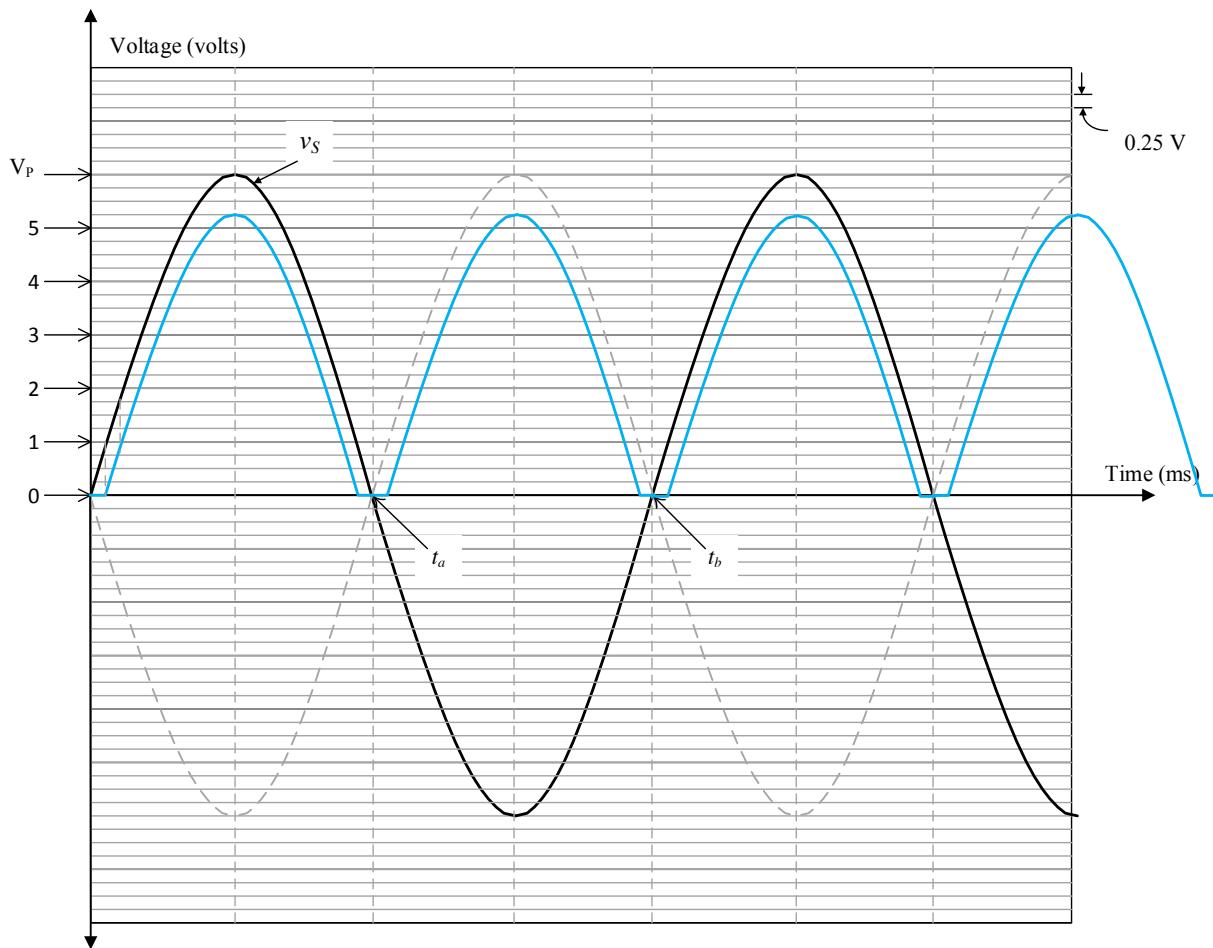
Figure 10:

(b) (6 points) On the same graph shown in Figure 10, sketch the waveform of the output voltage v_O.
IMPORTANT INSTRUCTIONS. Be very precise in your sketch, indicating clearly peak and cut-off points on the output voltage waveform. Lack of preciseness will be used as grounds to deduct marks for this part.

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You can continue the analysis steps on the back of this page . . .

(c) (16 points) Calculate the values corresponding to the entries in the following table, and write them on the corresponding table cells.

INSTRUCTIONS Calculations of the requested values may require some analysis steps. You MUST clearly display the steps by which you calculated those values. A failure to demonstrate the analysis steps, or any mismatch between the results of the steps and the numerical values entered on the table will lead to losing the marks corresponding to this part, even if the answer is numerically correct.

Peak value of the Output voltage v_O (Volts)	t_{D_1} First time instant at which D_1 starts conducting (in milliseconds)	t_{D_2} First time instant at which D_2 starts conducting (in milliseconds)	(PIV) Peak Inverse Voltage on the diodes (Volts)	v_O Average Value of the Output Voltage (Volts)
Solution: 5.25	**Solution:** 0.4	**Solution:** 10.4	**Solution:** 11.25	**Solution:** 3.1

Steps for finding t_{D_1}

Solution:

The supply voltage is given by

$$v_s(t) = 6 \times \sin\left(2\pi \times 60 \times t\right) \tag{27}$$

The diode starts conducting at $v_S(t) > V_{D,\mathrm{ON}}$, i.e., at $v_S(t) > 0.75$V. Thus, the point in time at which $v_s(t) = 0.75$ V marks the starting of conduction for the diode D_1. This is the time instant t_{D_1}. Hence,

$$
\begin{aligned}
v_s(t_{D_1}) &= 0.75 \tag{28}\\
0.75 &= 6 \times \sin\left(2\pi \times 50 \times t_{D_1}\right) \tag{29}\\
t_{D_1} &\approx 0.4 \text{ ms} \tag{30}
\end{aligned}
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Steps for finding t_{D_2}

Solution:

$$t_{D_2} = \frac{T}{2} + t_{D_1} \tag{31}$$

$$= 10.4 \text{ ms} \tag{32}$$

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Steps for finding PIV.

Solution:

$$\text{PIV} = 2 \times V_S - V_{D,\text{ON}} \tag{33}$$

$$= 2 \times 6 - 0.75 \tag{34}$$

$$= 11.25 \tag{35}$$

Steps for calculating average output voltage.

Solution: The output voltage can be expressed as

$$v_O \approx \begin{cases} v_S - V_{D,\text{ON}} & v_S > V_{D,\text{ON}} \\ 0 & -V_{D,\text{ON}} \leqslant v_S \leqslant V_{D,\text{ON}} \\ -(v_S - V_{D,\text{ON}}) & v_S \leqslant -V_{D,\text{ON}} \end{cases} \tag{36}$$

$$v_O(t) = \begin{cases} 0 & 0 \leqslant t < t_{D_1} \\ 6\sin(2\pi \times 50 \times t) - 0.75 & t_{D_1} \leqslant t < \frac{T}{2} - t_{D_1} \\ 0 & \frac{T}{2} - t_{D_1} \leqslant t < \frac{T}{2} + t_{D_1} \end{cases} \tag{37}$$

The average value of the output voltage is given by

$$\text{Average Output Voltage} = \frac{1}{T} \int_0^T v_O(t)\mathrm{d}t \tag{38}$$

Since the output voltage from 0 to $\frac{T}{2}$ is the same from $\frac{T}{2}$ to T, then

$$\text{Average Output Voltage} = 2 \times \frac{1}{T} \int_0^{T/2} v_O(t)\mathrm{d}t \tag{39}$$

However the output voltage is non-zero only when $t_{D_1} \leqslant t \leqslant \frac{T}{2} - t_{D_1}$. Therefore,

$$v_{O,\text{ave}} = 2 \times \frac{1}{T} \int_{t_{D_1}}^{T/2-t_{D_1}} (v_S - V_{D,\text{ON}})\,\mathrm{d}t \tag{40}$$

$$= \frac{2}{T}\left[\int_{t_{D_1}}^{T/2-t_{D_1}} 6 \times \sin(2\pi \times 50t)\right] - \frac{2}{T}V_{D,\text{ON}} \times \left(\frac{T}{2} - 2t_{D_1}\right) \tag{41}$$

$$= \frac{2}{T} - \frac{6}{2\pi \times 50} \times \cos(2\pi \times 50t)\Big|_{t=t_{D_1}}^{t=T/2-t_{D_1}} - \frac{2}{T}V_{D,\text{ON}} \times \left(\frac{T}{2} - 2t_{D_1}\right) \tag{42}$$

$$= 3.1 \text{ V} \tag{43}$$

(d) (10 points) Modify the circuit so that the output voltage, v_O, exhibits a maximum voltage drop of 0.1 V from its peak value.

IMPORTANT INSTRUCTIONS You **MUST** show your modifications on the circuit shown in Figure 11 (below). If your modifications necessitate adding new circuit elements, you must clearly indicate their values, and the analysis by which you arrived at the values of those elements. For example, if you think you need to add a resistor, then you must show clearly the nodes between which this resistor is to be connected and you must provide the steps by which you calculated the value of the resistance. Failure to adhere to those instructions will result in losing the mark corresponding to this part.
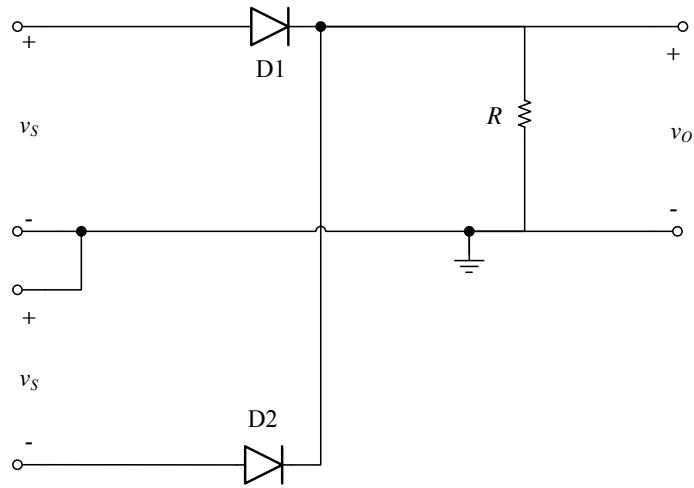

Figure 11:

Solution:

$$V_r = 0.1\text{V} \tag{44}$$

$$= \frac{V_S - 0.75}{2 \times 50 \times R \times C} \tag{45}$$

$$0.1 = \frac{5.25}{100 \times 10000 \times C} \tag{46}$$

$$C = 52.5\mu\text{F} \tag{47}$$

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———— *End of Examination.*
Do not write beyond this point. ————