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  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 18: Dynamics 4 (tides)

Tue 20th Apr

Outlook of the next few lectures

Dynamics important

- waves (this Lec. + 16, 18) and instabilities (Lec. 17)

  $\rightarrow$ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- how to describe waves (Lec. 15)
- types of waves (Lec. 16)

  $\rightarrow$ consequence + leading to instabilities
- instabilities (Lec. 17)

  $\rightarrow$ parcel-type (mechanistic) arguments for instability
- **tides (particularly as internal gravity waves)** (Lec. 18)

Outline

- tides

  → as deformation of sea surface arising from gravitational attraction **only** (rotation **leads** to evolution but does not **partake** in force balance)

  → observations and features

  → frequency + period

  → forcing

  → propagation

- internal tides (basically internal gravity waves)

  → dynamics and transfer

  → statistics (Garrett–Munk spectrum)

Key terms: high/low/spring/neap tides, tidal forcing, internal tides, wavenumber and length-scales Garrett–Munk spectrum

Recap: abyssal upwelling (Lec. 14)

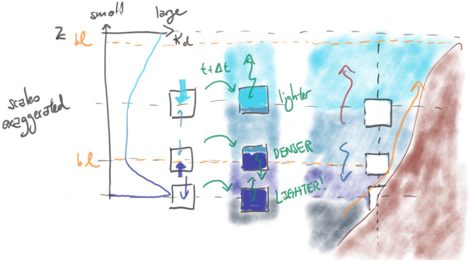

Figure: Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers

  → ~~broad diffusive~~ boundary intensified upwelling

what causes the abyssal intensification of κ_d? dynamics!

- internal gravity waves leading to shear instabilities which leads to diapycnal mixing

Recap: shear instabilities and diapycnal mixing (Lec. 17)

Figure: Schematic of mixing by (irreversible) wave breaking, with contours reconnecting leading to e.g. diapycnal mixing.

- growing disturbance, instability

  → mixing of material **across** isopycnals after reconnection, i.e. **diapycnal mixing**

- shear instabilities can be interpreted as a constructive inteference of waves

  → key role of vorticity and induced velocity

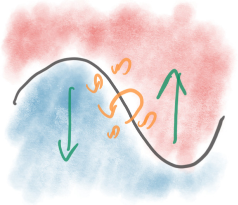

Figure: Velocity shear from waves can lead to mixing.

Recap: internal gravity waves (Lec. 16)

Recall that since $N \gg f$ on Earth, so internal inertia-gravity waves satisfy

$$\omega \approx \pm\sqrt{f^2 + \frac{N^2 k_x^2}{k_z^2}} \quad (\text{for } |k_z| \gg |k_x|)$$

- internal gravity waves **influenced by rotation**

  $\rightarrow$ main restoring force is gravity, with influences from Coriolis effect

  $\rightarrow$ remember $|f| \leq |\omega| \leq |N|$

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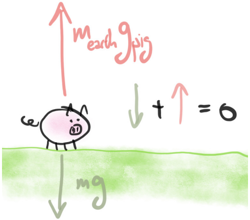
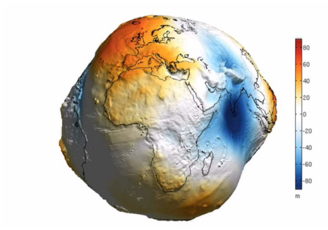
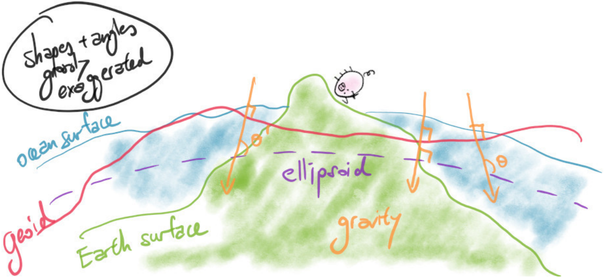
internal tides = internal gravity waves with tidal forcing

Recap: gravity and geoid (Lec. 7)







Tides: observations

Figure: Tides at (top) St. Michael Mount in Cornwall (near Penzance), and Holy Island causeway (Lindisfarne) in Northumberland, UK. Images by Michael Marten (`http://www.michaelmarten.com/`), image taken from (`twistedsifter.com`).

Tides: observations

Figure: High (or flood) and low (or ebb) tide at Tobermory, Isle of Mull, Scotland, using the pastel pink and red house as references. Modified images from `www.thechaoticscot.com` (left) and from myself (right).

- tidal signal most obvious in coastal sea level

  $\rightarrow$ but generally in SSH η (recall Lec. 7)

  $\rightarrow$ η/H largest near coasts (because of $H \ll 1$) (see Lec. 21 + 22)

Tides: observations

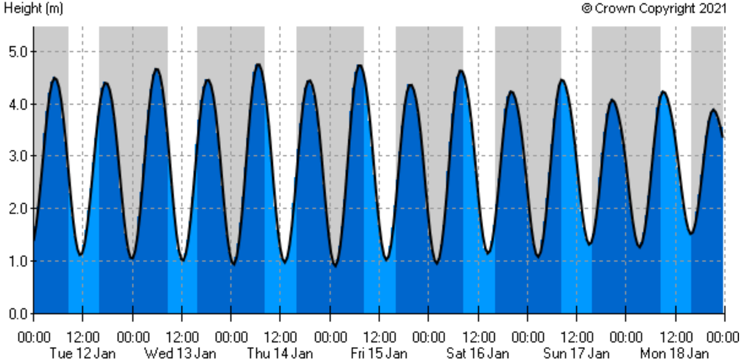


Figure: Predicted (!) tidal charts at Tobermory, Isle of Mull, Scotland between over a period of seven days. Taken from `http://www.ukho.gov.uk`.

- high and low tide at around 12 hr periods

  $\rightarrow$ longer term variablity (cf. wave packets, Lec. 15)

Tides: observations

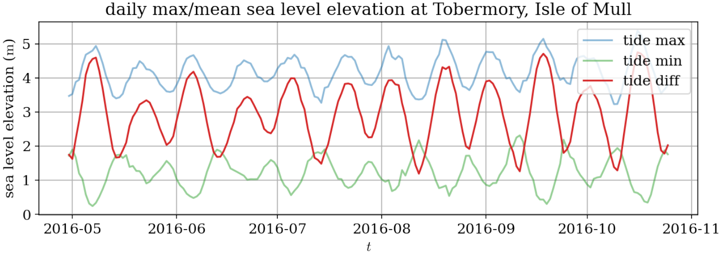


Figure: Daily maximum and minimum sea surface elevation (blue and green) and their difference (red) over a six month period. Data from BODC, see `tobermory_tides.ipynb`.

- longer term variablity
- oscillation of around **two weeks** between max and min difference

  $\rightarrow$ spring and neap tides

  (spring not to do with the season, more like the boing-boing mechanical spring)

Tides: observations

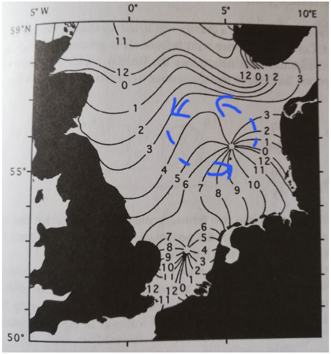


Figure: Phase lines (denoting peak of SSH, in hours in GMT) of the semi-diurnal tidal signal. Notice the anti-clockwise propagation (with boundary to the right). Diagram modified from Knauss (1997), Fig. 10.5.

- tides over the whole globe

  $\rightarrow$ wave-like structure
- propagation characterised by phase lines cf. c_p in Lec. 15)

  $\rightarrow$ **anti-clockwise** propagation in this NH example (cf. cyclonic)

  $\rightarrow$ note signal propagates with the boundary to the **right** of it

  $\rightarrow$ Coriolis effect again?

https://www.youtube.com/watch?v=u6ZNnGnc-cQ

Tides: forcing

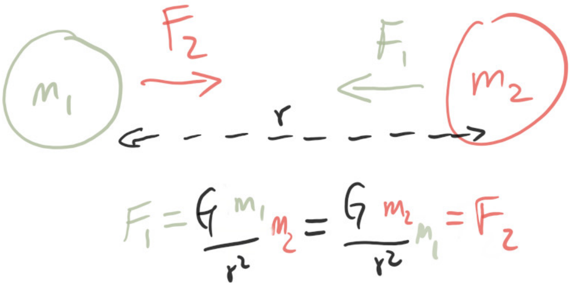


Figure: Schematic of gravitational attraction for two masses, from Lec. 7.

- driving and restoring force being gravity, what objects drive the tides then?

Esoteric pop culture interlude

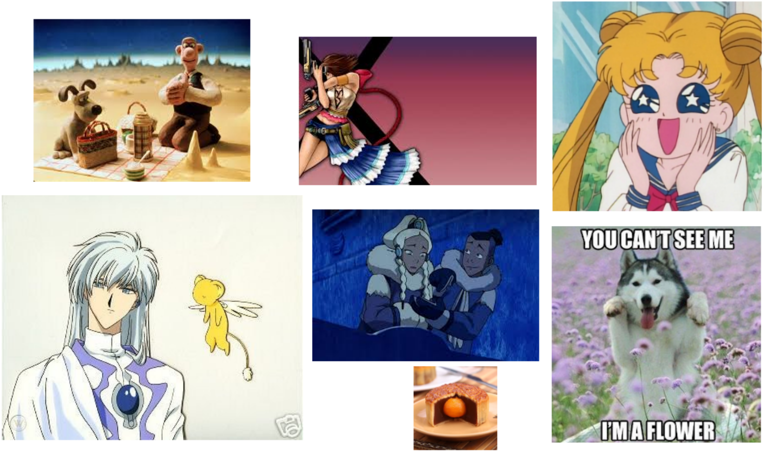


Figure: What is the thing that unites all of these? Bonus: can you name all of these? (Copyright probably with Aardman studios, Square Enix, Naoko Takeuchi + Toei animation, Clamp, Nickelodeon, and two not sures).

Tides: lunar forcing

Figure: Phases of the moon. From `www.dreamstime.com` (by user Miroslav Nemecek).

- the moon drives most of the Earth's tides
- Earth rotates with a period of a day

  $\rightarrow$ daily = diurnal
- principal tide around every 12 hours (semi-diurnal)
- moon's orbit of Earth around 28 days

Tides: forcing

Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here.

- suppose we have an Earth and moon(cake)

Tides: forcing

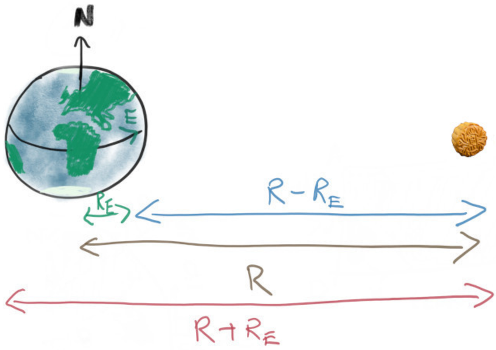


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here.

- define some distances

Tides: forcing

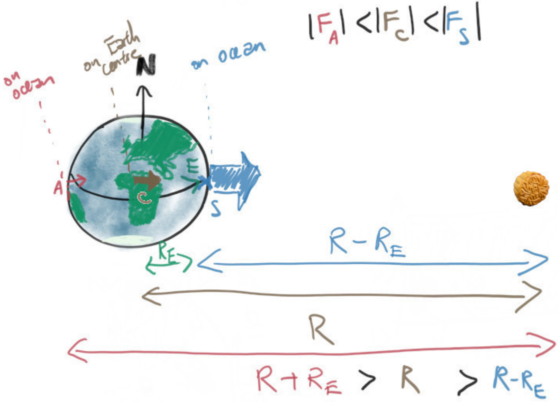


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here.

- differences in gravitational force because distances

Tides: forcing

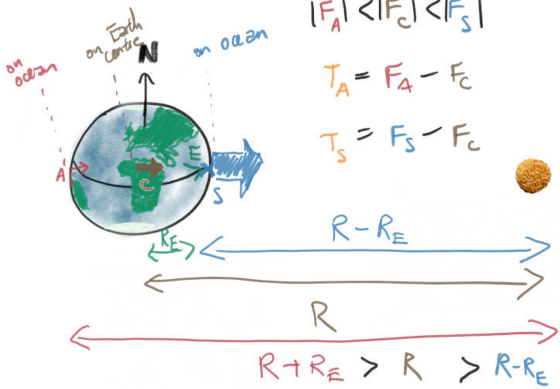


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here.

- tide generating force = force on ocean - force on center

Tides: forcing

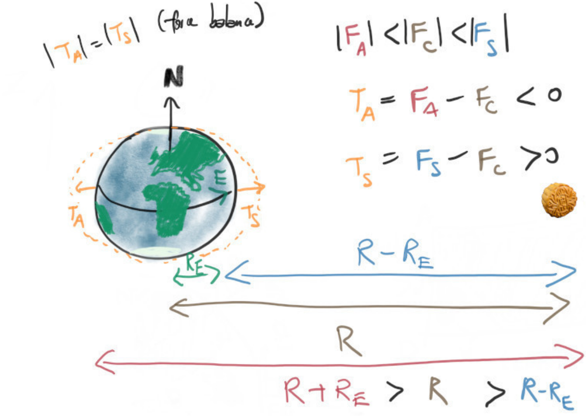


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here.

- force balance, equal and opposite, pile water ⇒ two bulges

Tides: forcing

- Earth rotates with period 24 hours, one bulge when moon overhead and one when on far side
  $\rightarrow$ i.e. **semi-diurnal** signal

- above apply equally to the Sun too
  $\rightarrow$ note while $M_{\text{Sun}} \gg M_{\text{moon}}$, $R_{\text{Sun-Earth}} \gg R_{\text{moon-Earth}}$, so since tidal acceleration is roughly

$$a_{\text{tide}} \sim \frac{M}{r^3},$$

  magnitude of Solar influence turns out to only be about half as strong (homework: show this explicitly)

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!!! above is for **equilibrium** theory, **dynamic** theory absolutely cares about rotation because then we need to care about wave propagation

Tides: forcing

symbol	period (in solar hrs)	rel. amp (to M_2)	name
M_2	12.42	1	principal lunar (semi-diurnal)
K_1	23.93	0.58	luni-solar (diurnal)
S_2	12.00	0.47	principal solar (semi-diurnal)
O_1	25.82	0.42	principal lunar (diurnal)
N_2	12.66	0.19	larger lunar elliptic (semi-diurnal)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Mf	327.85 ($\approx$ 14 days)	0.09	lunar fortnightly
Mm	661.30 ($\approx$ 28 days)	0.05	lunar monthly
SSa	4382.86	0.04	solar semi-annual

Table: Some sample tidal forcings sorted by relative amplitude to the M_2 tide (which is the largest forcing for Earth). Subset of Table 6.2 given in Wunsch (2015). The last few entries are weak and long term but they are there.

- M_2 and K_1 the dominant ones
  $\rightarrow$ usually do include these two in numerical models
- notice the periods are close to multiples of 12 hours

Tides: springs and neaps

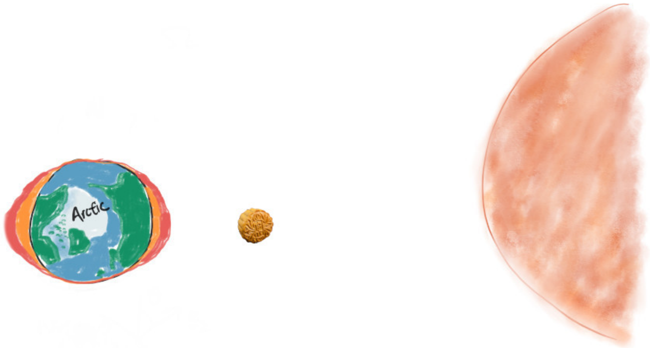

Figure: Schematic of spring and neap tides.

- moon(cake) and Sun in phase (or anti-phase)

Tides: springs and neaps

Figure: Schematic of spring and neap tides.

- spring tide: max difference in highs and lows

Tides: springs and neaps

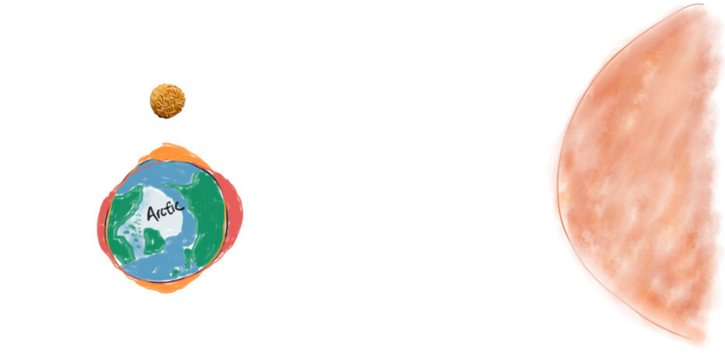

Figure: Schematic of spring and neap tides.

- moon(cake) and Sun in quadrature

Tides: springs and neaps

Figure: Schematic of spring and neap tides.

- neap tide: min difference in highs and lows

Tides: springs and neaps

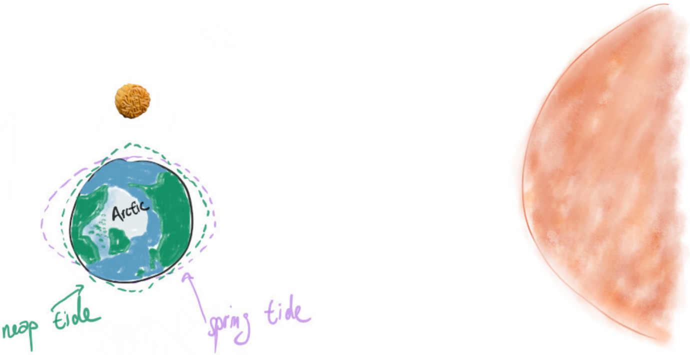

Figure: Schematic of spring and neap tides.

- occurs every two weeks or so (in line with lunar rotation)

Tides: propagation + Kelvin waves

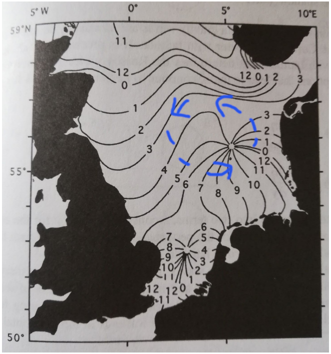

Figure: Phase lines (denoting peak of SSH, in hours in GMT) of the semi-diurnal tidal signal. Notice the anti-clockwise propagation (with boundary to the right). Diagram modified from Knauss (1997), Fig. 10.5.

- anti-clockwise signal in NH from SSH from Kelvin waves (see Lec 16)

  $\rightarrow$ Kelvin waves propagate cyclonically

  $\rightarrow$ distrubance from tide generating force
- fast propagation (because gravity wave speed)
- non-dispersive
- decays over Rossby deformation radius
- cares about rotation

  $\rightarrow$ **dynamic theory of tides**

Internal tides: generation

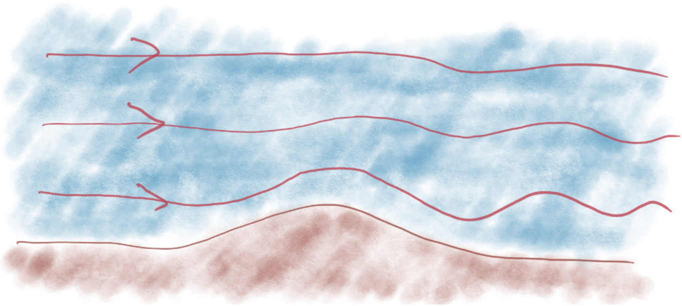

Figure: Flow over topography (e.g. tidal motion) leading to wave generation.

- flow over topography can lead to wave generation
  $\rightarrow$ internal gravity waves (Lec. 16)
- repeated "sloshing" by tide generating force
  $\rightarrow$ period of "sloshing" (e.g. M_2 is around 12 hrs)

Internal tides: decreasing in scale

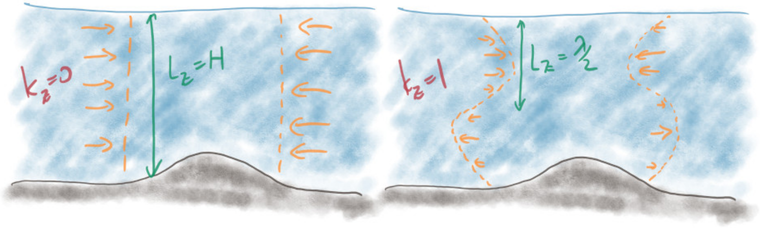

Figure: Motion associated to theoretical $k_z = 0$ and $k_z = 1$ (with boundary as ocean surface). Note associated with k_z is a length-scale.

- tidal forcing is to first approximation uniform over depth of ocean
  $\rightarrow$ think $k_z = 0$

  (commonly barotropic but I don't like "barotropic" used like this...)

- generating waves of increasing vertical variation (barotropic conversion, but, again...)

  $\rightarrow$ e.g. $k_z = 1$, increase in frequency, **decrease** in vertical wavelength or vertical scale

Internal tides: breaking

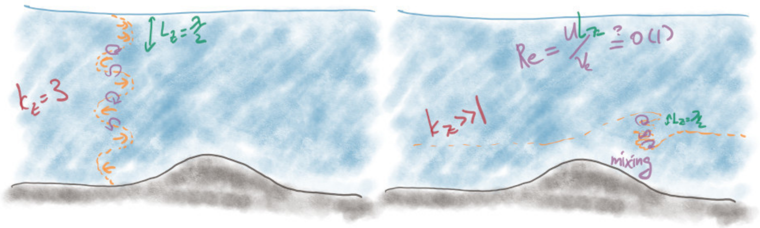

Figure: Motion associated to theoretical $k_z = 3$ and a more realistic case with nominal $k_z \gg 1$, via the observation that k_z can be linked to a length-scale.

- instabilities (Lec. 17) lead to more small-scales

  $\rightarrow$ e.g. k_z keeps increasing, so vertical scale keeps decreasing...
- eventually a dynamical Re (Lec. 10) starts becoming important

  $\rightarrow$ reconnecting isopycnals, diapycnal mixing (Lec. 10, 14, 16, 17)

Internal tides: statistics

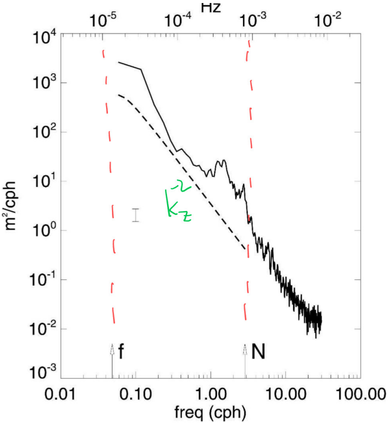

Figure: Observation of ocean displacement as a power spectrum, with the Garrett–Munk wavenumber (so frequency) spectrum dependence put in. Note also the frequency boundaries marked on since interal gravity waves should satisfy $f < \omega < N$. From Stevens *et al.* (2005), *J. Phys. Oceanogr.*, modified from their Fig. 3.

- general characteristics of internal waves?

  $\rightarrow$ Garrett–Munk spectrum

  (Garrett & Munk, 1972, *Geophys. Fluid. Dyn.*)

  $$E \sim \omega^{-2} \sim k_z^{-2} \quad (\omega \gg f)$$

- universal behaviour?
- initially fit to observations, but can derive from first principles?

  $\rightarrow$ e.g. wave turbulence formalism

  (way beyond here; see works by Victor L'vov or book by Sergey Nazarenko)

Summary

- tides as deformation arisng solely from gravitational attraction (Lec. 4 + 7)

  → tide generating force as a differential force
- signal particularly in SSH (Lec. 7), but also can generate internal waves (Lec. 16)

  → low mode forcing to high mode motion (decreasing length scale) (Lec. 15)

  → Garrett–Munk spectrum, $E \sim k_z^{-2}$

  → wave breaking (Lec. 17) lead to **diapycnal mixing** (Lec. 10), feedback onto **MOC** (Lec. 14)
- revisit this later in shelf and coastal dynamics (Lec. 21 + 22)

Summary

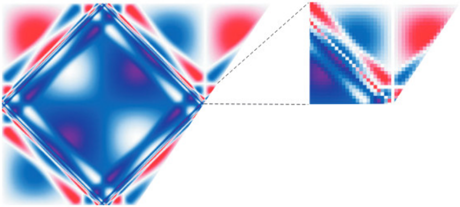

Figure: Internal gravity wave attractors in a uniformly stratified channel. Image from Maas (2005), *Int. J. Bifurcat. Chaos*, reworking of Maas *et al.* (1997), *Nature*.

- what do waves do when they break?
- where do wave break?
- how do waves break?
- how to model the above?
- implications for overall system?

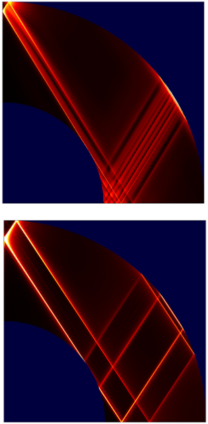

Figure: Inertial wave attractors in a homogeneous planetary interior at different tidal forcings. From Gordon Ogilvie (2009, *Mon. Not. Royal. Astro. Soc*).