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The repository principally contains the compiled products rather than the source for size reasons.

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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 6: Sea water properties (density)

Outline

- ▶ density $\rho = \rho(T, S)$
 - equation of state (EOS)
- ▶ pressure p
 - weight + hydrostatic balance (briefly, more in Lec. 7)
 - dynamical consequences
- ▶ in-situ vs. potential (vs. neutral) density
 - ρ vs. ρ_θ (vs. γ_n)
 - example and concept of work done
 - dynamical consequences

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Take home: it's **almost never** in-situ temp/dens we care about!

Key terms: EOS, hydrostatic balance, in-situ vs. potential/neutral density

Recap: parcel argument for buoyancy

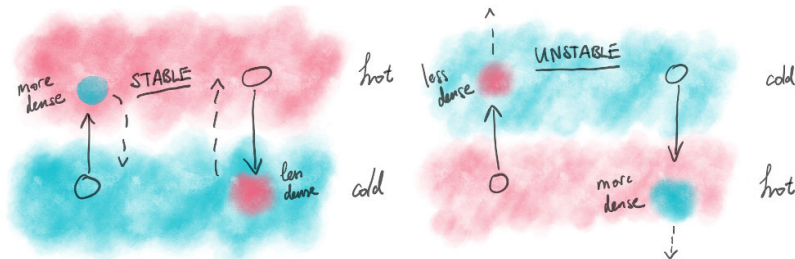


Figure: (Un)stable temperature configurations.

- **density** (!) ρ (units: kg m^{-3})
- **buoyancy** $b = -(\delta\rho/\rho_0)g$ (so units of...?)
 → how “floaty” something is (e.g. warm water, lighter density, **more** buoyant)

ultimately it's **density/buoyancy** we care about

Recap: equations of motion

Denoting $\mathbf{u} = (u, v)$ and $\mathbf{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} \right) = -\nabla p + \mathbf{F}_u + \mathbf{D}_u \quad (1)$$

$$\frac{\partial p}{\partial z} = -\rho g \quad (2)$$

$$\nabla \cdot \mathbf{u}_3 = 0 \quad (3)$$

$$\left(\frac{\partial T}{\partial t} + \mathbf{u}_3 \cdot \nabla T \right) = F_T + D_T \quad (4)$$

$$\left(\frac{\partial S}{\partial t} + \mathbf{u}_3 \cdot \nabla S \right) = F_S + D_S \quad (5)$$

$$\rho = \rho(T, S, p) \quad (6)$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Ocean density

- ▶ 10 m of seawater ≈ 1 atm (recall Lec. 1)
- ▶ over most of ocean, ρ varies from $\rho_0 = 1026 \text{ kg m}^{-3}$ by no more than **2%** (e.g. Gill, 1982)
→ small(!) but **CRUCIAL** variations! (see Lec. 7 + 8, 13 + 14)

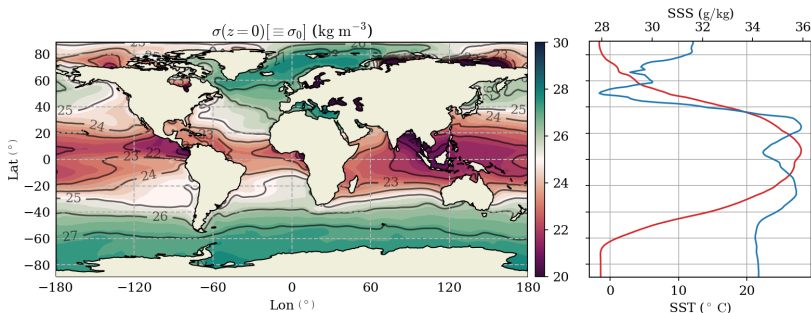
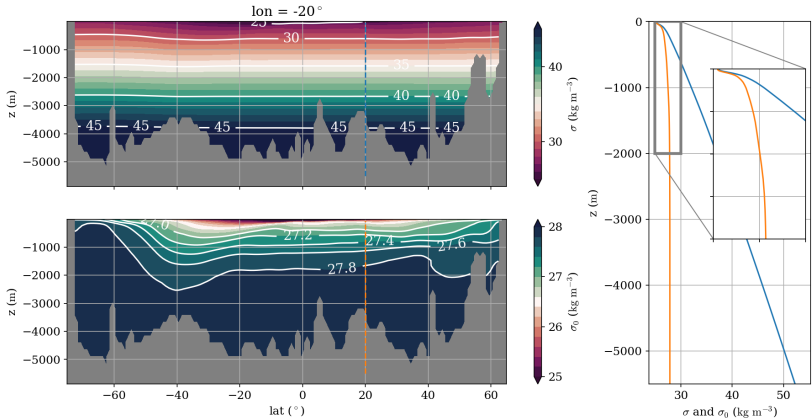


Figure: (left) Density at the surface (in-situ or referenced to sea surface) and (right) zonal averaged SST (red) and SSS (blue). Year-averaged data based on World Ocean Atlas 2013. See `plot_eos.ipynb`

Ocean density (more about this later...)

- ▶ **isopycnal** = lines/surfaces of constant density
- ▶ **pycnocline** = place/region below **mixed layer** where density gradient is largest (changes fastest)
→ cf. isotherm and thermocline (last Lec.)



Equation of State

T and S contribute to density, i.e. $\rho = \rho(T, S)$, and

$$\rho \nearrow \text{ as } T \searrow, \quad \rho \nearrow \text{ as } S \nearrow$$

Equation of State (EOS) is the actual $\rho = \rho(T, S)$

► $\rho \nearrow$ as $S \nearrow$ so possible example is

$$\rho \sim \beta S, \quad \beta \geq 0$$

► $\rho \nearrow$ as $T \searrow$ so could be

$$\rho \sim -\alpha T + \beta S, \quad \alpha, \beta \geq 0$$

→ can be negative as is...?

Equation of State

A linear EOS of seawater ($\alpha, \beta \geq 0$):

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

- expected behaviour with changing T and S
→ relative to **reference** T_0 and S_0

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- ▶ the “1” to show base density of ρ_0 (units: kg m^{-3})

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LHS and RHS units need to agree!

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LHS and RHS units need to agree!

- ▶ α has units $^{\circ}\text{C}^{-1}$ (or K^{-1}), **thermal coefficient of expansion**
- ▶ β has units $\text{g}^{-1} \text{ kg}$, **haline coefficient of contraction**

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

e.g. by hand:

$$\blacktriangleright \rho_0 = 1, T_0 = S_0 = 0, \alpha = \beta = 1,$$

$$\rho = 1 - T + S$$

so if $T = 10$, $S = 2$ then $\rho = \dots?$

\blacktriangleright “sensible” references are (partly taken from Rouquet et al. (2015), *J. Phys. Oceanogr.*)

$$T_0 = 10\text{ }^{\circ}\text{C}, \quad S_0 = 35\text{ g kg}^{-1}, \quad \rho_0 = 1026\text{ kg m}^{-3}$$

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

e.g. in Excel

| | A | B | C | D | E | F | G | H |
|----|-------|-------|------------|-------------|---|-------|------------|---|
| 1 | T | S | dens | Dens - 1000 | | rho0 | 1,026.0 | |
| 2 | 15.00 | 30.00 | 1,021.3448 | 21.3448 | | T0 | 10.0 | |
| 3 | 15.00 | 30.50 | 1,021.7276 | 21.7276 | | S0 | 35.0 | |
| 4 | 15.00 | 31.00 | 1,022.1103 | 22.1103 | | alpha | 0.00016131 | |
| 5 | 15.00 | 31.50 | 1,022.4931 | 22.4931 | | beta | 0.00074614 | |
| 6 | 15.00 | 32.00 | 1,022.8759 | 22.8759 | | | | |
| 7 | 15.00 | 32.50 | 1,023.2587 | 23.2587 | | | | |
| 8 | 15.00 | 33.00 | 1,023.6414 | 23.6414 | | | | |
| 9 | 15.00 | 33.50 | 1,024.0242 | 24.0242 | | | | |
| 10 | 15.00 | 34.00 | 1,024.4070 | 24.4070 | | | | |
| 11 | 15.00 | 34.50 | 1,024.7897 | 24.7897 | | | | |
| 12 | 15.00 | 35.00 | 1,025.1725 | 25.1725 | | | | |
| 13 | 15.00 | 35.50 | 1,025.5553 | 25.5553 | | | | |
| 14 | | | | | | | | |

Figure: Sample calculations of density using linear EOS in Excel (highly recommend you don't use Excel, because syntax is a bit messy...)

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

What does it look like? (define $\sigma = \rho - 1000$ here)

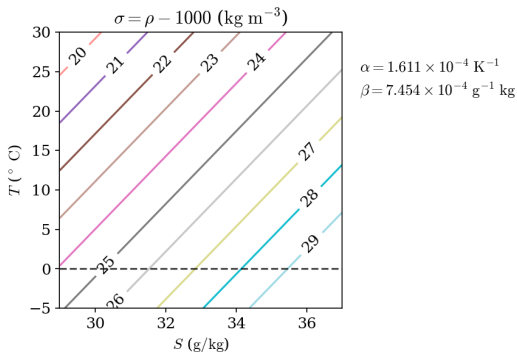


Figure: Linear EOS in TS space. See `plot.eos.ipynb`

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

Compare with “real” EOS? (TEOS-10 here using Fabien’s 75-term formula)

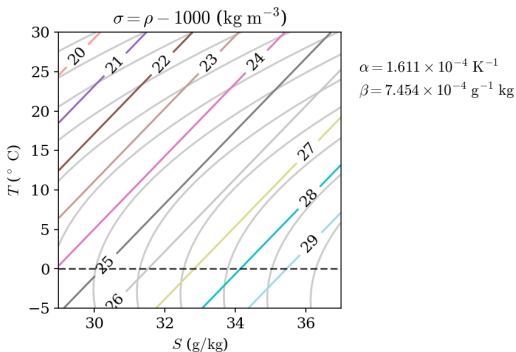


Figure: Linear EOS in TS space with TEOS10 as gray contours (same contour levels). See `plot_eos.ipynb`

“Mildly” Nonlinear EOS see Geoff Vallis’ (2006) book

$$\rho = \rho_0 \left[1 - \alpha \left(T_a + \frac{\lambda_1}{2} T_a^2 \right) + \beta \left(S_a - \frac{\lambda_2}{2} S_a^2 \right) - \nu T_a S_a \right]$$

► $T_a = T - T_0$ and $S_a = S - S_0$, the **anomalies**

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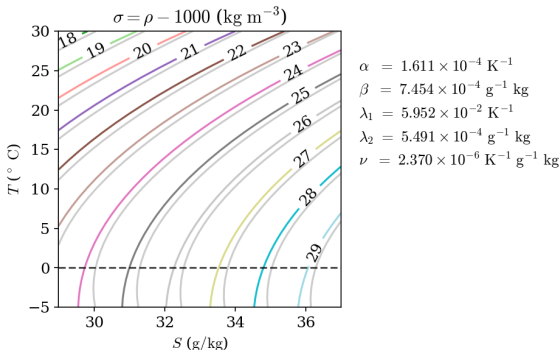


Figure: Toy nonlinear EOS (no **thermobaric effect**) in TS space with TEOS10 as gray contours (same contour levels).
See `plot_eos.ipynb`

Nonlinear EOS

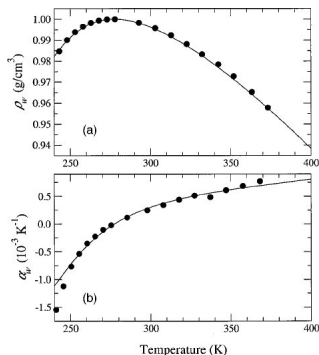


Figure: (top) $\rho = \rho(T)$ for pure water, (bot) $\alpha = \alpha(T)$. From Ashbaugh *et al.* (2002), *J. Chem. Phys.*.

- “real” EOS should be nonlinear
 - e.g. water densest around 4° C (so ice floats), i.e. cannot be linear in T
 - “real” EOS ongoing research (e.g. **TEOS-10**, works by Trevor McDougall)

Nonlinear EOS

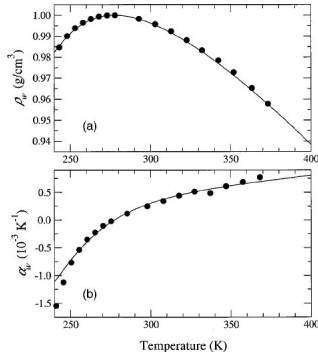


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- ▶ small discrepancies = can ignore?

Nonlinear EOS

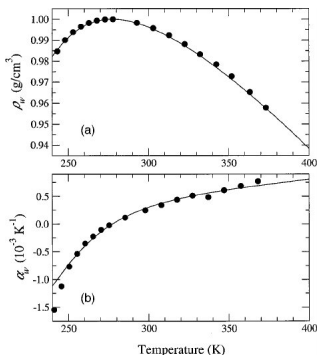


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→ “real” EOS ongoing research
(e.g. **TEOS-10**, works by Trevor McDougall)
- ▶ small discrepancies = can ignore?
→ **NO!** “small” difference really matter!
→ over most of ocean, ρ varies from $\rho_0 = 1026 \text{ kg m}^{-3}$ by no more than 2% (e.g. Gill, 1982)

“Full” Nonlinear EOS (e.g. Roquet et al. (2015), *J. Phys. Oceanogr.*)

- ▶ **TEOS-10** standard, polynomial with 75(!) terms chosen(!) to fit with real data
- ▶ note the change of sign in **gradient** close to freezing point
→ remember ice is **less dense** than water normally!

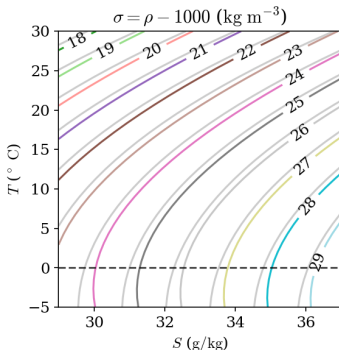


Figure: TEOS10 EOS (at surface) in TS space with toy nonlinear EOS as gray contours (same contour levels). See `plot.eos.ipynb`

Pressure + hydrostatic balance (briefly)

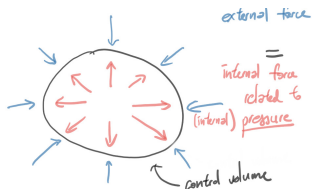


Figure: Fluid volume in force balance.

Consider a body (e.g. balloon) of fixed volume

- ▶ fixed volume \Rightarrow steady
- ▶ steady \Rightarrow in **force balance**, no net force

- ▶ **pressure** = force per area,

$$p = F/A, \quad \text{units: } \text{N m}^{-2} \equiv \text{Pa}$$

1 bar = 10^5 Pa (Pascals) (see e.g. Wikipedia for others)

\rightarrow cf. millibars (mbar) in atmosphere

\rightarrow lines of constant pressure = **isobar**

Pressure + hydrostatic balance (briefly)

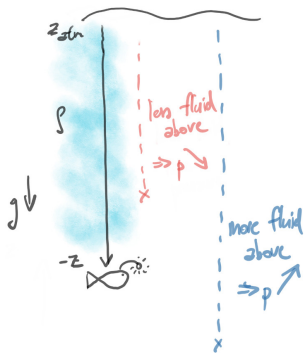


Figure: Schematic of hydrostatic pressure

- **hydrostatic approximation:**
pressure **equal** to weight above
when static
→ **weight** is $F = mg$ so for force
balance,

$$F = mg = g \int_{-z}^{z_{\text{atm}}} \rho \, dz = p ,$$

with $g \approx 9.81 \, \text{m s}^{-2}$

→ if $\rho = \text{const}$ then $p = \rho g z + p_{\text{atm}}$

Concept of work done

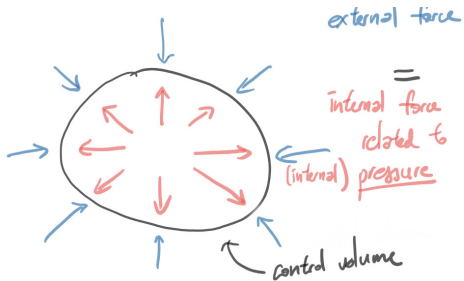


Figure: Working to compress a volume.

Concept of work done

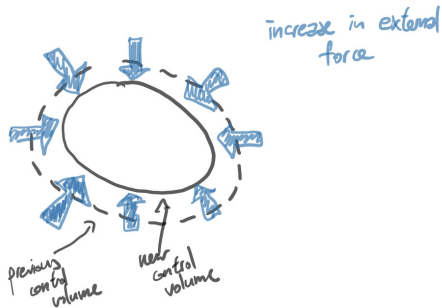


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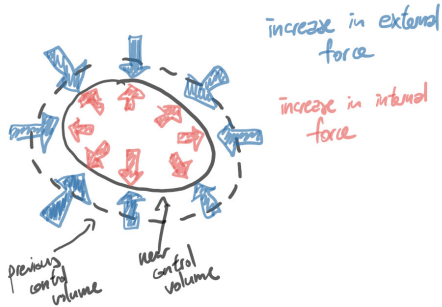


Figure: Working to compress a volume.

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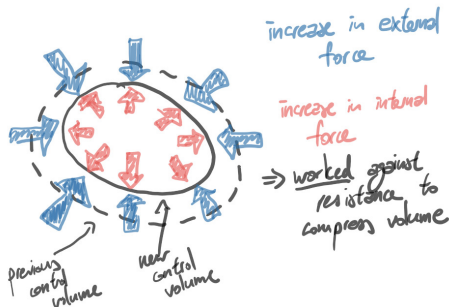


Figure: Working to compress a volume.

- ▶ to compress, need to **work** against something (in this case internal pressure)
- ▶ **energy** has to be put in (cf. $T \nearrow$ in fluid volume)

Work done and potential/neutral densities

Remember this graph from a few slides ago?

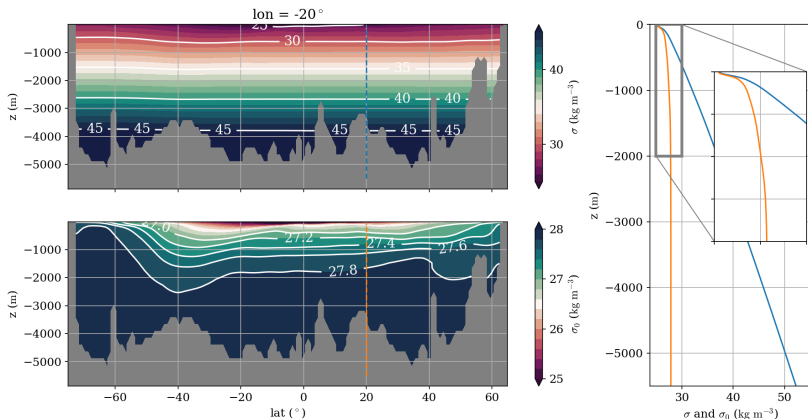


Figure: Meridional section in the Atlantic of (top left) in-situ density and (bot left) potential density **referenced to sea level**, with the corresponding vertical profiles plotted (right). See `plot_eos.ipynb`

Work done and potential/neutral densities

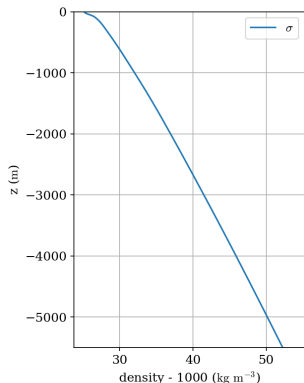


Figure: Vertical profile of in-situ density at the same location as in the previous graph. See `plot_eos.ipynb`

In-situ density $\rho(T, S, p)$

- increases with depth because more weight above (compression), but not necessarily dynamical!
→ abyssal increase of ρ mostly from p , but not necessarily from having to do **work**
- in-situ density profile is telling us there shouldn't be up/down motion, but we know we do have it!

Want something that adjusts for some/all of pressure contribution to density

Potential temperature and density

Potential temperature θ

- ▶ the temperature it will have if you take some water and move it to some **reference pressure** p_0 (usually p_{atm}) without exchange of heat and salt
 - without exchange of heat + mass = **adiabatic**
 - reference needed to set base line and **is a choice**
 - account for pressure contributions **relative to reference level**

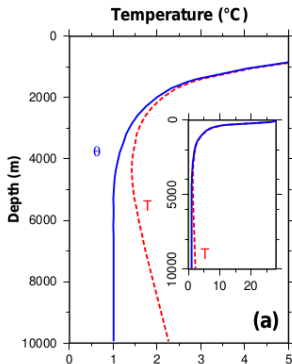


Figure: Vertical profile of in-situ temperature (red) and potential temperature (blue) in the Mariana Trench to highlight the differences. From Talley *et al.* (2011) Fig 4.10(a).

According to in-situ temperature, bottom water should be more buoyant (i.e. unstable + overturn)!

Concept of work done

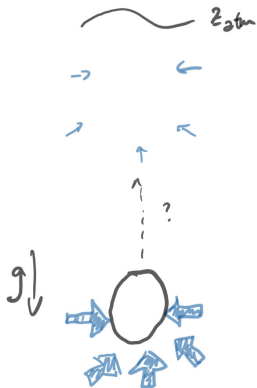


Figure: Volume change from pressure, but work done reversible if adiabatic.

Concept of work done

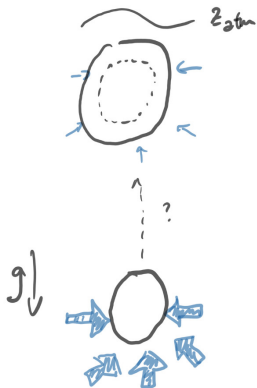
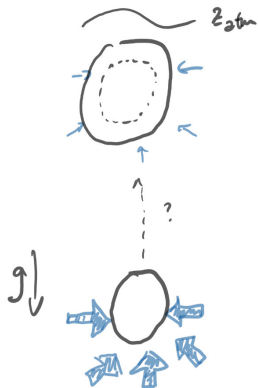


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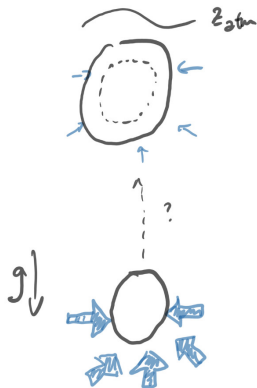
Concept of work done



- work is done by the system against the surroundings

Figure: Volume change from pressure, but work done reversible if adiabatic.

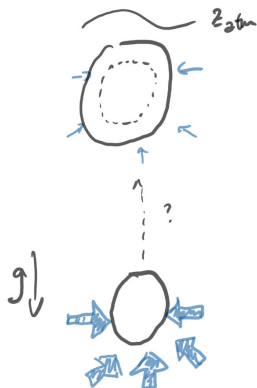
Concept of work done



- ▶ work is done by the system against the surroundings
- ▶ energy lost from system with expansion, change in
 - density (from volume since $\rho = m/V$)
 - temperature (less energy in system, cooling)

Figure: Volume change from pressure, but work done reversible if adiabatic.

Concept of work done



- ▶ work is done by the system against the surroundings
- ▶ energy lost from system with expansion, change in
 - density (from volume since $\rho = m/V$)
 - temperature (less energy in system, cooling)

but if *adiabatic*, then this process is **reversible** (no change in entropy), but we normally care about **irreversible** processes!

Figure: Volume change from pressure, but work done reversible if adiabatic.

Potential temperature and density

Potential density ρ_θ

- ▶ the density calculated using θ in EOS
 - reference dependent (from θ)
 - shallow region focus? choose sea level (0m) or 1000m
 - deeper region? choose 2/3/4000m
 - used to identify **water masses**
- ▶ crucial bit is that $\sigma_{0,1,2,3,4}$ has very **small gradients** (i.e. essentially **flat**) in the deep!

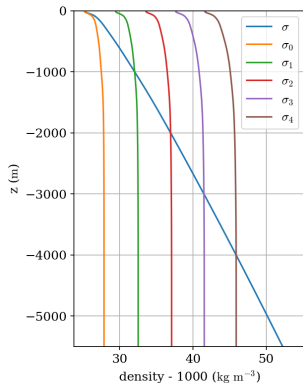


Figure: Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

Potential temperature and density

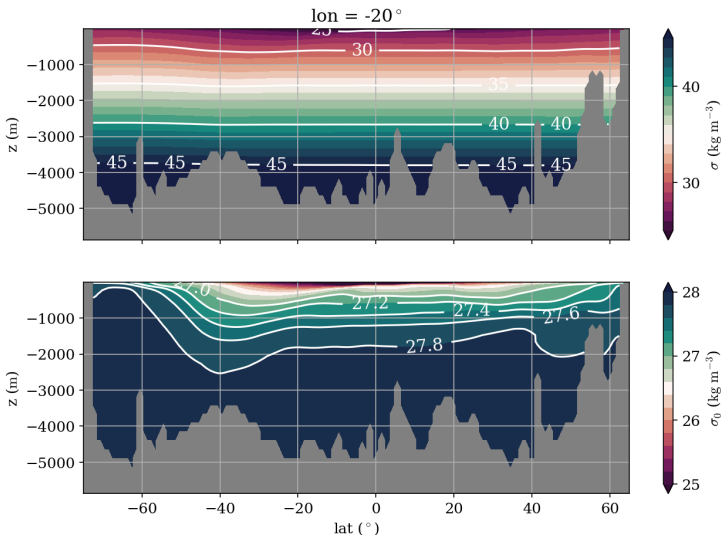


Figure: Meridional section in the Atlantic of (top) in-situ density and (bot) σ_0 . See `plot_eos.ipynb`

Potential temperature and density

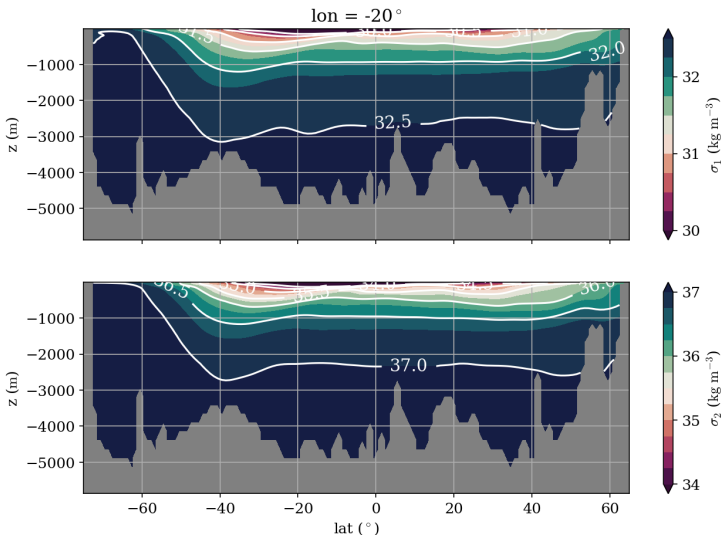


Figure: Meridional section in the Atlantic of (top) σ_1 and (bot) σ_2 . See `plot_eos.ipynb`

Potential temperature and density

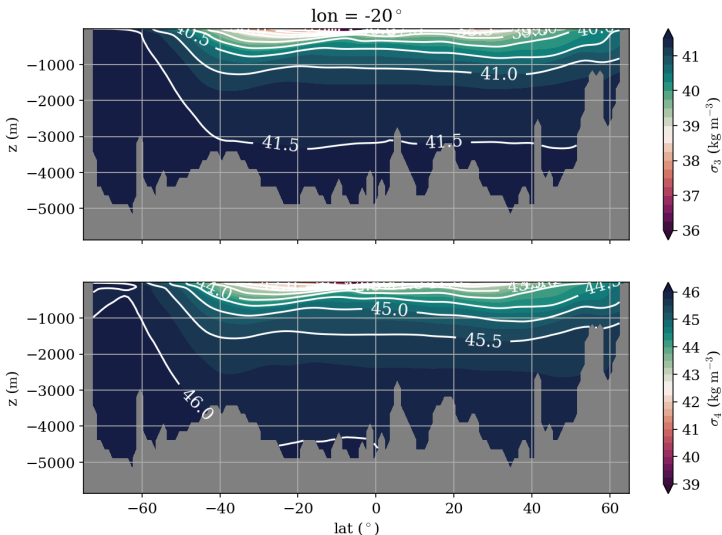


Figure: Meridional section in the Atlantic of (top) σ_3 and (bot) σ_4 . See `plot_eos.ipynb`

Neutral density

Potential density needs a reference

- ▶ **neutral density** γ_n in principle does not need a reference

→ complications with computing and/or existence of **neutral surfaces** (e.g.

Jackett & McDougall, 1997, *J. Phys. Oceanogr.*)

- ▶ can be used to identify **water masses** but restricted to present day ocean (cf. Lec.

5, see lec. 13 + 14)

→ e.g. not necessarily suitable for **paleoclimate** + very long-term climate change applications

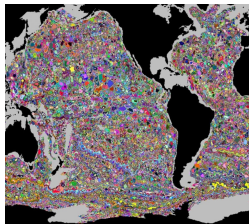


Figure: Topobaric surfaces, an almost neutral surface. From Stanley (2019), *Ocean. Modell.*, Figure 4.

Neutral density sample: Atlantic

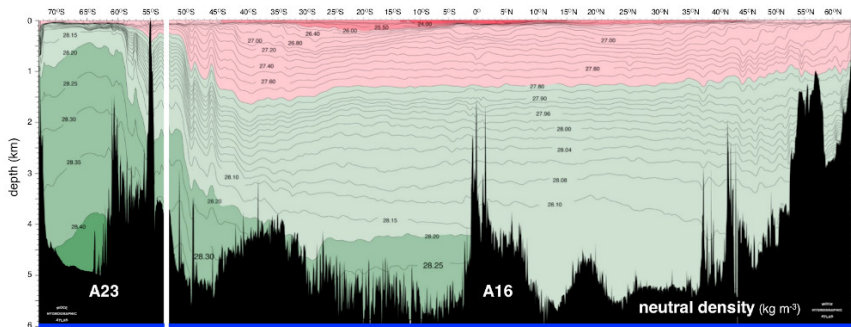


Figure: Neutral density (see later) meridional section in the Atlantic. From Koltermann *et al.* (2011), vol 3 of Hydrographic Atlas of the World Ocean Circulation Experiment (WOCE).

- flow largely along isopycnals (path of less resistance, not needing to do as much work) (revisit in Lec. 13, 14, 17)
 - consequence for **global meridional overturning circulation** (MOC)

Summary

- ▶ EOS to relate temperature and salinity to density
- ▶ hydrostatic approximation (more next Lec.)
 - weight to balance fluid above (related to density)
- ▶ in-situ vs. potential (vs. neutral) density
 - dynamics cares about work done
 - in-situ density σ has substantial pressure contributions
 - potential density σ_θ removes some pressure contribution (reference dependent)
 - neutral density γ_n removes more/all pressure contributions in principle

Take home: it's **almost never** in-situ temp/dens we care about!