OCES 2003 Assignment 3, Spring 2021

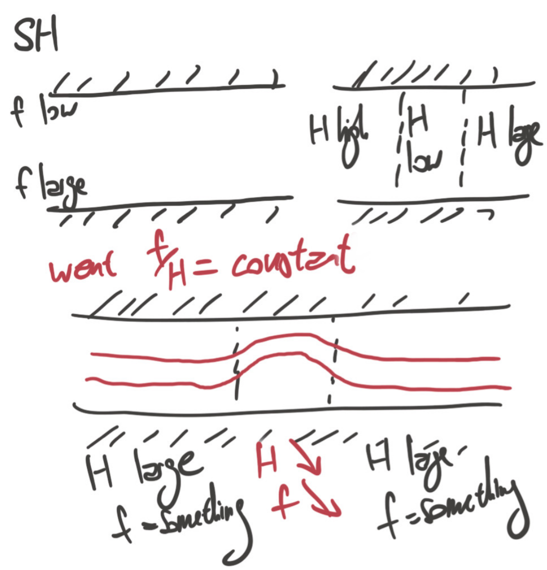
Inusuke Shibemoto

Set on: Tue 13th Apr; due: Tue 20th Apr

Model solutions and mark scheme

Problems

1. Pictorial argument as below:



If we are in the Northern Hemisphere, then you flip the top left diagram upside down (because the sign of f gets flipped), and find that the deflection as you go into the ridge is to the south, and reverting back to normal as you come out of the ridge. Either way, the deflection as you go into the ridge is *equator*-ward.

(1 mark for some sort of sensible argument, 1 mark for drawing some sensible looking contours, 1 mark for saying the contours flip north-south when in the Northern Hemisphere.)

2. (a) For L, T and M denoting units of length, time and mass, you have $c \sim L/T$, $\rho \sim M/L^3$, so

$$c = \sqrt{\frac{K}{\rho}} \quad \Rightarrow \quad \frac{L}{T} \sim \sqrt{\frac{K}{M/L^3}} \quad \Rightarrow \quad K \sim \frac{L^2}{T^2}\frac{M}{L^3} = ML^{-1}T^{-2},$$

so in standard units K would have units of $\mathrm{kg\ m^{-1}\ s^{-2}}$. If you looked this up and/or recognised it, this is actually equivalent to units of pressure (e.g. Pascals) because

$$\mathrm{Pa} = \mathrm{N\ m^{-2}} \quad \Rightarrow \quad \mathrm{Pa} \sim (MLT^{-2})L^{-2} = ML^{-1}T^{-2},$$

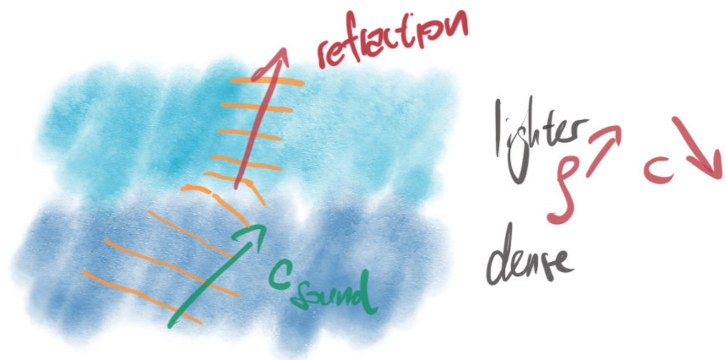
from recalling that $F = ma$ (force equals mass times acceleration).

(0.5 marks for some argument reaching to $ML^{-1}T^{-2}$ or something analogous, 0.5 marks for $\mathrm{kg\ m^{-1}\ s^{-2}}$*, since the question explicitly asks for standard units)*

(b) From the Newton–Laplace formula, $c \sim \rho^{-1/2}$, so if density increases the sound speed actually decreases, so it is actually the change in K that is leading to an increase in c.
(0.5 marks for noting $c \sim \rho^{-1/2}$ or analogous, 0.5 marks for saying it is the stiffness that leads to increases in c)

(c) For a non-dispersive wave there is no difference between the phase and group velocity, so it doesn't matter.
(0.5 mark for non-dispersive definition, 0.5 marks for saying it doesn't matter)

(d) We have refraction to the right diagram is wrong:



If c was lower in the other lighter medium for whatever reason, then the wave would refract to the left. You can work out the exact angles of refraction using *Snell's law*.
(1 mark for noting c is larger in lighter water since we are assuming K is fixed, 0.5 mark for having wave fronts turning left as it goes into lighter water, 0.5 marks for stating refraction)

(e) Recall from definition that $\gamma = c/\lambda$ (or whatever symbol you want to use for frequency). The speed c is given but units need converting:

$$c = 5220 \text{ km hr}^{-1} = 5200 \frac{1000 \text{ m}}{3600 \text{ s}^{-1}} = 1450 \text{ m s}^{-1}.$$

The standard modern concert pitch A is 440 Hz, so

$$\lambda = \frac{c}{\gamma} = \frac{1450}{440} = 3.30 \text{ m},$$

accurate to two decimal places. This is about four times larger than the quoted value of 78 cm = 0.78 m in air (e.g. `https://pages.mtu.edu/s̃uits/notefreqs.html`).
(0.5 marks for unit conversion in c, 0.5 marks for quoting frequency of concert A, 0.5 marks for the calculation of λ, 0.5 marks for degree of accuracy)

3. The v_g equation is a distraction in this bit, and it's really

$$\frac{\partial u_g}{\partial z} = \frac{g}{\rho_0 f} \frac{\partial \rho}{\partial y}.$$

In the atmosphere, you have meridional temperature gradients $\partial T / \partial y$, so this should imply a u_g by thermal wind balance, hence why this is called 'thermal wind balance', because you have a wind shear associated with temperature gradients.

The problem is of course the signs and thus the direction of flow. As you go polewards regardless of hemisphere, T decreases, so ρ increases, thus

- NH: $\partial T/\partial y < 0, \qquad \partial \rho/\partial y > 0, \qquad f > 0$
- SH: $\partial T/\partial y > 0, \qquad \partial \rho/\partial y < 0, \qquad f < 0.$

Overall you get $\partial u_g/\partial z > 0$ regardless of hemisphere. If flow is something small at the ground, it has to increase in the positive sense as you go higher up in altitude, so u_g should be large and positive at height, i.e. eastward.

The subpolar jet is stronger usually because $|\partial T/\partial y|$ is larger from mid-latitudes to poles than tropics to mid-latitudes (compensated accordingly by an increase of f but that's smaller). he resulting $\partial u_g/\partial z$ is larger, leading to a stronger flow (even if the subpolar jet is actually lower in altitude than the subtropical jet, suggesting there is less height for u_g to 'grow').

(1 mark for meridional temperature gradients and flow from thermal wind shear relation, 1 for signs of temperature/density gradients and f in BOTH hemisphere, 1 for noting both give $\partial u_g/\partial z > 0$, 1 for arguing growth of u_g with height so winds larger and eastward up high, 1 mark for temperature gradients larger in subpolar regions)

4. The four obvious ones I can think of are

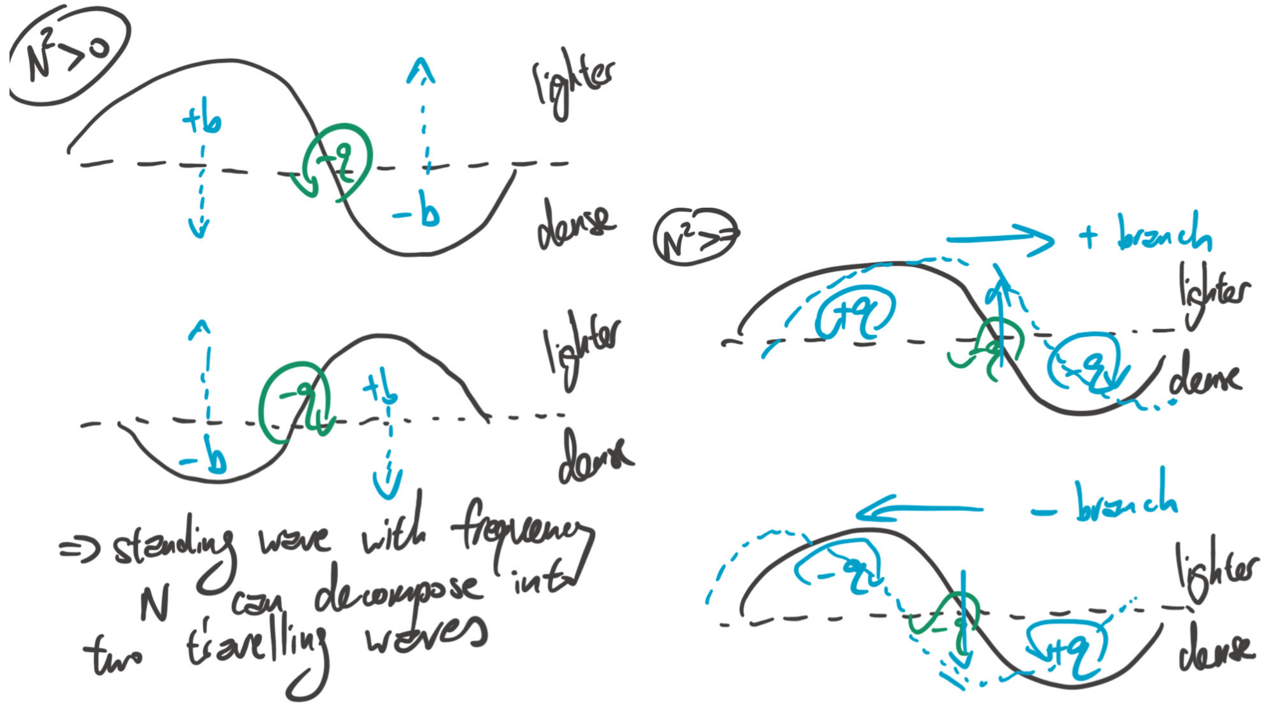
   - no ACC, revert to gyre dynamics
   - no strong circumpolar flow, reduction in isopycnal tilts in Southern Ocean by thermal wind shear
   - connectivity to basin leading to shoaling of global pycnocline
   - associated weakening of AMOC (less area to occupy so reduce volume transport)

   There are some other plausible ones

   - weakening of AMOC, ocean moves less heat
     → heat transport compensated by atmosphere? changing wind patterns?
     → changes to ice cover?
   - salinity signature might change, changes to cold/warm route into Atlantic and affect salt transport
   - ...

   (1 mark for each of the above points of plausible points that are appropriate justified/sourced, up to a maximum of 4 marks)

5. See below diagrams:



(3 marks available but be liberal, possibly 1 for noting buoyancy gradients leading to vorticity anomalies at nodes of standing wave, 1 for standing wave, 1 for decomposing standing wave into two travelling waves with appropriate vorticity anomalies)

bonus As the first diagram above, but when you swap the "dense" and "lighter" black labels, you keep the blue $\pm b$ but you have to change the blue arrow directions and flip the signs and orientation of the green vorticity label. Then you realise you can't even have a standing wave. You can keep the spatial structure but not the amplitude of the waveform, so you have an ever increasing wave pattern increasing in magnitude because of the centre vorticity anomaly, and is perhaps another way to interpret this static instability.

This is actually what you find with Rayleigh–Taylor instability if you do the mathematical linear analysis in full, where for a waveform $\exp[\mathrm{i}(kx - \omega t)] = \exp[\mathrm{i}k(x - ct)]$, the phase speed is $c = c_r + \mathrm{i}c_i$ with $c_r = 0$ (i.e. no propagation of the spatial form) but $c_i \sim N$, meaning the waveform grows like $\exp(\pm Nt)$.