Part 4 - Heuristics in the Adaptive A* [20 points]:

1) The project argues that "the Manhattan distances are consistent in gridworlds in which the agent can move only in the four main compass directions." Prove that this is indeed the case.

Using the definition of a heuristic is consistent if h(s) <= c(s,a) + h(s'). h(s) is the heuristic estimate of the cost from going to s to the target. c(s,a) is the cost of moving from s to s' and h(s') is the heuristic estimate from s' to the target.

Theorems

Because the state s is at coordinates (x,y) and the target is another set of (x',y') where x' is xgoal and y' is ygoal. Then h(s) = |x-x'| + |y-y'|

Because of this, the agent can move in 4 cardinals, with each move of 1 unit, where c(s,a) = 1

Proof

If we let s=(x,y), and s' be any neighboring state reached in any of the 4 directions, then moving east or west changes x by 1 unit, as h(s') = |(x +/- 1) - xgoal| + |y-ygoal|
When moving north or south, y coordinate changes by 1 as h(s') = |x-xgoal| + |(y+/- 1) - ygoal|

Since moving one step in any direction either gets you reduces or equal total cost to the target, by at most one, so h(s') >= h(s) -1 and h(s) <= h(s')+1
Since c(s,a) = 1, h(s) <= c(s,a) + h(s') , we satisfy the condition.

Conclusion: As manhattan distances satisfy the consistency condition and prove that the cost increase always be 1 or 0 in its travel to the target, Adaptive A* modifies this bc it updates h(s) by recognizing which are not going to break the consistency rule even if h(s) increases. This keeps the Manhattan distance valid and consistent through searching. In Adaptive A* it always finds the optimal path, and never revisits states unnecessarily This makes it efficient in its search. Overall, the Manhattan distance can only move in the main 4 compass directions, even when utilizing A*

2) Furthermore, it is argued that "The h-values $h_{new}(s)$... are not only admissible but also consistent." Prove that Adaptive A* leaves initially consistent h-values consistent even if action costs can increase.

The problem asks us to prove that the $h_{new}(s)$ in Adaptive A* will be consistent even if action costs go up.

Theorems and Definitions

h(s) is consistent when h(s) <= c(s,a) + h(s') ; h(s) is the heuristic estimate of the cost from going to s to the target. c(s,a) is the cost of moving from s to s' and h(s') is the heuristic estimate from s' to the target.

Proof

$h_{new}(s) = g(s_{goal})- g(s)$, where g(s) is the shortest path from start to s found in curr search and $g(s_{goal})$ is the actual shortest path cost to the target. This update ensures that at state s is either as accurate as the previous search or better.

$h_{new}(s) = g(s_{goal})- g(s)$
$h_{new}(s') = g(s_{goal})- g(s')$

Because the agent moves from s to s' with cost c(s,a): $g(s) <= g(s) + c(s,a)$
$g(s_{goal})- g(s) <= c(s,a) + g(s_{goal})- g(s')$
$h_{new}(s) <= c(s,a) + h_{new}(s')$

This holds to the previous statement that even if the action cost increases b/w searches the new search accounts for the increased cost. It will never decrease h- values as Adaptive A* ensures that future searches expand fewer state by making the heuristic more efficient