SCEPTIC Value-based compression

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I'm setting aside the complexity of the basis weights and eligibility function in the first equations to emphasize qualitative differences. Nevertheless, δ refers to the prediction error calculated from the usual eligibility-scaled PE equations from the main SCEPTIC model.

1 Spatial decay

1.1 Extant spatial decay model

This is the current winning model, using the factorized parameterization of learning rate versus decay. Here, I've simplified the notation to deemphasize basis functions, and e is the eligibility function on trial i given choice/RT t.

$$V_{i+1} = V_i + \alpha[e\delta_{(i|t)} - (1 - e)\gamma V_i] \tag{1}$$

2 Value compression models

Note that the value vector is renormalized after exponentiation to maintain the same min and max values pre- and post-compression. This is accomplished by:

$$V' = \exp\left(V/\phi\right) \tag{2}$$

$$V' = (\max(V) - \min(V)) \cdot \frac{V' - \min(V')}{\max(V') - \min(V')} + \min(V) \tag{3}$$

2.1 Variant 1: compression after update

Conceptually, this approach calculates the usual SCEPTIC update from PE, then uses exponential compression of the updated value vector (basis weights). Larger values of ϕ yield less compression, while small values shift the value function toward a narrow peak on RT_{Vmax}.

$$V_{i+1} = \exp(\frac{V_i + \alpha\delta_{(i|t)}}{\phi}) \tag{4}$$

In initial tests, this model has some parameter competition between α and ϕ, yielding negative correlations at subject level.

2.2 Variant 2: Updating against the compressed V

In this variant, the value vector is updated by exponential compression (applied to all basis weights), then the PE is calculated vis-a-vis this vector and added in the usual fashion. Conceptually, this is closer to the view that values are compressed offline (e.g., between trials), then updated by the surprise you experience of the current outcome against that compressed value function.

$$C(V_i) = \exp(\frac{V_i}{\phi}) \tag{5}$$

$$\delta_{(i|t)} = e\alpha(r_{(i|t)} - C(V_i)) \tag{6}$$

$$V_{i+1} = C(V_i) + \alpha\delta_{(i|t)} \tag{7}$$

2.3 Variant 3: Compression outside of eligibility function

Here, we only compress values that lie outside of the eligibility function. The big difference from variant 2 is that the prediction error is computed against the uncompressed value vector, then these are summed in the usual weighted fashion across basis weights.

Compressed values

$$C(V_i) = \exp(\frac{V_i}{\phi}) \tag{8}$$

Loss function

$$L(V_i) = V_i - C_i \tag{9}$$

$$\delta_{(i|t)} = e\alpha(r_{(i|t)} - V_i) \tag{10}$$

$$V_{i+1} = V_i + \delta_{(i|t)} - (1 - e)L(V_i) \tag{11}$$