Low-Complexity Passive Vehicle Suspension Design Based on Element-Number-Restricted Networks and Low-Order Admittance Networks

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This paper is concerned with the low-complexity passive suspension design problem, aiming at improving vehicle performance in the meanwhile maintaining simplicity in structure for passive suspensions. Two methods are employed to construct the low-complexity passive suspensions. Using the first method, the number of each element is restricted to one, and the performance for all networks with one inerter, one damper, and one spring is evaluated, where best configurations for different vehicle settings are identified. Using the second method, low-order admittance networks whose orders of admittance functions are no larger than three are utilized. Design methods are proposed by directly using the positive realness conditions imposed on the admittance functions. The effectiveness of the proposed methods is numerically demonstrated, and the comparison between these two constructing methods is conducted. [DOI: 10.1115/1.4040294]

1 Introduction

Vehicle suspensions are essential in determining the overall dynamics of a vehicle, contributing to improve the ride comfort and vehicle stability. Generally speaking, suspension systems can be classified into three categories, that is, passive, semi-active, and active suspensions, mainly based on the implemented devices (or actuators) [1,2]. Passive suspensions only use passive elements such as springs, dampers, and inerters [3]. High reliability and low cost are the main merits for passive suspensions. Semi-active suspensions utilize coefficient online controllable devices (or semi-active devices), such as semi-active dampers [4–6], semi-active springs [7], and semi-active inerters [8,9], to significantly improve vehicle performance without consuming much energy. Active suspensions not only provide best performance but also demand most energy due to force-generating actuators [10–14]. Although the performance of passive suspensions is limited due to the absence of feedback control actions, the current mainstream road vehicle suspensions are still working in a passive manner [15].

The structure for the traditional passive suspensions is mainly a parallel connection of a spring and a damper. The main task for passive suspension design is actually a parameter optimization problem in terms of the spring stiffness and the damping coefficient. However, due to the introduction of a new passive element called inerter [3], the structure for passive suspensions has been significantly changed, where various complex structures composed of springs, dampers, and inerters have been constructed [16–18]. As a result, the passive suspension design problem turns to be a structure design problem together with a parameter optimization problem. In Refs. [16–18], some given-structured networks were employed as passive suspension struts, where their performances were evaluated in a different manner. In Ref. [19], network synthesis was employed to simultaneously design the structure and the parameters for passive suspensions by using matrix inequalities. In Ref. [20], a mechatronic network strut was proposed for passive suspensions. In Ref. [21], the influence of inerter nonlinearities on vehicle suspensions was investigated. Moreover, the influence of inerter on natural frequencies of vibration systems has been studied [22]. Semi-active inerter concept (switching inertance) whose inertance can be adjusted online has been introduced [8], and the physical realization of semi-active inerter has been proposed in Ref. [9]. In Ref. [23], two types of energy-harvesting shock absorbers were proposed to simultaneously improve vehicle performances and regenerate energy, where inerter was utilized to model these energy-harvesting shock absorbers and to facilitate the analysis. Inerter has been applied in various mechanical control systems such as Refs. [24–36] and references therein.

Note that the structural simplicity is an important consideration for passive suspensions due to the limited working space and the difficulties in physical constructions for mechanical systems. However, such an issue has not been fully addressed in the existing investigations for inerter-based vehicle suspensions. Therefore, in this paper, low-complexity passive suspensions are studied to improve the suspension performance in the meanwhile maintaining structural simplicity. Here, the low-complexity passive suspensions are constructed by using two methods. Using the first method, the number of each element is restricted to one, and hence, the performance for all networks with one inerter, one damper, and one spring is evaluated. Using the second method, low-order admittance networks, the orders of which are no larger than three, are studied. Different from the matrix inequality optimization method in Ref. [19], in this paper, positive realness conditions imposed on the admittance functions are employed and optimized by using nonlinear optimization methods. Design methods are proposed for these two kinds of low-complexity passive suspensions, and the performances are evaluated and compared. In Ref. [37], a structure-immittance approach was proposed to design a full set of possible series-parallel networks with predetermined numbers of each element type for passive vibration control. The main differences between this paper and Ref. [37] are as

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follows: for the first method of this paper, the entire networks with one inerter, one damper, and one spring are emphasized, and for the second method of this paper, the order of the suspension admittance is restricted without constraining the suspension structure to be series-parallel type; while in Ref. [37], a full set of possible series-parallel networks with predetermined numbers of each element type is considered. These constitute the main contributions of this paper.

The structure of this paper is organized as follows: In Sec. 2, the vehicle model, the performance measures, and the constructing methods for the passive suspensions are introduced. In Sec. 3, the performance for all networks with one inerter, one damper, and one spring is evaluated. In Sec. 4, the performance for low-order admittance networks is evaluated. Conclusions are drawn in Sec. 5.

2 Problem Formulation

2.1 Vehicle Model and Performance Measures. A quarter-car vehicle model presented in Fig. 1 is employed, which consists of a sprung mass m_s, an unsprung mass m_u, and a tire with spring stiffness k_t [16–18]. The suspension strut is composed of a spring K and a parallel-connected passive network $W(s)$, where the passive network $W(s)$ incorporates inerters. Since the spring K provides static support for the vehicle body (sprung mass), it is predetermined during the design process as done in Refs. [16–18].

The suspension strut supplies an equal and opposite force on the sprung and unsprung masses, and $W(s)$ denotes the admittance of the passive network defined by the ratio of Laplace transformed force to relative velocity [16]. The methods on constructing the passive network $W(s)$ will be presented in Sec. 2.2.

The equations of motion in the Laplace domain are

$$m_s s^2 \hat{z}_s = -(K + sW(s))(\hat{z}_s - \hat{z}_u) \tag{1}$$

$$m_u s^2 \hat{z}_u = (K + sW(s))(\hat{z}_s - \hat{z}_u) + k_t(\hat{z}_r - \hat{z}_u) \tag{2}$$

For a suspension system, there are a number of design requirements such as passenger comfort, handling, tire normal loads, and limits on suspension travel. In the quarter-car model, these can be approximately quantified by the disturbance responses from z_r to z_s and z_u. Specially, the ride comfort and tire grip performances are employed in this paper. For the ride comfort, the root-mean-square (RMS) of the body vertical acceleration in response to road disturbances is employed and defined as J_1 as follows:

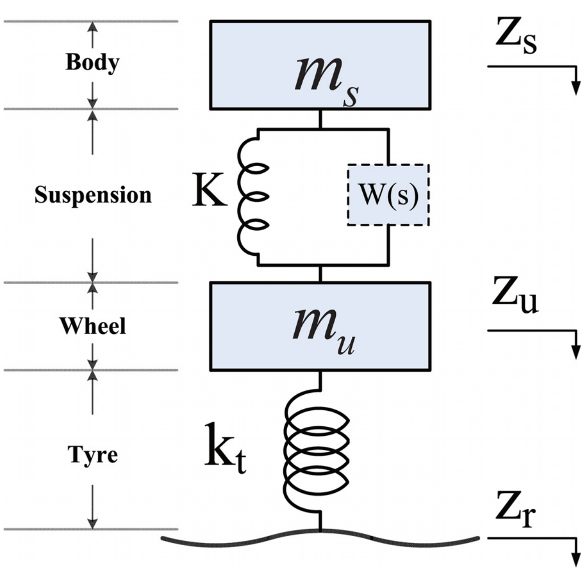

Fig. 1 A quarter-car vehicle model

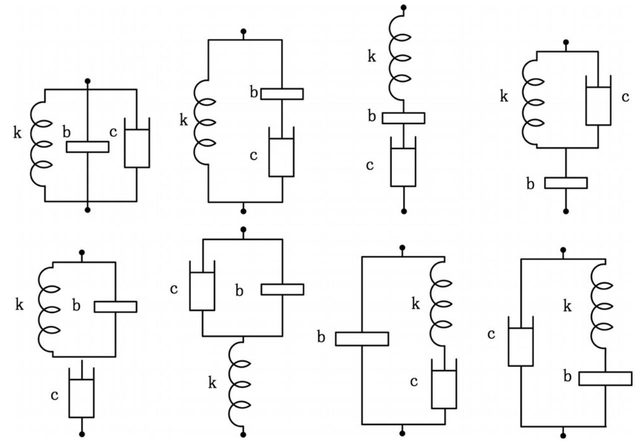

Fig. 2 Explicit networks with one inerter, one damper, and one spring. The first row from left to right denotes *C*1, *C*2, *C*3, and *C*4 and the second row from left to right denotes *C*5, *C*6, *C*7, and *C*8.

$$J_1 = 2\pi(V\kappa)^{1/2}\|sT_{\hat{z}_r \to \hat{z}_s}\|_2 \tag{3}$$

where V is the speed of the car, κ is the road roughness parameter. $T_{x\to y}$ denotes the transfer function from x to y, and $\|\cdot\|_2$ is the standard H_2 norm. The RMS tire grip parameter J_3 is defined as

$$J_3 = 2\pi(V\kappa)^{1/2}\left\|\frac{1}{s}T_{\hat{z}_r \to k_t(\hat{z}_u - \hat{z}_r)}\right\|_2 \tag{4}$$

The detailed derivations on the performance measures of ride comfort J_1 and tire grip J_3 are given in Ref. [16].

2.2 Methods of Constructing the Passive Network $W(s)$. In this paper, the passive network $W(s)$ in Fig. 1 can be constructed by using two methods, that is, by using element-number-restricted networks and by using low-order admittance networks, respectively.

2.2.1 Element-Number-Restricted Networks. For the first method, the number of each element is restricted to one to maintain simplicity of the suspension strut, and then all networks with one inerter, one damper, and one spring as shown in Fig. 2 are investigated. The admittances for the networks shown in Fig. 2 are given in Table 1.

Due to the specific structure of the suspension strut in Fig. 1, the spring stiffness k for networks $C1$ and $C2$ in Fig. 2 can be included in the parallel spring K. In this way, networks $C1$ and $C2$ can be simplified by removing the spring k. Note that networks $C1$, $C2$, $C3$, and $C4$ have been investigated in Refs. [16–18].

Table 1 Admittances for the networks in Fig. 2

$W_1(s) = \dfrac{k}{s} + bs + c$	$W_2(s) = \dfrac{k}{s} + \dfrac{1}{\dfrac{1}{bs} + \dfrac{1}{c}}$
$W_3(s) = \dfrac{1}{\dfrac{s}{k} + \dfrac{1}{bs} + \dfrac{1}{c}}$	$W_4(s) = \dfrac{1}{\dfrac{1}{\dfrac{k}{s} + c} + \dfrac{1}{bs}}$
$W_5(s) = \dfrac{1}{\dfrac{1}{\dfrac{k}{s} + bs} + \dfrac{1}{c}}$	$W_6(s) = \dfrac{1}{\dfrac{1}{c + bs} + \dfrac{s}{k}}$
$W_7(s) = bs + \dfrac{1}{\dfrac{s}{k} + \dfrac{1}{c}}$	$W_8(s) = c + \dfrac{1}{\dfrac{s}{k} + \dfrac{1}{bs}}$

However, in this paper, we provide a comprehensive evaluation for the performance of all the networks with one inerter, one damper, and one spring.

2.2.2 Low-Order Admittance Networks. The number of each element for the element-number-restricted networks in Fig. 2 is restricted to one, and hence, the advantage of simplicity in suspension structure is maintained. However, the structures for these given networks are not guaranteed to be the optimal structures. Therefore, in this section, the passive networks $W(s)$ are deemed as "black box," and in this way, both the structure and the parameters are optimized simultaneously.

For the second method, network synthesis methods are utilized to obtain the passive network $W(s)$. Since the realized networks are generally more complex if higher-order admittances are considered, then to keep simplicity of the obtained suspension strut, only the admittances up to third-order are investigated in this paper.

It is well known that the mechanical network $W(s)$ is passive if and only if its admittance is positive real [3,38]. In addition, for any real-rational function $W(s)$ which is positive real, there exists a one-port mechanical network whose admittance equals $W(s)$ which consists of a finite interconnection of springs, dampers, and inerters [3].

Therefore, the condition for the considered admittances to be realized as passive networks is that the admittance is positive real. Since only admittances up to third-order are considered in this paper, the conditions for checking positive realness given in Ref. [39] are employed.

LEMMA 1. *Any first-order real-rational function in the form of*

$$W_{1\text{st}}(s) = \frac{\alpha_1 s + \alpha_0}{\beta_1 s + \beta_0} \tag{5}$$

where $\alpha_i \geq 0$ and $\beta_i \geq 0$ (not all $\beta_i = 0$) is positive real.

Lemma 1 can be easily obtained by using the definition of positive-realness [39,40].

LEMMA 2 (Corollary 11 in Ref. [39]). *A second-order (bi-quadratic) real-rational function in the form of*

$$W_{2\text{nd}}(s) = \frac{\alpha_2 s^2 + \alpha_1 s + \alpha_0}{\beta_2 s^2 + \beta_1 s + \beta_0} \tag{6}$$

where $\alpha_i \geq 0$ and $\beta_i \geq 0$ (not all $\beta_i = 0$) is positive real if and only if $\alpha_1\beta_1 \geq (\sqrt{\alpha_0\beta_2} - \sqrt{\beta_0\alpha_2})^2$.

LEMMA 3 (Theorem 13 in Ref. [39]). *Consider a third-order (bi-cubic) real-rational function in the form of*

$$W_{3\text{rd}}(s) = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0}{\beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0} \tag{7}$$

where $\alpha_i \geq 0$, $\beta_i \geq 0$ (not all $\beta_i = 0$). Denote $a_0 := \alpha_0\beta_0$, $a_1 := \alpha_1\beta_1 - \alpha_0\beta_2 - \alpha_2\beta_0$, $a_2 := \alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1$, and $a_3 := \alpha_3\beta_3$. Thus, $W_{3\text{rd}}(s)$ is positive real if and only if

(1) $(\alpha_1 + \beta_1)(\alpha_2 + \beta_2) \geq (\alpha_0 + \beta_0)(\alpha_3 + \beta_3)$;
(2) *one of the following holds:*
  (a) $a_3 = 0$, $a_2 \geq 0$, $a_0 \geq 0$, and $-a_1 \leq 2\sqrt{a_0 a_2}$;
  (b) $a_3 > 0$, $a_0 \geq 0$, and $(b1)$ or $(b2)$ holds:
    (b1) $a_1 \geq 0$ and $-a_2 \leq \sqrt{3a_1 a_3}$;
    (b2) $a_2^2 > 3a_1 a_3$ and $2a_2^3 - 9a_1 a_2 a_3 + 27 a_0 a_3^2 \geq 2(a_2^2 - 3a_1 a_3)^{3/2}$.

Then, there are two steps in obtaining the passive network $W(s)$ by using network synthesis:

(1) For a given performance measure, optimize the admittances in Eqs. (5)–(7), separately.
(2) Construct specific networks by using network synthesis based on the obtained admittances.

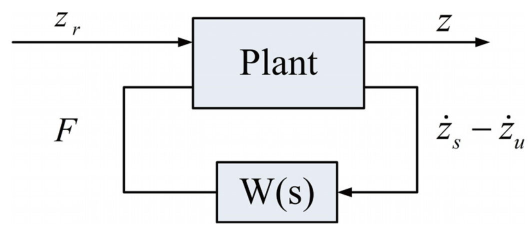

Fig. 3 Control diagram description

From system control point of view, this problem is a passive controller design problem as shown in Fig. 3, where the vehicle except $W(s)$ is seen as the plant and the passive admittance $W(s)$ is seen as the passive controller, the external disturbance is the road vertical displacement, and the sprung mass acceleration and the dynamic tire load are the controlled outputs.

Then, the problem can be transformed into an optimization problem as follows:

$$\min \|T_{z_r \to z}\|_2,$$

such that $W(s)$ is one of the low-order positive-real admittances shown in Eqs. (5)–(7).

The ride comfort and tire grip performance measures defined in Eqs. (3) and (4) are considered as the objective functions, which can be analytically calculated by using the method in Ref. [17]. For the first-order admittance, the problem can be transformed into an unconstrained nonlinear optimization problem by replacing the decision variables as those squares, and then the unconstrained nonlinear optimization solver *fminsearch* in MATLAB can be employed. For the second-order admittance, a constrained optimization problem can be obtained and solved by using the solver *fmincon* in MATLAB. For the third-order admittance, it is generally a constrained nonlinear optimization problem. However, the constraints are very complex involving logic operations which make it very difficult to solve. In this paper, the constraints are separated into three categories with respect to constraints (a), $(b1)$, and $(b2)$ in Lemma 3, respectively. Three optimization problems are formulated with these three kinds of constraints. The optimal solution is the best solution among these three optimization problems. In particular, constraint (a) contains an equality constraint, which can be further separated as two cases with $\alpha_3 = 0$ and $\beta_3 = 0$, respectively. In this way, the number of decision variables is reduced by one, which will improve the efficiency of optimization. These three optimization problems can be solved by using the constrained nonlinear optimization solver *fmincon* in MATLAB.

3 Performance Evaluation for Element-Number-Restricted Networks

In this section, the performance for all the networks containing one inerter,one damper, and one spring is evaluated. The vehicle and road parameters used in this paper are given in Table 2. The traditional passive struts without inerters as shown in Fig. 4 are employed for comparison.

An optimization problem can be formulated, where the performance measures J_1 and J_3 given in Eqs. (3) and (4) are the

Table 2 Vehicle and road parameters

Description	Value
Sprung mass, m_s	250 (kg)
Unsprung mass, m_u	35 (kg)
Tire stiffness, k_t	150 (kN·m^{-1})
Static stiffness, K	10–120 (kN·m^{-1})
Road roughness, κ	5×10^{-7} (m^3 cycle^{-1})
Vehicle forward speed, V	25 (ms^{-1})

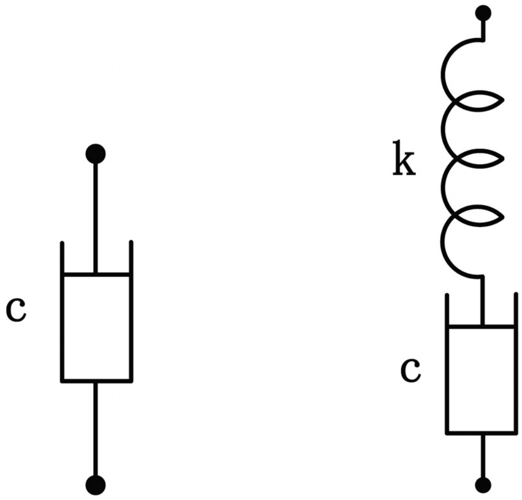

Fig. 4 Traditional passive struts without inerters. From left to right denotes *TC*1 and *TC*2.

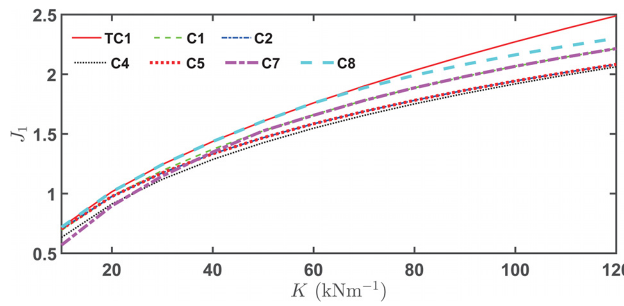

Fig. 5 Optimal ride comfort performance comparison for all networks with one inerter, one damper, and one spring

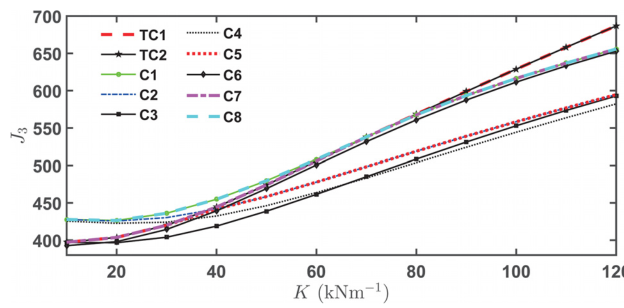

Fig. 6 Optimal tire grip performance comparison for all networks with one inerter, one damper, and one spring

objective functions, and the element coefficients (inertance b, spring stiffness k, and damping coefficient c) for the networks in Fig. 2 are the decision variables. The performance measures J_1 and J_3 can be analytically calculated by using the method given in Refs. [17] and [18], and in Ref. [17], the analytical solutions for networks $TC1$, $TC2$, and $C1-C4$ have been derived. In this section, solutions for networks $C5$–$C8$ are numerically solved by using Nelder–Mead method.

The simulation results are given in Figs. 5–9. The numerical results show that for the optimal ride comfort case, $TC2$, $C3$, and $C6$ reduce to $TC1$, $C2$, and $C1$, respectively. Therefore, the results for these networks with respect to ride comfort are not presented in Fig. 5, and the left figures in Figs. 7 and 8.

From Fig. 5, one sees that all networks with inerters are superior to the traditional strut $TC1$. In addition, for soft suspensions,

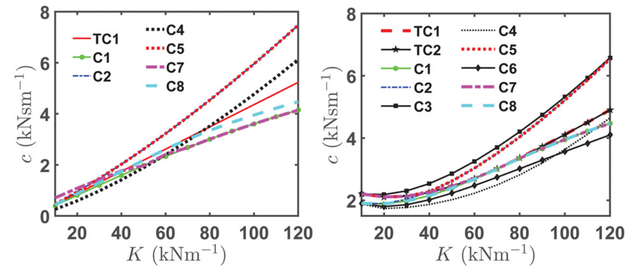

Fig. 7 Optimal damping coefficients *c*: left figure: for optimal ride comfort and right figure: for optimal tire grip

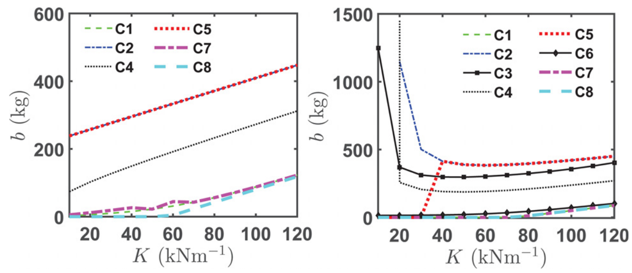

Fig. 8 Optimal inertances *b*: left figure: for optimal ride comfort and right figure: for optimal tire grip

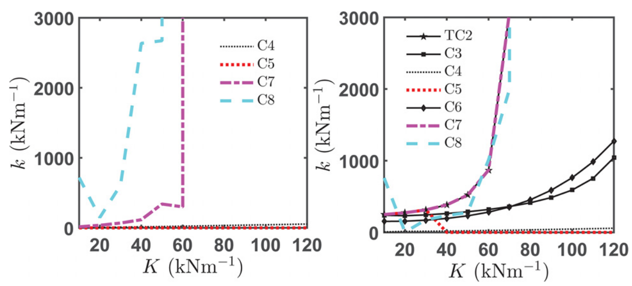

Fig. 9 Optimal stiffnesses *k*: left figure: for optimal ride comfort and right figure: for optimal tire grip

i.e., the suspensions with a small static stiffness K (around 10 kN/m to 25 kN/m), the best network for ride comfort is $C7$; while for stiff suspensions (around 25 kN/m to 120 kN/m), the best network for ride comfort is $C4$. Similarly, in terms of tire grip performance, the networks $C3$ and $C4$ are the best networks for soft and stiff suspensions, respectively. The optimal damping coefficients, inertances, and stiffnesses are shown in Figs. 7–9, where one sees that the series-connected inerters require larger inertances than parallel-connected ones. Moreover, Fig. 9 shows that $C7$ and $C8$ demand a very large stiffness k.

4 Performance Evaluation for Low-Order Admittance Networks

In this section, the simulation results are provided by using the vehicle and road parameters in Table 2.

The case where the static stiffness K is 80 kN/m is used to illustrate the proposed design method, and the results are given in Table 3. The optimal admittances are obtained by using network synthesis method. Specifically, for the first- and second-order admittances with respect to optimal ride comfort, the realized networks are straightforwardly obtained; for the second-order admittance with respect to optimal tire grip, a biquadratic minimal function [41] is obtained and realized by using the Bott–Duffin method [42]; and for the third-order admittance, the obtained impedance (the reciprocal of admittance) for the optimal ride

Table 3 Results for the case where $K = 80$ kN/m

		First-order	Second-order	Third-order
Ride comfort	J_1	1.78	1.74	1.67
	Admittance	$\frac{4537.44s}{s+12.22}$	$\frac{26.66s^2+4034.84s}{s+9.65}$	$\frac{37.05s(s+47.62)(s+21.60)}{s^2+11.43s+169.90}$
	Structure	Figure 14 (left)	Figure 14 (middle)	Figure 14 (right)
	Parameters	$b = 371.30$ kg; $c = 4537.44$ Ns/m	$b_1 = 417.95$ kg; $b_2 = 28.48$ kg; $c = 4309.79$ N s/m	$b_1 = 224.25$ kg; $b_2 = 44.38$ kg; $c = 3071.62$ N s/m; $k = 45639.10$ N/m
Tire grip	J_3	518.85	503.08	498.83
	Admittance	$\frac{4033.57s}{s+10.18}$	$\frac{2903.93(s+8.49)(s+0.42)}{s^2+11.44s+143.96}$	$\frac{550359.91(s+5.82)(s+0.82)}{(s^2+12.35s+133.58)(s+166.09)}$
	Structure	Figure 10 (left)	Figure 10 (middle)	Figure 10 (right)
	Parameters	$b = 396.10$ kg; $c = 4033.57$ N s/m	$k_1 = 985.49$ N/m; $c_1 = 72.50$ Ns/m; $b_1 = 6.76$ kg; $k_2 = 153.76$ N/m; $k_3 = 31147.40$ N/m; $b_2 = 213.64$ kg; $c_2 = 2903.93$ N s/m; $b_3 = 1369.30$ kg;	$k = 1203.37$ N/m; $k_1 = 549.16$ kN/m; $c_1 = 1574.40$ Ns/m; $c_2 = 1615.16$ Ns/m; $b = 127.29$ kg;

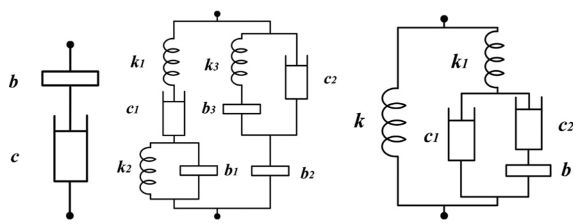

Fig. 10 The optimal networks for tire grip performance where $K = 80$ kN/m: from left to right: first-order, second-order, and third-order admittances

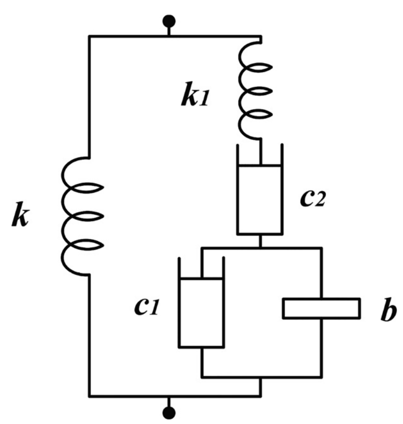

Fig. 11 An alternative realization of the third-order admittance for the optimal tire grip performance where $K = 80$ kN/m: $k = 1203.37$ N/m, $k_1 = 549.16$ kN/m, $c_1 = 3109.08$ Ns/m, $c_2 = 3189.56$ Ns/m, and $b = 496.40$ kg

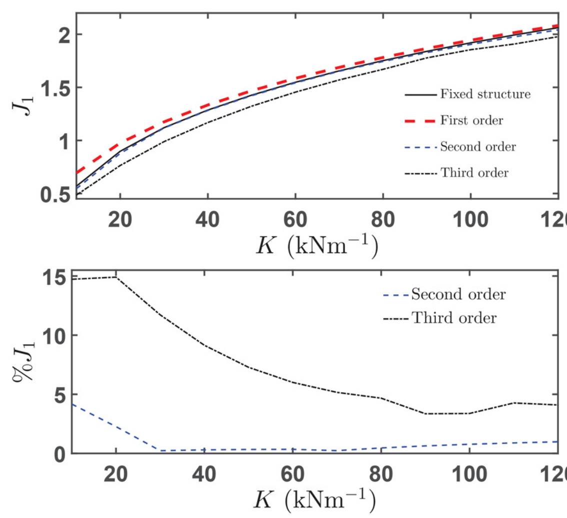

Fig. 12 The ride comfort performance comparison between low-order admittance networks and the networks with one inerter, one damper, and one spring: upper figure: J_1 and lower figure: percentage improvements over the networks with one inerter, one damper, and one spring

comfort case is a special third-order positive-real function [3,27], which can realized by using the method in Ref. [3]. The obtained third-order admittance for the optimal tire grip is also a special third-order positive-real function [3,27], but different from the optimal ride comfort case, this admittance can be realized in two ways as shown in the right figure in Figs. 10 and 11 by using the method in Ref. [3]. Since the inertance and damping coefficient for the realization in the right figure of Fig. 10 is relatively small, in this sense, such a realization is better than the one in Fig. 11.

The optimization results for different static stiffnesses are given in Figs. 12 and 13, where it is shown in that the performance (both ride comfort and tire grip) can be improved by using higher-order admittances. The case labeled as "fixed structure" in Figs. 12 and 13 denotes the optimal performance of the networks with one inerter, one damper, and one spring, which can be derived from Figs. 5 and 6. For example, in Fig. 12, for the case where the static stiffness K is 20 kN/m, the "fixed strucutre" refers to $C7$, as $C7$ in Fig. 5 provides the best performance when $K = 20$ kN/m. Since all the admittances of these networks in Fig. 2 are second-order admittances, the performance for these networks is better than the first-order admittance networks. However, from Figs. 12 and 13, one sees that although the performance of given-structured networks in Fig. 2 is close to the optimal second-order admittance networks, the given-structured networks in Fig. 2 are not optimal second-order admittance networks. From the results of the case with $K = 80$ kN/m shown in Table 3, one sees that the

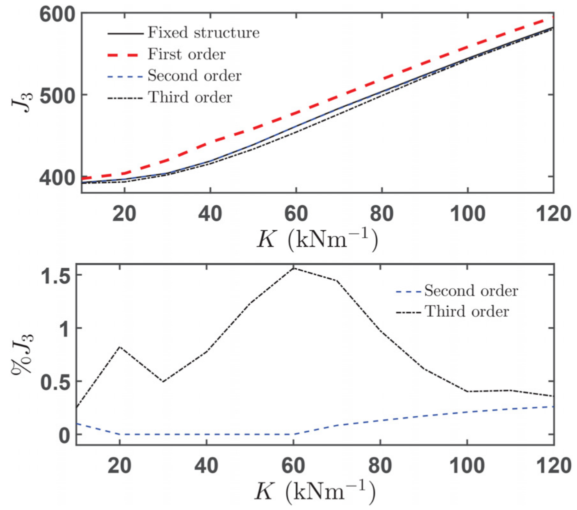

Fig. 13 The tire grip performance comparison between low-order admittance networks and the networks with one inerter, one damper, and one spring: upper figure: J_3 and lower figure: percentage improvements over the networks with one inerter, one damper, and one spring

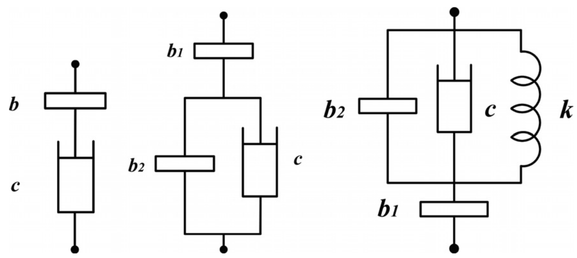

Fig. 14 The optimal networks for ride comfort performance where $K = 80$ kN/m: from left to right: first-order, second-order, and third-order admittances

optimal structure may be very complex which is difficult to be implemented in practice.

However, by comparing Figs. 10 and 14, one sees that for the optimal ride comfort case, the optimal third-order network only contains an extra spring compared with the optimal second-order network; for the optimal tire grip case, the optimal third-order network has less element and simpler structure compared with the optimal second-order network. Such a fact indicates that the optimal higher-order admittance networks are not always more complex than the optimal lower-order ones. In some circumstances, by using the network synthesis method and considering high-order admittances, the performance can be improved in the meanwhile maintaining simplicity in structure.

5 Conclusions

In this paper, the performance evaluation problem for low-complexity passive suspensions with inerters has been investigated, where two methods were employed to construct the suspension strut. Using the first method, all the networks with one inerter, one damper, and one spring were considered. By restricting the number of each element to one, the structural simplicity for suspension struts was maintained. The best networks for soft and stiff suspensions have been identified via numerical simulation. Using the second method, low-order admittance networks were investigated, where the order was restricted to no larger than three. Similarly, the structural simplicity for suspension struts was maintained by restricting the order of admittances. A two-step design method was proposed, and a comparison between the element-number-restricted networks (in the first method) and the low-order admittance networks was conducted. The optimal first-order, second-order, and third-order admittance networks were obtained, and it was shown that the given-structured networks in the first method were not the optimal second-order admittance, though the given-structured networks in the first method possess second-order admittances. It was also demonstrated that normally high-order admittance networks improve suspension performances at the expense of using more complex structures, but in some circumstances, suspension performances can be improved by considering high-order admittance networks in the meanwhile maintaining simplicity in structure.

Note that in this paper, the models for the vehicle and elements (springs, dampers, and inerters) are ideal without considering the nonlinearities such as friction and backlash, and the effectiveness is demonstrated numerically. In the further work, the practical implementation issues will be investigated by involving more realistic vehicle and element models, and experimental research will be conducted to further evaluate the performance of the proposed methods.

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