Forecasting Chaotic time series by a Neural Network

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Abstract: This paper examines how efficient neural networks are relative to linear and polynomial approximations to forecast a time series that is generated by the chaotic Mackey-Glass differential delay equation. The forecasting horizon is one step ahead. A series of regressions with polynomial approximators and a simple neural network with two neurons is taking place and compare the multiple correlation coefficients. The neural network, a very simple neural network, is superior to the polynomial expansions, and delivers a virtually perfect forecasting. Finally, the neural network is much more precise, relative to the other methods, across a wide set of realizations.

Keywords: neural network forecasting, chaos forecasting, Mackey-Glass forecasting, time series forecasting.

1. Introduction

Over the last decade, nonlinear prediction of chaotic time series has been a popular subject for many scientists. It is very troubling to deal with the nonlinear time series with a high degree of non-stationary. In practice, non-stationary time series are very common in as diverse fields as geophysics, finance, and biological sciences. Other examples can be found in weather forecast [1], speech coding [2] and noise cancellation [3], etc. One view in financial economics is that market prices are random and that past prices cannot be used as a guide for the price behaviour in the future. Chaos theory, however, suggests that a seemingly random process may in fact have been generated by a deterministic function that is not random. Moreover, chaos theory suggests that a time series which seems to be random may be generated by a deterministic function. An obvious example of such processes is random numbers generated by computers. When a time series is chaotic, it implies that the laws underlying the time series can be expressed as a deterministic dynamical system. Chaotic time series are considered as the outputs of nonlinear dynamic systems. If one cannot specify the initial condition with infinite precision, the long time future behaviour of these time series is unpredictable. Most of the work on the prediction of chaotic behaviour has been done in the context of time series analysis. One assumes that the state space of the dynamical systems is not directly observable and that the dynamics is not known (which is the case in most practical situation). However, these deterministic equations are not usually given explicitly. Predictions rely on the empirical regularities derived from the experimental observations of the real system.

In time series prediction, a challenge is to learn fast dynamics (high frequencies in the linear case) and to cancel noise simultaneously. This challenge is related directly to the under-fitting, over-fitting and trade-off. Indeed, learning noise causes potentially over-fitting, whereas, forgetting fast dynamics leads to under-fitting. Time series prediction is a very important practical problem with several of applications from economic and business planning to signal processing and control. A difficulty with a chaotic series is that linear models such as time series or regressions cannot capture regularities in such a series. If the data is not generated by a high dimensional process, it should have short-term predictability, but not with the use of linear forecasting models. In this case a non-linear model such as a neural network is appropriate. As far as forecasting is concerned, the most difficult part is modelling the chaotic time series.

Today, there are popular tools such as neural networks [1] [2] [3] neuro-fuzzy system [4] and evidence combination [5], which are mathematical models that can deal with nonlinearity [20] [21]. Their fault-tolerant characteristic makes them suitable for time series prediction. Many researchers have been greatly interested in the comparability of chaotic behaviour and phenomena of character. This is due to the irregular oscillations and fluctuations often observed in real data such as prices, employment, inflation, investment figures, etc. [6] [7]. Also the nonlinear chaotic models offer a way to produce this behaviour that has attracted attention as a possible alternative explanation for the fluctuations phenomena in the real world without the introduction of stochastic elements. In addition, Many researchers have been greatly interested in comparability of chaotic behaviour and phenomena of character [8] [9] [10]. Several mathematical models have been generated for the forecasting of these nonlinear time series problems, including the exponential model, nearest neighbour regression, and the feed forward network model. It is the purpose of this paper to address the above problems using a radically different approach.

Instead of calculating invariants, a construction of a predictive model directly from time series data is attempted. A related problem is, given a time series generated from a finite-dimensional strange attractor of a specified partial differential equation, construct an ordinary differential equation describing the dynamics restricted to the strange attractor.

Many forecasting algorithms have been developed based upon the theory of dynamic reconstruction from a scalar time series. One can easily find that almost all techniques for predicting chaotic time series have been previously developed for approximating general continuous functions, such as local linear approximation, polynomial approximation, radial basis functions, and neural networks. The effectiveness of these techniques has been confirmed by testing a large number of examples. Of course, there also exist some limitations for each technique, especially, the power of prediction is still quite limited. Obviously, it is related to the sensitive dependence on initial conditions in chaotic systems. Computational resources also hamper obtaining a good prediction to some extent. In fact, chaotic behaviour is quite irregular so that the traditional mature algorithms might be inappropriate for predicting them. The irregularity is related to the fact that chaotic attractors possess very complicated geometric structures.

2. Related research

There exists a vast literature on comparing time series forecasts from neural network and chaotic time series.

For dynamic systems with complex, ill-conditioned, or nonlinear characteristics, the fuzzy modelling method is very effective to describe the properties of the systems. Takagi–Sugeno's model (T–S model) was proposed [14]. The main idea of the fuzzy modelling methods is to construct a set of local linear equations to form a global function approximation based on fuzzy reasoning rules. The objectives of the fuzzy modelling methods are embodied in two aspects. On the one hand, it is important to develop the fast and practical fuzzy modelling methods with a simple structure and better robustness. On the other hand, it is essential that the resulting fuzzy model is easy to understand, that is, the resulting fuzzy model do not lose its interpretability.

Yuehui Chen [2005] introduces a new time-series forecasting model based on the flexible neural tree (FNT) [15]. The FNT model is generated initially as a flexible multi-layer feed-forward neural network and evolved using an evolutionary procedure. Very often it is a difficult task to select the proper input variables or time-lags for constructing a time series model. The research demonstrates that the FNT model is capable of handing the task automatically. The performance and effectiveness of the proposed method are evaluated using time series prediction problems and compared with those of related methods.

Y.M. Chen [2007] proposes a dynamic evolving computation system (DECS) model, for adaptive on-line learning, and its application for dynamic time series prediction. DECS evolve through evolving clustering method and evolutionary computation for structure learning, Levenberg–Marquardt method for parameter learning, learning and accommodate new input data [16]. DECS is created and updated during the operation of the system. At each time moment the output of DECS is calculated through a knowledge rule inference system based on m-most activated fuzzy rules which are dynamically chosen from a fuzzy rule set. An approach is proposed for a dynamic creation of a first order Takagi–Sugeno type fuzzy rule set for the DECS model. The fuzzy rules can be inserted into DECS before, or during its learning process, and the rules can also be extracted from DECS during, or after its learning process. It is demonstrated that DECS can effectively learn complex temporal sequences in an adaptive way and outperform some existing models.

E. Gomez-Ramırez [2007] proposes a Polynomial artificial neural network (PANN) which has been shown to be powerful for forecasting nonlinear time series [17]. The training time is small compared to the time used by other algorithms of artificial neural networks and the capacity to compute relations between the inputs and outputs represented by every term of the polynomial. In this paper a new structure of polynomial is presented that improves the performance of this type of network considering only non-integers exponents. The architecture adaptation uses genetic algorithm (GA) to find the optimal architecture for every example.

Tai-Yue Wang [2007] proposes a Back-propagation network (BPN), the most often used ANN, has been employed to solve a number of real problems[18]. It has a tendency to be trapped at local minima, i.e. non-global optimal solutions as applying to the forecasting problems according to the past experience. These limitations cause the BPN to become inconsistent and unpredictable. On the other hand, because the oscillations and irregular motions have often been observed in real time series, deterministic equilibrium models could not describe the phenomena of real data in forecasting modelling. Consequently, the nonlinear chaotic model has attracted researcher's attention as a possible alternative explanation for the fluctuations phenomena in the real world because of its ability to offer a way to produce mentioned above behaviour without the introduction of stochastic elements. This study will use the genetic algorithm to optimize BPN in order to forecast the chaotic time series problem truthfully. The experimental results confirm that the genetic algorithm performs well as a global search algorithm. Furthermore, it is shown that designating the topology and parameters of the neural network as decision variables simultaneously and then using the genetic algorithms to determine their values can improve the forecasting effectiveness of the resulting BPN when applied to a chaotic time series problem.

D. Kugiumtzis [1998], analyze a Local linear prediction, based on the ordinary least squares (OLS) approach, which is one of several methods that have been applied to prediction of chaotic time series [19]. Apart from potential linearization errors, a drawback of this approach is the high variance of the predictions under certain conditions. Here, a different set of so-called linear regularization techniques, originally derived to solve ill-posed regression problems, are compared to OLS for chaotic time series corrupted by additive measurement noise. These methods reduce the variance compared to OLS, but introduce more bias. A main tool of analysis is the singular value decomposition (SVD), and a key to successful regularization is to damp the higher order SVD components. Several of the methods achieve improved prediction compared to OLS for synthetic noise-corrupted data from well-known chaotic systems. Similar results were found for real-world data from the R-R intervals of ECG signals. Good results are also obtained for real sunspot data, compared to published predictions using nonlinear techniques.

3. Model Presentation

In this paper, it is proposed a chaotic time series model, which predicts a time series one step ahead, which is generated by the following Mackey-Glass (MG) time-delay differential equation.

$$\dot{x}(t) = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.1x(t)$$

This time series is chaotic, and so there is no clearly defined period. The series will not converge or diverge, and the trajectory is highly sensitive to initial conditions.
This is a benchmark problem in the neural network and fuzzy modelling research communities. To obtain the time series value at integer points, the fourth-order Runge-Kutta method to find the numerical solution is applied (Figure 1). The algorithm that it is used proves how efficient neural networks are relative to linear and polynomial approximations.
A series of regressions with polynomial approximators and a simple neural network with two neurons take place and compare the multiple correlation coefficients.

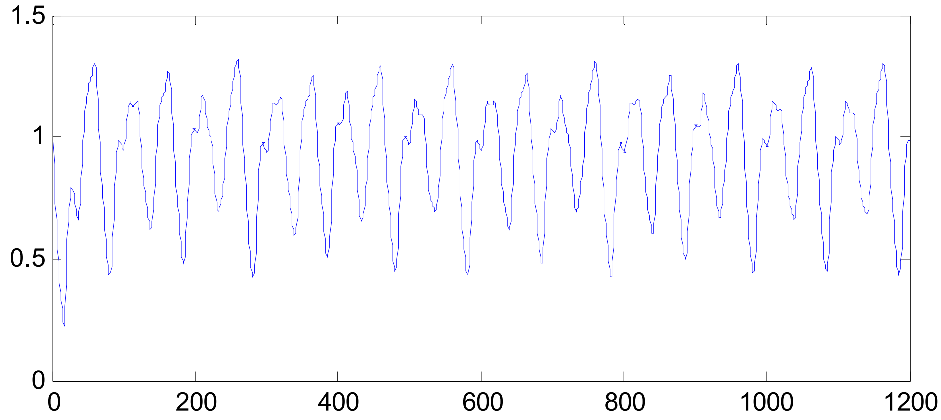

Figure 1. Mackey-Glass (MG) chaotic time series

The neural network is genetically evolved as it is using genetic algorithms for training. In time-series prediction, known values of the time series up to the point in time are used to predict the value at some point in the future.

3. Results

Table 1 depicts the results for the goodness of fit or R^2 statistics for this base set of realizations. Second-order polynomials are compared with a simple network with two neurons.

Table 1: Results

Approximation	R^2
Linear	0.65
Polynomial-Order 2	0.89
Tchebeycheff Polynomial-Order 2	0.89
Hermite-Order 2	0.89
Legendre-Order 2	0.89
Laguerre-Order 2	0.89
Neural Network: FF, 2 neurons, 1 layer	0.98

This table shows several important results for prediction a chaotic time series. First, there are definite improvements in abandoning pure linear approximation. Second, the power polynomial and the orthogonal polynomials give the same prediction results.

There is no basis for preferring one over the other. Third, the neural network, a very simple neural network genetically evolved, is superior to the polynomial expansions, and delivers a very good result. Finally, the neural network is much more precise, relative to the other methods, across a wide set of realizations.

5. Conclusion

As is easily understandable from the above, the introduction of nonlinearity makes the estimation problem much more challenging and time-consuming than the case of the standard linear model. But it also makes the estimation process much more interesting. Given that we can converge to many different results or parameter values, we have to find ways to differentiate the good from the bad, or the better from a relatively worse set of estimates. This paper is an effort to apply an existing genetically evolved global search method for neural network estimation in order to modelling a time series avoid the high risk of falling into locally optimal results.

5. References

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