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- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 :
Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 16: Dynamics 2 (waves and dynamic mechanisms)

Thur 8^{th} Apr

Outlook of the next few lectures

Dynamics important, next few lectures on

- waves (this Lec. + 16, 18) and instabilities (Lec. 17)

  $\rightarrow$ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- how to describe waves (Lec. 15)
- **types of waves** (Lec. 16)

  $\rightarrow$ consequence + leading to instabilities
- instabilities (Lec. 17)

  $\rightarrow$ parcel-type (mechanistic) arguments for instability
- tides (particularly as internal gravity waves) (Lec. 18)

Outline

- gravity waves

  → gravity/buoyancy as restoring mechanism ($\sqrt{gH}$)
- inertial waves

  → Coriolis as restoring mechanism (f)

  → e.g. Rossby waves, Kelvin waves
- inertial-gravity + internal waves ($\sqrt{gH}$ or N, and f)

  → extra depth dimension to deal with

  → Brunt–Väisälä or buoyancy frequency N

- propagation mechanism (Rossby wave example)

  → kinematic argument with **vorticity**

Key terms: buoyancy frequency, gravity waves, inertial waves, Rossby waves, Kelvin waves, vorticity inversion

Recap: what goes down must come up

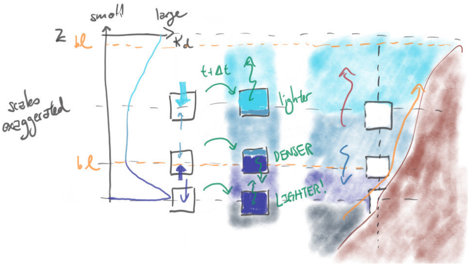

Figure: Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers

$\rightarrow$ ~~broad diffusive~~ boundary intensified upwelling

what causes the bounary intensification of κ_d? dynamics!

- at the surface, lots of things... (convection, waves, Langmuir turbulence etc.)
- at the bottom, probably tidal conversion (Lec. 18) $\rightarrow$ internal gravity waves (Lec. 16) $\rightarrow$ shear instabilities (Lec. 17)

Recap: waves and dispersion relation

- waves are ubiquitous physical features

  → depends on physics
- wave described by the dispersion relation $\omega = \mathcal{F}(k)$

  → physics dictate the form of $\mathcal{F}$

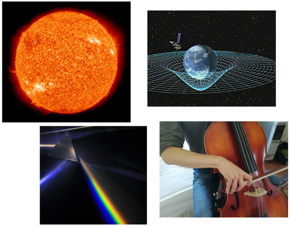

Figure: Examples of systems supporting waves. All figures from Wikipedia except the cello one.

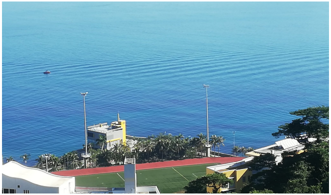

Figure: Gravity waves with signal at the sea surface (as darker and lighter bands). Taken at HKUST.

- (linear) waves can interfere with each other

  → constructive or destructive

  → interference can lead to steepening and breaking ("becoming" nonlinear)

Recap: wave propagation

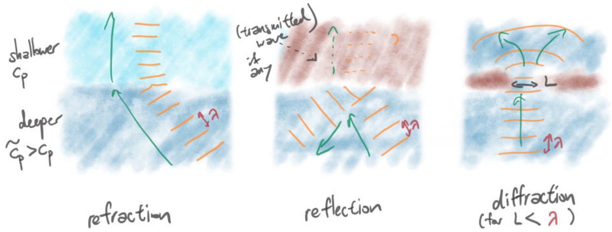

Figure: Schematic of refraction, reflection (and transmission), and diffraction. See Lec. 15.

- phase speed (in a direction) and group velocity as (note $\omega = \mathcal{F}(k)$)

$$c_{p,x} = \frac{\omega}{k}, \qquad c_{g,x} = \frac{\partial \omega}{\partial k}$$

$\rightarrow$ individual wave vs. wavepacket behaviour

$\rightarrow$ contribute to wave phenomenon (e.g. refraction from $c_p = c_p(\boldsymbol{x})$)

Wave steepening and breaking

Figure: Schematic of mixing by (irreversible) wave breaking, with contours reconnecting leading to e.g. diapycnal mixing.

- growing waves by instability

  → convective and/or shear (see Lec. 17)

  → mixing of material **across** isopycnals after reconnection, leading to diapycnal mixing
- feedback onto MOC (see Lec. 14)

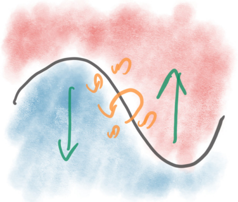

Figure: Velocity shear from waves can lead to mixing.

Gravity waves

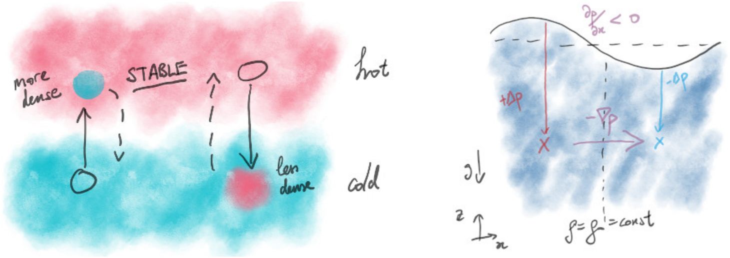

Figure: Gravity as restoring force. Pictures adapted from ones used in Lec. 7.

- deviation from resting **isopycnal** experiences restoring force from gravity (buoyancy)

  → left case: internal isopycnal (as an **isotherm**)

  → right case: sea surface is the isopycnal
- weak damping, restoring force, overshoots ⇒ oscillatory motion (up and down in this case)

Gravity waves

For simplicity, consider a **homogeneous** (i.e. $\rho = \text{const}$) layer of fluid (cf. Lec. 11 + 12, right case in previous Figure) until we get to internal waves

Gravity waves

For simplicity, consider a **homogeneous** (i.e. $\rho = \text{const}$) layer of fluid (cf. Lec. 11 + 12, right case in previous Figure) until we get to internal waves

In this instance, dispersion relation for gravity waves is given by (without derivation)

$$\omega^2 = gk \tanh(kH) \qquad \Rightarrow \qquad \omega = \pm\sqrt{gk \tanh(kH)}$$

- H is water depth, and tanh = hyperbolic tangent, goes from -1 to 1

  $\rightarrow$ note symmetry in both directions (the $\pm$ sign)

  ($\omega \leq 0$ cases are just **shifts** in the **phase**)

Gravity waves

$$\omega = \pm\sqrt{gk \tanh(kH)}$$

- for **deep** water waves ($kH \gg 1$) and **shallow** water waves ($kH \ll 1$),

$$\omega_{\text{deep}} = \pm\sqrt{gk}, \qquad \omega_{\text{shallow}} = \pm k\sqrt{gH}$$

  $\rightarrow kH \gg 1$ so $\tanh(kH) \rightarrow 1$

  $\rightarrow kH \ll 1$ with $\tanh(kH) \approx kH + O((kH)^3)$ (do a Taylor expansion)
- deep water waves are depth-independent and dispersive
- shallow water waves are slower in shallow waters ($c_p \sim \sqrt{H}$) and non-dispersive ($c_p = c_g$)

Gravity waves

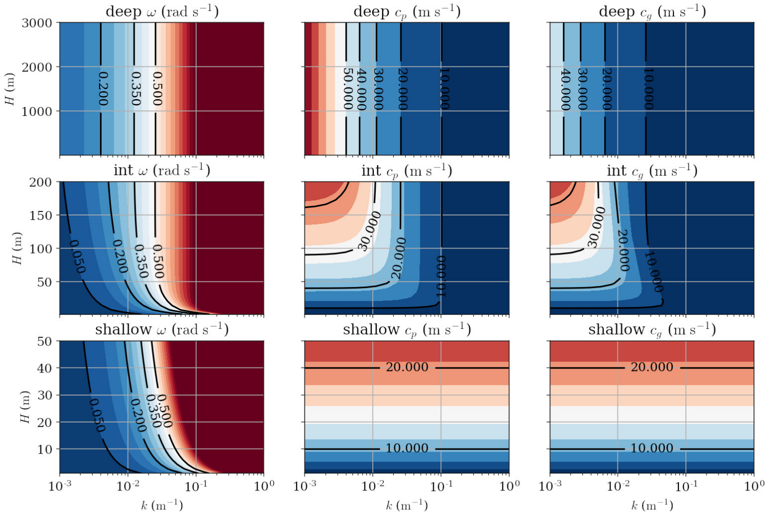

Figure: Water wave ω, c_p and c_g over (k,H) space, with k shown on a log axis. k chosen so wavelengths are roughly between 50 m to 5 km (recall $k = 2\pi/\lambda$). Also note the transitions from shallow to intermediate to deep are really to do with $kH \sim H/\lambda$. See `waves.ipynb`.

Inertial waves

Inertial waves has the Coriolis “force” act as the restoring force

- generic for rotating systems

  → planetary interiors, stars, galactic disks

- mostly arise in context of internal waves

  → at surface buoyancy effects can dominate

- limited in frequency by inertial frequency f_0 (cf. Coriolis parameter)

  → revisit later when talking about inertia-gravity waves

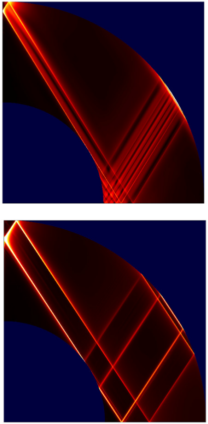

Figure: Inertial wave attractors in a homogeneous planetary interior at different tidal forcings. From Gordon Ogilvie (2009, *Mon. Not. Royal. Astro. Soc*).

Inertial-Gravity waves

In reality Coriolis and buoyancy effects both contribute

- large-scale and/or slow $\Rightarrow$ Coriolis important (because Ro $\ll 1$), classify as inertial waves

  $\rightarrow$ e.g. Rossby waves
- small-scale and/or fast $\Rightarrow$ Coriolis unimportant

  $\rightarrow$ e.g. internal gravity waves
- somewhere in between? Poincaré or inertia-gravity waves

$$\boxed{\omega = \pm\sqrt{f_0^2 + gH(k_x^2 + k_y^2)}}$$

$\rightarrow$ for $gH(k_x^2 + k_y^2) \gg f_0$, recover gravity waves

$\rightarrow$ for $gH(k_x^2 + k_y^2) \ll f_0$, recover inertial waves

Rossby deformation radius

$$\omega = \pm\sqrt{f_0^2 + gH(k_x^2 + k_y^2)}$$

- boundary between gravity and inertial regimes is roughly

$$\boxed{L_d = \frac{\sqrt{gH}}{f_0}}$$

- the Rossby deformation radius (for shallow water system)

  $\rightarrow$ roughly also the boundary where geostrophic approximation should hold (see Lec. 8 + 13)

  $\rightarrow$ estimates in a few slides

Internal waves

- introduce a useful quantity

$$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$$

- Brunt–Väisälä or buoyancy frequency (units: s^{-1})

  $\rightarrow N^2$ normally used

  $\rightarrow$ note $\partial\rho/\partial z < 0$ for stable stratification, i.e. $N^2 > 0$

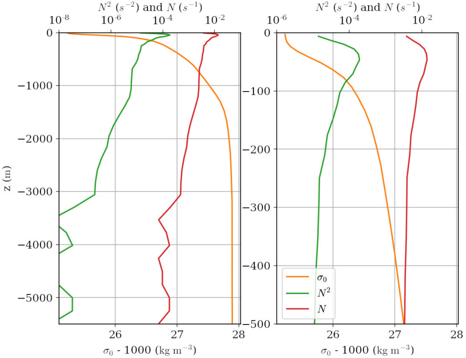

Figure: σ_0 (see Lec. 6) and the associated N^2 and N. $N^2 \ll 1$ means weakly stratified (weak density gradients), whilst $N^2 < 0$ shows unstable stratification (none in this case, but see Lec. 17). See `plot_eos.ipynb`.

simplistic view (!): $\sqrt{gH} \rightarrow N$

Internal waves

Generally then, **internal** inertia-gravity waves described by

$$\omega = \pm\sqrt{\frac{f_0^2 k_z^2 + N^2 k_x^2}{k_x^2 + k_z^2}} \approx \pm\sqrt{f_0^2 + \frac{N^2 k_x^2}{k_z^2}} \quad \text{(for } |k_z| \gg |k_x|\text{)}$$

- atmosphere and ocean has $N/f_0 = O(10^1 \text{ to } 10^2)$
  $\rightarrow$ so really we have gravity waves **influenced by rotation**
  $\rightarrow$ refer to them here as internal waves

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- note that $|f_0| \leq |\omega| \leq |N|$
  $\rightarrow$ since $0 \leq k_{x,z}^2/(k_x^2 + k_z^2) \leq 1$
  $\rightarrow$ frequency is **much lower** than gravity waves

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internal tides to be seen as internal waves (Lec. 18)

Internal waves

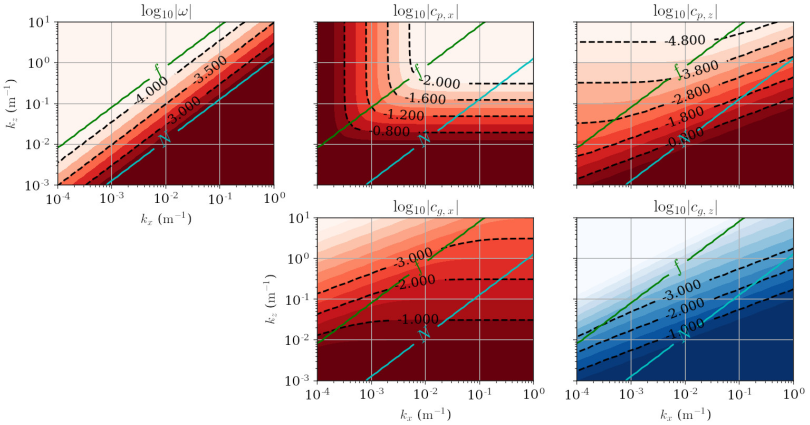

Figure: Inertial-gravity waves (with the $k_z \gg k_x$ approximation) ω, $c_{p,x}$, $c_{p,y}$, $c_{g,x}$ and $c_{p,y}$ as a log-log plot in (k_x, k_z) space, with $f = 5 \times 10^{-5}$ and $N = 3 \times 10^{-3}$ (oceanic relevant values). The contours denote the exponent x of $|10^x|$ and the colour shading denotes the sign (more blue = more negative *actual* values rather than exponents, more red = more positive *actual* values rather than exponents); since k_x and k_z is chosen to be positive, everything except $c_{g,z}$ is positive. Contours of f and N plotted with an offset plotted to show the boundary beyond which everything is either gravity waves or inertial oscillations. See `waves.ipynb`.

Internal waves

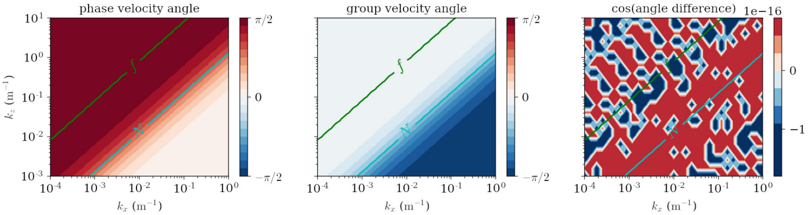

Figure: Inertial-gravity waves (with the $k_z \gg k_x$ approximation) phase velocity $\boldsymbol{c}_p$ angles and group velocity $\boldsymbol{c}_g$ angles (in radians, relative to the horizontal, and note $\pi/2 = 90^\circ$). The final panel shows $\boldsymbol{c}_p \cdot \boldsymbol{c}_g = |\boldsymbol{c}_p||\boldsymbol{c}_g| \cos\theta$ (which is zero up to rounding errors). Contours of f and N plotted with an offset plotted as in the previous diagram. See `waves.ipynb`.

- note that, for inertial-gravity waves (left as a bonus exercise),

$$\boldsymbol{c}_p \cdot \boldsymbol{c}_g = 0$$

$\rightarrow$ i.e. phase and group velocities are **perpendicular** to each other (see Lec. 4)

Deformation radius

- boundary given again by the Rossby deformation radius (for the continuously stratified case)

$$L_d = \frac{NH}{f}$$

  $\rightarrow L_{d,\text{atmos}} = O(1000\text{ km})$, scale of cyclones and anti-cyclones, i.e. weather systems form (synoptic structures)

  $\rightarrow L_{d,\text{ocean}} = O(50\text{ km})$, scale of ocean eddies
- latitude (through f) and H dependent

  $\rightarrow$ smaller L_d for **high** latitudes and **shallow** regions

  $\rightarrow$ consequence for **geostrophic approximation**? (e.g. shelves and coasts, see Lec. 21 + 22)

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- internal L_d defined analogously (normally smaller than above)

Kelvin waves (more on this in Lec. 18, 21 + 22)

A type of boundary wave

- need f and a **boundary**

  $\rightarrow$ could be land (coastal Kelvin waves) (see Lec. 18, 21 + 22)

  $\rightarrow$ could be a wave guide (e.g. equator where f changes sign, equatorial Kelvin waves) (see OCES 4001, El-Niño, QBO etc.)

- needs f but propagates at the gravity wave speed, with

$$\omega = k\sqrt{gH}$$

  $\rightarrow$ non-dispersive

  $\rightarrow$ fairly fast (gravity wave speed)

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- **NOTE the lack of $\pm$!**

Kelvin waves (more on this in Lec. 18, 21 + 22)

- boundary introduces asymmetry in this case: general solution like

$$\eta \sim \mathrm{e}^{\pm f_0 y/\sqrt{gH}} \cos(kx - \omega t)$$

$\rightarrow$ take $y \leq 0$ to be **boundary**, if $f_0 > 0$ (NH), need minus sign, and vice-versa

$\rightarrow$ wave propagates cyclonically (same sign as f)

- taking $f_0 > 0$ (NH),

$$\eta \sim \mathrm{e}^{-y/L_d} \cos(kx - \omega t),$$

so decay over the $L_d = \sqrt{gH}/f_0$

Rossby waves (more on this later)

A (particularly important) type of **inertial** wave

- requires a **gradient** in background vorticity

  $\rightarrow \partial f / \partial y = \beta$ (planetary case)

  $\rightarrow$ background flow $-\partial U / \partial y \sim \nabla \times \boldsymbol{u}$ (see later and Lec. 17)
- dispersion relation given by (on β-plane)

$$\boxed{\omega = -\frac{\beta k_x}{k_x^2 + k_y^2}}$$

$\rightarrow$ note that Rossby waves propagate to the **west** (more generally, **retrograde** or against the mean flow) since

$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0,$$

and **long** waves ($k_x \ll 1$) are **fast**(er)

Rossby waves (more on this later)

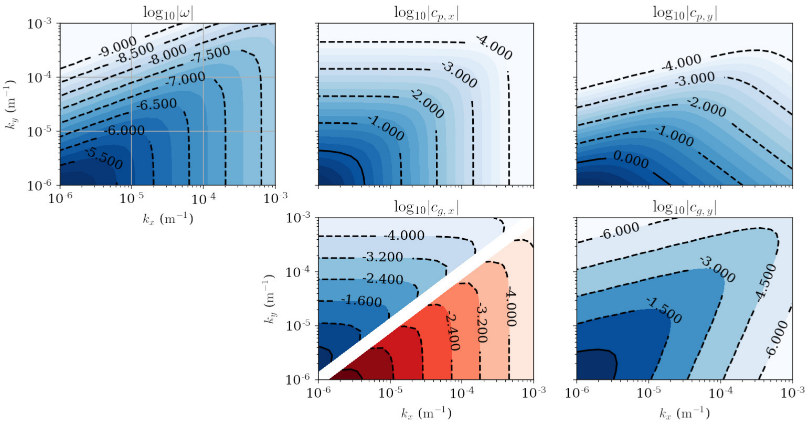

Figure: Rossby waves ω, $c_{p,x}$, $c_{p,y}$, $c_{g,x}$ and $c_{p,y}$ as a log-log plot in (k_x, k_y) space, with magnitude also as logs. The contours denote the exponent x of $|10^x|$ and the colour shading denotes the sign (more blue = more negative *actual* values, more red = more positive *actual* values); since k_x and k_y is chosen to be positive, everything except $c_{g,x}$ is negative. Choice of k_x and k_y correspond to wavelengths roughly between 6 km to 6000 km (Rossby waves are usually seen as planetary-scale waves). See `waves.ipynb`.

Propagation mechanism: Rossby waves

Rossby waves propagate **west**-ward (or, more generally, retrograde)

$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0$$

why?

Propagation mechanism: Rossby waves

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$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0$$

why?

Key bits to the pictorial/parcel (cf. Lec 5 for temperature) argument:

- the initial wave conserves and carries vorticity (spini-ness, recall Lec. 4, 11, 12) into the external environments

  $\rightarrow$ these are now vorticity anomalies
- vorticity anomalies induces a velocity/flow (because spini-ness)
- induced flow seen to self-advect the wave and move it to the **West** (retrograde in the general case)

Propagation mechanism: Rossby wave example

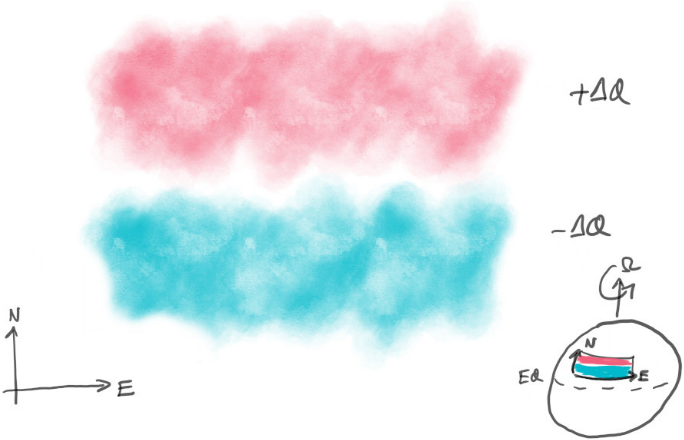

Figure: Rossby wave propagation schematic.

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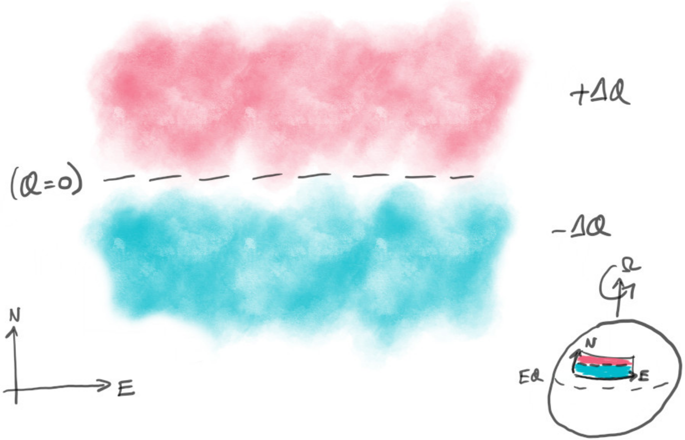

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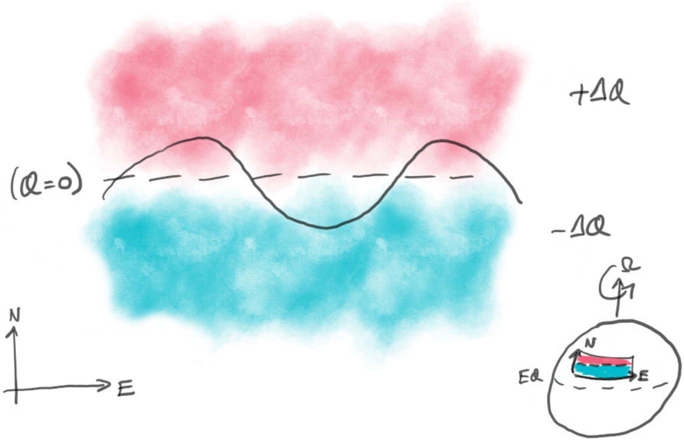

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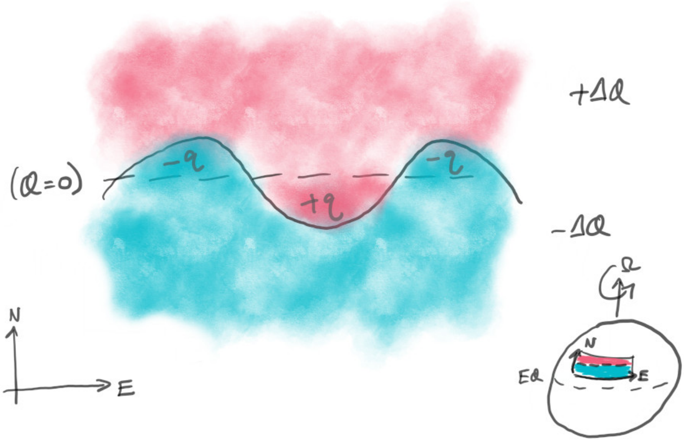

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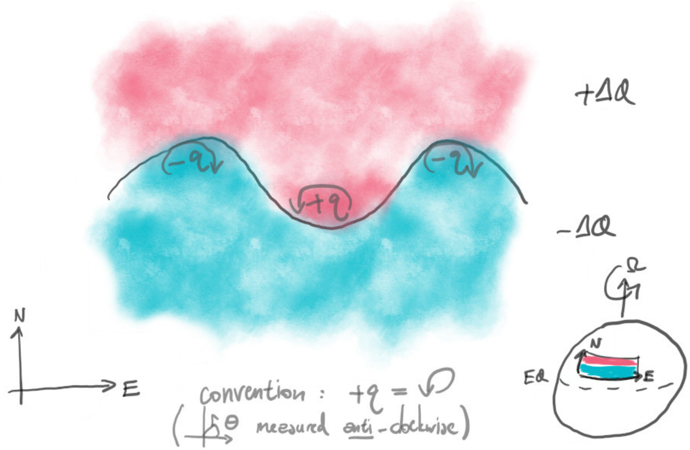

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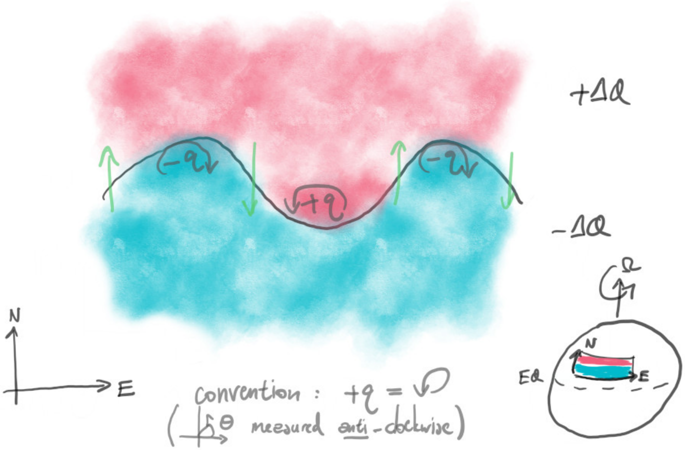

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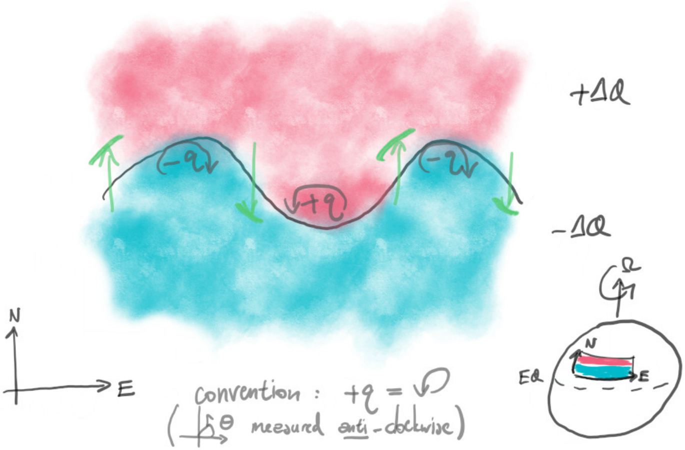

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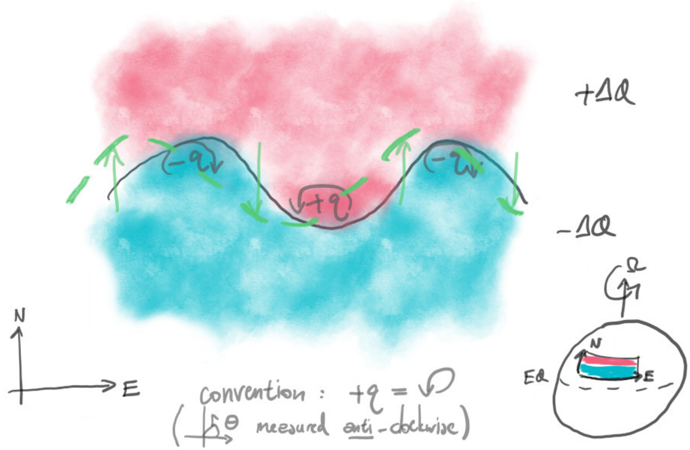

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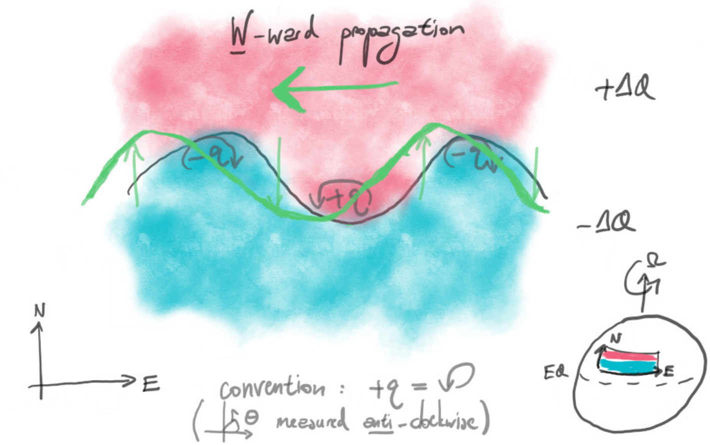

Figure: Rossby wave propagation schematic.

Summary

- gravity waves (gravity/buoyancy)
- inertial waves (Coriolis)
- inertial-gravity waves (general)

  $\rightarrow$ internal waves have $|f| \leq |\omega| \leq |N|$

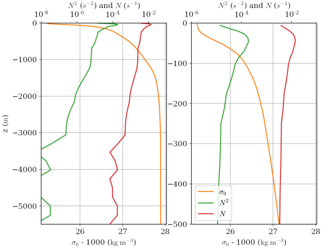

Figure: σ_0 (see Lec. 6) and the associated N^2 and N. See `plot_eos.ipynb`.

- Brunt–Väisälä or buoyancy frequency N

$$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$$

$\rightarrow$ measure of stratification strength (see also Lec. 17)

Summary

- parcel argument for **west**-ward Rossby wave propagation

  → conservation of vorticity

  → vorticity anomalies induces flow

  → self-advecting

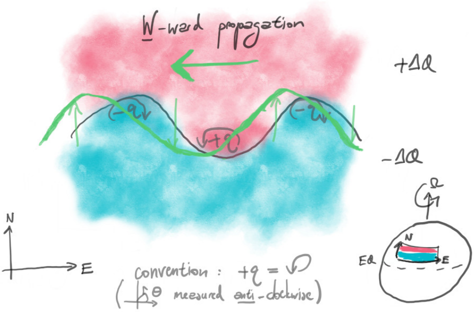

Figure: Rossby wave propagation schematic.

- generalisations exist (e.g. internal gravity waves in Harnik *et al.*, 2008, *J. Atmos. Sci*)

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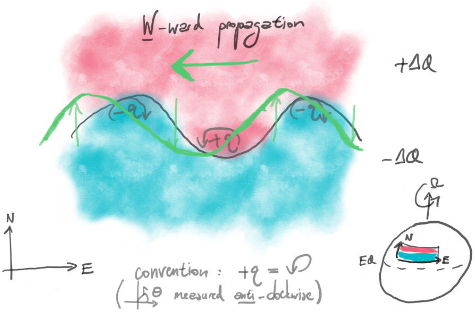

Figure: Rossby wave propagation schematic.

- generalisations exist (e.g. internal gravity waves in Harnik *et al.*, 2008, *J. Atmos. Sci*)
- two such waves interacting? (see Lec. 17)

  → potential for instabilities