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# OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

## Lecture 7: Mechanical forcing 1 (pressure and gravity)

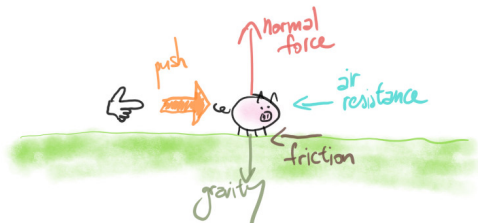
# Outline

- ▶ recall forcing on ocean
  - thermodynamic ( $T$  and  $S \Rightarrow \rho$  and buoyancy)
  - mechanical (wind, gravity, pressure, rotation etc.)
- ▶ gravity + pressure (alluded to last Lec.)
  - geoid (see also Lec. 18)
  - sea surface height (SSH)
  - weight
  - hydrostatic pressure
  - some consequences for flow

**Key terms:** geoid, SSH, hydrostatic pressure

## Recap: forces

- Newton's second law: objects are in **steady state** (at **rest** or **steady speed**) unless there is a **net force**



**Figure:** Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vaccum because we have air resistance + abuse of animal rights).

- ▶ thermodynamic forcing: affects  $T$  and  $S$
- ▶ mechanical forcing: affects momentum

thermodynamic variables affects momentum via pressure

## Recap: buoyancy forces

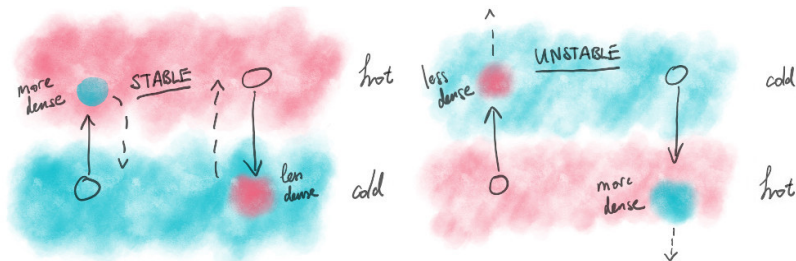
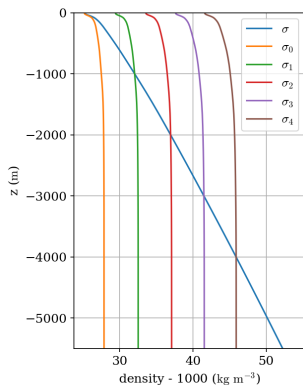


Figure: (Un)stable temperature configurations.

- **buoyancy** of fluid ultimately depends on **density**
  - lighter density water, more 'floaty'
  - heavier things (less buoyant water) = more **weight**, imbalance and sinks

# Recap: in-situ vs. potential/neutral density

- ▶  $\rho = \rho(T, S, p)$  via the EOS, but want to neglect  $p$  contribution to  $\rho$  because **non-dynamic** (from a **work done** point of view)
- ▶ **in-situ density**  $\rho$  says you basically have no up-down motion in the deep  
→ but we know we have a bit!  
→ contributions from  $p$  included here
- ▶ **potential densities** referenced to different levels says you can  
→ **some**  $p$  contribution removed
- ▶ **you want the  $p$  resulting from  $\rho$  but without  $p$  in the  $\rho$**  (otherwise, a circular argument?)



**Figure:** Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

## Recap: equations of motion

Denoting  $\mathbf{u} = (u, v)$  and  $\mathbf{u}_3 = (u, v, w)$ , to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} \right) = -\nabla p + F_u + D_u \quad (1)$$

$$\frac{\partial p}{\partial z} = -\rho g \quad (2)$$

$$\nabla \cdot \mathbf{u}_3 = 0 \quad (3)$$

$$\left( \frac{\partial T}{\partial t} + \mathbf{u}_3 \cdot \nabla T \right) = F_T + D_T \quad (4)$$

$$\left( \frac{\partial S}{\partial t} + \mathbf{u}_3 \cdot \nabla S \right) = F_S + D_S \quad (5)$$

$$\rho = \rho(T, S, p) \quad (6)$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

# Gravity

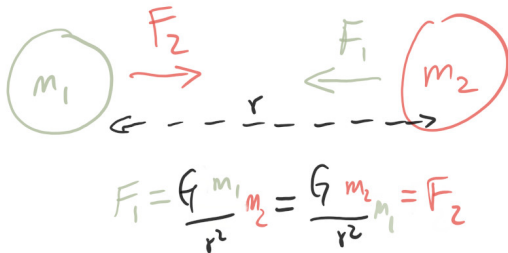
- **attraction** between bodies of different **masses**

→ note it is a purely attractive force (cf. magnetism)

$$F = G \frac{m_1 m_2}{r^2}, \quad G = 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

→  $G$  the **gravitational constant**

(exercise: check the LHS and RHS units agree)



**Figure:** Schematic of gravitational attraction for two masses. If  $m_1 \gg m_2$  (e.g. Earth and a pig) then forces on each body are equal, but its effect on one the pig is much larger than it is for the Earth (recall  $F = ma$ ).



# Gravity and weight

- Let's take Earth as an example:

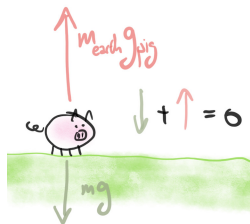
$$F = G \frac{m_{\text{earth}}}{r_{\text{earth}}^2} m,$$

taking (units!)

$$G = 6 \times 10^{-11}$$

$$m_{\text{earth}} = 6 \times 10^{24}$$

$$r_{\text{earth}} = 6400 \text{ km} \approx 6 \times 10^6 \text{ m}$$

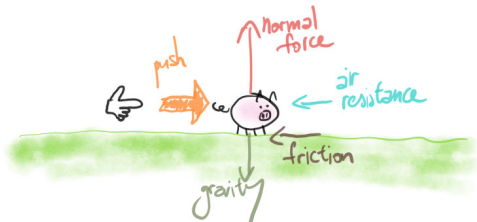


**Figure:** Gravity as applied on Earth. Note that  $g_{\text{pig}}$  is tiny (exercise: make an estimate of  $g_{\text{pig}}$ ).

$$F = 6 \times 10^{-11} \frac{6 \times 10^{24}}{(6 \times 10^6)^2} m = \frac{6^2}{6^2} \times 10^{-11+24-12} m = 10m \equiv mg$$

- the **gravitational acceleration on Earth** (recall  $F = ma$ ) is  
 $g \approx 10 \text{ m s}^{-2}$  (exercise: don't drop the decimal places like I did above and repeat the calculation)

# Mass vs. weight

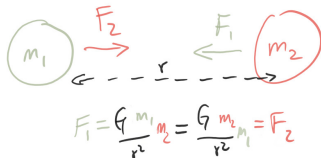


**Figure:** Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vacuum because we have air resistance + abuse of animal rights).

- ▶ the pig above with **mass**  $m$  has **weight**  $mg$ 
  - **mass** is how much 'stuff' a body has
  - **weight** is a **force** and dependent on value of  $g$
  - e.g. pig has same mass on moon but **weighs less** there because  $g_{\text{moon}} \approx (1/6)g$  (exercise: why is  $g_{\text{moon}}$  smaller?)

# Geoid

- picture for spherical bodies with uniform mass (then  $r$  is distance between centre of gravity), but Earth is not quite spherical...



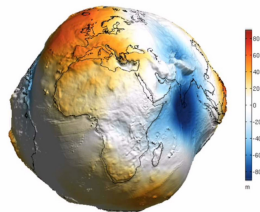
**Figure:** Schematic of gravitational attraction for two masses.



**Figure:** Cartoon of spherical vs. ellipsoid earth (it's inflated slightly at the Equator from Earth's spinning). Modified picture from NASA.

# Geoid

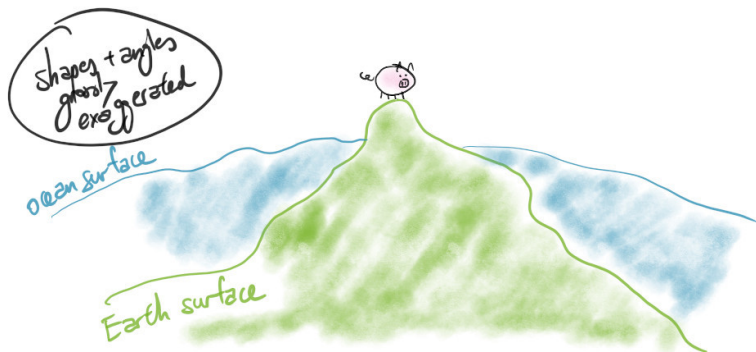
- ...nor is the mass uniformly distributed!  
→ where there is more mass there is more gravitational attraction
- **geoid** is the surface that the ocean surface would trace out if we only had **gravity** and **rotation**



**Figure:** The “lumpy potato” Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.

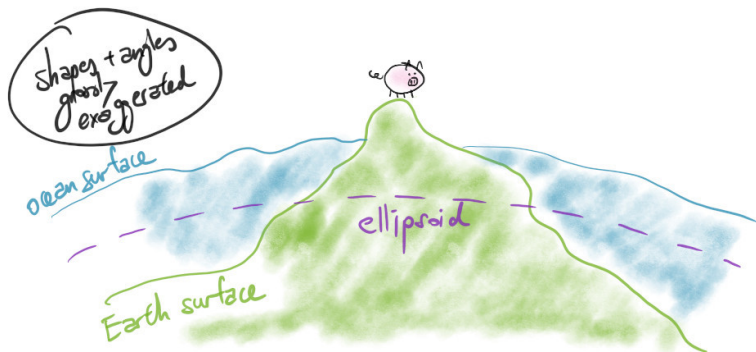
- or, geoid is the surface where gravity is everywhere **perpendicular** to it (I like this one more...)
- wind and tidal action move sea surface around
- important concept in dynamics + **sea level science**

# Geoid



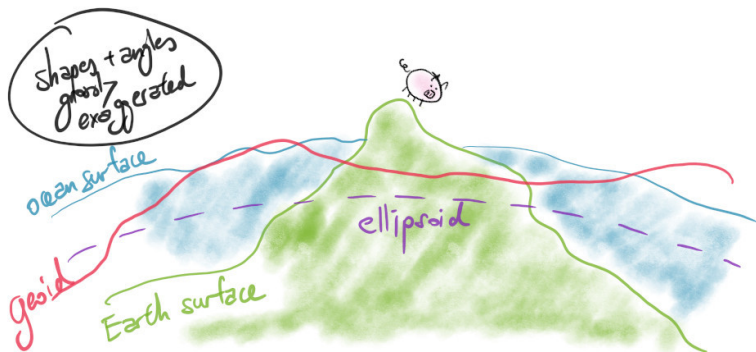
**Figure:** Schematic of the ellipsoid and geoid.

# Geoid



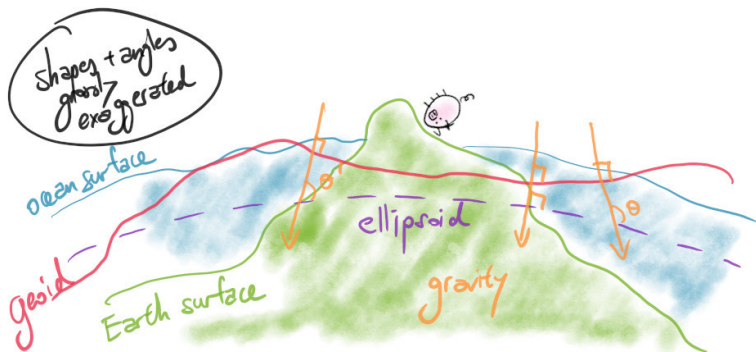
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**Figure:** Schematic of the ellipsoid and geoid.



## Gravity + weight

- ▶ differences greatly exaggerated above, in reality gravity variations are very small
  - that's why it was very difficult to get the geoid!
  - really needed **satellites** (see Lec. 20)

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- ▶ for most intents and purposes we can take  $g$  to be a constant
  - remember the ocean is quite thin ( $H/L \ll 1$ )
- ▶ it does matter when we are talking about things like **sea level** (see OCES 4001)
  - sea level change but **relative to what?**
  - e.g. ellipsoid? ground? geoid?

**SSH** to be instantaneous height relative to ellipsoid

# Pressure (recall last Lec.)

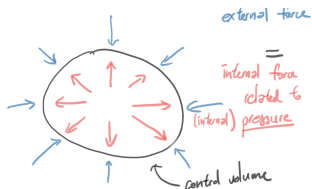


Figure: Fluid volume in force balance.

Consider a body (e.g. balloon) of fixed volume

- fixed volume  $\Rightarrow$  steady
- steady  $\Rightarrow$  in **force balance**, no net force

- **pressure** = force per area,

$$p = F/A, \quad \text{units: } \text{N m}^{-2} \equiv \text{Pa}$$

1 bar =  $10^5$  Pa (Pascals) (see e.g. Wikipedia for others)

→ cf. millibars (mbar) in atmosphere

→ lines of constant pressure = **isobar**

# Pressure: atmospheric example

日期/Date: 14.10.2020 香港時間/HK Time: 14:00 香港天文台 Hong Kong Observatory

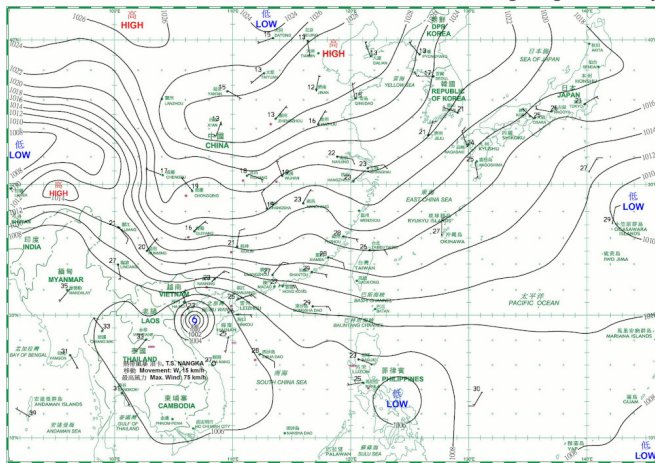


Figure: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

# Hydrostatic balance (recall last Lec.)

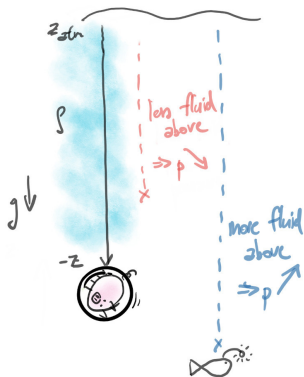


Figure: Schematic of hydrostatic pressure

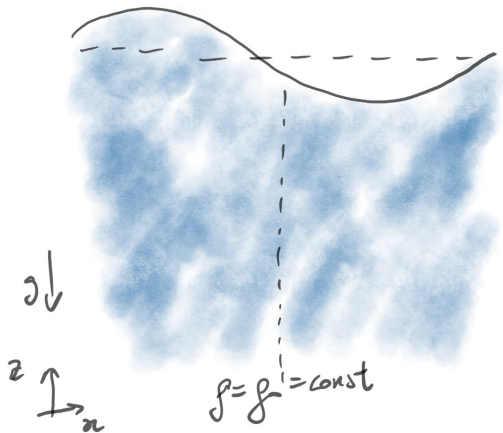
- **hydrostatic approximation:**  
pressure **approximately equal** to weight above when static  
→ **weight** is  $F = mg$  so for force balance,

$$F = mg = g \int_{-z}^{z_{\text{atm}}} \rho(z') \, dz' = p,$$

with  $g \approx 9.81 \, \text{m s}^{-2}$

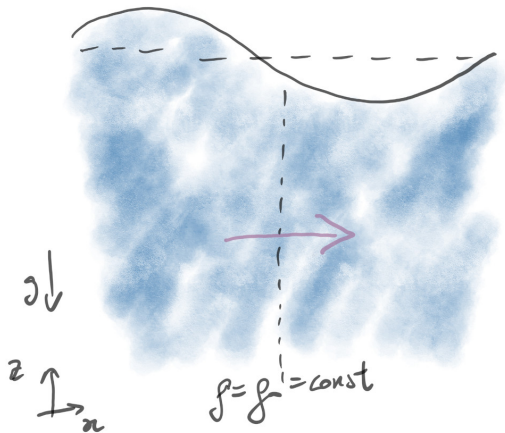
→ if  $\rho = \text{const}$  then  $p = \rho g z + p_{\text{atm}}$

# Horizontal effect?



**Figure:** Horizontal effect because of hydrostatic pressure.

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# Horizontal effect?

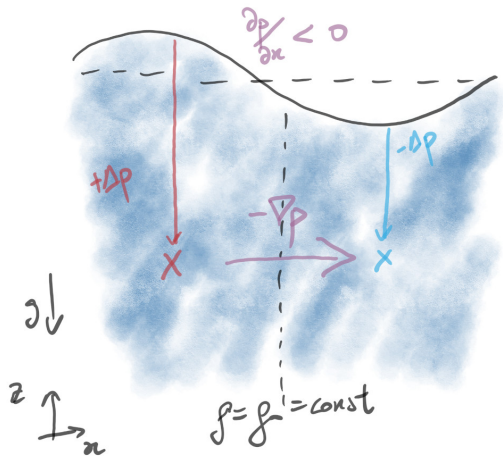
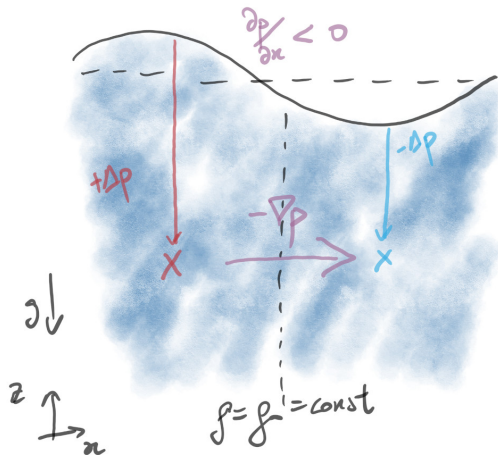


Figure: Horizontal effect because of hydrostatic pressure.

## Horizontal effect?



- assuming hydrostatic balance, water moves from  $+\Delta p$  to  $-\Delta p$  because there is a **net force** (negative pressure gradient  $-\nabla p$ )  
→ important later for **geostrophic flows** (see next Lec.)

Figure: Horizontal effect because of hydrostatic pressure.

# Atmospheric example revisited

日期/Date: 14.10.2020 香港時間/HK Time: 14:00 香港天文台 Hong Kong Observatory

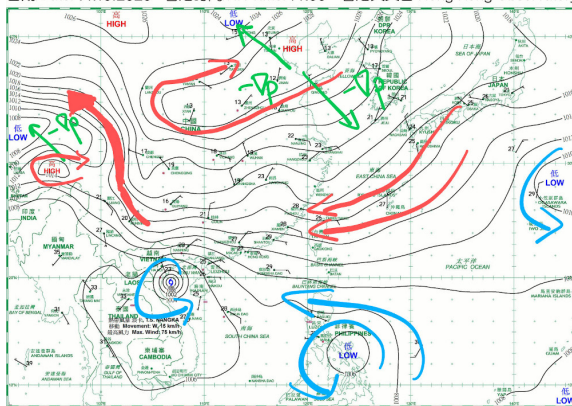
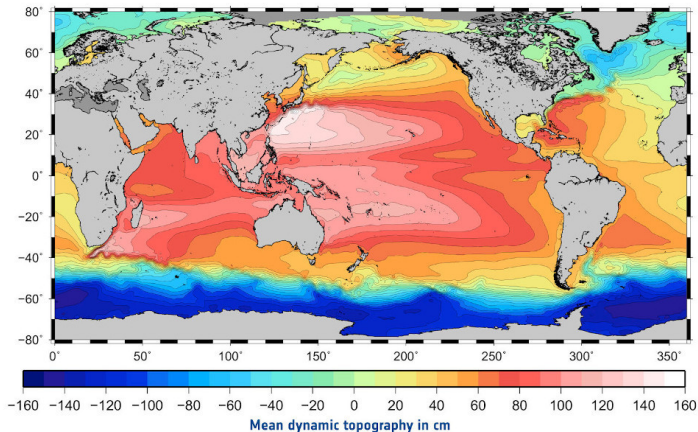


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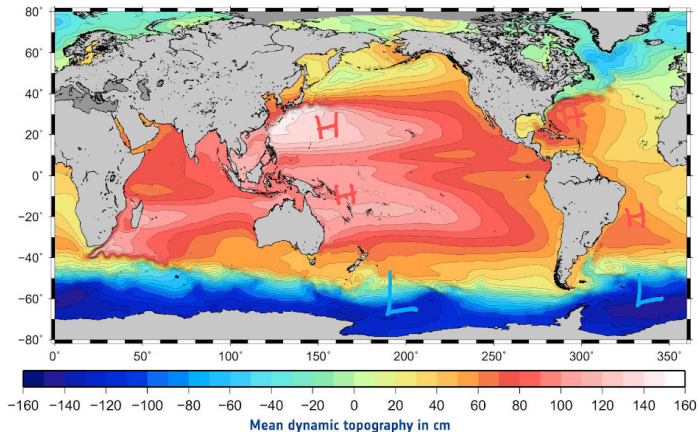
- note that flow doesn't go in the direction of  $-\nabla p!$   
→ **along** rather than **across** isobars (Coriolis effect, see next Lec.)

# Oceanic example



**Figure:** Time-mean global SSH (also called **mean dynamic topography**), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

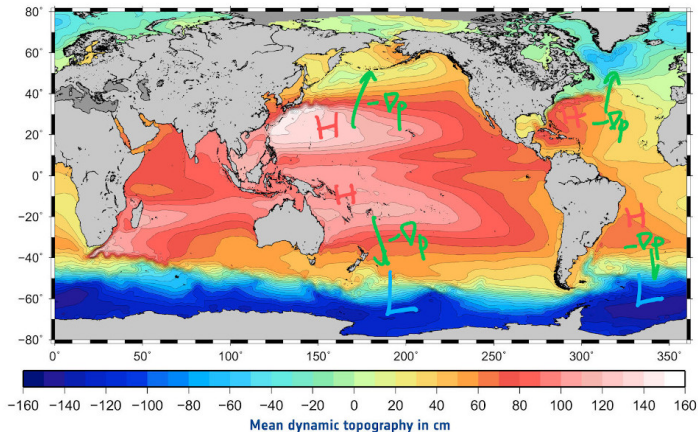
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- contours of SSH related to isobars via **hydrostatic balance**

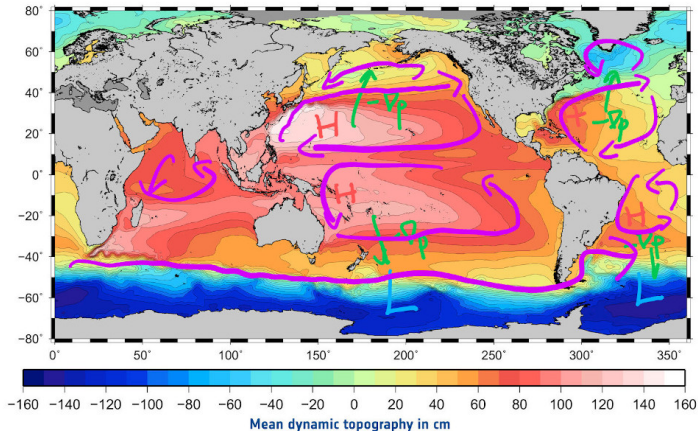
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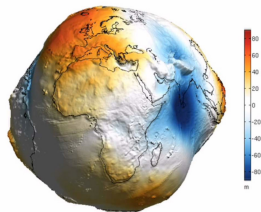


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- contours of SSH related to isobars via **hydrostatic balance**  
→ flow is **along** rather than **across** isobars (Coriolis effect, see next Lec.)

# Summary

- ▶ gravity + **geoid**
  - astronomical forcing on ocean  
(see Lec. 18)
  - geoid important for e.g. **sea level change** (see Lec. 18 + OCES 4001)
- ▶ **hydrostatic pressure**
  - pressure proportional to weight of fluid above
- ▶ **buoyancy** (thermodynamic stuff) affects **pressure**...
- ▶ ...leading to pressure **gradients** (mechanical force) driving a **flow**...
  - ...but **rotation** can influence resulting flow! (see next Lec.)



**Figure:** The “lumpy potato” Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.