

Question 1 Réponse

$$(s^3+3s^2+5s+1)Y(s) = (s^3+4s^2+6s+8)X(s).$$

$$\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}$$

Note: $1 + \frac{s^2+s+7}{s^3+3s^2+5s+1}$

Question 2 Réponse

Réponse de partie (a) :

$$(s^2+5s+10)X(s) = 7F(s)$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 10x = 7f$$

Réponse de partie (b) :

$$(s^2+21s+110)X(s) = 15F(s)$$

$$\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 110x = 15f$$

Question 3 Réponse

$$\frac{C(s)}{R(s)} = \frac{s^5 + 2s^4 + 4s^3 + s^2 + 4}{s^6 + 7s^5 + 3s^4 + 2s^3 + s^2 + 5}$$

$$(s^6+7s^5+3s^4+2s^3+s^2+5)C(s) = (s^5+2s^4+4s^3+s^2+4)R(s)$$

$$\frac{d^6c}{dt^6} + 7\frac{d^5c}{dt^5} + 3\frac{d^4c}{dt^4} + 2\frac{d^3c}{dt^3} + \frac{d^2c}{dt^2} + 5C = \frac{d^5r}{dt^5} + 2\frac{d^4r}{dt^4} + 4\frac{d^3r}{dt^3} + \frac{d^2r}{dt^2} + 4r$$

Question 4 Réponse

$$F(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+3)^2} + \frac{D}{(s+3)}$$

$$A = \frac{10}{(s+2)(s+3)^2} \Big|_{s \rightarrow 0} = \frac{5}{9} \quad B = \frac{10}{s(s+3)^2} \Big|_{s \rightarrow -2} = -5$$

$$C = \frac{10}{s(s+2)} \Big|_{s \rightarrow -3} = \frac{10}{3}, \text{ and } D = (s+3)^2 \frac{dF(s)}{ds} \Big|_{s \rightarrow -3} = \frac{40}{9}$$

$$f(t) = \frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} + \frac{40}{9}e^{-3t}$$

$$s(s+2)|_{s \rightarrow -3} = 5 \quad \int_{-\infty}^{\infty} e^{-3t} dt = \lim_{s \rightarrow -3} \frac{ds}{s^2} = 9$$

$$f(t) = \frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} + \frac{40}{9}e^{-3t}$$

Question 5 Réponse

$$s^2X(s) - 4s + 4 + 2sX(s) - 8 + 2X(s) = \frac{2}{s^2 + 2^2}$$

$$X(s) = \frac{4s^3 + 4s^2 + 16s + 18}{(s^2 + 4)(s^2 + 2s + 2)}$$

$$X(s) = -\left(\frac{1}{5}\right)\frac{s + \frac{1}{2}}{s^2 + 2^2} + \left(\frac{1}{5}\right)\frac{21(s+1) + 2}{(s+1)^2 + 1}$$

$$x(t) = \frac{1}{5} \left[21e^{-t} \cos t + \frac{2}{21}e^{-t} \sin t - \frac{1}{2} \sin 2t - \cos 2t \right]$$

$$= \frac{(2 \sin(t) + 21 \cos(t)) e^{-t}}{5} - \frac{\sin(2t)}{10} - \frac{\cos(2t)}{5}$$