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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 10: Mechanical forcing 4 (diffusion and friction)

Outline

- ▶ energy + momentum
 - recap of concept
 - sources and sinks
- ▶ concept of diffusion and friction
 - inherently small-scale
 - largely driven by dynamics!
 - Reynolds/Ekman number (cf. Rossby number)
 - Ekman boundary layers revisited

Key terms: friction, diffusion, viscosity/diffusivity, boundary layers, Reynolds/Ekman number

Recap: equations of motion

Denoting $\mathbf{u} = (u, v)$ and $\mathbf{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} \right) = -\nabla p + F_u + \mathbf{D}_u \quad (1)$$

$$\frac{\partial p}{\partial z} = -\rho g \quad (2)$$

$$\nabla \cdot \mathbf{u}_3 = 0 \quad (3)$$

$$\left(\frac{\partial T}{\partial t} + \mathbf{u}_3 \cdot \nabla T \right) = F_T + \mathbf{D}_T \quad (4)$$

$$\left(\frac{\partial S}{\partial t} + \mathbf{u}_3 \cdot \nabla S \right) = F_S + \mathbf{D}_S \quad (5)$$

$$\rho = \rho(T, S, p) \quad (6)$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Recap: wind forcing



Figure: Schematic of Ekman layer (boundary denoted by orange).

- ▶ wind a **source** of **momentum** for the ocean
- ▶ but influence has vertical limit
- ▶ direct influence only over the **Ekman (boundary) layer**
- ▶ difference in wind/current speed \Rightarrow **transfer** of momentum ocean (usually **into** ocean and hence **source**; why?)
 - \rightarrow molecular **diffusive** rate \Rightarrow very slow! (see next Lec.)
 - \rightarrow **instabilities** \Rightarrow much faster (because on dynamical time-scales; see Lec. 17)

Wind forcing the chief source of **momentum** into ocean, but where is the **sink**? (vertical transfer in Lec 13)

Recap: wind forcing

- ▶ Ekman transport (mass flux) perpendicular to wind vector
- ▶ Ekman suction/pumping (i.e. up/downwelling) related to wind stress curl

$$w_e \sim \frac{1}{f} \mathbf{e}_z \cdot (\nabla \times \boldsymbol{\tau})$$

→ related to fluid
divergence $\nabla \cdot \mathbf{u}$

→ analogous relations with vertical
component of fluid vorticity

$$\omega = \mathbf{e}_z \cdot (\nabla \times \mathbf{u})$$

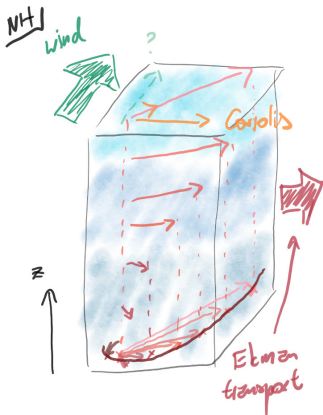


Figure: Schematic of Ekman spiral.

What sets Ekman layer depth?

Recap (?): momentum and energy

Recall that (Lec. 4)

- ▶ (linear) **momentum** $p = mv$ (units = ???)
→ here it will be $p = \rho u$
- ▶ Newton's law refers to **momentum** not velocity

Recap (?): momentum and energy

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- ▶ **kinetic energy** is $K = (\rho/2)|\mathbf{u}|^2 = (\rho/2)\mathbf{u} \cdot \mathbf{u}$ (units = ???)
→ note this is a scalar (why?)
→ might sometimes see it without a factor of ρ (units = ???)
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dynamics \leftrightarrow mass/momentum/energy transfer + conservation

Diffusion

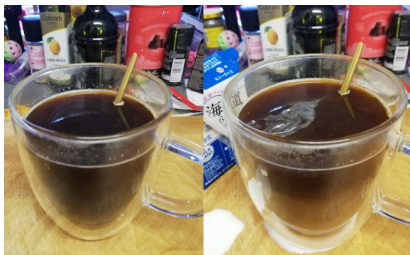


Figure: Making a mess of coffee + milk...

Beyond **advection** (more later), momentum, energy and other **tracers** (e.g. temperature, salinity, oxygen) can transfer by **diffusion**

- ▶ if no **stirring** then, milk will gradually **spread**
→ but very **slowly**!

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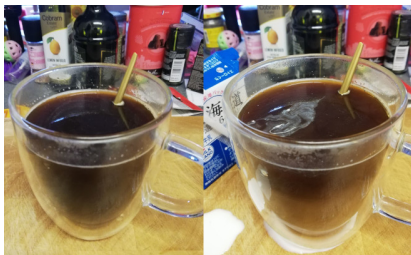


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- ▶ if no **stirring** then, milk will gradually **spread**
→ but very **slowly**!
- ▶ tendency to spread = **diffusion**
→ diffusion aims to erase **gradients** of “stuff”
→ a **redistribution**, i.e. total amount of “stuff” **conserved**

Diffusion

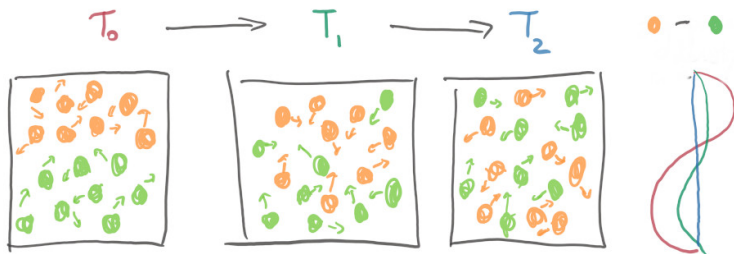


Figure: Schematic of **microscopic** motion leading to **macroscopic** diffusion. Here it is actual particle types, but can also imagine particles carrying “stuff”, bumping into each other and transferring “stuff”, and eventually the distribution of “stuff” evens out (but the total is conserved).

- ▶ at small-scale (!), “random” motion of particles (**Brownian motion**)
- ▶ collision \Rightarrow transfers \Rightarrow spreading
 \rightarrow physics (!) \Rightarrow **gradient erasing** transfers more likely than **gradient increasing ones** (e.g. **entropy**)
- ▶ **molecular diffusion**, inherently **small-scale**

Diffusion

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but called **viscosity**, denoted ν or $\mu = \rho\nu$ (I’m going to use ν , the **kinematic** viscosity)
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- ▶ **molecular** diffusivities/viscosities (units of $\text{m}^2 \text{s}^{-1}$) are substance dependent and **known**

Diffusion

material	κ_T	κ_S	ν
seawater	10^{-7}	10^{-9}	10^{-6}
air	10^{-5}	—	10^{-5}
honey	10^{-6}	—	10^{-2}
lava	10^{-7} (!)	—	depends, 10^0 ?
steel	10^1	—	big (!?)

Table: Table of molecular diffusivity/viscosity values at some control conditions. All numerical entries have units of $\text{m}^2 \text{s}^{-1}$.

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- κ has units of $\text{m}^2 \text{s}^{-1}$, define a **diffusion time** as

$$t_m = L^2 / \kappa$$

→ time t_m to travel distance L is transport is only by molecular diffusion with diffusivity κ

→ for sea water, if $L = 100 \text{ m}$ (surface to thermocline say), temperature diffusion time is about **3000 yrs!** (homework)

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- **stirring** leads to larger-than-molecular transport, spreading and erasing gradients (!!! eventually anyway...)
→ quantify as an **effective diffusivity** $\kappa_e \gg \kappa_{\text{molecular}}$

But we don't know κ_e , because it is dynamics dependent!

Diffusion

- ▶ resulting **dynamic time-scales** $t_d = \kappa_e \ll t_m$
→ e.g., for $\kappa_{e,z} = O(10^{-2} \text{ m}^2 \text{ s}^{-1})$, then for example above
 t_d is about **10 days** ($\ll t_m = 3000 \text{ yrs!}$)

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Understanding dynamics fundamental in getting ocean transport/circulation right! (see Lec. 15 – 20)

Friction

Friction = resistance to **relative** motion

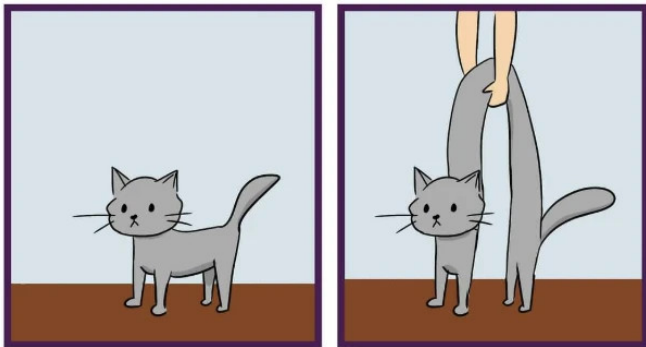


Figure: Cat physics. Picture from Meowingtons.

Friction

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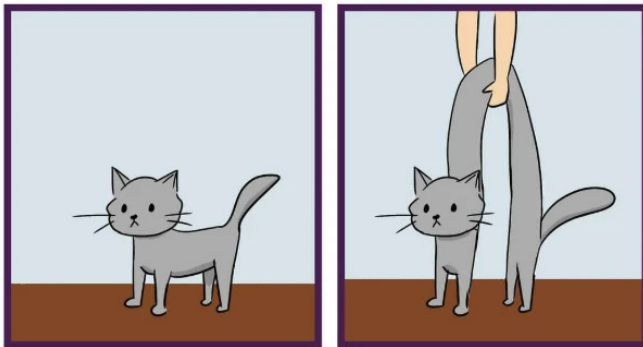


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- resistance to **relative** motion \Rightarrow needs **motion**
 - \rightarrow function of velocity \mathbf{u} (κ can sometimes be function of \mathbf{u} too...)
 - \rightarrow e.g. **linear drag** $-\mathbf{r}\mathbf{u}$ (moving a pig along the floor)
 - \rightarrow e.g. **quadratic drag** $-C_d|\mathbf{u}|\mathbf{u}$ (air resistance after a certain point)

Friction and diffusion

- ▶ **internal stresses** of object resisting relative motion
→ contributes to fluid viscosity (cats are viscous then?)
- ▶ viscosity **redistributes** momentum (but total momentum conserved), and friction **removes** velocity differences

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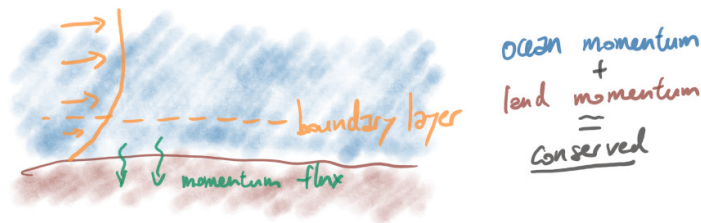


Figure: Momentum diffusion and friction. Friction arises because there is relative motion, so it might look like there is momentum loss if we are looking only at the ocean as it's own system. But really what is happening is that the ocean is transferring momentum into the land, and from the ocean + earth system point of view there is no momentum lost.

Diffusion and friction

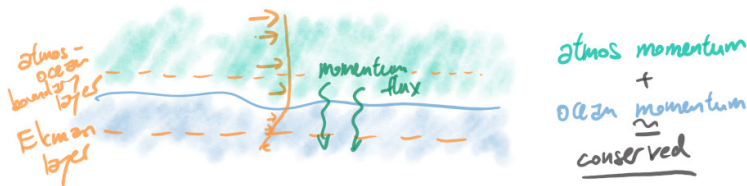


Figure: Same as above but for atmosphere and land. Atmosphere acts as a **source** of momentum for ocean most of the time (equivalently, the ocean acts as a **sink** of momentum for the atmosphere as well as a **drag** for the atmospheric winds).

- similar picture but between fluid of different density too
→ more details about interfacial + topographic **form stress** later (see Lec 13)

Diffusion and friction

Some key points:

- ▶ **diffusion** reduces gradients + **variance** of “stuff”, but conserves “stuff”
→ e.g. viscosity reduces K and ∇p (bit sloppy here with notation), but conserves total p
- ▶ effective diffusion = molecular + dynamics, dynamics usually the most important contribution across the length-scales (see **Reynolds number** later)
- ▶ **friction** removes velocity differences (and therefore K)
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Fundamentally dependent on **dynamics!**

Boundary layers



Figure: Schematic of Ekman layer (boundary denoted by orange).

- momentum input by atmosphere via **frictional** stress
→ details actually complicated and current research topic
(e.g. search for **bulk formulas**, **mixed layer dynamics** etc.)

Q. what sets the **boundary layer** extent?

Q. where/when is **diffusion** (and thus **friction**) important?

Non-dimensional numbers

Recall $Ro = (U/L)/f$ is a measure of relative importance of advection and rotation

- ▶ $Ro \ll 1$ means rotation important
→ could use it as “how small does L need to be for rotation to not matter” (cf. diffusion t_m above)

Non-dimensional numbers

Recall $Ro = (U/L)/f$ is a measure of relative importance of advection and rotation

- ▶ $Ro \ll 1$ means rotation important
→ could use it as “how small does L need to be for rotation to not matter” (cf. diffusion t_m above)
- ▶ for general diffusion the **Péclet number** is defined as

$$Pe = \frac{UL}{\kappa} = \frac{\text{advective transport}}{\text{diffusive transport}}$$

Reynolds + Ekman number

- ▶ when dealing with **viscosity** it is the **Reynolds number**

$$\text{Re} = \frac{UL}{\nu}$$

→ if $\text{Re} \ll 1$ (e.g. L and/or U small, ν large) then **viscous** effects are important

- ▶ for rotating fluid systems the **Ekman number**

$$\text{Ek} = \frac{\nu}{fL^2} = \frac{(U/L)/f}{UL/\nu} = \frac{\text{Ro}}{\text{Re}}$$

is usually the important one

→ might see other conventions and analogues (e.g. plasma physics?)

Boundary layers

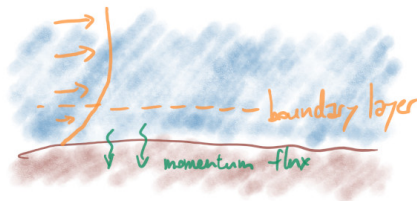


Figure: Schematic of a Prandtl boundary layer.

- boundary layer is where diffusive effects important, define it as e.g. $Re \approx 1$, i.e.

$$Re = \frac{UL}{\nu} \quad \Rightarrow \quad L_{bl} \sim \frac{\nu}{U}$$

→ complication since U depends on L

→ one standard solution has $L_{bl} \sim 1/\sqrt{Re}$, i.e. boundary layer width **decreases** with increasing Reynolds number

Boundary layers

- ▶ similarly, the Ekman layer depth could be when $Ek \approx 1$, i.e.

$$Ek = \frac{\nu}{fL^2} \quad \Rightarrow \quad L_{ek} \sim \sqrt{\frac{\nu}{f}}$$

→ since (!) $f \sim 10^{-4} \text{ s}^{-1}$, if we take $\nu = \nu_{e,z} = 10^{-2} \text{ m}^2 \text{ s}^{-1}$, we have

$$L_{ek} \sim \sqrt{\frac{10^{-2}}{10^{-4}}} = \sqrt{10^2} = 10 \text{ m}$$

- ▶ if we use the molecular ν the Ekman layer depth as well as Prandtl boundary layer width will be tiny (homework)
 - $Re \gg 1$ for geophysical flows
 - $Ek \ll 1$ because ν and f small

Summary

- ▶ **diffusion** is **spreading** (erasing **gradients**)
 - **redistributing** things (**variance** decreasing but conserved total)
 - depends on magnitude of gradients and **diffusivity** κ
 - molecular vs. effective (latter **dynamics** dependent)
- ▶ **friction** resists **relative** motion
 - motion dependent
 - **sink/source** of momentum/energy of ocean (to land/atmosphere) (see Lec. 13)
- ▶ inter-system transfers through **boundary layers**
 - where Re and/or $Ek = O(1)$
 - these are usually **thin** (important consequences!)

another reason why **dynamics is important!**