OCES 2003 Assignment 4, Spring 2021

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Set on: Thur 29th$^{\text{th}}$ Apr; due: Thur 6$^{\text{th}}$ May

Model solutions and mark scheme

Problems

1. The main equation we are using will be

$$f = \left(1 + \frac{\Delta v}{c}\right) f_0, \qquad \Delta f = \frac{\Delta v}{c} f_0$$

   (a) Just put some numbers in:

$$\Delta f_{24} = \frac{-0.1}{3 \times 10^8} 24 \times 10^9 = -8 \text{ Hz}, \qquad \Delta f_{32} = \frac{-0.1}{3 \times 10^8} 32 \times 10^9 = -11 \text{ Hz}.$$

   (1 mark each, take 0.5 marks off for each answer not given in the specified accuracy)

   (b) This is a red shift, i.e. reduction in frequency, either directly from the sign of Δf and/or noting that things are moving apart from each other.

   (0.5 mark for red shift and 0.5 mark for justification)

   (c) While acceleration is normally $a = \mathrm{d}v/\mathrm{d}t$, since we have uniform acceleration, this is just $a = \delta v/\delta t = 0.1/1 = 0.1$ m s^{-2}. The sign should be positive because it is Jerry pulling away, so it should be a positive acceleration.

   (0.5 mark for the 0.1 numerical value, and 0.5 marks for the positive sign and units)

   (d) We have generally $F = ma$, but by assumption the only change in the force is from gravity, so we have $F = ma = mg$, so by equating we have $\delta g = a = 0.1$ m s^{-2} (notice the mass plays no role in determining g here).

   (0.5 mark for the 0.1 numerical value [so allow error carried forward from previous question], and 0.5 marks for the argument and units)

   (e) Denoting the uncertainty by little δ ('delta'), then

$$f + \delta f = \left(1 + \frac{\Delta v + \delta v}{c}\right) f_0.$$

   There are no δc and δf_0 because we assumed we have perfect knowledge of those. Then since we have an expression for f is already, we can eliminating terms accordingly to give

$$\delta f = \frac{\delta v}{c} f_0 \qquad \Rightarrow \qquad \delta v = \frac{c \delta f}{f_0} = \frac{3 \times 10^8 \times (\pm 1)}{32 \times 10^9} = 0.0094 \text{ m s}^{-1}.$$

   (0.5 marks for the 0.0094 and 0.5 marks for the units. Give 0.5 marks for some vaguely sensible argument if the answer is not correct.)

   (f) By same logic as in part (d, e), there is no δt, so $\delta g = \delta a = \delta v/\Delta t = \delta v/1 = 0.0094$ m s^{-2}.

   (0.5 marks for the 0.0094 and 0.5 marks for the units. Give some 0.5 marks for some vaguely sensible argument if the answer is not correct.)

(g) If we use a lower frequency, the same amount of uncertainty δf translates to a larger δv and thus δg, since the Doppler shift factor is still then same size. Or, put another way, we need an instrument with more accurate sensors if we use a lower frequency to achieve the same degree of uncertainty.

(0.5 marks for noticing the larger uncertainty, and 0.5 marks for some argument to control uncertainty. There might be others that I haven't thought of, maybe let me know about those)

(h) The energy of an electromagnetic wave goes like $E = hf$ where h is a constant (the Planck's constant), so higher frequency requires more energy to excite, so is costly from an operational point of view, possibly reducing the life of the satellite.

(0.5 marks for noting there is more energy being used, and 0.5 marks for that affecting the satellite operation. There might be others that I haven't thought of, maybe let me know about those)

(i) Any thing with mass has a gravitational field, and thus an associated tide generating force. While these satellites are tide raising in that sense, they have a relatively small mass so the associated forces and accelerations are tiny.

(0.5 marks for yes there is a tide, and 0.5 marks for no it probably doesn't matter because the associated effects will be tiny)

2. The numbers of interest are as follows:

	G	m	r_{Earth}	r	$a_{\text{tide}} \approx 2Gm(r_{\text{Earth}}/r^3)$
Sun	6.7×10^{-11}	2.0×10^{30}	6.4×10^{6}	1.5×10^{11}	5.1×10^{-7}
Moon	6.7×10^{-11}	7.3×10^{22}	6.4×10^{6}	3.8×10^{8}	1.1×10^{-6}
Jupiter	6.7×10^{-11}	1.9×10^{27}	6.4×10^{6}	8.4×10^{11}	2.7×10^{-12}

(a) The numbers are not the same as the ones on the wikipedia page, partly because these "errors" are amplified substantially by the $1/r^3$ term.

(1 mark each for each of the Sun and Moon answer at the right accuracy. 0.5 marks for anything deviating from the given numerical answer [the calculation needs to be done in one go and not sequentially, since that compounds a lot of rounding errors])

(b) So we want to work out the r such that the Moon has a tidal acceleration 5.9×10^{-7}. Rearranging gives

$$r = \left(\frac{2Gmr_{\text{Earth}}}{a_{\text{tide}}}\right)^{1/3} = \left(\frac{2Gmr_{\text{Earth}}}{5.1 \times 10^{-7}}\right)^{1/3} \approx 5.0 \times 10^{8}\ \text{m}.$$

Notice this new r is not that much bigger than the current distance of the moon at 3.9×10^8 m. This is partly to do with the $1/r^3$ scaling of a_{tide}.

(1 mark for answer, take 0.5 marks for dropping units and/or degree of accuracy)

(c) See value for Jupiter above, and note that it is tiny compared to the corresponding numbers for the Moon and Sun.

Note: we need Jupiter to calculate for orbits because Jupiter is no negligible for gravitational attraction. The scaling for the force goes like $1/r^2$, but the scaling for tidal acceleration goes like $1/r^3$, so the force is important but the tidal acceleration may not be.

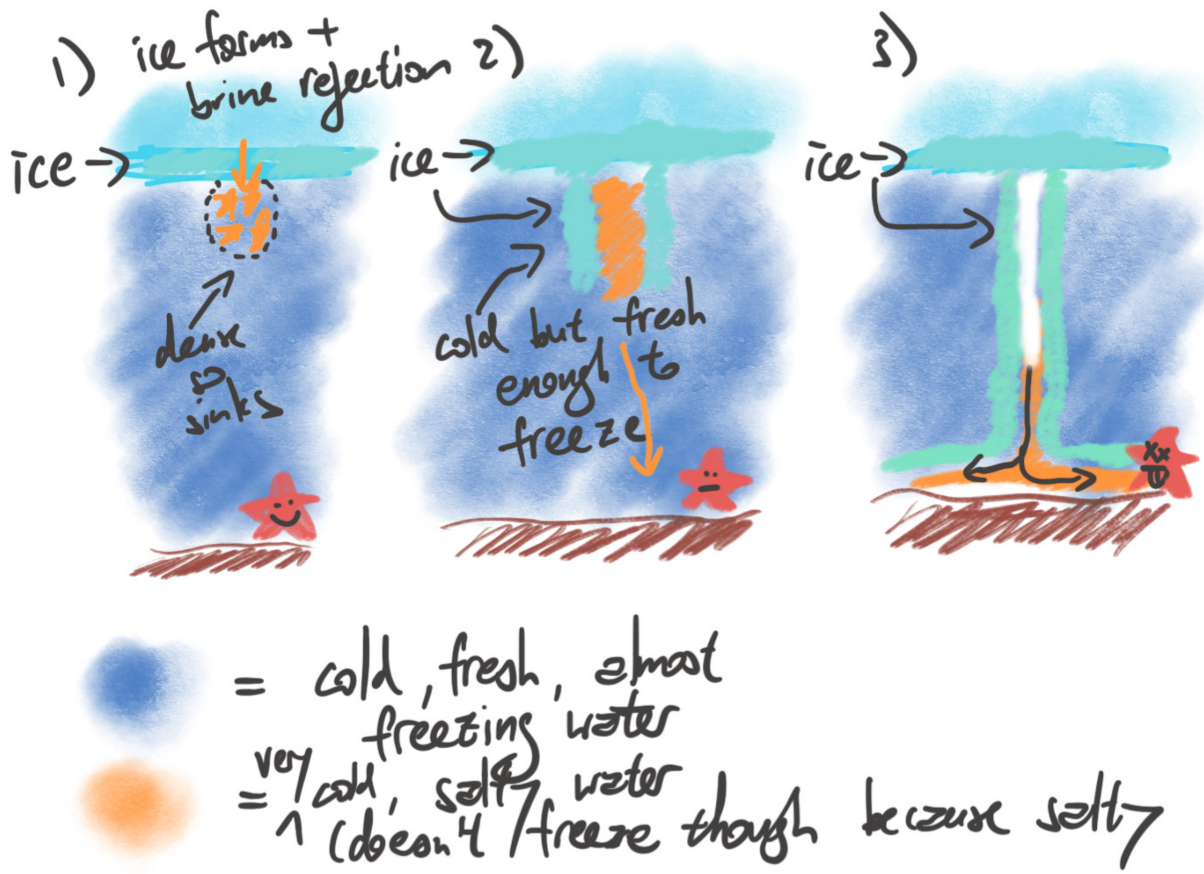
(0.5 mark for some sort of answer, 0.5 marks for noting it is tiny compared to the corresponding values for the Moon and the Sun)

(d) There is no expected change in the frequency and amplitude. If the single moon had half the mass, it would have a gravitational field that is half the magnitude, so the tidal amplitude would be halved, but you still get two bulges 180° apart from consideration of the tidal generating force. If you now have two moons of half the mass each but 180° apart, then two the bulges add up to the original bulge since the individual effectives are in phase with each other.

The orbital distance might actually evolve in this instance but we made the assumptions that the distances are the same as before.

(0.5 mark for no change in amplitude, 0.5 marks for no change in frequency, 1 mark for some argument relating to the tidal generating force [give 0.5 marks as appropriate])

3. My attempt as below; no actual starfish was harmed in the making of the schematic.



1) When ice forms, salt gets kicked out of the ice by brine rejection, so the very cold water below becomes salty, making it dense and statically unstable, and it sinks as a plume.
2) The sinking plume is very cold but salty so it doesn't freeze (it is *supercooled*). The surrounding water is fresher but on the verge of freezing, so when touched by the very cold plume the surrounding water freezes.
3) Freezing water can kick out more salt and the plume can self-sustain, and sink all the way to the bottom and spread, all the while freezing its surroundings. This forms icicle-like structures (hence brinicle).

There are other details (e.g. saline water attracts/sucks in surrounding water, forming of insulating layers to prevent warming of plume, structure etc.) but the above is the gist of it. Brinicles were hypothesised in the 60s but only filmed for the first time in 2011 (sources: Wikipedia).

(4 marks for some sort of sensible drawing, 2 marks for the supporting text. Give 1 mark if need be if references are used, but give no more than 6 marks for the question)

bonus Some of the ones I can immediately see wrong/confusing with that (old?) NOAA and/or NASA entry on tides:

- no mention of inertial or non-inertial frame
- presumably non-inertial, but no mention of center of rotation (center of Earth vs. barycenter etc.; probably want the latter)
- no definition of tides as such (if tides are to be the deformation arising from gravitational attraction, then the "inertial" stuff has nothing to do with tides by definition)
- the inertial forces are really the centrifugal forces, but centrifugal forces are not really tide raising forces
- the diagram seems to imply a bulge arising directly from the forces (it is the collective *convergence* of the water piling up which leads to the bulge)
- ...