Quiz 5

Quelques formules:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\omega})e^{j\omega n}d\omega \qquad X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\frac{2\pi}{N})$$

$$x[n-n_0] \xleftrightarrow{DTFT} e^{-j\omega n_0}X(e^{j\omega}) \qquad a^n u[n] \quad |a|<1 \xleftrightarrow{DTFT} 1/(1-ae^{-j\omega}) \qquad \delta[n] \xleftrightarrow{DTFT} 1$$

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \quad |a|<1 \qquad \sum_{k=0}^{n_1} a^k = \frac{1-a^{n_1+1}}{1-a} \quad a\neq 1,\ n_1 \geq 0 \qquad \sum_{k=n_1}^{\infty} a^k = \frac{a^{n_1}}{1-a} \quad |a|<1 \qquad \sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1}-a^{n_2+1}}{1-a} \quad a\neq 1,\ n_2 \geq n_1$$

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Question 1 Quelle est la transformée de Fourier du signal : $x[n] = (1/4)^n u[n+2] + 4\delta[n-3]$?

$$\sum_{n=-2}^{\infty} \left(\frac{1}{4}\right)^n u[n+2]\, e^{-j\omega n} = \sum_{n=-2}^{\infty} \left(\frac{1}{4}e^{-j\omega}\right)^n = \frac{\left(\frac{1}{4}e^{-j\omega}\right)^{-2}}{1-\frac{1}{4}e^{-j\omega}} = 16\, e^{j2\omega}\frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

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$$\sum_{n=-2}^{+\infty} \left(\frac{1}{4}\right)^n u[n+2]\, e^{-j\omega n} = \underbrace{\left(\frac{1}{4}\right)^{-2} e^{j2\omega}}_{n=-2} + \underbrace{\left(\frac{1}{4}\right)^{-1} e^{j\omega}}_{n=-1} + \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n e^{-j\omega n}}_{1/(1-\frac{1}{4}e^{-j\omega})}$$

$$= 16\, e^{j2\omega} + 4e^{j\omega} + \frac{1}{1-\frac{1}{4}e^{-j\omega}} = \frac{16\, e^{j2\omega}(1-\frac{1}{4}e^{-j\omega}) + 4e^{j\omega}(1-\frac{1}{4}e^{-j\omega}) + 1}{1-\frac{1}{4}e^{-j\omega}} = \frac{16\, e^{j2\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{16\, e^{j2\omega}}{1-\frac{1}{4}e^{-j\omega}} + F.T.\{4\delta[n-3]\} = \frac{16\, e^{j2\omega}}{1-\frac{1}{4}e^{-j\omega}} + 4e^{-j3\omega}$$

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$$\left(\frac{1}{4}\right)^n u[n] \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{4}e^{-j\omega}} \qquad \left(\frac{1}{4}\right)^{n+2} u[n+2] \xleftrightarrow{DTFT} \frac{e^{j2\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

$$\underbrace{\left(\frac{1}{4}\right)^{-2}}_{(\frac{1}{4})^n}\left(\frac{1}{4}\right)^{n+2} u[n+2] \xleftrightarrow{DTFT} \frac{16\, e^{j2\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

Question 2 Quel est le signal $x[n]$ dont la transformée de Fourier est $X(e^{j\omega}) = \dfrac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})}$?

$$X(e^{j\omega}) = \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{1-\frac{1}{3}e^{-j\omega}}$$

$$A = \left.\frac{2}{1-\frac{1}{3}e^{-j\omega}}\right|_{e^{j\omega}=2} = \frac{2}{1-\frac{1}{3}\cdot 2} = 6$$

$$B = \left.\frac{2}{1-\frac{1}{2}e^{-j\omega}}\right|_{e^{j\omega}=3} = \frac{2}{1-\frac{1}{2}\cdot 3} = -4$$

$$X(e^{j\omega}) = \frac{6}{1-\frac{1}{2}e^{-j\omega}} - \frac{4}{1-\frac{1}{3}e^{-j\omega}} \qquad x[n] = 6\left(\frac{1}{2}\right)^n u[n] - 4\left(\frac{1}{3}\right)^n u[n]$$