Practice for Final Examination

1. Explain the algorithm analysis framework; how to analyze algorithms; how to design algorithms.
2. Prove $f(n)=n^{1.5}$ does not belong to $O(n \log n)$.
3. Find the upper bound of the number of nodes in a binary tree with a height of h.
4. Explain data structures stack and queue; explain FIFO rule and FILO rule.
5. Explain data structure heap.
6. List all of main data structures and explain their definitions and operations.
7. List all of main algorithm design paradigms and strategies and their corresponding problems.
8. When we use a max heap to implement a priority queue, what are the time complexity of both the add and delete operations?
9. Find the explicit expression of $T(n)$ if $T(n) = T(n-1)+n$, $T(1)=1$.
10. In a circular doubly linked list with 10 nodes, we will need to change 4 links if we want to delete a node other than the head node. Point out which four links.
11. Prove $f1(n)\times f2(n)=\Omega(g1(n)\times g2(n))$ if $f1(n)=\Omega(g1(n))$and $f2(n)=\Omega(g2(n))$.
12. When using linked list to perform insertion sort, each time we remove an element from the input list and insert it to the correct position in the linked list. Assume that we have n numbers to be sorted, prove that the time complexity of the insertion sort algorithm is $O(n^2)$.
13. The Tower of Hanoi is a mathematical game. The game consists of 3 rods, and n disks of different sizes which can slide onto any rod. The puzzle starts with the disks stack in ascending order of size on rod number1 (the smallest disk is on the top). The objective of the game is to move the entire stack to rod number 3, obeying the following rules:
    1) Only one disk can be moved at a time;
    2) Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod;
    3)No disk may be placed on top of a smaller disk.

    In this problem, please write an algorithm using the recursive technique to output a series of moves which solve the Tower of Hanoi problem. The input and output are as follows:

    Input: one single number which represents the number of disks on the rod

    Output: Each line of output consists of a pair(i, j), which represents a move from rod number i to rod number j. The sequence of moves should move the entire stack from rod number 1 to rod number 3.
14. In the following questions, consider the list of numbers: 62, 31, 70, 91, 25, 11, 9, 61, 73, 6.

1). Show the result of inserting the numbers in the list in the same order specified above into an initially empty minimum heap. Note that you need to show how the heap looks like after each number is inserted.

2). Show the result of inserting the numbers in the list in the same order specified above into an initially empty binary search tree. Note that you need to show how the binary search tree looks like after each number is inserted.

3). Use the binary search tree you created in question 2. What are the two possible binary search trees after 62 is deleted?

4). Explain how you can utilize a minimum heap to sort the list of numbers in descending order. (3 points) Let n be the number of elements in the list. What is the time complexity of your sorting algorithm?

15. In this problem, we consider logic expressions with the following operands and operators:

    Operands: 0 and 1, which represents false and true, respectively.

    Operators: &(and), |(or), and!(not).

    Note that you also have to consider left and right parenthesis. The precedences of the three operators are ! > & > |. Draw the logical expression tree of the expression!(0&!1&0|0)&0.

Since!(not) is an unary operator, we ask you to put its only operand to its right child. Write down its prefix expression.

16. Which of the following sorting algorithms does not require $O(n^2)$ steps in the worst case?
A. insertion sort
B. selection sort
C. heap sort
D. bubble sort
E. Quick sort

17. Suppose that you need to maintain a collection of data whose contents are fixed—i.e., you need to search for and retrieve existing items, but never need to add or delete items. Although the collection of data may be quite large, you may assume that it can fit in the computer's memory. Which of the following data structures is the most efficient one to use for this task?
A. a sorted array
B. a linked list
C. a binary search tree
D. a queue
E. all of the above perform the same in this case

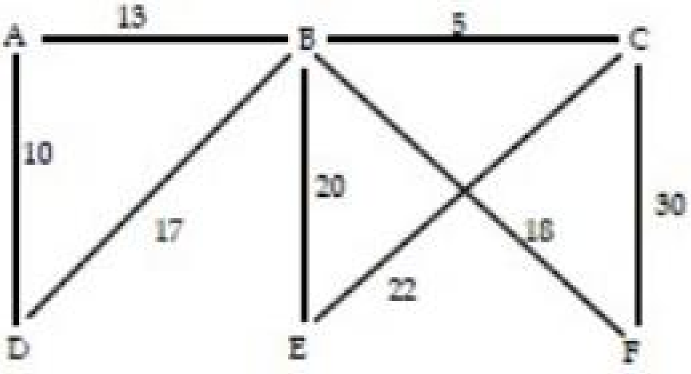
18. If a binary search tree is not allowed to have duplicates, there is more than one way to delete a node in the tree when that node has two children. One way involves choosing a replacement node from the left sub-tree. If this is done, which node are we looking for?
A. the largest node in the sub-tree
B. the smallest node in the sub-tree
C. the root of the left sub-tree
D. the next-to-smallest node in the sub-tree
E. it doesn't matter – any node in the left sub-tree will do

19. Which of the following data structures is most appropriate for situations in which you need to efficiently manage (key, value) pairs that are stored on disk?
A. an array
B. a linked list
C. a binary search tree
D. a 2-3 tree
E. a B-tree

20. Which of the following state-space search methods makes the most efficient use of memory?
A. breadth-first search
B. depth-first search
C. greedy search
D. A* search
E. they are all equivalent

21.
Asymptotic notations(O, Ω , Θ);
Time Efficiency of Algorithms;
Data structures(array, list, queue, stack, tree, graph);
Sorting(bubble, selection, merge,quick,heap);
Binary tree and AVL tree;
Random Number Generation;
Greedy Algorithms;
Numerical Computing;
Backtracking, branch and bound.

II-4. Graph Algorithms II ***(15 points total)***



a. *6 points*
Suppose that Prim's algorithm has been executed, starting from node F, up to the point at which there are four edges selected for inclusion in the minimal spanning tree. List these four edges in the order that they are selected for inclusion, using notation similar to (A, B) to specify an edge.

b. *6 points*
Suppose that the MST2 algorithm from Problem Set 5 has been executed, up to the point where there are four edges selected for inclusion in the minimal spanning tree. List these four edges in the order that they are selected for inclusion.

c. *3 points*
Show an example of a spanning tree that is *not* minimal by darkening the appropriate edges on the diagram below: