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  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 6: Sea water properties (density)

Thur 18th Feb

Outline

- density $\rho = \rho(T, S)$

  $\rightarrow$ equation of state (EOS)
- pressure p

  $\rightarrow$ weight + hydrostatic balance (briefly, more in Lec. 7)

  $\rightarrow$ dynamical consequences
- in-situ vs. potential (vs. neutral) density

  $\rightarrow$ ρ vs. ρ_θ (vs. γ_n)

  $\rightarrow$ example and concept of work done

  $\rightarrow$ dynamical consequences

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Take home: it's **almost never** in-situ temp/dens we care about!

Key terms: EOS, hydrostatic balance, in-situ vs. potential/neutral density

Recap: parcel argument for buoyancy

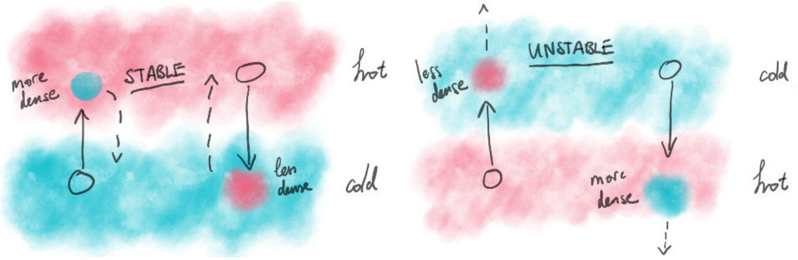


Figure: (Un)stable temperature configurations.

- density (!) ρ (units: kg m^{-3})
- buoyancy $b = -(\delta\rho/\rho_0)g$ (so units of...?)

  $\rightarrow$ how "floaty" something is (e.g. warm water, lighter density, **more** buoyant)

ultimately it's density/buoyancy we care about

Recap: equations of motion

Denoting $\boldsymbol{u} = (u, v)$ and $\boldsymbol{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F}_{\boldsymbol{u}} + \boldsymbol{D}_{\boldsymbol{u}} \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2}$$

$$\nabla \cdot \boldsymbol{u}_3 = 0 \tag{3}$$

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T \right) = F_T + D_T \tag{4}$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S \right) = F_S + D_S \tag{5}$$

$$\rho = \rho(T, S, p) \tag{6}$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Ocean density

- 10 m of seawater $\approx$ 1 atm (recall Lec. 1)
- over most of ocean, ρ varies from $\rho_0 = 1026\ \mathrm{kg\ m^{-3}}$ by no more than **2%** (e.g. Gill, 1982)

  $\rightarrow$ small(!) but **CRUCIAL** variations! (see Lec. 7 + 8, 13 + 14)

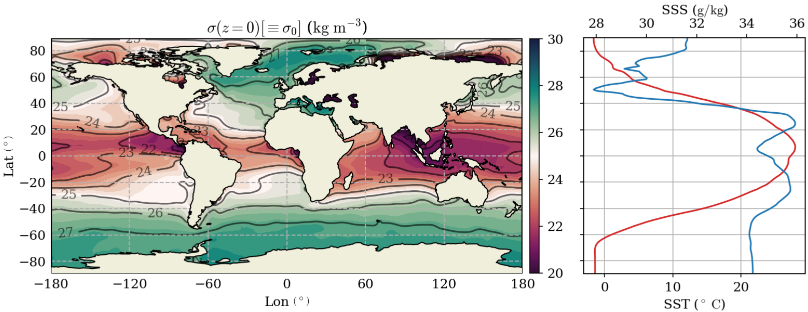
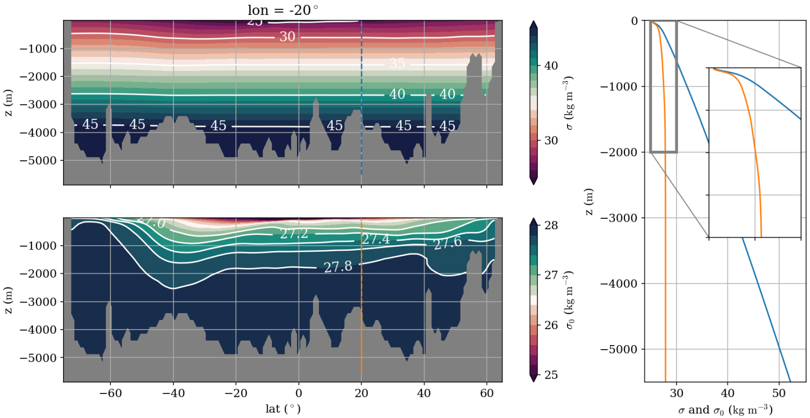


Figure: (left) Density at the surface (in-situ or referenced to sea surface) and (right) zonal averaged SST (red) and SSS (blue). Year-averaged data based on World Ocean Atlas 2013. See `plot_eos.ipynb`

Ocean density (more about this later...)

- isopycnal = lines/surfaces of constant density
- pycnocline = place/region below mixed layer where density gradient is largest (changes fastest)

  → cf. isotherm and thermocline (last Lec.)



Equation of State

T and S contribute to density, i.e. $\rho = \rho(T, S)$, and

$$\rho \nearrow \text{ as } T \searrow, \qquad \rho \nearrow \text{ as } S \nearrow$$

Equation of State (EOS) is the actual $\rho = \rho(T, S)$

- $\rho \nearrow$ as $S \nearrow$ so possible example is

$$\rho \sim \beta S, \qquad \beta \geq 0$$

- $\rho \nearrow$ as $T \searrow$ so could be

$$\rho \sim -\alpha T + \beta S, \qquad \alpha, \beta \geq 0$$

$\rightarrow$ can be negative as is...?

Equation of State

A linear EOS of seawater ($\alpha, \beta \geq 0$):

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

- expected behaviour with changing T and S
  $\rightarrow$ relative to reference T_0 and S_0

Equation of State

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- the "1" to show base density of ρ_0 (units: kg m^{-3})

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LHS and RHS units need to agree!

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- the "1" to show base density of ρ_0 (units: kg m^{-3})

LHS and RHS units need to agree!

- α has units $^\circ$C^{-1} (or K^{-1}), thermal coefficient of expansion
- β has units g^{-1} kg, haline coefficient of contraction

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

e.g. by hand:

- $\rho_0 = 1$, $T_0 = S_0 = 0$, $\alpha = \beta = 1$,

  $$\rho = 1 - T + S$$

  so if $T = 10$, $S = 2$ then $\rho = \ldots$?

- "sensible" references are (partly taken from Rouquet et al. (2015), *J. Phys. Oceanogr.*)

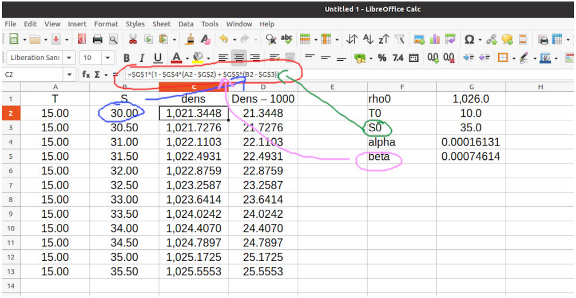
  $$T_0 = 10\ ^\circ\mathrm{C}, \quad S_0 = 35\ \mathrm{g\ kg^{-1}}, \quad \rho_0 = 1026\ \mathrm{kg\ m^{-3}}$$

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

e.g. in Excel


Untitled 1 - LibreOffice Calc

C2 =G1*(1 - G4*(A2 - G2) + G5*(B2 - G3))

	A	B	C	D	E	F	G	H
1	T	S	dens	Dens – 1000		rho0	1,026.0	
2	15.00	30.00	1,021.3448	21.3448		T0	10.0	
3	15.00	30.50	1,021.7276	21.7276		S0	35.0	
4	15.00	31.00	1,022.1103	22.1103		alpha	0.00016131	
5	15.00	31.50	1,022.4931	22.4931		beta	0.00074614	
6	15.00	32.00	1,022.8759	22.8759				
7	15.00	32.50	1,023.2587	23.2587				
8	15.00	33.00	1,023.6414	23.6414				
9	15.00	33.50	1,024.0242	24.0242				
10	15.00	34.00	1,024.4070	24.4070				
11	15.00	34.50	1,024.7897	24.7897				
12	15.00	35.00	1,025.1725	25.1725				
13	15.00	35.50	1,025.5553	25.5553				
14								

Figure: Sample calculations of density using linear EOS in Excel (highly recommend you don't use Excel, because syntax is a bit messy...)

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

What does it look like? (define $\sigma = \rho - 1000$ here)

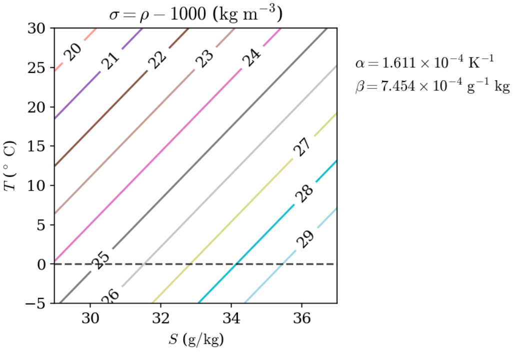


Figure: Linear EOS in *TS* space. See `plot_eos.ipynb`

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

Compare with "real" EOS? (TEOS-10 here using Fabien's 75-term formula)

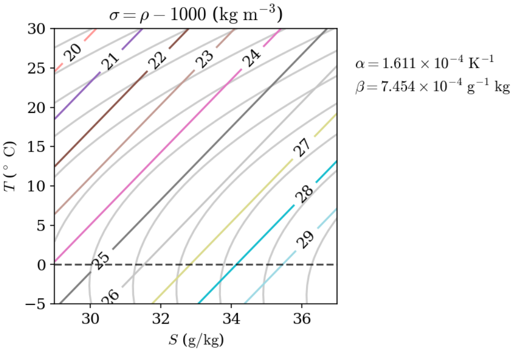


Figure: Linear EOS in *TS* space with TEOS10 as gray contours (same contour levels). See `plot_eos.ipynb`

"Mildly" Nonlinear EOS see Geoff Vallis' (2006) book

$$\rho = \rho_0 \left[1 - \alpha \left(T_a + \frac{\lambda_1}{2} T_a^2 \right) + \beta \left(S_a - \frac{\lambda_2}{2} S_a^2 \right) - \nu T_a S_a \right]$$

- $T_a = T - T_0$ and $S_a = S - S_0$, the anomalies

"Mildly" Nonlinear EOS see Geoff Vallis' (2006) book

$$\rho = \rho_0 \left[1 - \alpha \left(T_a + \frac{\lambda_1}{2} T_a^2 \right) + \beta \left(S_a - \frac{\lambda_2}{2} S_a^2 \right) - \nu T_a S_a \right]$$

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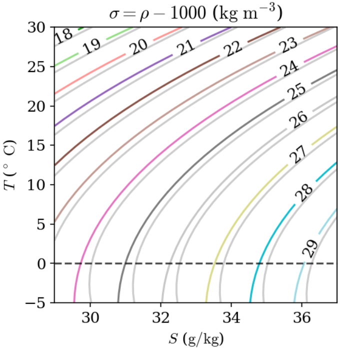


$$\begin{aligned} \alpha &= 1.611 \times 10^{-4}\ \mathrm{K}^{-1} \\ \beta &= 7.454 \times 10^{-4}\ \mathrm{g}^{-1}\ \mathrm{kg} \\ \lambda_1 &= 5.952 \times 10^{-2}\ \mathrm{K}^{-1} \\ \lambda_2 &= 5.491 \times 10^{-4}\ \mathrm{g}^{-1}\ \mathrm{kg} \\ \nu &= 2.370 \times 10^{-6}\ \mathrm{K}^{-1}\ \mathrm{g}^{-1}\ \mathrm{kg} \end{aligned}$$

Figure: Toy nonlinear EOS (no thermobaric effect) in *TS* space with TEOS10 as gray contours (same contour levels). See `plot_eos.ipynb`

Nonlinear EOS

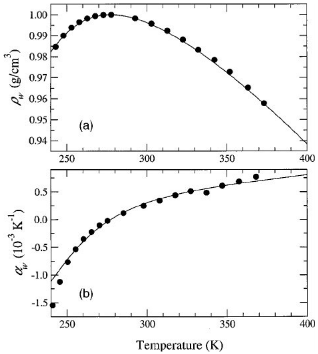


Figure: (top) $\rho = \rho(T)$ for pure water, (bot) $\alpha = \alpha(T)$. From Ashbaugh *et al.* (2002), *J. Chem. Phys.*.

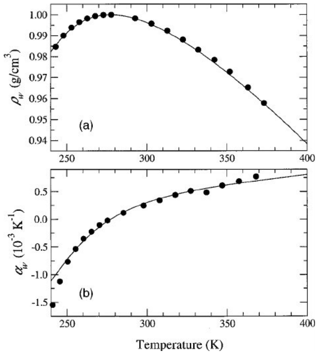
- "real" EOS should be nonlinear

  $\rightarrow$ e.g. water densest around 4° C (so ice floats), i.e. cannot be linear in T

  $\rightarrow$ "real" EOS ongoing research

  (e.g. TEOS-10, works by Trevor McDougall)

Nonlinear EOS



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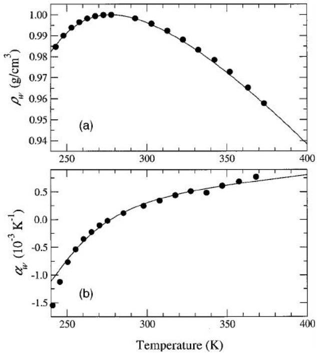
- small discrepancies = can ignore?

Nonlinear EOS



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- small discrepancies = can ignore?

  $\rightarrow$ **NO**! "small" difference really matter!

  $\rightarrow$ over most of ocean, ρ varies from $\rho_0 = 1026\mathrm{kg\ m}^{-3}$ by no more than 2% (e.g. Gill, 1982)

"Full" Nonlinear EOS (e.g. Roquet et al. (2015), *J. Phys. Oceanogr.*)

- TEOS-10 standard, polynomial with 75(!) terms chosen(!) to fit with real data
- note the change of sign in gradient close to freezing point $\rightarrow$ remember ice is **less dense** than water normally!

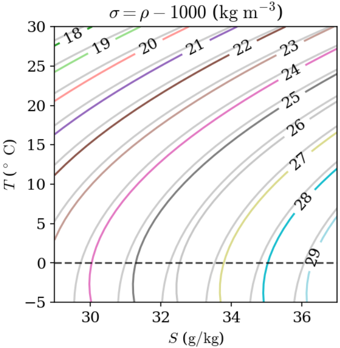


Figure: TEOS10 EOS (at surface) in *TS* space with toy nonlinear EOS as gray contours (same contour levels). See `plot_eos.ipynb`

Pressure + hydrostatic balance (briefly)

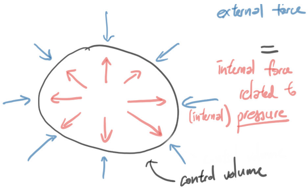


Figure: Fluid volume in force balance.

Consider a body (e.g. balloon) of fixed volume

- fixed volume $\Rightarrow$ steady
- steady $\Rightarrow$ in force balance, no net force

- pressure = force per area,

$$p = F/A, \qquad \text{units: N m}^{-2} \equiv \text{Pa}$$

1 bar = 10^6 Pa (Pascals) (see e.g. Wikipedia for others)

$\rightarrow$ cf. millibars (mbar) in atmosphere

$\rightarrow$ lines of constant pressure = isobar

Pressure + hydrostatic balance (briefly)

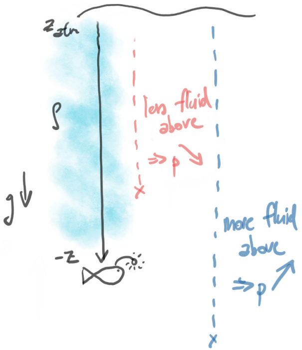


Figure: Schematic of hydrostatic pressure

- hydrostatic approximation: pressure **equal** to weight above when static

  $\rightarrow$ weight is $F = mg$ so for force balance,

$$F = mg = \boxed{g \int_{-z}^{z_{\mathrm{atm}}} \rho \, \mathrm{d}z = p},$$

with $g \approx 9.81\ \mathrm{m\ s^{-2}}$

$\rightarrow$ if $\rho = \mathrm{const}$ then $p = \rho g z + p_{\mathrm{atm}}$

Concept of work done

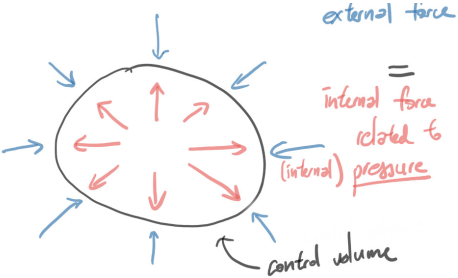


Figure: Working to compress a volume.

Concept of work done

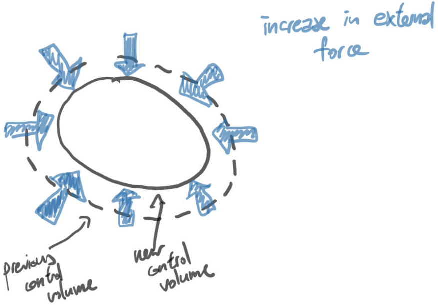


Figure: Working to compress a volume.

Concept of work done

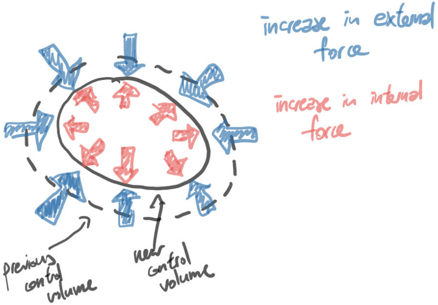


Figure: Working to compress a volume.

Concept of work done

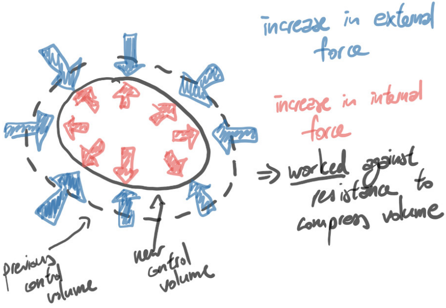


Figure: Working to compress a volume.

- to compress, need to work against something (in this case internal pressure)
- energy has to be put in (cf. $T \nearrow$ in fluid volume)

Work done and potential/neutral densities

Remember this graph from a few slides ago?

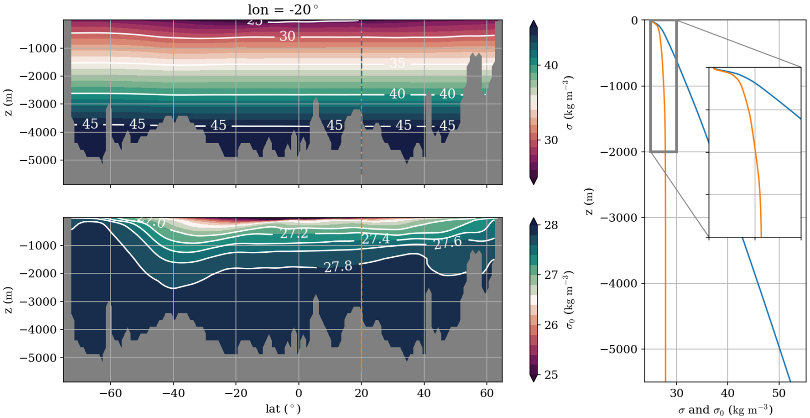


Figure: Meridional section in the Atlantic of (top left) in-situ density and (bot left) potential density referenced to sea level, with the corresponding vertical profiles plotted (right). See `plot_eos.ipynb`

Work done and potential/neutral densities

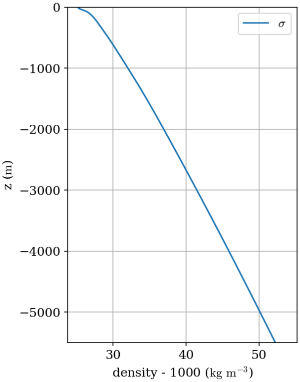


Figure: Vertical profile of in-situ density at the same location as in the previous graph. See `plot_eos.ipynb`

In-situ density $\rho(T, S, p)$

- increases with depth because more weight above (compression), but not necessarily dynamical!

  $\rightarrow$ abyssal increase of ρ mostly from p, but not necessarily from having to do work
- in-situ density profile is telling us there shouldn't be up/down motion, but we know we do have it!

Want something that adjusts for some/all of pressure contribution to density

Potential temperature and density

Potential temperature θ

- the temperature it will have if you take some water and move it to some reference pressure p_0 (usually p_{atm}) without exchange of heat and salt

  $\rightarrow$ without exchange of heat + mass = adiabatic

  $\rightarrow$ reference needed to set base line and **is a choice**

  $\rightarrow$ account for pressure contributions **relative to reference level**

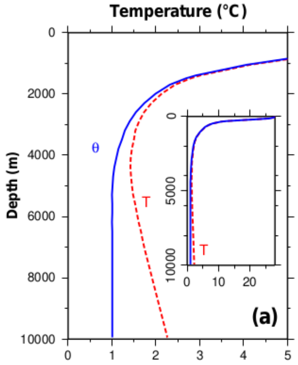


Figure: Vertical profile of in-situ (red) and potential temperature (blue) in the Mariana Trench to highlight the differences. From Talley *et al.* (2011) Fig 4.10(*a*).

According to in-situ temperature, bottom water should be <u>more</u> buoyant (i.e. <u>unstable</u> + overturn)!

Concept of work done

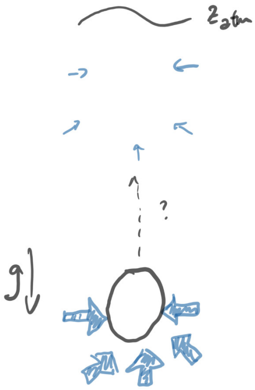
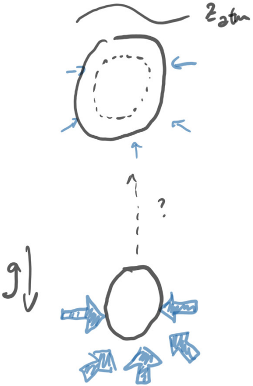


Figure: Volume change from pressure, but work done reversible if adiabatic.

Concept of work done



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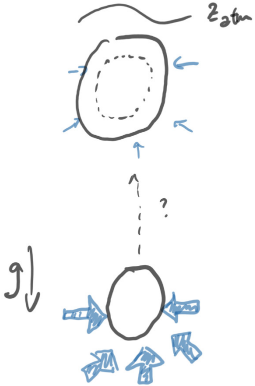


Figure: Volume change from pressure, but work done reversible if adiabatic.

- work is done by the system against the surroundings

Concept of work done

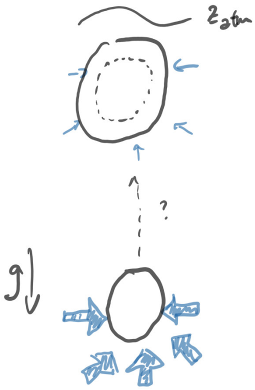


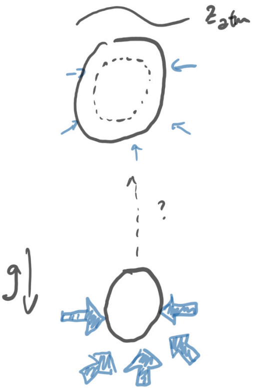
Figure: Volume change from pressure, but work done reversible if adiabatic.

- work is done by the system against the surroundings
- energy lost from system with expansion, change in

  $\rightarrow$ density (from volume since $\rho = m/V$)

  $\rightarrow$ temperature (less energy in system, cooling)

Concept of work done



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- work is done by the system against the surroundings
- energy lost from system with expansion, change in

  $\rightarrow$ density (from volume since $\rho = m/V$)

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but if *adiabatic*, then this process is **reversible** (no change in **entropy**), but we normally care about **irreversible** processes!

Potential temperature and density

Potential density ρ_θ

- the density calculated using θ in EOS

  $\rightarrow$ reference dependent (from θ)

  $\rightarrow$ shallow region focus? choose sea level (0m) or 1000m

  $\rightarrow$ deeper region? choose 2/3/4000m

  $\rightarrow$ used to identify water masses

- crucial bit is that $\sigma_{0,1,2,3,4}$ has very **small gradients** (i.e. essentially **flat**) in the deep!

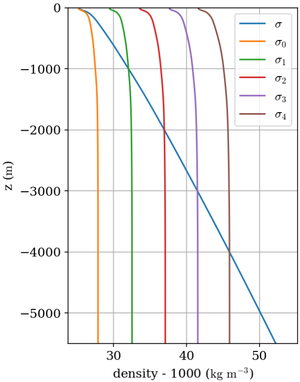

Figure: Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

Potential temperature and density

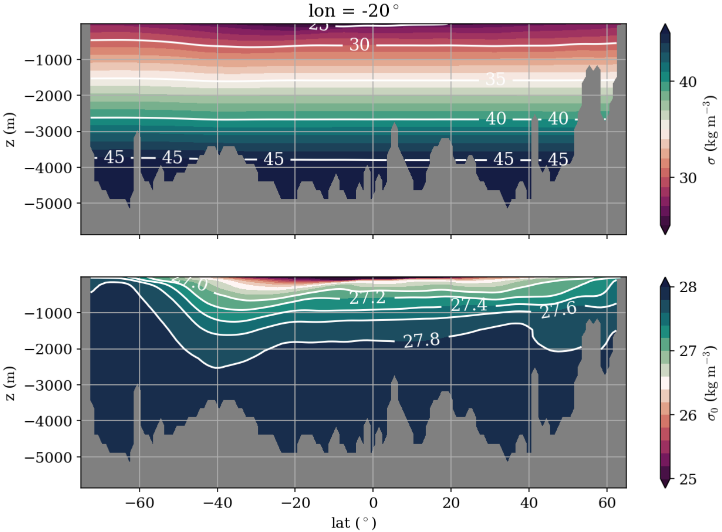

Figure: Meridional section in the Atlantic of (top) in-situ density and (bot) σ_0. See `plot_eos.ipynb`

Potential temperature and density

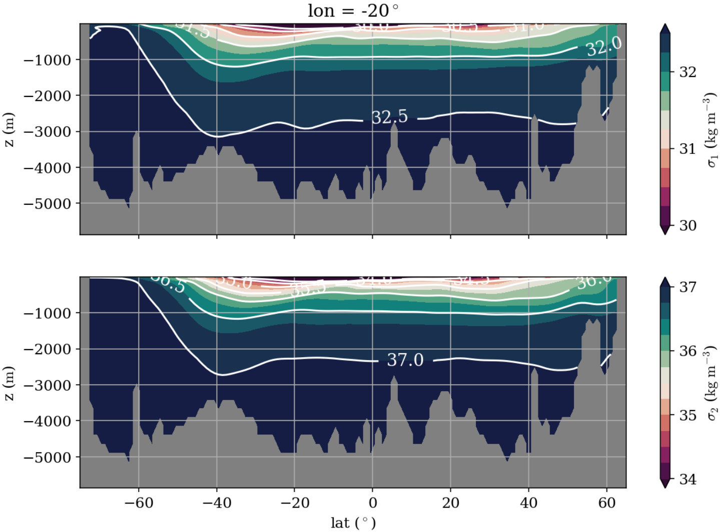

Figure: Meridional section in the Atlantic of (top) σ_1 and (bot) σ_2. See `plot_eos.ipynb`

Potential temperature and density

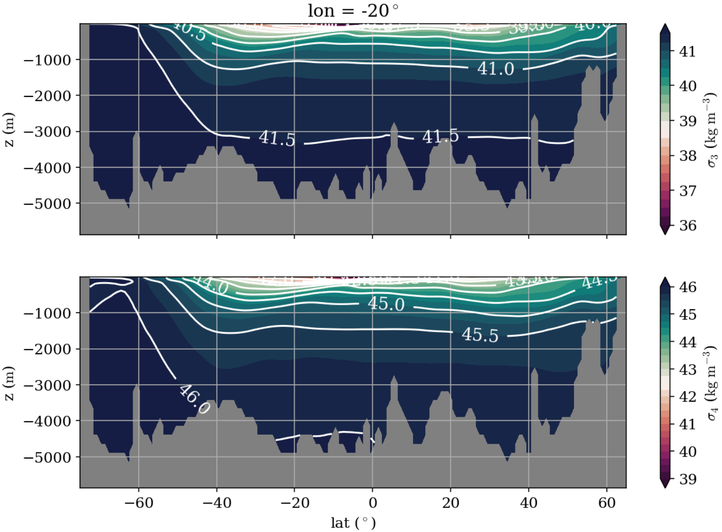

Figure: Meridional section in the Atlantic of (top) σ_3 and (bot) σ_4. See `plot_eos.ipynb`

Neutral density

Potential density needs a reference

- neutral density γ_n in principle does not need a reference

  $\rightarrow$ complications with computing and/or existence of neutral surfaces (e.g. Jackett & McDougall, 1997, *J. Phys. Oceangr.*)

- can be used to identify water masses but restricted to present day ocean (cf. Lec. 5, see lec. 13 + 14)

  $\rightarrow$ e.g. not necessarily suitable for paleoclimate + very long-term climate change applications

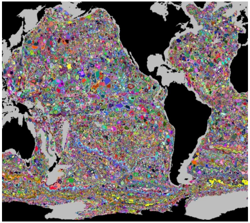

Figure: Topobaric surfaces, an almost neutral surface. From Stanley (2019), *Ocean. Modell.*, Figure 4.

Neutral density sample: Atlantic

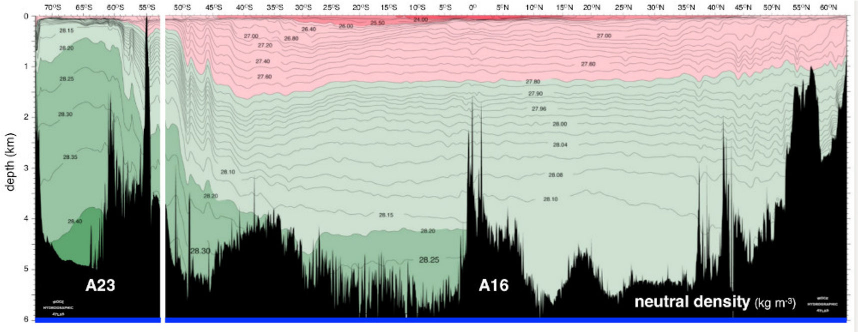

Figure: Neutral density (see later) meridional section in the Atlantic. From Koltermann *et al.* (2011), vol 3 of Hydrographic Atlas of the World Ocean Circulation Experiment (WOCE).

- flow largely along isopycnals (path of less resistance, not needing to do as much work) (revisit in Lec. 13, 14, 17)

  → consequence for global meridional overturning circulation (MOC)

Summary

- EOS to relate temperature and salinity to density
- hydrostatic approximation (more next Lec.)

  $\rightarrow$ weight to balance fluid above (related to density)
- in-situ vs. potential (vs. neutral) density

  $\rightarrow$ dynamics cares about work done

  $\rightarrow$ in-situ density σ has substantial pressure contributions

  $\rightarrow$ potential density σ_θ removes some pressure contribution (reference dependent)

  $\rightarrow$ neutral density γ_n removes more/all pressure contributions in principle

Take home: it's **almost never** in-situ temp/dens we care about!