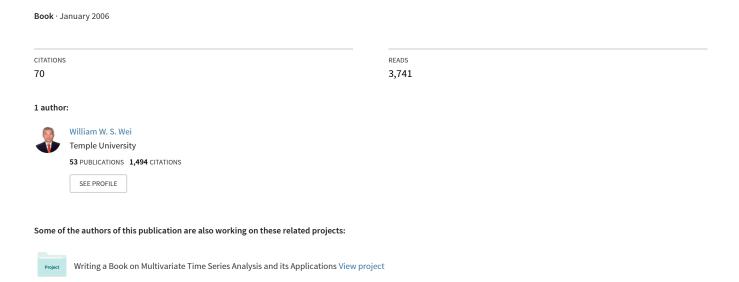
#### Time Series Analysis: Univariate and Multivariate Methods, 2nd edition, 2006



## Time Series Analysis

#### Univariate and Multivariate Methods

SECOND EDITION

#### William W. S. Wei

Department of Statistics The Fox School of Business and Management Temple University



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The cover image appears as Figure 17.1 within the text and embodies all the concepts of time series analysis. For instance, the thin solid line represents stationary series as discussed in Chapters 2 and 3; the heavy solid line is nonstationary series as discussed in Chapter 4. As you learn more, you may also see the concepts of a second nonstationary series, forecasting, backcast, seasonal series, heterocedasticity and GARCH models, input and output series, noise series, vector time series, nonlinear time series, and cointegration in the cover image.

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William W. S. Wei is Professor of Statistics at Temple University in Philadelphia, PA. He earned his B.A. in Economics from the National Taiwan University (1966), B.A. in Mathematics from the University of Oregon (1969), and M.S. (1972) and Ph.D. (1974) in Statistics from the University of Wisconsin-Madison. From 1982-87, he was the Chair of the Department of Statistics at Temple University. His research interest includes time series analysis, forecasting methods, statistical modeling, and applications of statistics in business and economics. He has developed new methodology in seasonal adjustment, aggregation and disaggregation, outlier detection, robust estimation, and vector time series analysis. Some of his most significant contributions include extensive research on the effects of aggregation, methods of measuring information loss due to aggregation, new stochastic procedures of performing data disaggregation, model-free outlier detection techniques, robust methods of estimating autocorrelations, and statistics for analyzing multivariate time series. He is a Fellow of the American Statistical Association, a Fellow of the Royal Statistical Society, and an Elected Member of the International Statistical Institute. He is an Associate Editor of the Journal of Forecasting and the Journal of Applied Statistical Science.

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