KAON 2022, September 16th 2022

$K \to \mu^+\mu^-$ as a third kaon

golden mode

Avital Dery

AD, Ghosh, Grossman, Schacht,
https://arxiv.org/pdf/2104.06427.pdf

The message of this talk:

A measurement of time dependence in $K \to \mu^+\mu^-$ is sensitive to the CKM parameter $\bar{\eta}$ with theory uncertainty of $\mathcal{O}(1\%)$

$K_L \to \pi^0 \nu\bar{\nu}$

$K^+ \to \pi^+ \nu\bar{\nu}$

Theoretically clean, sensitive to functions of $V_{ts}^* V_{td}$, $V_{cs}^* V_{cd}$.

What about $K_{L,S} \to \mu^+ \mu^-$?

K_L

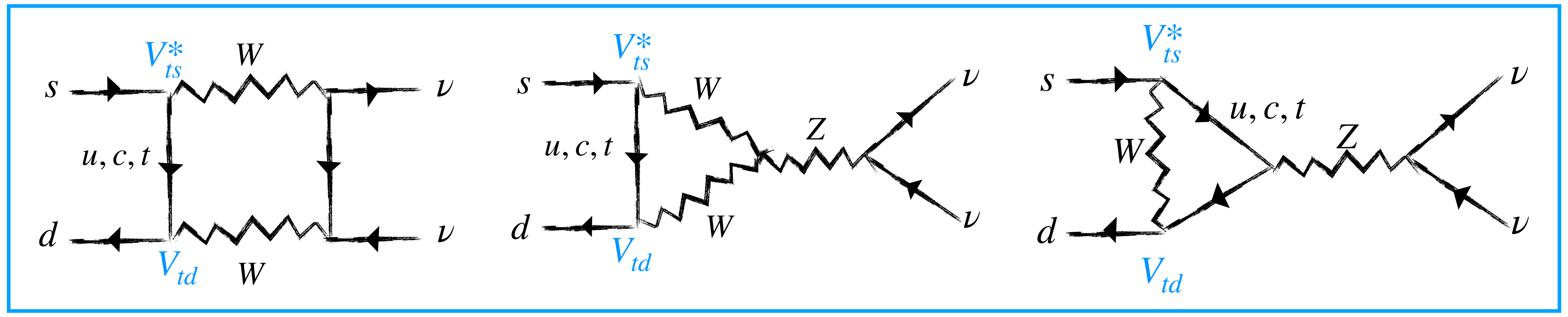
K_S

- Experimentally straightforward
- Similar weak Hamiltonian (also sensitive to $V_{ts}^* V_{td}$, $V_{cs}^* V_{cd}$)

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \sum_{l=e,\mu,\tau} \left(V_{cs}^* V_{cd} X_{NL}^l + V_{ts}^* V_{td} X(x_t) \right) (\bar{s}d)_{V-A} (\bar{\nu}_l \nu_l)_{V-A}$$

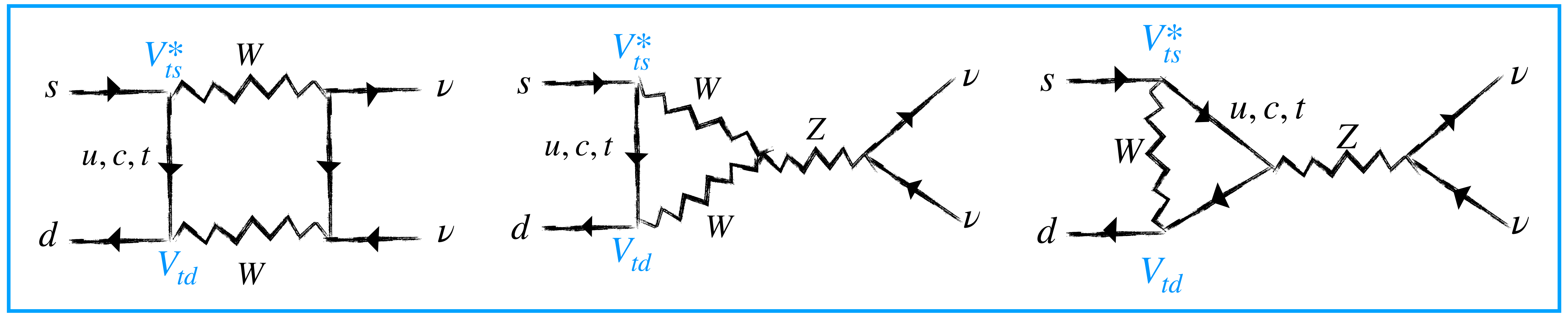
$K_L \to \pi^0 \nu\bar{\nu}$

- Within the SM determined entirely from the weak effective Hamiltonian (short-distance physics)
- Purely CP-violating



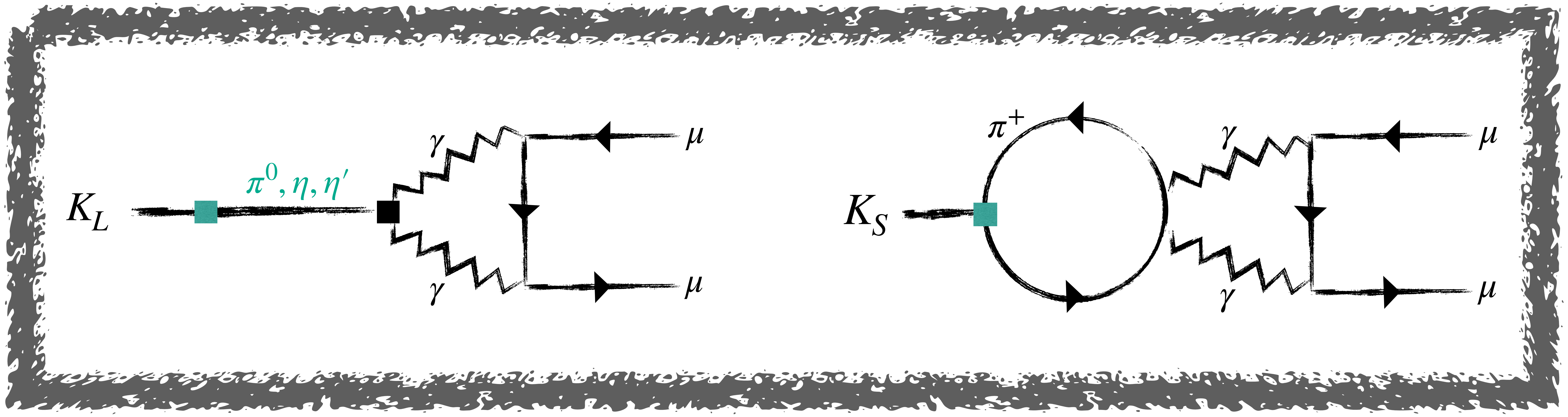
$K_L \to \pi^0 \nu\bar{\nu}$

- Within the SM determined entirely from the weak effective Hamiltonian (short-distance physics)
- Purely CP-violating



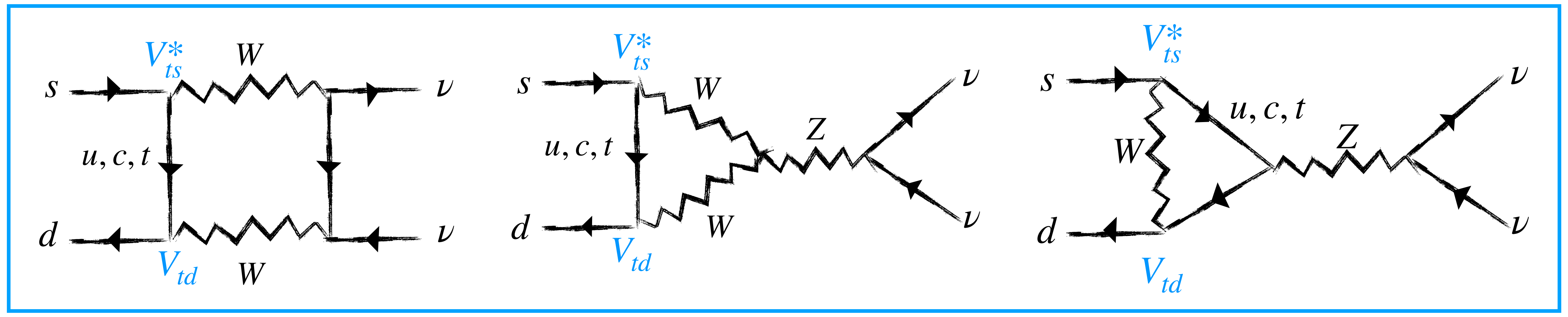
$K_{L,S} \to \mu^+\mu^-$

Dominated by Long-distance effects



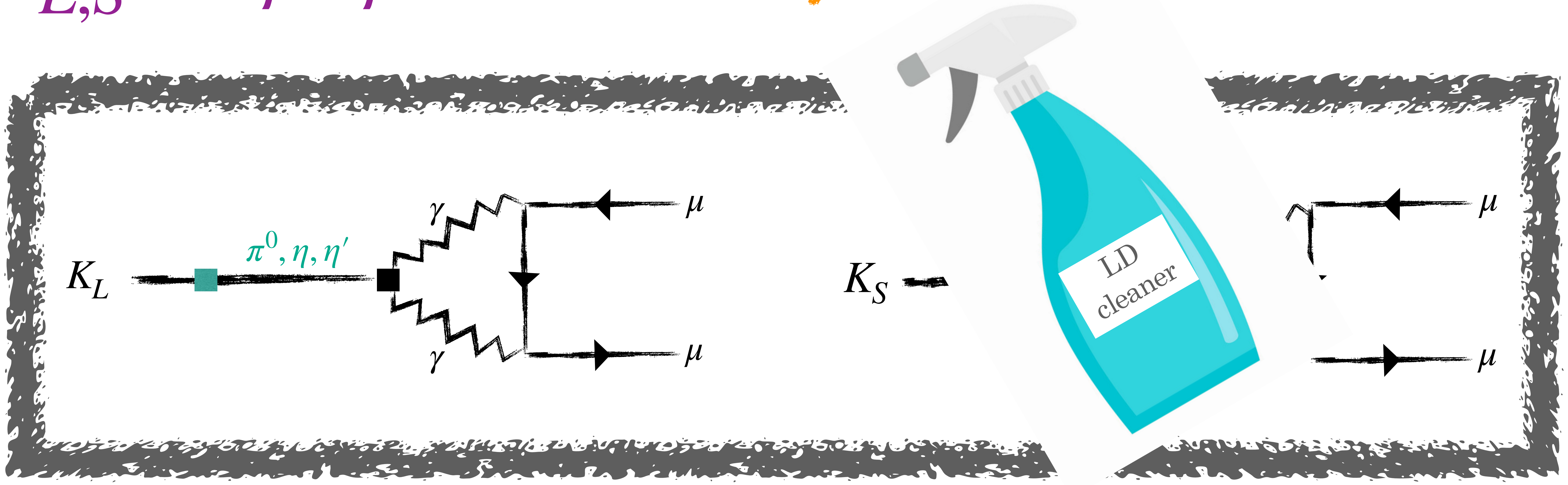
$K_L \to \pi^0 \nu\bar{\nu}$

- Within the SM determined entirely from the weak effective Hamiltonian (short-distance physics)
- Purely CP-violating



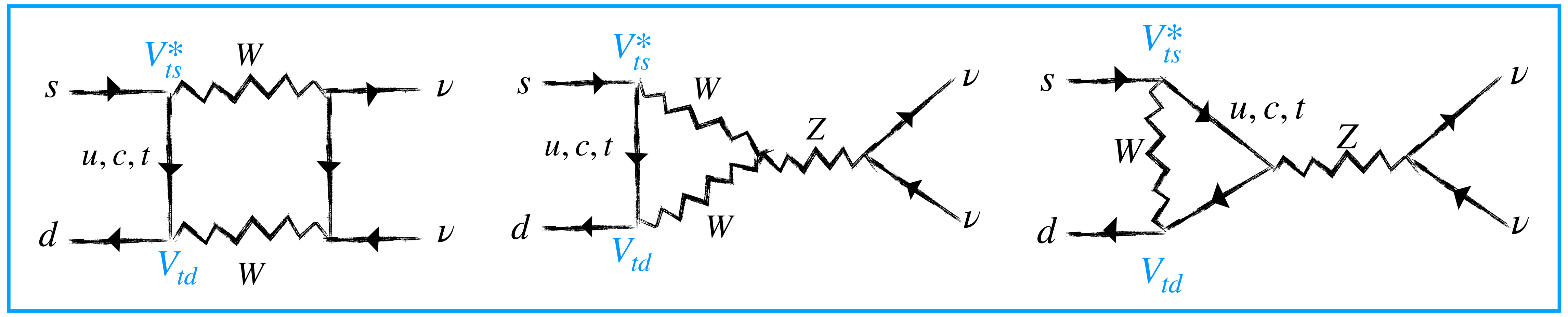
$K_{L,S} \to \mu^+\mu^-$

Dominated by Long-d effects



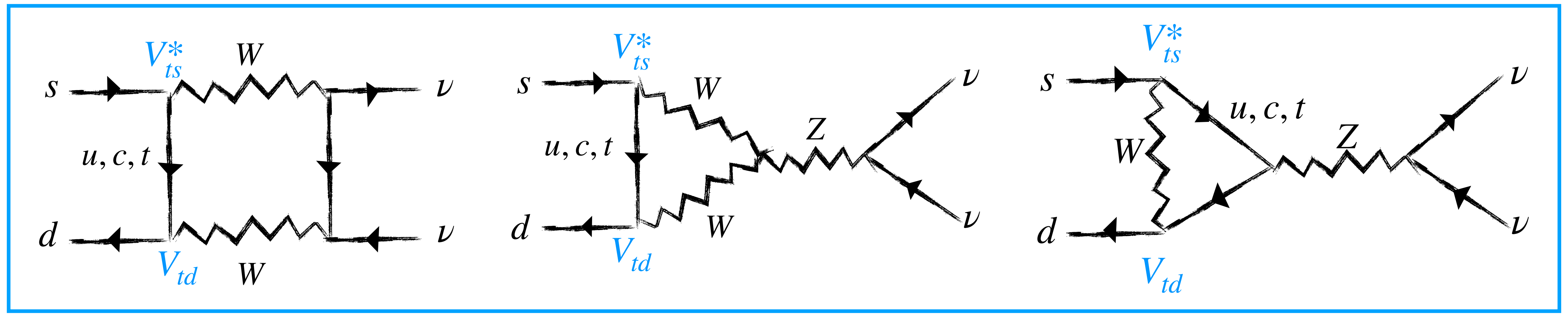
$K_L \to \pi^0 \nu \bar{\nu}$

- Within the SM determined entirely from the weak effective Hamiltonian (short-distance physics)
- Purely CP-violating



$K_{L,S} \to \mu^+ \mu^-$

Exactly the same CKM dependence



In numbers

$$\mathscr{B}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$$

$$\mathscr{B}(K_S \to \mu^+\mu^-)_{\text{LHCb}} < 2.1 \cdot 10^{-10}$$

$$\boxed{\mathscr{B}(K_L \to \mu^+\mu^-)_{\text{SM}} = \begin{cases} (6.85 \pm \underbrace{0.80}_{\text{LD}} \pm \underbrace{0.06}_{\text{SD}}) \times 10^{-9}\,(+) \\ (8.11 \pm 1.49 \pm 0.13) \times 10^{-9}\,(-) \end{cases}}$$

$$\boxed{\mathscr{B}(K_S \to \mu^+\mu^-)_{\text{SM}} = (5.18 \pm \underbrace{1.50}_{\text{LD}} \pm \underbrace{0.02}_{\text{SD}}) \times 10^{-12}}$$

How can we get rid of the long-distance contributions?

The key: CPV comes from the weak (SD) diagrams

Framework

We work under the following approximations -

1. We neglect CPV in mixing, negligible for our purposes, $\mathcal{O}(\varepsilon_K) \sim 10^{-3}$
2. We assume that the leptonic current is vectorial - within the SM this is an approximation, good to $\mathcal{O}(m_K^2/m_W^2)$
3. We neglect CPV in the long-distance physics, of $\mathcal{O}(\lambda^4) \sim 10^{-3}$

All fulfilled in the SM to $\mathcal{O}(10^{-3})$

CP analysis of $K \to \mu^+\mu^-$

Initial state: kaon mass eigenstates are CP eigenstates,

$$\boxed{\underset{\text{CP-odd}}{K_L}\,, \qquad \underset{\text{CP-even}}{K_S}}$$

Final state: since the kaon has $J = 0$, the dimuon state can have either $S = 0, \ell = 0$

or $S = 1, \ell = 1$

corresponding to final states:

$$\boxed{\underset{\text{CP-odd}}{(\bar{\mu}\mu)_{\ell=0}}, \qquad \underset{\text{CP-even}}{(\bar{\mu}\mu)_{\ell=1}}}$$

CP analysis of $K \to \mu^+\mu^-$

Initial state: kaon mass eigenstates are CP eigenstates,

$$\underset{\text{CP-odd}}{K_L}\,, \qquad \underset{\text{CP-even}}{K_S}$$

Final state: since the kaon has $J = 0$, the dimuon state can have either $S = 0, \ell = 0$ or $S = 1, \ell = 1$

corresponding to final states:

$$\underset{\text{CP-odd}}{(\bar{\mu}\mu)_{\ell=0}}, \qquad \underset{\text{CP-even}}{(\bar{\mu}\mu)_{\ell=1}}$$

In practice, we measure the incoherent sums,

$$\Gamma(K_S \to \mu^+\mu^-)_{meas.} = \Gamma(K_S \to (\mu^+\mu^-)_{\ell=0}) + \Gamma(K_S \to (\mu^+\mu^-)_{\ell=1})$$

$$\Gamma(K_L \to \mu^+\mu^-)_{meas.} = \Gamma(K_L \to (\mu^+\mu^-)_{\ell=0}) + \Gamma(K_L \to (\mu^+\mu^-)_{\ell=1})$$

If we could extract the CPV modes, we would have a similar situation (theoretically) to $K_L \to \pi^0\nu\bar{\nu}$

Time dependent rate

G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

$\Gamma_S \simeq 500\,\Gamma_L$

$\Gamma = \underbrace{\frac{\Gamma_L + \Gamma_S}{2}}\simeq \frac{\Gamma_S}{2}$

$$\left(\frac{d\Gamma}{dt}\right) = N_f\, f(t)\,, \qquad f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\, C_{Int.} \cos(\Delta m t - \varphi_0) e^{-\Gamma t}\,,$$

4 Experimental parameters

$\{C_L, C_S, C_{Int.}, \varphi_0\}$

Time dependent rate

G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

$$\left(\frac{d\Gamma}{dt}\right) = N_f\, f(t)\,, \qquad f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\, C_{Int.} \cos(\Delta m t - \varphi_0) e^{-\Gamma t}\,,$$

4 Experimental parameters

Theory parameters: 4 amplitudes and 2 phases

SD $|A(K_S \to (\mu^+\mu^-)_{\ell=0})|$

$|A(K_S \to (\mu^+\mu^-)_{\ell=1})|$ $\qquad \varphi_0 \equiv \arg(A(K_S)_0^* A(K_L)_0)$

$|A(K_L \to (\mu^+\mu^-)_{\ell=0})|$ $\qquad \varphi_1 \equiv \arg(A(K_S)_1^* A(K_L)_1)$

SD $|A(K_L \to (\mu^+\mu^-)_{\ell=1})|$

A priori, 6 theory parameters

Time dependent rate

G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

$$\left(\frac{d\Gamma}{dt}\right) = N_f\, f(t)\,, \qquad f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\, C_{Int.} \cos(\Delta m t - \varphi_0) e^{-\Gamma t}\,,$$

4 Experimental parameters
$\{C_L, C_S, C_{Int.}, \varphi_0\}$

Theory parameters: 4 amplitudes and 2 phases

SD $|A(K_S \to (\mu^+\mu^-)_{\ell=0})|$
$|A(K_S \to (\mu^+\mu^-)_{\ell=1})|$
$|A(K_L \to (\mu^+\mu^-)_{\ell=0})|$
SD $\cancel{|A(K_L \to (\mu^+\mu^-)_{\ell=1})|}$

$\varphi_0 \equiv \arg(A(K_S)_0^* A(K_L)_0)$
$\cancel{\varphi_1 \equiv \arg(A(K_S)_1^* A(K_L)_1)}$

Under our approximations two are negligible

A priori, ~~6~~ theory parameters
4, 1 of which is pure SD

$\{\, |A(K_S)_0|, |A(K_L)_0|, |A(K_S)_1|, \arg(A(K_S)_0^* A(K_L)_0)\}$

Entire system can be solved

4 Experimental parameters

$\{C_L, C_S, C_{Int.}, \varphi_0\}$ $\longleftrightarrow$ 4 theory parameters $\{\,|A(K_S)_0|, |A(K_L)_0|, |A(K_S)_1|, \arg(A(K_S)_0^* A(K_L)_0)\}$

$$
\begin{aligned}
C_L &= |A(K_L)_0|^2 \\
C_S &= |A(K_S)_0|^2 + \beta_\mu^2 |A(K_S)_1|^2 \\
C_{Int.} &= D\,|A(K_S)_0 A(K_L)_0| \\
\varphi_0 &= \arg\left(A(K_S)_0 A(K_L)_0\right)
\end{aligned}
$$

$$D = \frac{N_{K^0} - N_{\overline{K}^0}}{N_{K^0} + N_{\overline{K}^0}}$$

In particular, we can solve for the CPV amplitude $|A(K_S)_0|$

$$\boxed{\frac{1}{D^2}\frac{C_{Int.}^2}{C_L} = |A(K_S)_0|^2}$$

SM prediction

G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

$$|A(K_S)_0|^2 = \left| \frac{G_F}{2} \frac{2\alpha_{em} m_K m_\mu Y(x_t)}{\pi \sin^2\theta_W} \times f_K \times V_{cs}V_{cd}\,\mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{cs}^* V_{cd}}\right) \right|^2$$

SM prediction

G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

Only hadronic parameter, $\mathcal{O}(1\%)$
uncertainty from isospin breaking

$$\boxed{|A(K_S)_0|^2 = \left| \frac{G_F}{2} \frac{2\alpha_{em} m_K m_\mu Y(x_t)}{\pi \sin^2 \theta_W} \times f_K \times V_{cs} V_{cd} \, \mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{cs}^* V_{cd}} \right) \right|^2}$$

SM prediction

G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

Only hadronic parameter, $\mathcal{O}(1\%)$ uncertainty from isospin breaking

$$\boxed{|A(K_S)_0|^2 = \left| \frac{G_F}{2} \frac{2\alpha_{em} m_K m_\mu Y(x_t)}{\pi \sin^2\theta_W} \times f_K \times \underbrace{V_{cs}V_{cd}\,\mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{cs}^* V_{cd}}\right)}_{} \right|^2}$$

$$\bar{\eta}_{ds} = A^2\lambda^4\bar{\eta} + \mathcal{O}(\lambda^7)$$

SM prediction

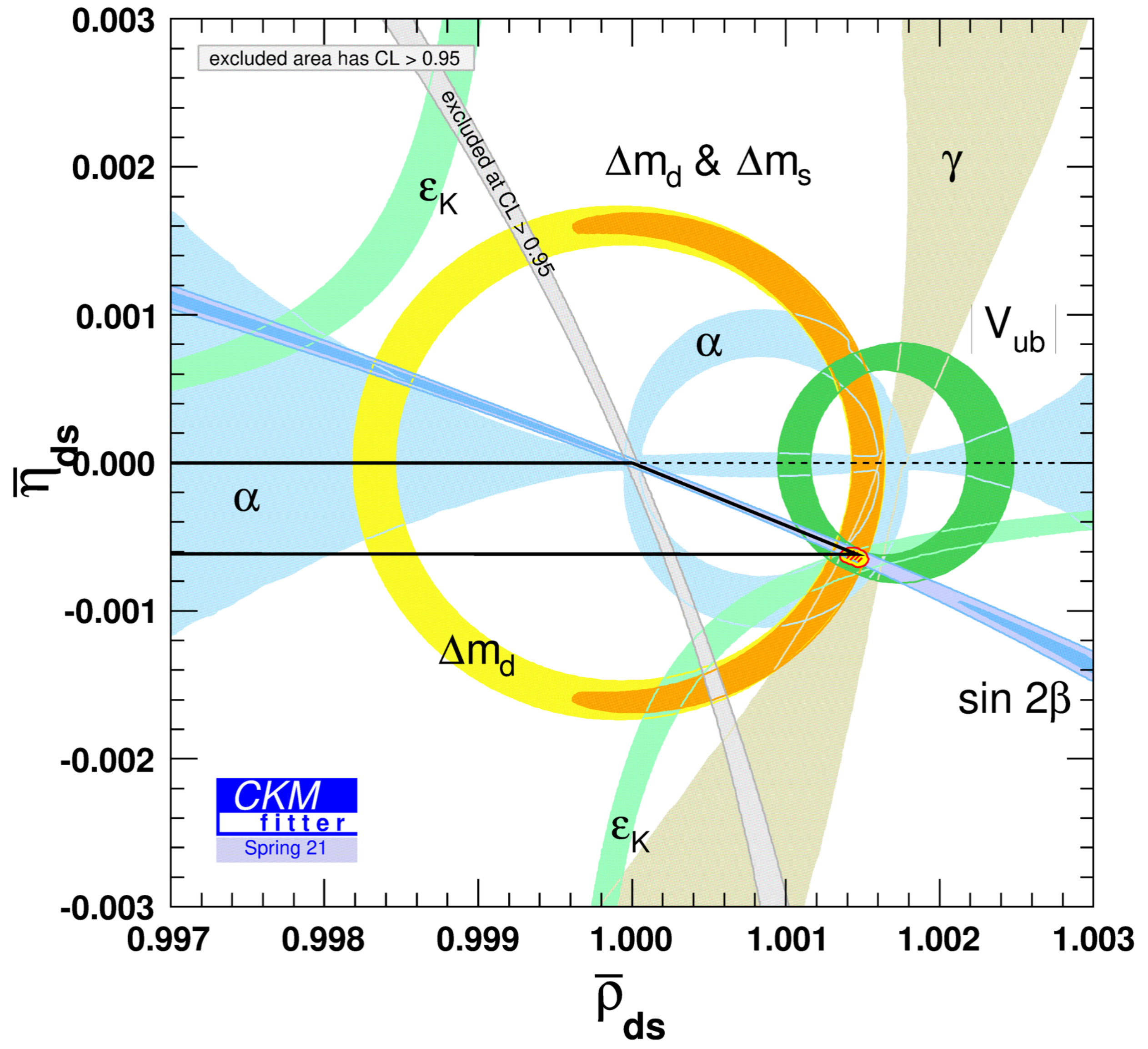
G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

Only hadronic parameter, $\mathcal{O}(1\%)$
uncertainty from isospin breaking

$$|A(K_S)_0|^2 = \left| \frac{G_F}{2} \frac{2\alpha_{em} m_K m_\mu Y(x_t)}{\pi \sin^2\theta_W} \times f_K \times V_{cs}V_{cd}\, \underbrace{\mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{cs}^* V_{cd}}\right)} \right|^2$$

$$\bar{\eta}_{ds} = A^2\lambda^4\bar{\eta} + \mathcal{O}(\lambda^7)$$

Current error on $\bar{\eta}_{ds}$
from B physics is $\mathcal{O}(5\%)$



SM prediction

[G. D'Ambrosio and T. Kitahara [arXiv:1707.06999]
AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]]

Only hadronic parameter, $\mathcal{O}(1\%)$
uncertainty from isospin breaking

$$|A(K_S)_0|^2 = \left| \frac{G_F}{2} \frac{2\alpha_{em} m_K m_\mu Y(x_t)}{\pi \sin^2\theta_W} \times f_K \times \underbrace{V_{cs}V_{cd}\,\mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{cs}^* V_{cd}}\right)} \right|^2$$

$\bar{\eta}_{ds} = A^2\lambda^4\bar{\eta} + \mathcal{O}(\lambda^7)$

THE message for this talk:

A measurement of $\Gamma(K \to \mu^+\mu^-)(t)$ would provide a clean determination of $\bar{\eta}$ from kaon physics, with ultimate theory uncertainty under 1 %

→ golden mode

Complementarity with $K_L \to \pi^0 \nu \bar{\nu}$

$$R_{\rm SL} = \frac{\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SD}}{\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})} = 1.55 \times 10^{-2} \left[\frac{\lambda}{0.225}\right]^2 \left[\frac{Y(x_t)}{X(x_t)}\right]^2$$

[A. J. Buras and E. Venturini, [arXiv:2109.11032]]

Eliminates the dependence on $|V_{cb}|^4$

Complementary probe of NP models

[**AD**, M. Ghosh [arXiv:2112.05801]]

How feasible is this experimentally?

Next talk by Rado Marchevski!

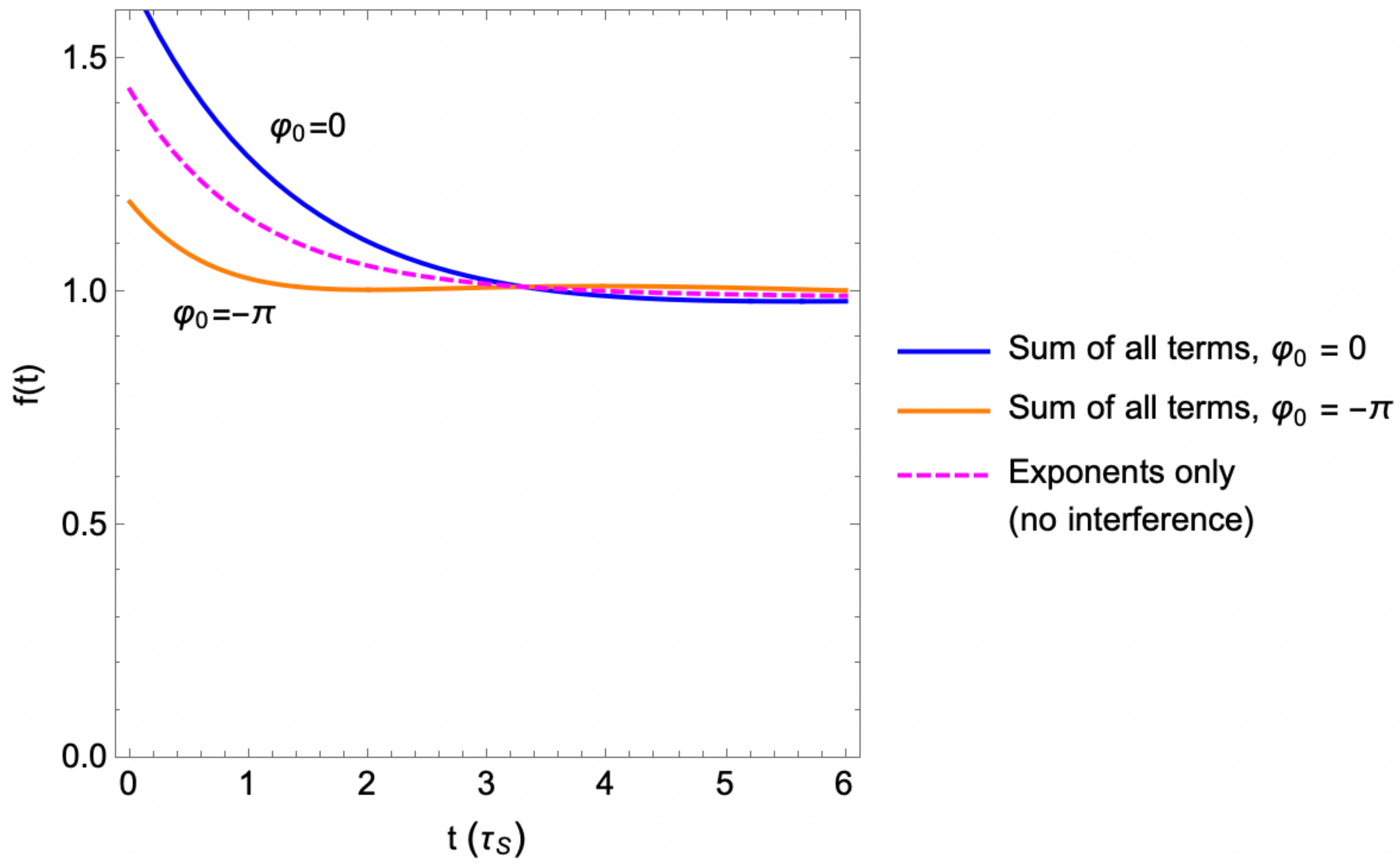
Some theory input on feasibility

AD, M. Ghosh, Y. Grossman, S. Schacht
(to appear)

What is the value of φ_0?

Will affect the sensitivity of the measurement

$$f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\, C_{Int.} \cos(\Delta m t - \varphi_0) e^{-\Gamma t},$$



Some theory input on feasibility

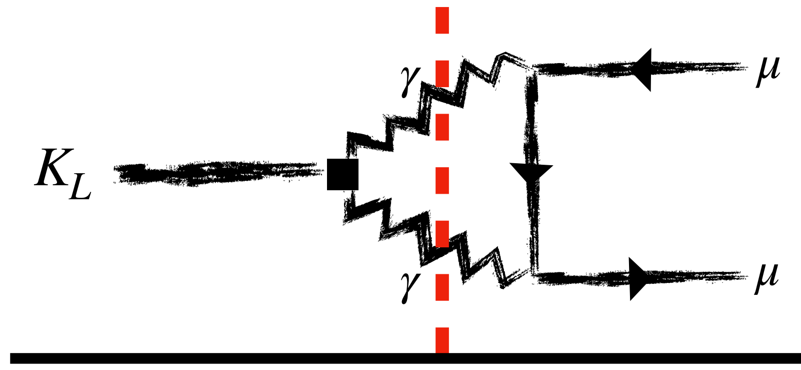
AD, M. Ghosh, Y. Grossman, S. Schacht
(to appear)

What is the value of φ_0?

$$f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\, C_{Int.} \cos(\Delta m t - \varphi_0) e^{-\Gamma t},$$

Will affect the sensitivity of the measurement

$$\cos^2 \varphi_0 = \frac{\left(\mathrm{Im} A(K_L)\right)^2}{|A(K_L)|^2} =$$



$$\overline{\mathscr{B}(K_L \to \mu^+\mu^-)} = C_{\mathrm{QED}} \frac{\mathscr{B}(K_L \to \gamma\gamma)}{\mathscr{B}(K_L \to \mu^+\mu^-)}$$

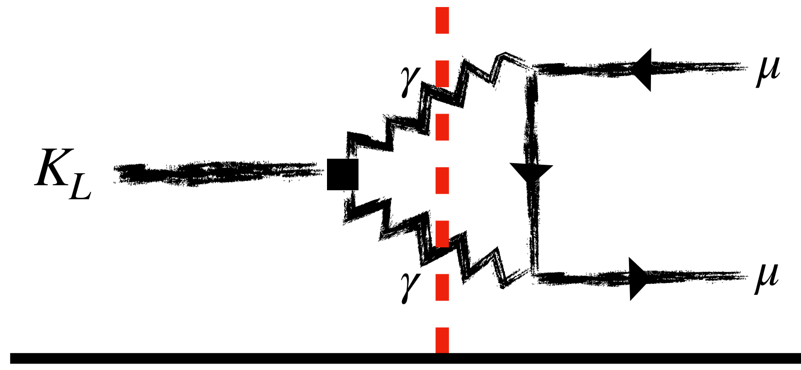
Some theory input on feasibility

[**AD**, M. Ghosh, Y. Grossman, S. Schacht (to appear)]

What is the value of φ_0?

$$f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\, C_{Int.} \cos(\Delta m t - \varphi_0) e^{-\Gamma t},$$

Will affect the sensitivity of the measurement



$$\cos^2\varphi_0 = \frac{\left(\mathrm{Im}A(K_L)\right)^2}{|A(K_L)|^2} = \frac{[\text{diagram}]}{\mathcal{B}(K_L \to \mu^+\mu^-)} = C_{\mathrm{QED}} \frac{\mathcal{B}(K_L \to \gamma\gamma)}{\mathcal{B}(K_L \to \mu^+\mu^-)}$$

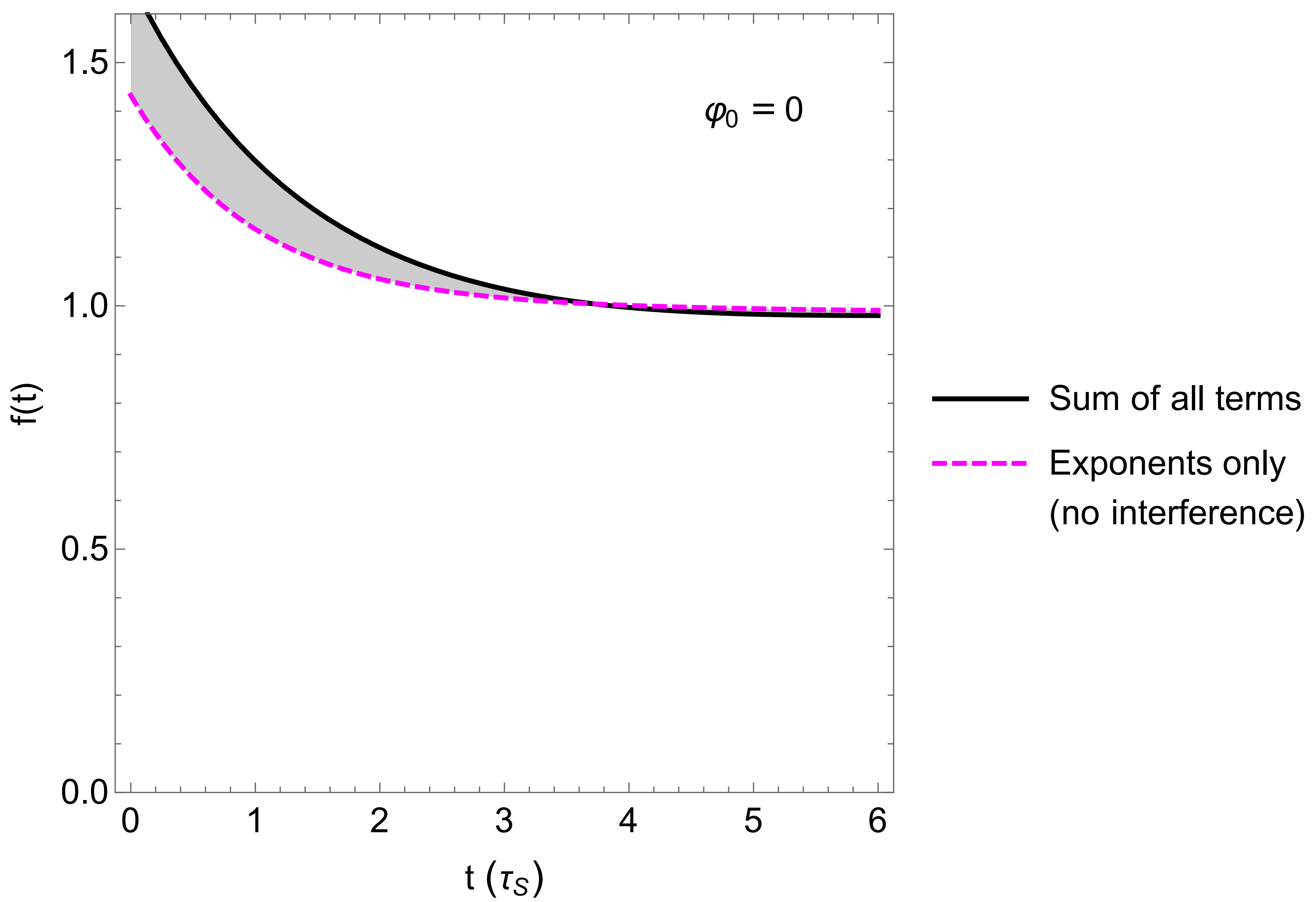
$$\left[\cos\varphi_0\right]_{\mathrm{SM}} = 0.978 \pm 0.009$$

Close to maximal

Some theory input on feasibility

[**AD**, M. Ghosh, Y. Grossman, S. Schacht (to appear)]

$$f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\, C_{Int.}\, \cos(\Delta m t - \varphi_0) e^{-\Gamma t},$$



Approximate SM prediction, $D = 1$:

$(C_L)_{\rm SM} \equiv 1$

$(C_S)_{\rm SM} \approx 0.43$

$(C_{Int.})_{\rm SM} \approx 0.12$

Naive estimate - $\mathcal{O}(10^{13})$ Kaons are needed

Summary and Conclusions

- The time-dependent rate in $K \to \mu^+\mu^-$ is a complementary golden mode, allowing for a clean independent determination of CKM parameters from kaon physics.

- The ultimate theoretical uncertainty is of $\mathcal{O}(1\%)$

- The relevant CKM parameter is $\bar{\eta}_{ds} \approx A^2\lambda^4\bar{\eta}$, the same as for $K_L \to \pi^0\bar{\nu}\nu$.

- The ratio $\mathcal{B}(K_S \to \mu^+\mu^-)_{\ell=0} / \mathcal{B}(K_L \to \pi^0\bar{\nu}\nu)$ is an extremely clean SM observable, dependent only on the parameters λ, m_t .

  [A. J. Buras and E. Venturini, [arXiv:2109.11032]]

- The same $K \to \mu^+\mu^-$ observable is also a sensitive probe of NP

  [**AD**, M. Ghosh [arXiv:2112.05801]]

- The phase shift in the oscillating rate, φ_0, is an additional clean SM prediction, and implies close to maximal integrated event rate.

  [**AD**, Ghosh, Grossman, Schacht [in preparation]]

- A theorist's estimation - we need $\mathcal{O}(10^{13})$ neutral kaons, with $\mathcal{O}(1)$ Dilution factor

Thank you