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  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 7: Mechanical forcing 1 (pressure and gravity)

Tue 23rd Feb

Outline

- recall forcing on ocean
  $\rightarrow$ thermodynamic (T and $S \Rightarrow \rho$ and buoyancy)
  $\rightarrow$ mechanical (wind, gravity, pressure, rotation etc.)

- gravity + pressure (alluded to last Lec.)
  $\rightarrow$ geoid (see also Lec. 18)
  $\rightarrow$ sea surface height (SSH)
  $\rightarrow$ weight
  $\rightarrow$ hydrostatic pressure
  $\rightarrow$ some consequences for flow

Key terms: geoid, SSH, hydrostatic pressure

Recap: forces

- Newton's second law: objects are in steady state (at rest or steady speed) unless there is a net force

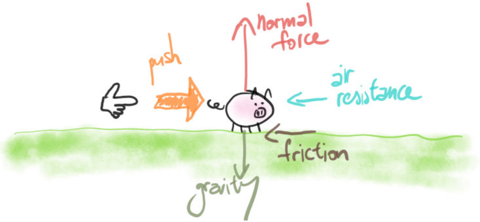


Figure: Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vaccum because we have air resistance + abuse of animal rights).

- thermodynamic forcing: affects T and S
- mechanical forcing: affects momentum

thermodynamic variables affects momentum via pressure

Recap: buoyancy forces

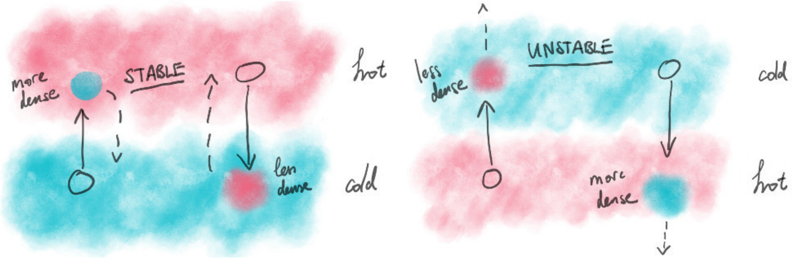


Figure: (Un)stable temperature configurations.

- buoyancy of fluid ultimately depends on density

  $\rightarrow$ lighter density water, more 'floaty'

  $\rightarrow$ heavier things (less buoyant water) $=$ more weight, imbalance and sinks

Recap: in-situ vs. potential/neutral density

- $\rho = \rho(T, S, p)$ via the EOS, but want to neglect p contribution to ρ because non-dynamic (from a work done point of view)
- in-situ density ρ says you basically have no up-down motion in the deep

  $\rightarrow$ but we know we have a bit!

  $\rightarrow$ contributions from p included here
- potential densities referenced to different levels says you can

  $\rightarrow$ **some** p contribution removed
- **you want the p resulting from ρ but without p in the ρ** (otherwise, a circular argument?)

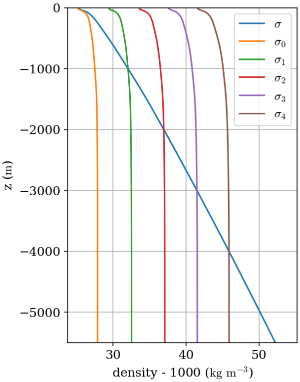


Figure: Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

Recap: equations of motion

Denoting $\boldsymbol{u} = (u, v)$ and $\boldsymbol{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F}_{\boldsymbol{u}} + \boldsymbol{D}_{\boldsymbol{u}} \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2}$$

$$\nabla \cdot \boldsymbol{u}_3 = 0 \tag{3}$$

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T \right) = F_T + D_T \tag{4}$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S \right) = F_S + D_S \tag{5}$$

$$\rho = \rho(T, S, p) \tag{6}$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Gravity

- attraction between bodies of different masses
  $\rightarrow$ note it is a purely attractive force (cf. magnetism)

$$F = G\frac{m_1 m_2}{r^2}, \qquad G = 6.674 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$$

  $\rightarrow$ G the gravitational constant
  (exercise: check the LHS and RHS units agree)

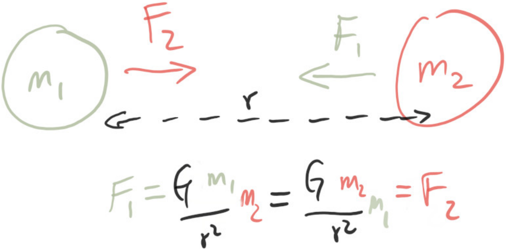


Figure: Schematic of gravitational attraction for two masses. If $m_1 \gg m_2$ (e.g. Earth and a pig) then forces on each body are equal, but its effect on one the pig is much larger than it is for the Earth (recall $F = ma$).

Gravity and weight

- Let's take Earth as an example:

$$F = G\frac{m_{\text{earth}}}{r^2_{\text{earth}}}m,$$

taking (units!)

$G = 6 \times 10^{-11}$

$m_{\text{earth}} = 6 \times 10^{24}$

$r_{\text{earth}} = 6400 \text{ km} \approx 6 \times 10^{6} \text{ m}$

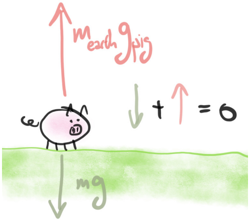


Figure: Gravity as applied on Earth. Note that g_{pig} is tiny (exercise: make an estimate of g_{pig}).

$$F = 6 \times 10^{-11}\frac{6 \times 10^{24}}{(6 \times 10^{6})^2}m = \frac{6^2}{6^2} \times 10^{-11+24-12}m = 10m \equiv mg$$

- the gravitational acceleration **on Earth** (recall $F = ma$) is $g \approx 10 \text{ m s}^{-2}$ (exercise: don't drop the decimal places like I did above and repeat the calculation)

Mass vs. weight

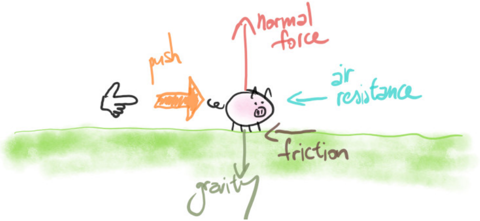


Figure: Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vaccum because we have air resistance + abuse of animal rights).

- the pig above with mass m has weight mg

  $\rightarrow$ mass is how much 'stuff' a body has

  $\rightarrow$ weight is a **force** and dependent on value of g

  $\rightarrow$ e.g. pig has same mass on moon but **weighs less** there because $g_{\text{moon}} \approx (1/6)g$ (exercise: why is g_{moon} smaller?)

Geoid

- picture for spherical bodies with uniform mass (then r is distance between centre of gravity), but Earth is not quite spherical...

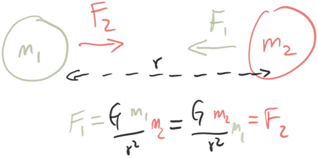


Figure: Schematic of gravitational attraction for two masses.

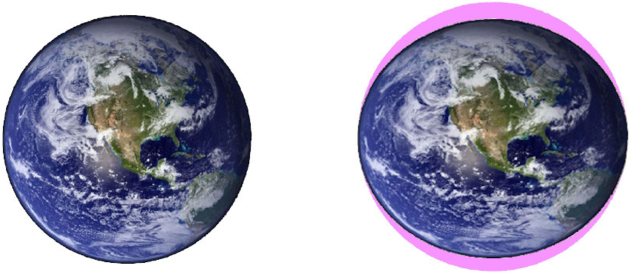


Figure: Cartoon of spherical vs. ellipsoid earth (it's inflated slightly at the Equator from Earth's spinning). Modified picture from NASA.

Geoid

- ...nor is the mass uniformly distributed!

  $\rightarrow$ where there is more mass there is more gravitational attraction

- geoid is the surface that the ocean surface would trace out if we only had gravity and rotation

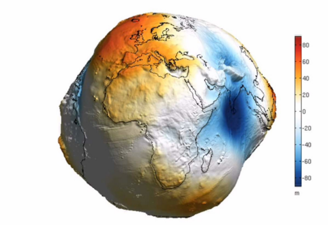


Figure: The "lumpy potato" Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.

$\rightarrow$ or, geoid is the surface where gravity is everywhere perpendicular to it (I like this one more...)

$\rightarrow$ wind and tidal action move sea surface around

$\rightarrow$ important concept in dynamics + sea level science

Geoid

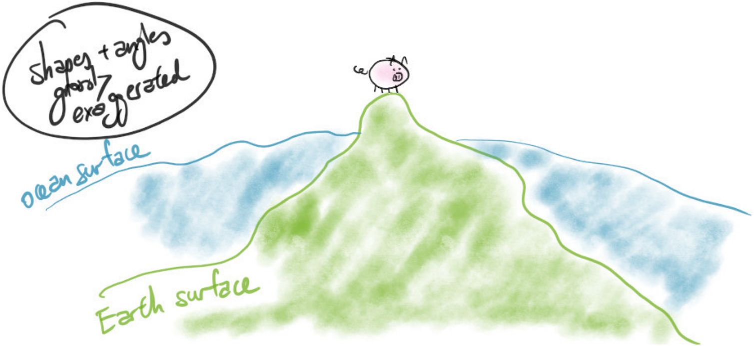
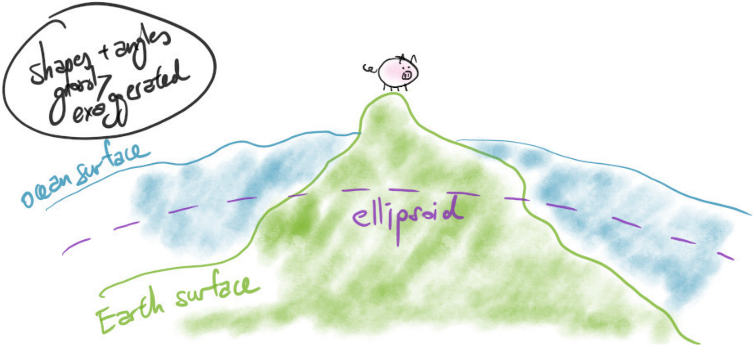


Figure: Schematic of the ellipsoid and geoid.

Geoid



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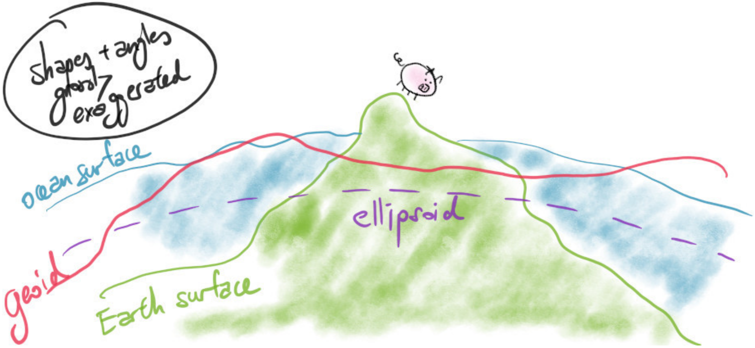

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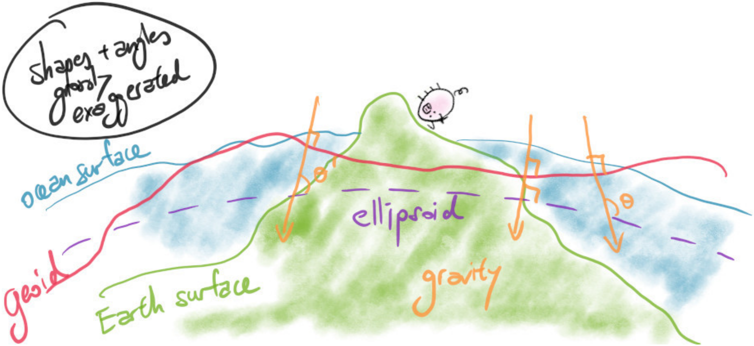

Figure: Schematic of the ellipsoid and geoid.

Gravity + weight

- differences greatly exaggerated above, in reality gravity variations are very small

  → that's why it was very difficult to get the geoid!

  → really needed satellites (see Lec. 20)

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  $\rightarrow$ remember the ocean is quite thing ($H/L \ll 1$)

- it does matter when we are talking about things like sea level (see OCES 4001)

  $\rightarrow$ sea level change but **relative to what**?

  $\rightarrow$ e.g. ellipsoid? ground? geoid?

SSH to be instantaneous height relative to ellipsoid

Pressure (recall last Lec.)

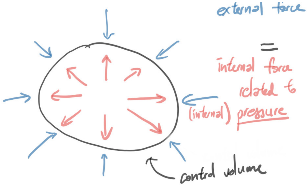


Figure: Fluid volume in force balance.

Consider a body (e.g. balloon) of fixed volume

- fixed volume $\Rightarrow$ steady
- steady $\Rightarrow$ in force balance, no net force

- pressure = force per area,

$$p = F/A, \qquad \text{units: N m}^{-2} \equiv \text{Pa}$$

1 bar = 10^6 Pa (Pascals) (see e.g. Wikipedia for others)

$\rightarrow$ cf. millibars (mbar) in atmosphere

$\rightarrow$ lines of constant pressure = isobar

Pressure: atmospheric example

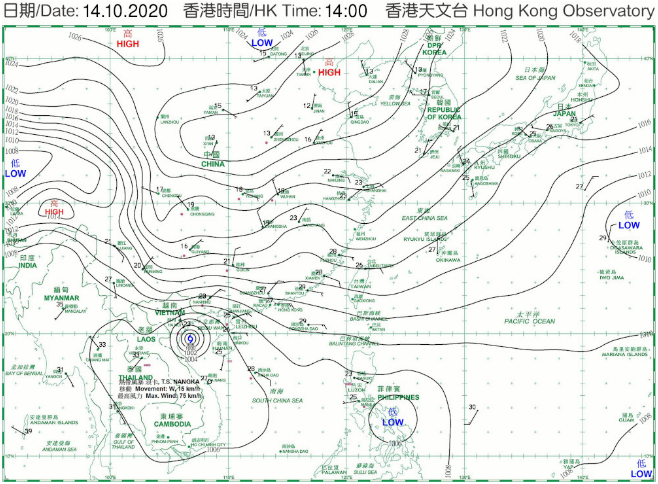


Figure: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

Hydrostatic balance (recall last Lec.)

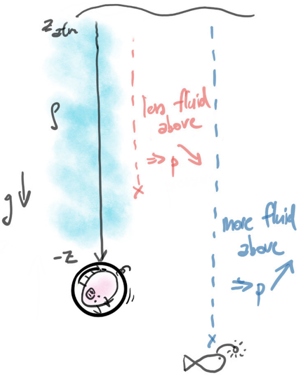


Figure: Schematic of hydrostatic pressure

- hydrostatic approximation: pressure **approximately equal** to weight above when static

  $\rightarrow$ weight is $F = mg$ so for force balance,

$$F = mg = \boxed{g \int_{-z}^{z_{\mathrm{atm}}} \rho(z')\, \mathrm{d}z' = p},$$

with $g \approx 9.81\ \mathrm{m\ s^{-2}}$

$\rightarrow$ if $\rho = \mathrm{const}$ then $p = \rho g z + p_{\mathrm{atm}}$

Horizontal effect?

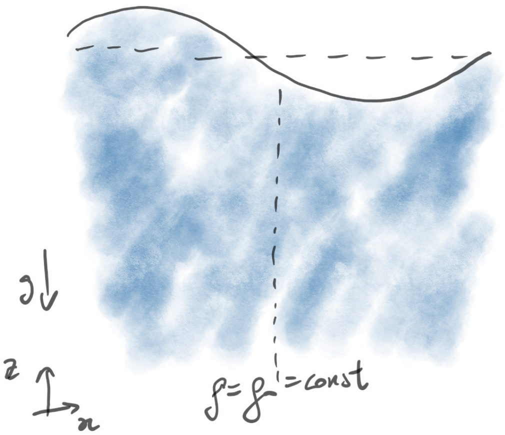
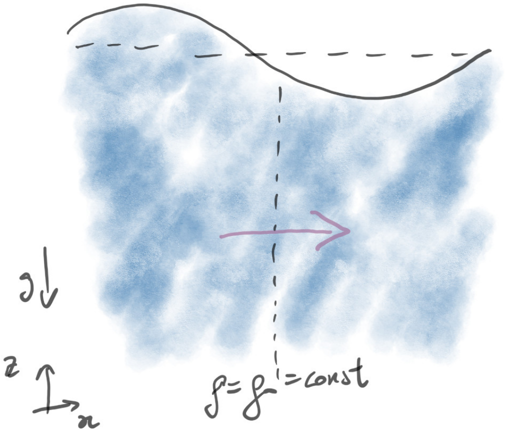


Figure: Horizontal effect because of hydrostatic pressure.

Horizontal effect?



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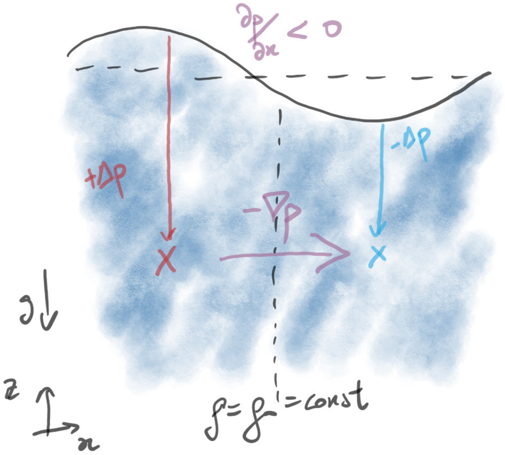

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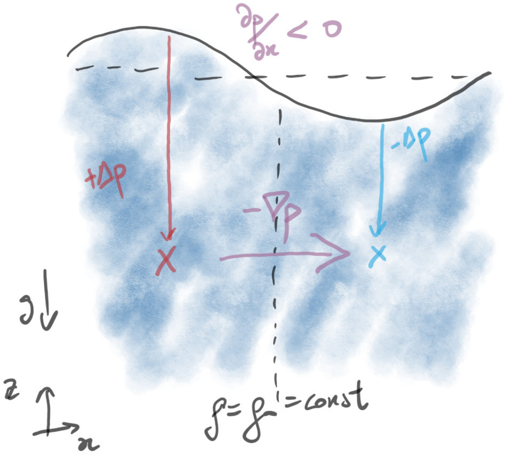


Figure: Horizontal effect because of hydrostatic pressure.

- assuming hydrostatic balance, water moves from $+\Delta p$ to $-\Delta p$ because there is a **net force** (negative pressure gradient $-\nabla p$)

  $\rightarrow$ important later for geostrophic flows (see next Lec.)

Atmospheric example revisited

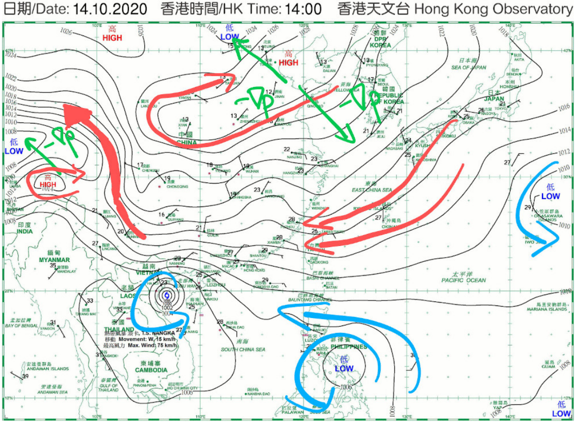


Figure: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

- note that flow doesn't go in the direction of $-\nabla p$!

  $\rightarrow$ **along** rather than **across** isobars (Coriolis effect, see next Lec.)

Oceanic example

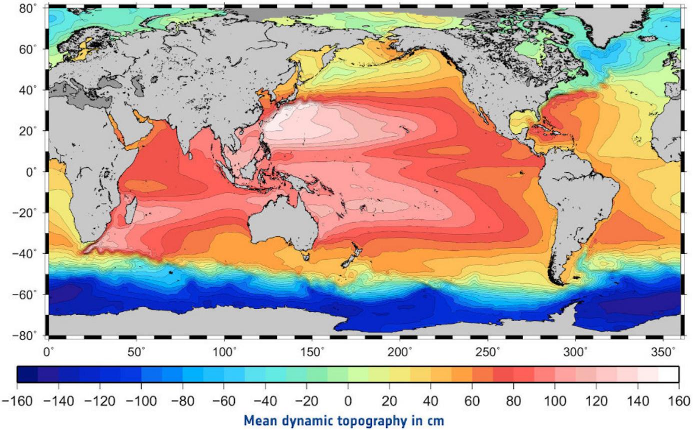
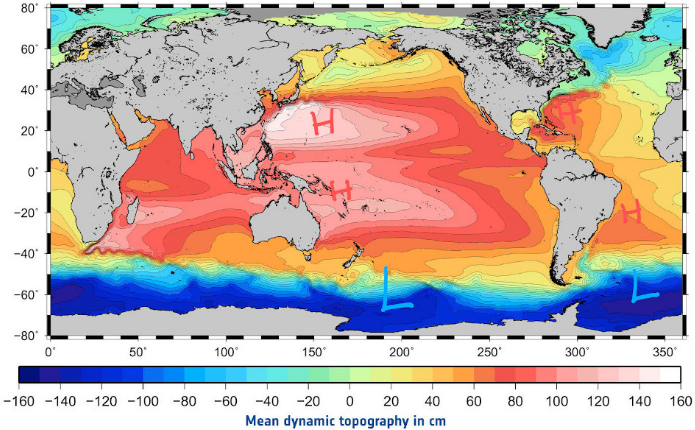


Figure: Time-mean global SSH (also called mean dynamic topography), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

Oceanic example



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- contours of SSH related to isobars via hydrostatic balance

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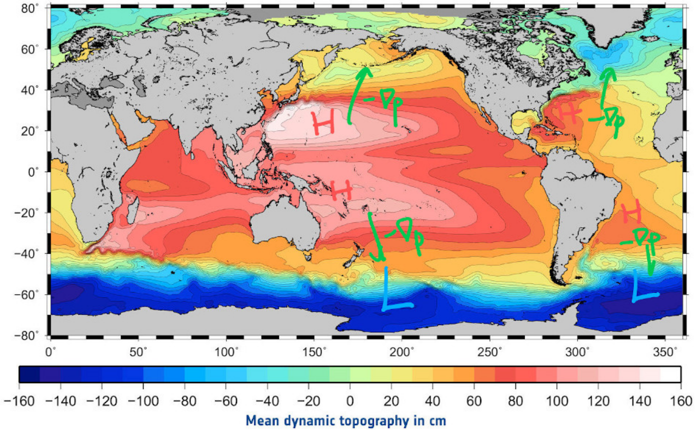

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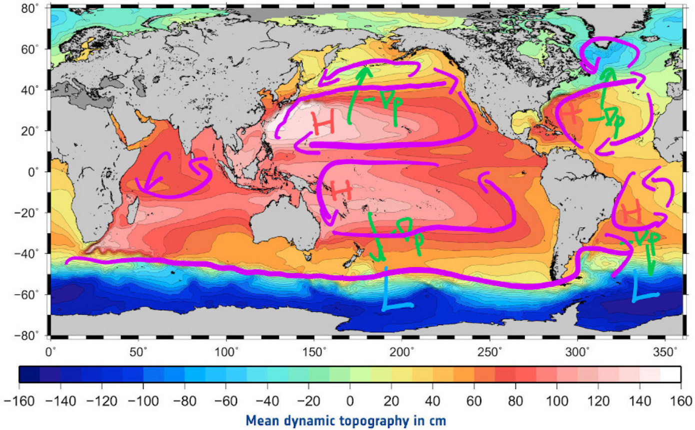

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- contours of SSH related to isobars via hydrostatic balance

  → flow is **along** rather than **across** isobars (Coriolis effect, see next Lec.)

Summary

- gravity + geoid

  → astronomical forcing on ocean

  (see Lec. 18)

  → geoid important for e.g. sea level change (see Lec. 18 + OCES 4001)
- hydrostatic pressure

  → pressure proportional to weight of fluid above

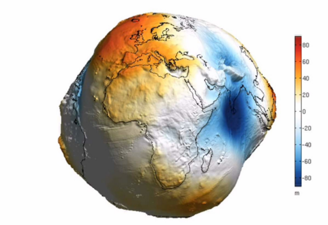

Figure: The "lumpy potato" Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.

- buoyancy (thermodynamic stuff) affects pressure...
- ...leading to pressure **gradients** (mechanical force) driving a **flow**...

  → ...but rotation can influence resulting flow! (see next Lec.)