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PHY 2723

HIVER 23

Devoir 1

1-a Question 1

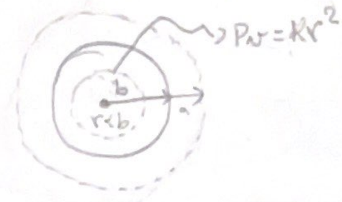
a) Calculer la densité de flux électrique ($\vec{D}$) à $r = 0.02$ m

$P_v = Kr^2$

$K = 4\times10^{-5}\ C/m^5$

$\varepsilon_0 = 8.85\times10^{-12}$

$b = 0.1$ m



Avec le théorème de Gauss sur une surface de Gauss sphérique.

• $\oint \vec{E}\cdot d\vec{s} = \dfrac{Q_{int}}{\varepsilon_0}$ Par symétrie $\vec{D}(\vec{r}) = D(r)\hat{r}$

$\Rightarrow \oint_S E\cdot ds = \int_0^{2\pi}\int_0^{\pi} E(r)\, r^2 \sin\theta\, d\theta\, d\varphi$

$= E_r(r)\cdot r^2 \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \underbrace{\int_0^{\pi} \sin\theta\, d\theta}_{2}$

$\boxed{\oint E\cdot ds = E_r(r)\, 4\pi r^2}$ ← côté gauche de l'équat°

• $\dfrac{Q_{int}}{\varepsilon_0} = \iiint P_v\, dv = \int_0^{r}\int_0^{2\pi}\int_0^{\pi} Kr'^2 \times r'^2 \sin'\theta\, d\theta' d\varphi' dr'$

$\vec{D} = \dfrac{Q_{int}}{4\pi R^2}$

$= K\int_0^{0,02} r'^4 dr' \int_0^{2\pi} d\varphi' \int_0^{\pi} \sin'\theta\, d\theta'$

$= K \times \dfrac{r'^5}{5} \times \underbrace{\varphi\Big|_0^{2\pi}}_{2\pi} \times \underbrace{(-\cos'\theta)\Big|_0^{\pi}}_{2}$

$\boxed{\dfrac{Q_{int}}{\varepsilon_0} = \dfrac{4\pi K r'^5}{5\varepsilon_0}}$ ← côté droit de l'équat°

d'où $E_r(r)\cdot 4\pi r^2 = \dfrac{Q_{int}}{\varepsilon_0} = E_r(r) = \dfrac{4\pi K r'^5}{5\varepsilon_0\, 4\pi r'^2} \Rightarrow E_r(r) = \dfrac{Kr'^3}{5\varepsilon_0}$ or $\vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0 \times E_r(r)$

d'où $\vec{D} = \dfrac{Kr'^3}{5\varepsilon_0}\times\varepsilon_0 \Rightarrow \vec{D} = \dfrac{Kr'^3}{5} = \dfrac{4\times10^{-5}\times(0.02)^3}{5} \Rightarrow \boxed{\vec{D} = 6.4\times10^{-11}}$

b) Déterminons la distance r.

$r > b$: extérieur de la sphère

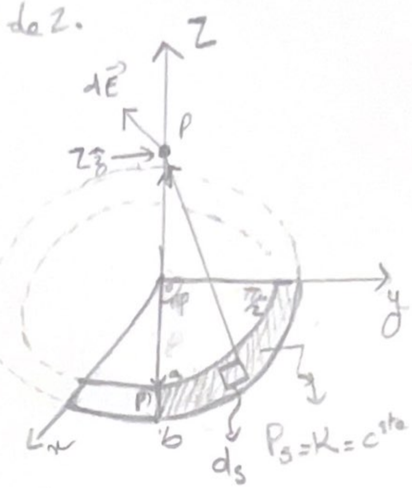
$Q_{int} = \int_0^{b}\int_0^{2\pi}\int_0^{\pi} Kr'^4 \sin\theta' d\theta' d\varphi' dr' = K\int_0^{b} r'^4 dr' \int_0^{2\pi} d\varphi' \int \sin\theta' d\theta' = \dfrac{4\pi K b^5}{5}$.

$\vec{D}(r>b) = \dfrac{4\pi K b^5}{5\times4\pi\times r^2} = \boxed{\dfrac{Kb^5}{5r^2}}$ donc $r^2 = \dfrac{Kb^5}{5\vec{D}(r>b)}$ avec $\vec{D}(r>b) = \vec{D}(r<b)$ et $b = 0.1$

$r = \sqrt{\dfrac{4\times10^{-5}\times0.1^5}{5\times6.4\times10^{-11}}} = \boxed{r = 1.12\ m}$

Exercice 2

Trouvons la composante Z du champs électriq sur n'importe quel point de Z.



La distance vectorielle de la charge jusqu'au point P sur l'axe Z

est $\vec{R} = -\rho'\hat{\rho} + z\hat{z}$ d'où $R = \sqrt{\rho'^2 + z^2} = (\rho'^2 + z^2)^{1/2}$

L'intensité du champ électriq au point $P(0,0,Z)$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_s' |\vec{r} - \vec{r}'|}{|\vec{r} - \vec{r}'|^3} ds'$$

$$= \frac{\rho_s'}{4\pi\varepsilon_0}\int_a^b\int_0^{\pi/2} \frac{ds'}{R^3}\vec{R}$$

$$= \frac{\rho_s'}{4\pi\varepsilon_0}\int_a^b\int_0^{\frac{\pi}{2}} \frac{\rho' d\rho' d\varphi'}{(\rho'^2 + z^2)^{3/2}}(-\rho'\hat{\rho} + z\hat{z})$$

En coordonnées cylindriq

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ Z = Z \end{cases} \quad (\rho, \phi, Z)$$

$ds' = \rho' d\rho' d\varphi'$

$\hat{\rho} = \cos\varphi + \sin\varphi.$

de plus, $\int_0^{\frac{\pi}{2}} \hat{\rho}\, d\varphi' = \int_0^{\frac{\pi}{2}} (\cos\varphi + \sin\varphi)\, d\varphi$

$$= \sin\varphi - \cos\varphi \Big|_0^{\frac{\pi}{2}} = 1 - (-1) = 2.$$

$$\Rightarrow \frac{\rho_s'}{4\pi\varepsilon_0}\int_a^b\int_0^{\frac{\pi}{2}} \frac{\rho' d\rho' d\varphi'}{(\rho'^2 + z^2)^{3/2}}(-\rho'\hat{\rho} + z\hat{z})$$

$$\Rightarrow \vec{E} = \frac{\rho_s'}{4\pi\varepsilon_0}\left[\int_a^b\int_0^{\frac{\pi}{2}} \frac{-\rho'^2\hat{\rho}\, d\rho' d\varphi'}{(\rho'^2 + z^2)^{3/2}} + \int_a^b\int_0^{\frac{\pi}{2}} \frac{\rho' d\rho' d\varphi' z}{(\rho'^2 + z^2)^{3/2}}\hat{z}\right]$$

Seule la composante en Z nous intéresse d'où

$$\vec{E_Z} = \frac{\rho_s' Z}{4\pi\varepsilon_0}\left[\int_a^b\int_0^{\frac{\pi}{2}} \frac{\rho' d\rho' d\varphi'}{(\rho'^2 + z^2)^{3/2}}\hat{z}\right]$$

$$= \frac{\rho_s' Z}{4\pi\varepsilon_0}\underbrace{\int_0^{\frac{\pi}{2}} d\varphi'}_{\frac{\pi}{2}} \times \underbrace{\int_a^b \frac{\rho' d\rho'}{(\rho'^2 + z^2)^{3/2}}}_{} \times \hat{z}$$

Par la division en fraction partielle

$$\boxed{\vec{E_Z} = \frac{\rho_s' Z}{8\varepsilon_0}\left[\frac{1}{(a^2 + Z^2)^{1/2}} - \frac{1}{(b^2 + Z^2)^{1/2}}\right]\hat{Z}}$$

~~Ques~~

Exercice 3

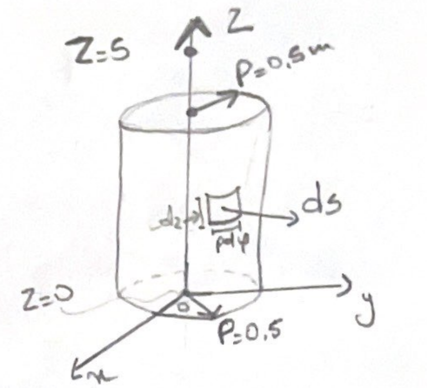
Calculons le flux électrique causé par $\vec{D}$

$\vec{D} = (45\hat{y} + 30\hat{z})\ C.m^{-2}$

$0 < z < 5m$

$\rho = 0{,}5m$

$0 < \phi < \pi$



$$\Psi = \oint \vec{D} \cdot d\vec{S}$$

$$d\vec{S} = \rho\, d\phi\, dz\, \hat{\rho}$$

$$\Rightarrow \Psi = \iint (45\hat{y} + 30\hat{z})(\rho\, d\phi\, dz\, \hat{\rho}) \quad \text{avec} \begin{cases} \hat{y} = \sin\phi\,\hat{\rho} + \cos\phi\,\hat{\phi} \\ \hat{z} = 1\hat{z} \end{cases}$$

$$= \iint \left(45(\sin\phi\,\hat{\rho} + \cos\phi\,\hat{\phi}) + 30\hat{z}\right)(\rho\hat{\rho}\, d\phi\, dz)$$

$$= \rho \iint (45\sin\phi\,\hat{\rho} + 45\cos\phi\,\hat{\phi} + 30\hat{z})(\hat{\rho}\, d\phi\, dz)$$

$$= \rho \int_0^5 \int_0^{\pi} 45 \sin\phi\, d\phi\, dz$$

$$= 0{,}5 \times \int_0^5 dz \times \int_0^{\pi} 45 \sin\phi\, d\phi$$

$$= 0{,}5 \times 5 \times 45 \times (-\cos(\phi))\Big|_0^{\pi} = 0{,}5 \times 5 \times 45 \times (-\cos\pi - (-\cos 0))$$

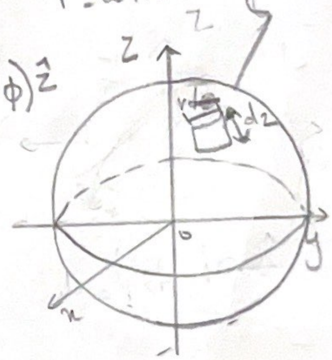
$$\boxed{\Psi = 225\ C}$$

Question 4 (Exercice 4)

Déterminons une équation du champs $\vec{E}$

$\rho_S = \sigma_0 \cos\theta$

$r = 2m$



Par la loi de Coulomb, on obtient

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\rho_S(\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3} dS' = \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{\pi} \frac{\rho_S(\vec{r'})(\vec{r} - \vec{r'})}{\rho'^3} (\rho'^2 \sin\theta' d\theta' d\phi)\hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{\pi} \frac{\sigma_0 \cos\theta' (-\rho'\hat{\rho})}{\cancel{\rho'^3}} (\cancel{\rho'^2} \sin\theta' d\theta' d\phi)\hat{z}$$

$$(\sin\theta'\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y})\,d\theta' -$$

$$= \frac{-\sigma_0}{4\pi\varepsilon_0} \int_0^{2\pi} d\phi' \int_0^{\pi} \hat{z} \cos\theta' \sin\theta' (\sin\theta' \cos\phi\,\hat{x} + \sin\theta \sin\phi\,\hat{y})\,d\theta' -$$

$$= \frac{-\sigma_0 \hat{z}}{4\pi\varepsilon_0} \underbrace{\int_0^{2\pi} d\phi'}_{2\pi} \underbrace{\int_0^{\pi} \cos^2\theta \sin\theta\, d\theta}_{\frac{2}{3}}$$

$$\boxed{E_{\hat{z}} = \frac{-\sigma_0}{3\varepsilon_0}\hat{z}}$$

$dS = \rho'^2 \sin\theta' d\theta' d\phi'$

$\vec{r'} = \vec{R} = \rho'\hat{\rho}$

$\vec{r} = \vec{0}$

$\vec{r} - \vec{r'} = -\rho'\hat{\rho}$

$|\vec{r} - \vec{r'}| = \rho'$