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# OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 11: Gyres 1 (overview + Sverdrup balance)

Tue 9<sup>th</sup> Mar

# Outline

- ▶ (wind driven?) gyres and features
  - subtropical/subpolar gyres
  - Western Boundary Currents (WBCs) (e.g. Gulf stream, Kuroshio)
- ▶  $\beta$ -plane + Sverdrup balance
  - simple model of wind-driven gyre (on  $\beta$ -plane)
  - wind balancing gradient of Coriolis
- ▶ depth-independent model with no topography: Sverdrup balance
  - interior dynamics

**Key terms:** subtropical/subpolar gyres,  $f = f_0 + \beta y$ , Sverdrup balance

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1. recall material on **gyres** (Lec 2)
2. construct model (wind, rotation, friction; Lec. 4, 7-10)
  - **$\beta$ -plane**
  - assumption for density (but see next Lec.) (Lec. 5 + 6)
3. analyse and deduce via balance arguments
  - **Sverdrup balance** (wind + rotation; Lec. 8 + 9)
  - **mass conservation** (Lec. 4?)
  - **vorticity balance** (wind + friction; Lec. 9 + 10)

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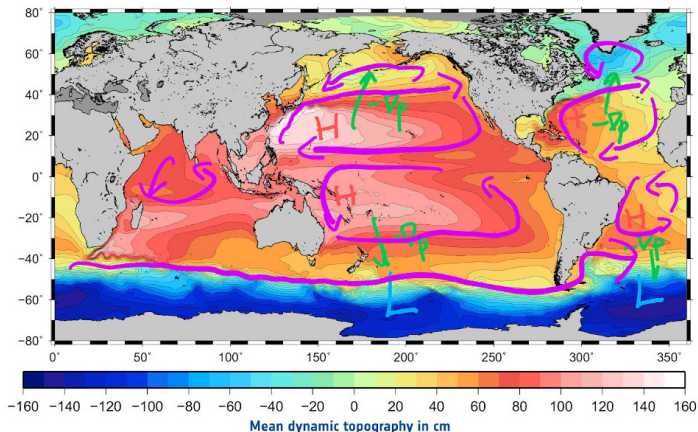
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**“All models are wrong, but some are useful”**

– attributed to George Box

## Recap: gyres

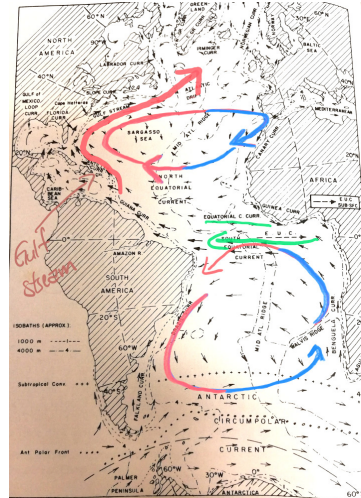


**Figure:** Time-mean global SSH (also called **mean dynamic topography**, with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al* (2011), J. Geophys. Res. Oceans.

- contours of SSH + geostrophic balance  $\Rightarrow$  flow, important part of **MOC**

# Recap: gyres + WBCs

- ▶ **subtropical and subpolar gyres** (former shown here)
  - **anti-cyclonic and cyclonic** respectively (in both hemispheres) (see Lec 8 + 9)
- ▶ **Western Boundary Current** as a part of system
  - Gulf stream here, Kuroshio in Pacific
  - transports **tropical + warm** water towards high latitudes

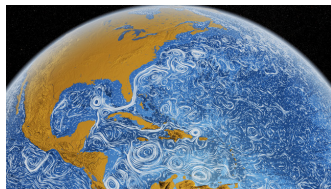
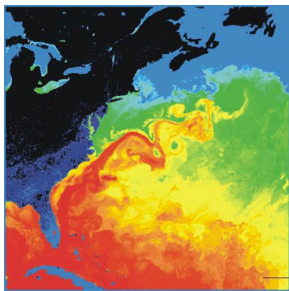


**Figure:** Features in the Atlantic Ocean. Modified Figure 7.9 from Pickard & Emery (1990), 5th edn.



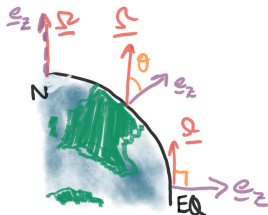
## Recap: gyres + WBCs

- Q. why Western and not Eastern? (see next Lec.)
- Q. processes leading to eddies? (see Lec. 17)
- Q. fluctuations + role in climate? (see Lec. 17 + OCES 4001)



Gulf stream in temperature (left) and surface current speed, from NASA

## Recall: Coriolis effect and parameter (Lec. 8)



- ▶ for a spherical Earth we take rotation axis to be z-axis, i.e.  $\underline{\Omega} = \Omega \underline{e}_z$  (this a vector), but locally, z is depth...
- ▶ introduce the latitudinally varying **Coriolis parameter**

**Figure:** Mis-alignment of  $\underline{\Omega}$  and  $\underline{e}_z$  used locally for depth.

$$f = 2\Omega \sin(\text{latitude})$$

- ▶ want to further simplify this, spherical (i.e. (lon, lat, depth)) to Cartesian geometry (i.e.  $(x, y, z)$ ) (cf. Lec 8, when rationalising Coriolis effect)

# $\beta$ -plane

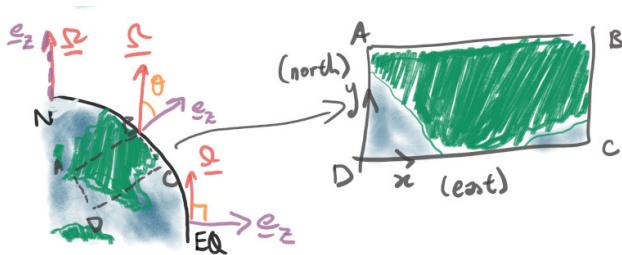


Figure:  $\beta$ -plane schematic.  $(\text{lon}, \text{lat}) \rightarrow (x, y)$  with  $f = f_0 + \beta y$  on the plane.

- ▶  $x \leftrightarrow \text{lon}, y \leftrightarrow \text{lat}, z \leftrightarrow \text{depth}$
- ▶  $f = 2\Omega \sin(\text{lat}) \leftrightarrow f = f_0 + \beta y$ 
  - $f_0$  (units:  $\text{s}^{-1}$ , same as  $f$ ) the uniform rotation frequency
  - $\beta = \partial f / \partial y$  (units: exercise)
  - **not** haline contraction (context should be clear)

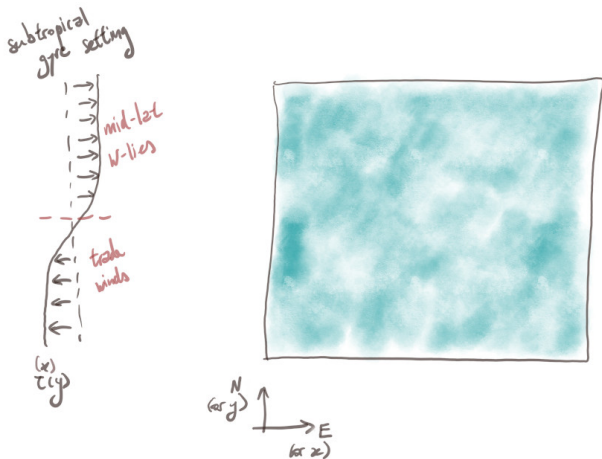
**will use  $\beta$ -plane extensively**

# Model preliminaries

For simplification, going to assume:

- ▶ NH  $\beta$ -plane ( $f = f_0 + \beta y > 0$ ), domain is square
- ▶ subtropical wind profile (for subtropical gyre)
  - only for simplificty, will extend
  - note: negative **wind stress curl**
- ▶ assume **depth-indepence** (either  $\rho = \rho_0$  or vertically integrate)
  - sometimes **barotropic** (I don't like this term for technical reasons)
- ▶ lateral frictional boundary layers
  - main **sink** of stuff is going to be over boundary layers

# Single (subtropical) gyre: schematic



**Figure:** Schematic of wind-drive model (NH, assume subtropical wind profile,  $\beta$ -plane, square, homogeneous in density)

# Equations

Original equations something like (in vertical vorticity  $\omega$ ):

$$\frac{\partial \omega}{\partial t} + \underbrace{\mathbf{u} \cdot \nabla \omega}_{\text{inertia}} + \underbrace{\beta v}_{\text{Coriolis}} = \underbrace{-r\omega}_{\text{drag}} + \underbrace{F_\tau(x, y)}_{\text{wind}}$$

- ▶ 2d equations in  $(x, y)$  (assumed no vertical variation)
- ▶ no pressure gradients (took a  $\nabla \times$  of momentum equation)
- ▶ Coriolis effect appears through  $\beta v$  (as  $f = f_0 + \beta y$ )  
→ it is **meridional** velocity  $v$  that shows up here
- ▶  $F_\tau = \mathbf{e}_z \cdot \nabla \times \boldsymbol{\tau}$  is **wind stress curl**  
→  $F_\tau < 0$  for wind profile in previous slide
- ▶ friction **parameterised** as **linear drag**  
→ important near boundaries

# Sverdrup balance

Throw away time derivative and inertia term (cf.  $Ro \ll 1$ ) gives

**Stommel's model** (Stommel, 1948, *Trans. Amer. Geophys. Union*)

$$\underbrace{r\omega}_{\text{drag}} + \underbrace{\beta v}_{\text{rotation}} = \underbrace{F_{\tau}(x, y)}_{\text{wind}}$$

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In **interior** friction is unimportant, so we have **Sverdrup balance**

$$\beta v \sim F_{\tau}(x, y)$$

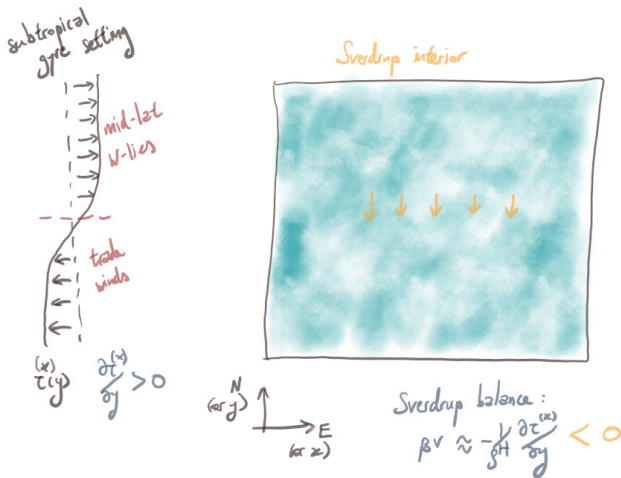
- Coriolis balancing wind stress curl  
→ geostrophy is Coriolis balancing **pressure gradient** (but no pressure in vorticity equation)

$\beta > 0$  so  $v$  related to wind stress curl in interior

Note: The standard derivation involves looking at Ekman up/downwellings associated with the wind, then imply the Sverdrup interior

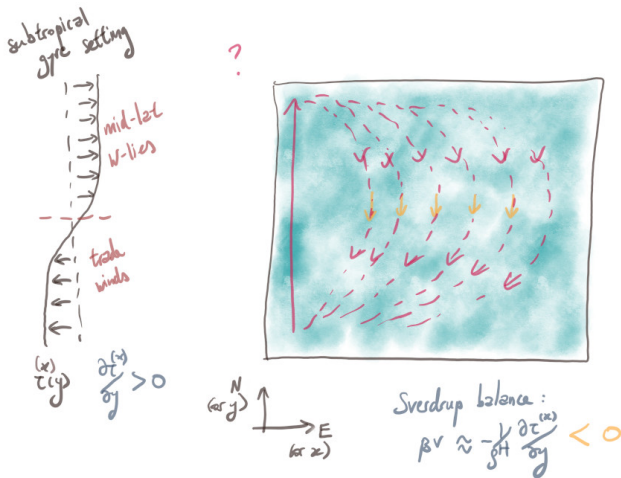


# Single (subtropical) gyre: Sverdrup interior



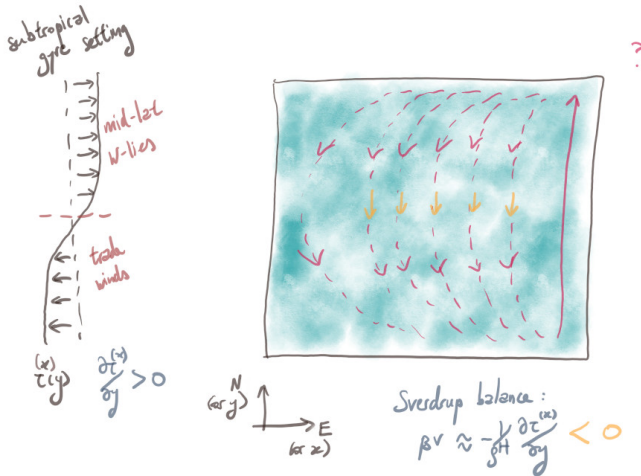
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# Single (subtropical) gyre: mass conservation



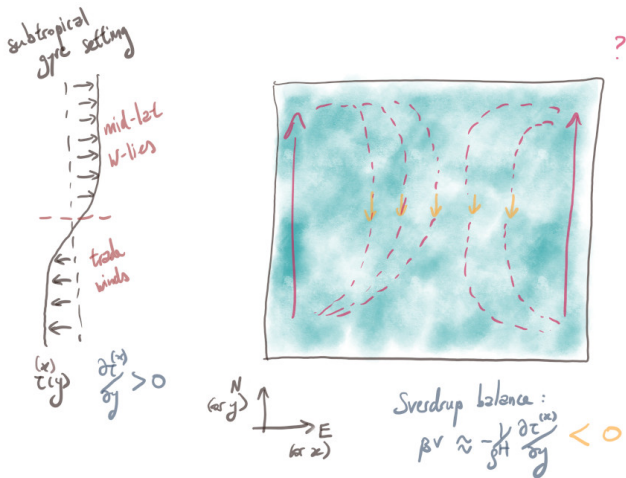
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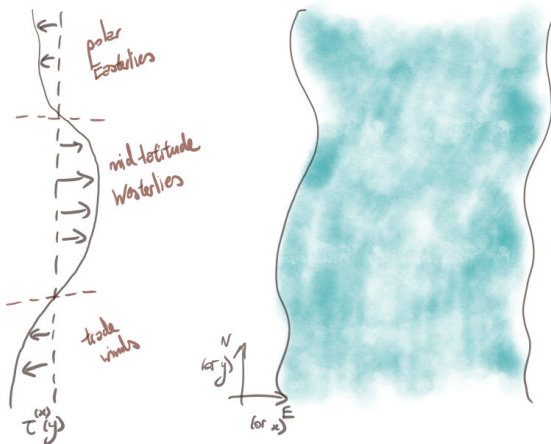
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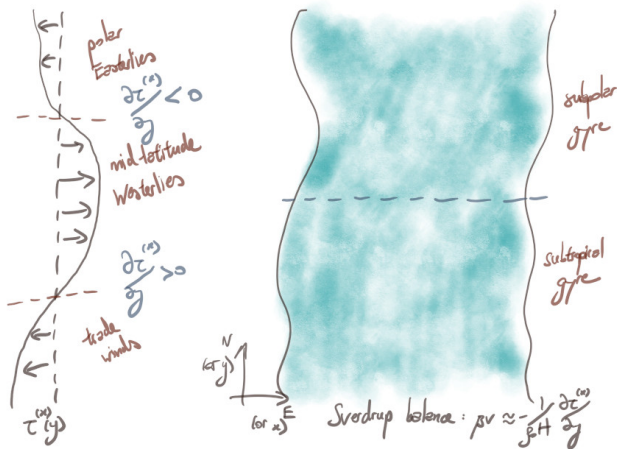
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# Double (subtropical + subpolar) gyre



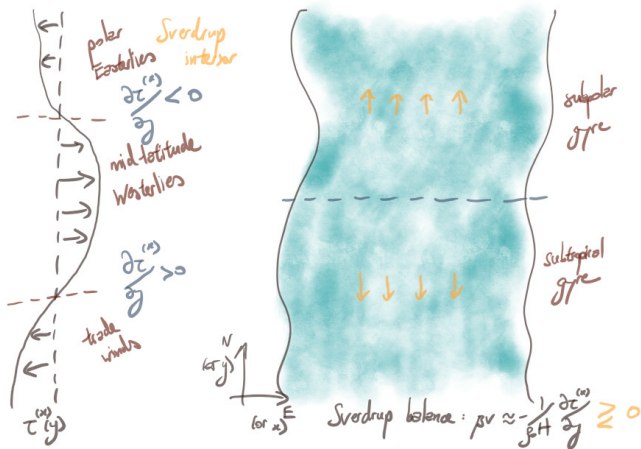
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# Double (subtropical + subpolar) gyre



**Figure:** Schematic of wind-drive model (NH,  $\beta$ -plane, homogeneous in density)

# Summary

## ► Sverdrup balance:

$$\beta v \sim F_{\tau}(x, y)$$

→ **Sverdrup interior**,  
meridional flow related to  
wind stress curl

## ► mass balance + continuity implies essentially two possibilities

## ► intuition: flow should be **western intensified**

→ energetic argument, flow should go in direction of wind

**next lecture: support intuition by **vorticity balance** argument**

