Boring but important disclaimers:

- If you are not getting this from the GitHub repository or the associated Canvas page, you are probably getting the substandard version of these slides (e.g. CourseHero, Chegg etc.) Don't pay money for those, because you can get the most updated version for free at

  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
- I do not claim the compiled products and/or code are completely mistake free (e.g. I know I don't write Pythonic code). Use the material however you like, but use it at your own risk.
- As said on the repository, I have tried to honestly use as content that is self made, open source or explicitly open for fair use, and citations should be there. If however you are the copyright holder and you want the material taken down, please flag up the issue accordingly and I will happily try and swap out the relevant material.

OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 16: Dynamics 2 (waves and dynamic mechanisms)

Thur 8th Apr

Outlook of the next few lectures

Dynamics important, next few lectures on

- waves (this Lec. + 16, 18) and instabilities (Lec. 17)

  $\rightarrow$ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- how to describe waves (Lec. 15)
- **types of waves** (Lec. 16)

  $\rightarrow$ consequence + leading to instabilities
- instabilities (Lec. 17)

  $\rightarrow$ parcel-type (mechanistic) arguments for instability
- tides (particularly as internal gravity waves) (Lec. 18)

Outline

- gravity waves

  → gravity/buoyancy as restoring mechanism ($\sqrt{gH}$)
- inertial waves

  → Coriolis as restoring mechanism (f)

  → e.g. Rossby waves, Kelvin waves
- inertial-gravity + internal waves ($\sqrt{gH}$ or N, and f)

  → extra depth dimension to deal with

  → Brunt–Väisälä or buoyancy frequency N

- propagation mechanism (Rossby wave example)

  → kinematic argument with **vorticity**

Key terms: buoyancy frequency, gravity waves, inertial waves, Rossby waves, Kelvin waves, vorticity inversion

Recap: what goes down must come up

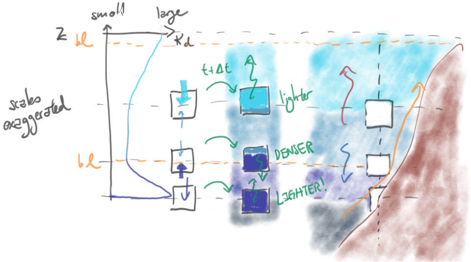


Figure: Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers

  $\rightarrow$ ~~broad diffusive~~ boundary intensified upwelling

what causes the bounary intensification of κ_d? dynamics!

- at the surface, lots of things... (convection, waves, Langmuir turbulence etc.)
- at the bottom, probably tidal conversion (Lec. 18) $\rightarrow$ internal gravity waves (Lec. 16) $\rightarrow$ shear instabilities (Lec. 17)

Recap: waves and dispersion relation

- waves are ubiquitous physical features

  $\rightarrow$ depends on physics
- wave described by the dispersion relation $\omega = \mathcal{F}(k)$

  $\rightarrow$ physics dictate the form of $\mathcal{F}$

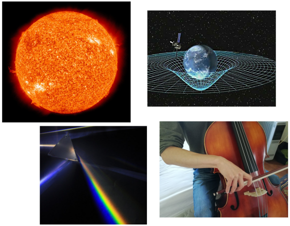


Figure: Examples of systems supporting waves. All figures from Wikipedia except the cello one.

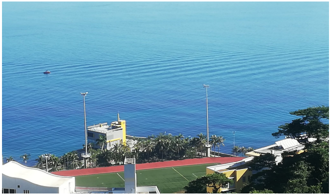


Figure: Gravity waves with signal at the sea surface (as darker and lighter bands). Taken at HKUST.

- (linear) waves can interfere with each other

  $\rightarrow$ constructive or destructive

  $\rightarrow$ interference can lead to steepening and breaking ("becoming" nonlinear)

Recap: wave propagation

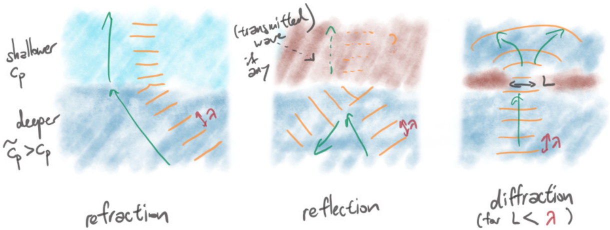


Figure: Schematic of refraction, reflection (and transmission), and diffraction. See Lec. 15.

- phase speed (in a direction) and group velocity as (note $\omega = \mathcal{F}(k)$)

$$c_{p,x} = \frac{\omega}{k}, \qquad c_{g,x} = \frac{\partial \omega}{\partial k}$$

$\rightarrow$ individual wave vs. wavepacket behaviour

$\rightarrow$ contribute to wave phenomenon (e.g. refraction from $\boldsymbol{c}_p = \boldsymbol{c}_p(\boldsymbol{x})$)

Wave steepening and breaking

Figure: Schematic of mixing by (irreversible) wave breaking, with contours reconnecting leading to e.g. diapycnal mixing.

- growing waves by instability

  → convective and/or shear (see Lec. 17)

  → mixing of material **across** isopycnals after reconnection, leading to diapycnal mixing
- feedback onto MOC (see Lec. 14)

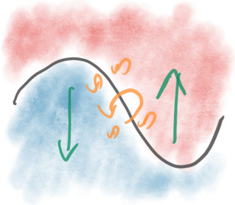


Figure: Velocity shear from waves can lead to mixing.

Gravity waves

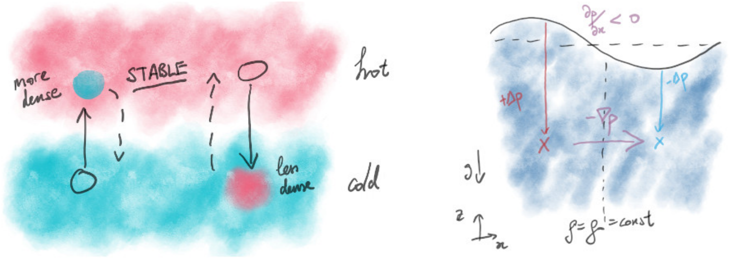


Figure: Gravity as restoring force. Pictures adapted from ones used in Lec. 7.

- deviation from resting **isopycnal** experiences restoring force from gravity (buoyancy)

  $\rightarrow$ left case: internal isopycnal (as an **isotherm**)

  $\rightarrow$ right case: sea surface is the isopycnal
- weak damping, restoring force, overshoots $\Rightarrow$ oscillatory motion (up and down in this case)

Gravity waves

For simplicity, consider a **homogeneous** (i.e. $\rho = \text{const}$) layer of fluid (cf. Lec. 11 + 12, right case in previous Figure) until we get to internal waves

Gravity waves

For simplicity, consider a **homogeneous** (i.e. $\rho = \text{const}$) layer of fluid (cf. Lec. 11 + 12, right case in previous Figure) until we get to internal waves

In this instance, dispersion relation for gravity waves is given by (without derivation)

$$\omega^2 = gk \tanh(kH) \qquad \Rightarrow \qquad \omega = \pm\sqrt{gk \tanh(kH)}$$

- H is water depth, and tanh = hyperbolic tangent, goes from -1 to 1

  $\rightarrow$ note symmetry in both directions (the $\pm$ sign)

  ($\omega \leq 0$ cases are just **shifts** in the **phase**)

Gravity waves

$$\omega = \pm\sqrt{gk \tanh(kH)}$$

- for **deep** water waves ($kH \gg 1$) and **shallow** water waves ($kH \ll 1$),

$$\omega_{\text{deep}} = \pm\sqrt{gk}, \qquad \omega_{\text{shallow}} = \pm k\sqrt{gH}$$

  $\rightarrow kH \gg 1$ so $\tanh(kH) \rightarrow 1$

  $\rightarrow kH \ll 1$ with $\tanh(kH) \approx kH + O((kH)^3)$ (do a Taylor expansion)

- deep water waves are depth-independent and dispersive
- shallow water waves are slower in shallow waters ($c_p \sim \sqrt{H}$) and non-dispersive ($c_p = c_g$)

Gravity waves

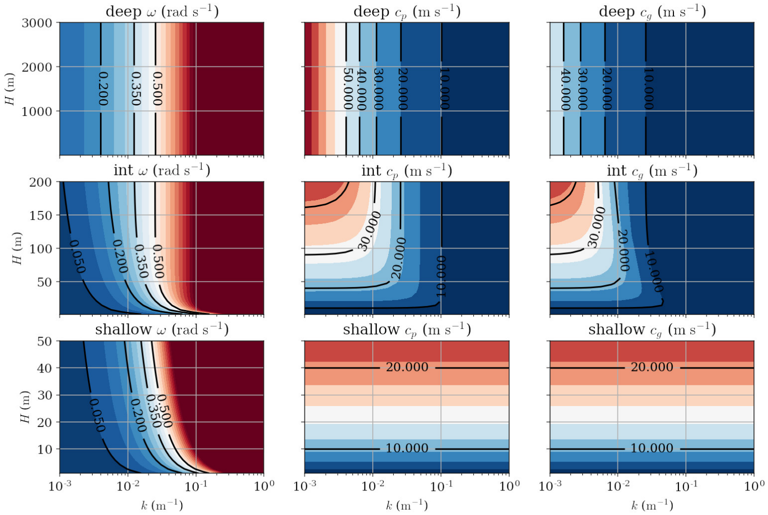


Figure: Water wave ω, c_p and c_g over (k,H) space, with k shown on a log axis. k chosen so wavelengths are roughly between 50 m to 5 km (recall $k = 2\pi/\lambda$). Also note the transitions from shallow to intermediate to deep are really to do with $kH \sim H/\lambda$. See `waves.ipynb`.

Inertial waves

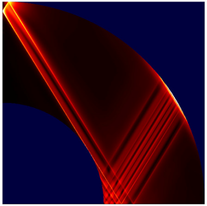
Inertial waves has the Coriolis "force" act as the restoring force

- generic for rotating systems

  → planetary interiors, stars, galactic disks
- mostly arise in context of internal waves

  → at surface buoyancy effects can dominate
- limited in frequency by inertial frequency f_0 (cf. Coriolis parameter)

  → revisit later when talking about inertia-gravity waves



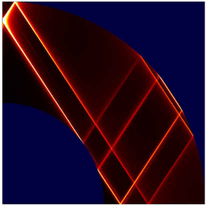


Figure: Inertial wave attractors in a homogeneous planetary interior at different tidal forcings. From Gordon Ogilvie (2009, *Mon. Not. Royal. Astro. Soc*).

Inertial-Gravity waves

In reality Coriolis and buoyancy effects both contribute

- large-scale and/or slow $\Rightarrow$ Coriolis important (because Ro $\ll 1$), classify as inertial waves

  $\rightarrow$ e.g. Rossby waves
- small-scale and/or fast $\Rightarrow$ Coriolis unimportant

  $\rightarrow$ e.g. internal gravity waves

- somewhere in between? Poincaré or inertia-gravity waves

$$\omega = \pm\sqrt{f_0^2 + gH(k_x^2 + k_y^2)}$$

$\rightarrow$ for $gH(k_x^2 + k_y^2) \gg f_0$, recover gravity waves

$\rightarrow$ for $gH(k_x^2 + k_y^2) \ll f_0$, recover inertial waves

Rossby deformation radius

$$\omega = \pm\sqrt{f_0^2 + gH(k_x^2 + k_y^2)}$$

- boundary between gravity and inertial regimes is roughly

$$\boxed{L_d = \frac{\sqrt{gH}}{f_0}}$$

- the Rossby deformation radius (for shallow water system)

  $\rightarrow$ roughly also the boundary where geostrophic approximation should hold (see Lec. 8 + 13)

  $\rightarrow$ estimates in a few slides

Internal waves

- introduce a useful quantity

$$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$$

- Brunt–Väisälä or buoyancy frequency (units: s^{-1})

  $\rightarrow N^2$ normally used

  $\rightarrow$ note $\partial\rho/\partial z < 0$ for stable stratification, i.e. $N^2 > 0$

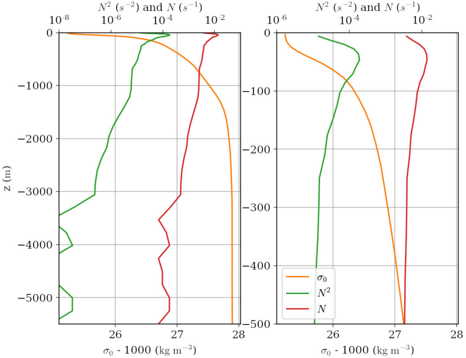


Figure: σ_0 (see Lec. 6) and the associated N^2 and N. $N^2 \ll 1$ means weakly stratified (weak density gradients), whilst $N^2 < 0$ shows unstable stratification (none in this case, but see Lec. 17). See `plot_eos.ipynb`.

simplistic view (!): $\sqrt{gH} \rightarrow N$

Internal waves

Generally then, **internal** inertia-gravity waves described by

$$\omega = \pm\sqrt{\frac{f_0^2 k_z^2 + N^2 k_x^2}{k_x^2 + k_z^2}} \approx \pm\sqrt{f_0^2 + \frac{N^2 k_x^2}{k_z^2}} \quad (\text{for } |k_z| \gg |k_x|)$$

- atmosphere and ocean has $N/f_0 = O(10^1 \text{ to } 10^2)$
  $\rightarrow$ so really we have gravity waves **influenced by rotation**
  $\rightarrow$ refer to them here as internal waves

Internal waves

Generally then, **internal** inertia-gravity waves described by

$$\omega = \pm\sqrt{\frac{f_0^2 k_z^2 + N^2 k_x^2}{k_x^2 + k_z^2}} \approx \pm\sqrt{f_0^2 + \frac{N^2 k_x^2}{k_z^2}} \quad (\text{for } |k_z| \gg |k_x|)$$

- atmosphere and ocean has $N/f_0 = O(10^1 \text{ to } 10^2)$

  $\rightarrow$ so really we have gravity waves **influenced by rotation**

  $\rightarrow$ refer to them here as internal waves
- note that $|f_0| \leq |\omega| \leq |N|$

  $\rightarrow$ since $0 \leq k_{x,z}^2/(k_x^2 + k_z^2) \leq 1$

  $\rightarrow$ frequency is **much lower** than gravity waves

Internal waves

Generally then, **internal** inertia-gravity waves described by

$$\omega = \pm\sqrt{\frac{f_0^2 k_z^2 + N^2 k_x^2}{k_x^2 + k_z^2}} \approx \pm\sqrt{f_0^2 + \frac{N^2 k_x^2}{k_z^2}} \quad (\text{for } |k_z| \gg |k_x|)$$

- atmosphere and ocean has $N/f_0 = O(10^1 \text{ to } 10^2)$
  $\rightarrow$ so really we have gravity waves **influenced by rotation**
  $\rightarrow$ refer to them here as internal waves
- note that $|f_0| \leq |\omega| \leq |N|$
  $\rightarrow$ since $0 \leq k_{x,z}^2/(k_x^2 + k_z^2) \leq 1$
  $\rightarrow$ frequency is **much lower** than gravity waves

internal tides to be seen as internal waves (Lec. 18)

Internal waves

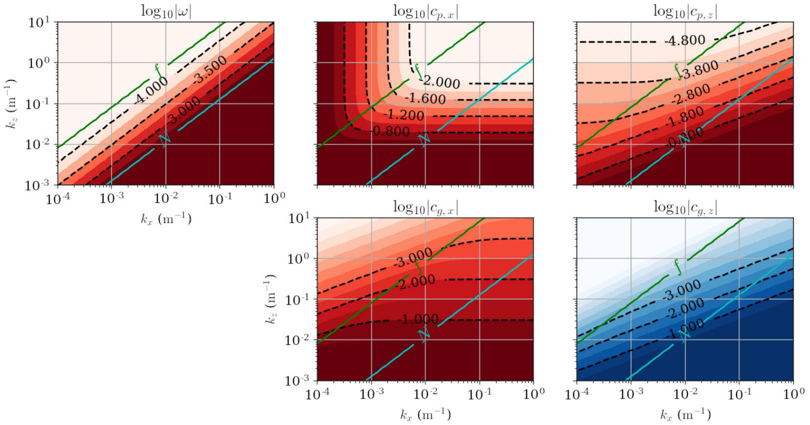


Figure: Inertial-gravity waves (with the $k_z \gg k_x$ approximation) ω, $c_{p,x}$, $c_{p,y}$, $c_{g,x}$ and $c_{p,y}$ as a log-log plot in (k_x, k_z) space, with $f = 5 \times 10^{-5}$ and $N = 3 \times 10^{-3}$ (oceanic relevant values). The contours denote the exponent x of $|10^x|$ and the colour shading denotes the sign (more blue = more negative *actual* values rather than exponents, more red = more positive *actual* values rather than exponents); since k_x and k_z is chosen to be positive, everything except $c_{g,z}$ is positive. Contours of f and N plotted with an offset plotted to show the boundary beyond which everything is either gravity waves or inertial oscillations. See `waves.ipynb`.

Internal waves

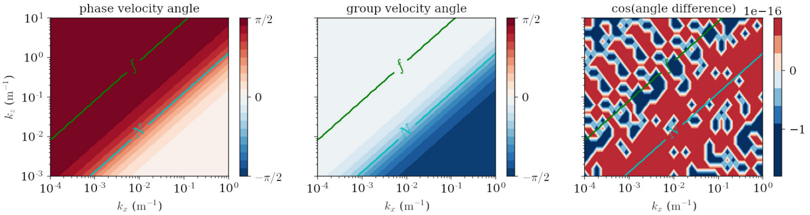


Figure: Inertial-gravity waves (with the $k_z \gg k_x$ approximation) phase velocity $\boldsymbol{c}_p$ angles and group velocity $\boldsymbol{c}_g$ angles (in radians, relative to the horizontal, and note $\pi/2 = 90^\circ$). The final panel shows $\boldsymbol{c}_p \cdot \boldsymbol{c}_g = |\boldsymbol{c}_p||\boldsymbol{c}_g| \cos\theta$ (which is zero up to rounding errors). Contours of f and N plotted with an offset plotted as in the previous diagram. See `waves.ipynb`.

- note that, for inertial-gravity waves (left as a bonus exercise),

$$\boldsymbol{c}_p \cdot \boldsymbol{c}_g = 0$$

$\rightarrow$ i.e. phase and group velocities are **perpendicular** to each other (see Lec. 4)

Deformation radius

- boundary given again by the Rossby deformation radius (for the continuously stratified case)

$$L_d = \frac{NH}{f}$$

  $\rightarrow L_{d,\text{atmos}} = O(1000 \text{ km})$, scale of cyclones and anti-cyclones, i.e. weather systems form (synoptic structures)

  $\rightarrow L_{d,\text{ocean}} = O(50 \text{ km})$, scale of ocean eddies

- latitude (through f) and H dependent

  $\rightarrow$ smaller L_d for **high** latitudes and **shallow** regions

  $\rightarrow$ consequence for **geostrophic approximation**? (e.g. shelves and coasts, see Lec. 21 + 22)

Deformation radius

- boundary given again by the Rossby deformation radius (for the continuously stratified case)

$$L_d = \frac{NH}{f}$$

  $\rightarrow L_{d,\text{atmos}} = O(1000\text{ km})$, scale of cyclones and anti-cyclones, i.e. weather systems form (synoptic structures)

  $\rightarrow L_{d,\text{ocean}} = O(50\text{ km})$, scale of ocean eddies
- latitude (through f) and H dependent

  $\rightarrow$ smaller L_d for **high** latitudes and **shallow** regions

  $\rightarrow$ consequence for **geostrophic approximation**? (e.g. shelves and coasts, see Lec. 21 + 22)

- internal L_d defined analogously (normally smaller than above)

Kelvin waves (more on this in Lec. 18, 21 + 22)

A type of boundary wave

- need f and a **boundary**

  $\rightarrow$ could be land (coastal Kelvin waves) (see Lec. 18, 21 + 22)

  $\rightarrow$ could be a wave guide (e.g. equator where f changes sign, equatorial Kelvin waves) (see OCES 4001, El-Niño, QBO etc.)

- needs f but propagates at the gravity wave speed, with

$$\omega = k\sqrt{gH}$$

  $\rightarrow$ non-dispersive

  $\rightarrow$ fairly fast (gravity wave speed)

Kelvin waves (more on this in Lec. 18, 21 + 22)

A type of boundary wave

- need f and a **boundary**

  $\rightarrow$ could be land (coastal Kelvin waves) (see Lec. 18, 21 + 22)

  $\rightarrow$ could be a wave guide (e.g. equator where f changes sign, equatorial Kelvin waves) (see OCES 4001, El-Niño, QBO etc.)
- needs f but propagates at the gravity wave speed, with

$$\omega = k\sqrt{gH}$$

  $\rightarrow$ non-dispersive

  $\rightarrow$ fairly fast (gravity wave speed)
- **NOTE the lack of $\pm$!**

Kelvin waves (more on this in Lec. 18, 21 + 22)

- boundary introduces asymmetry in this case: general solution like

$$\eta \sim \mathrm{e}^{\pm f_0 y/\sqrt{gH}} \cos(kx - \omega t)$$

$\rightarrow$ take $y \leq 0$ to be **boundary**, if $f_0 > 0$ (NH), need minus sign, and vice-versa

$\rightarrow$ wave propagates cyclonically (same sign as f)

- taking $f_0 > 0$ (NH),

$$\eta \sim \mathrm{e}^{-y/L_d} \cos(kx - \omega t),$$

so decay over the $L_d = \sqrt{gH}/f_0$

Rossby waves (more on this later)

A (particularly important) type of **inertial** wave

- requires a **gradient** in background vorticity

  $\rightarrow \partial f / \partial y = \beta$ (planetary case)

  $\rightarrow$ background flow $-\partial U / \partial y \sim \nabla \times \boldsymbol{u}$ (see later and Lec. 17)

- dispersion relation given by (on β-plane)

$$\boxed{\omega = -\frac{\beta k_x}{k_x^2 + k_y^2}}$$

$\rightarrow$ note that Rossby waves propagate to the **west** (more generally, **retrograde** or against the mean flow) since

$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0,$$

and **long** waves ($k_x \ll 1$) are **fast**(er)

Rossby waves (more on this later)

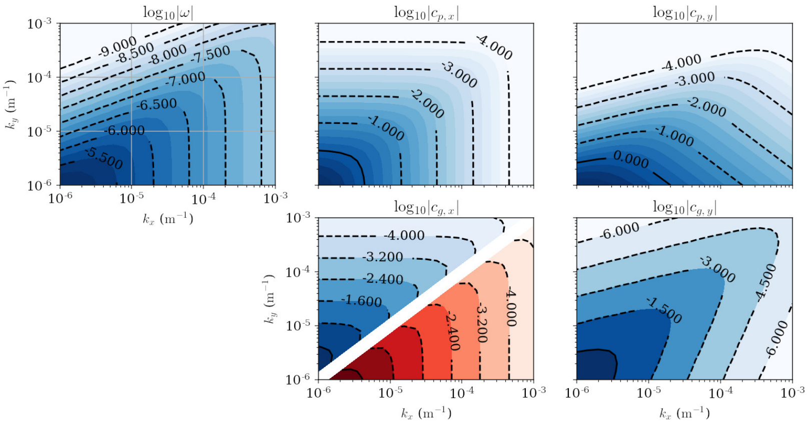


Figure: Rossby waves ω, $c_{p,x}$, $c_{p,y}$, $c_{g,x}$ and $c_{p,y}$ as a log-log plot in (k_x, k_y) space, with magnitude also as logs. The contours denote the exponent x of $|10^x|$ and the colour shading denotes the sign (more blue = more negative *actual* values, more red = more positive *actual* values); since k_x and k_y is chosen to be positive, everything except $c_{g,x}$ is negative. Choice of k_x and k_y correspond to wavelengths roughly between 6 km to 6000 km (Rossby waves are usually seen as planetary-scale waves). See `waves.ipynb`.

Propagation mechanism: Rossby waves

Rossby waves propagate **west**-ward (or, more generally, retrograde)

$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0$$

why?

Propagation mechanism: Rossby waves

Rossby waves propagate **west**-ward (or, more generally, retrograde)

$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0$$

why?

Key bits to the pictorial/parcel (cf. Lec 5 for temperature) argument:

- the initial wave conserves and carries vorticity (spini-ness, recall Lec. 4, 11, 12) into the external environments

  $\rightarrow$ these are now vorticity anomalies
- vorticity anomalies induces a velocity/flow (because spini-ness)
- induced flow seen to self-advect the wave and move it to the **West** (retrograde in the general case)

Propagation mechanism: Rossby wave example

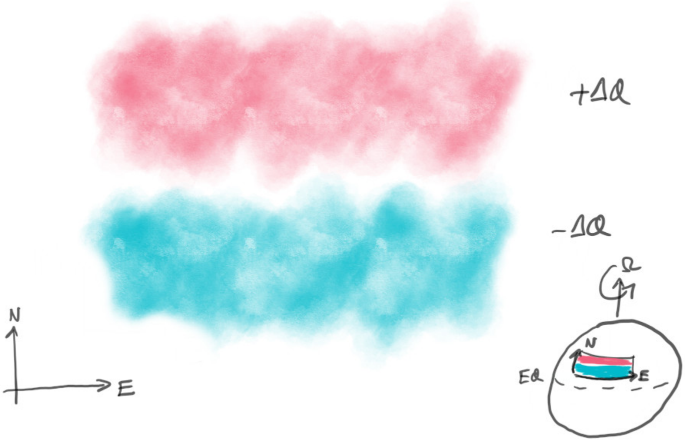


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

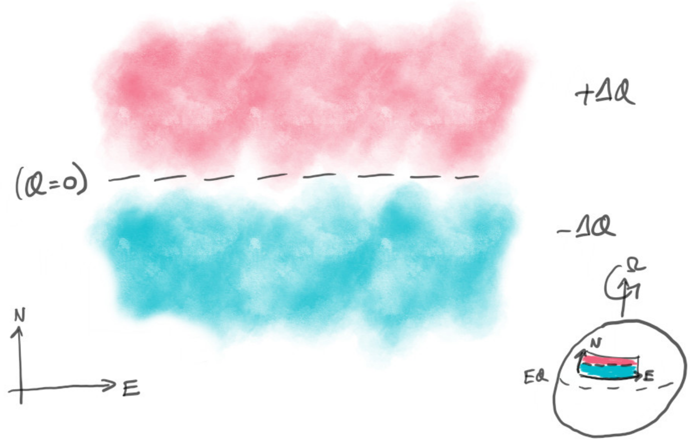


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

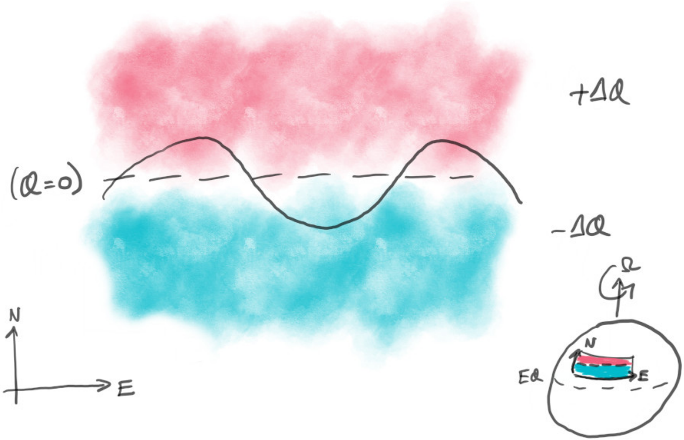


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

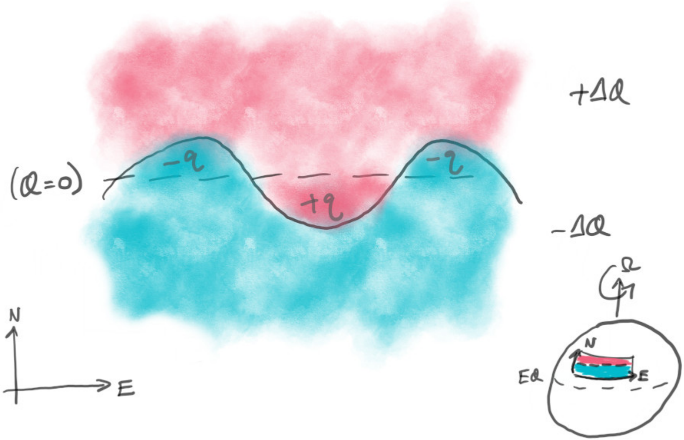


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

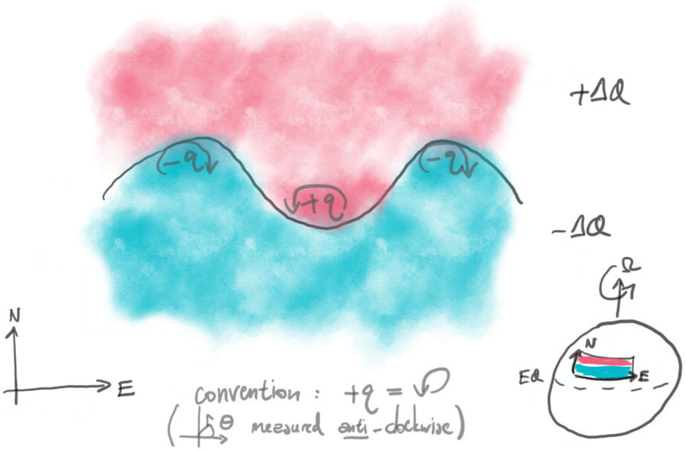


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

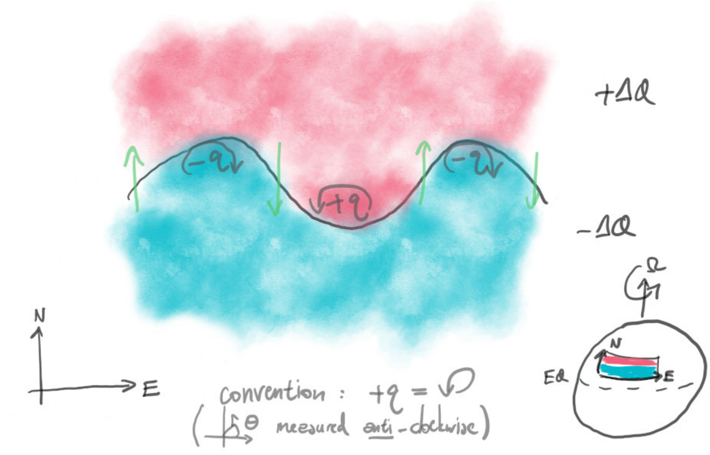


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

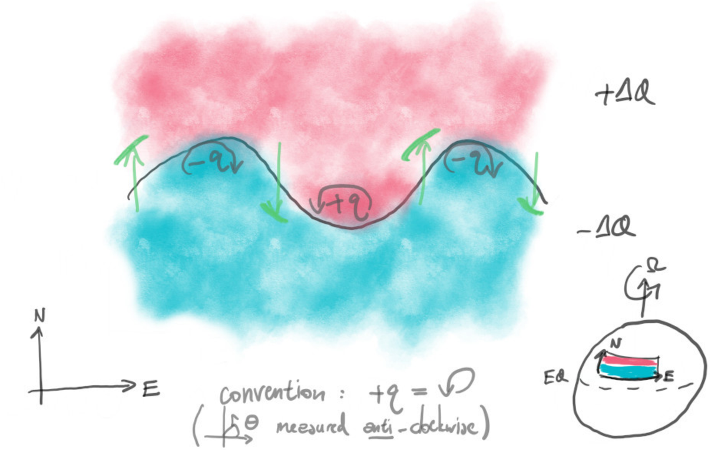


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

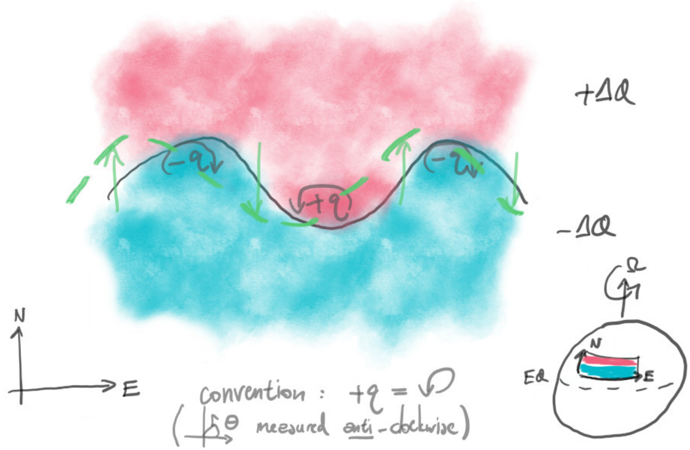


Figure: Rossby wave propagation schematic.

Propagation mechanism: Rossby wave example

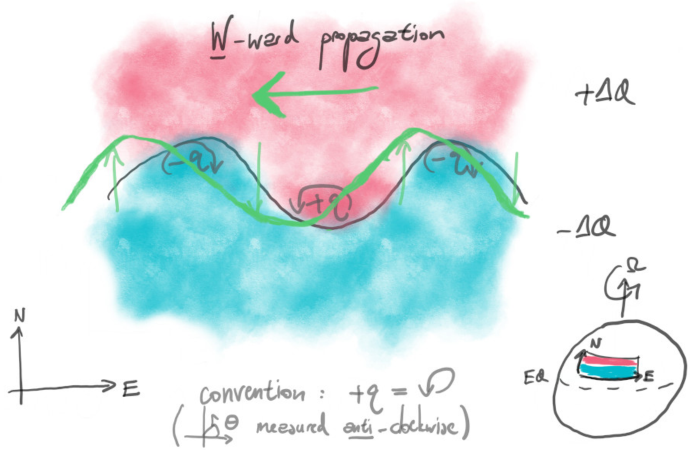


Figure: Rossby wave propagation schematic.

Summary

- gravity waves (gravity/buoyancy)
- inertial waves (Coriolis)
- inertial-gravity waves (general)

  → internal waves have $|f| \leq |\omega| \leq |N|$

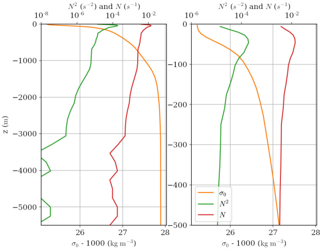


Figure: σ_0 (see Lec. 6) and the associated N^2 and N. See `plot_eos.ipynb`.

- Brunt–Väisälä or buoyancy frequency N

$$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$$

→ measure of stratification strength (see also Lec. 17)

Summary

- parcel argument for **west**-ward Rossby wave propagation

  → conservation of vorticity

  → vorticity anomalies induces flow

  → self-advecting

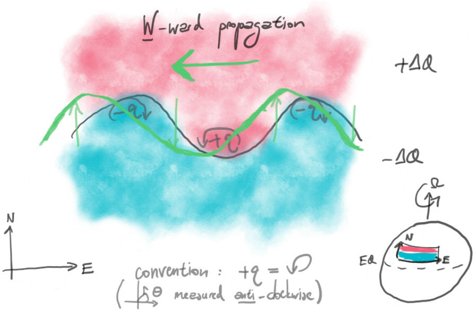


Figure: Rossby wave propagation schematic.

- generalisations exist (e.g. internal gravity waves in Harnik *et al.*, 2008, *J. Atmos. Sci*)

Summary

- parcel argument for **west**-ward Rossby wave propagation

  → conservation of vorticity

  → vorticity anomalies induces flow

  → self-advecting

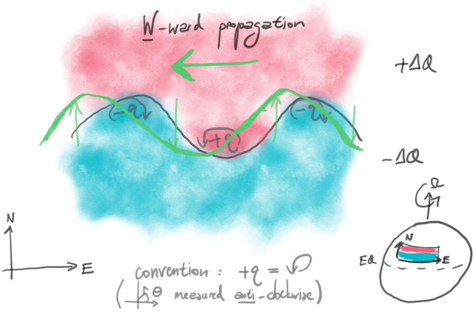


Figure: Rossby wave propagation schematic.

- generalisations exist (e.g. internal gravity waves in Harnik *et al.*, 2008, *J. Atmos. Sci*)
- two such waves interacting? (see Lec. 17)

  → potential for instabilities