
1: The First Problem

if $A \rightarrow B$ then through augmentation rule $ACDE \rightarrow BCDE$

if $ACDE \rightarrow BCDE$ is valid in R and it covers entire relational schema then its left hand side is a minimal key (A,C,D,E)

if $E \rightarrow CD$ then through augmentation rule $ABE \rightarrow ABCD$

if $ABE \rightarrow ABCD$ is valid in R and it covers entire relational schema then its left hand side is a minimal key (A,B,E)

2: The second problem

if $AB \rightarrow DE$ then through augmentation rule $ABC \rightarrow CDE$

if $ABC \rightarrow CDE$ is valid in R and it covers entire relational schema then its left hand side is a minimal key (A,B,C)

if $D \rightarrow ABC$ then through augmentation rule $DE \rightarrow ABCE$

if $DE \rightarrow ABCE$ is valid in R and it covers entire relational schema then its left hand side is a minimal key (D,E)

3: The third problem

if $A \rightarrow CE$ and $CE \rightarrow BD$ then through transitivity rule $A \rightarrow BD$

if $A \rightarrow CE$ and $A \rightarrow BD$ then through union rule $A \rightarrow BCDE$

if $A \rightarrow BCDE$ is valid in R and it covers entire relational schema then its left hand side is a minimal key (A)

4: The fourth problem

if $A \rightarrow B$ then through augmentation rule $ACDE \rightarrow BCDE$

if $ACDE \rightarrow BCDE$ is valid in R and it covers entire relational schema then its left hand side is a minimal key (A,C,D,E)

if $B \rightarrow A$ then through augmentation rule $BCDE \rightarrow ACDE$

if $BCDE \rightarrow ACDE$ is valid in R and it covers entire relational schema then its left hand side is a minimal key (B,C,D,E)