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# Time Series Analysis

## *Univariate and Multivariate Methods*

SECOND EDITION

**William W. S. Wei**

*Department of Statistics*

*The Fox School of Business and Management*

*Temple University*



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William W. S. Wei is Professor of Statistics at Temple University in Philadelphia, PA. He earned his B.A. in Economics from the National Taiwan University (1966), B.A. in Mathematics from the University of Oregon (1969), and M.S. (1972) and Ph.D. (1974) in Statistics from the University of Wisconsin–Madison. From 1982–87, he was the Chair of the Department of Statistics at Temple University. His research interest includes time series analysis, forecasting methods, statistical modeling, and applications of statistics in business and economics. He has developed new methodology in seasonal adjustment, aggregation and disaggregation, outlier detection, robust estimation, and vector time series analysis. Some of his most significant contributions include extensive research on the effects of aggregation, methods of measuring information loss due to aggregation, new stochastic procedures of performing data disaggregation, model-free outlier detection techniques, robust methods of estimating autocorrelations, and statistics for analyzing multivariate time series. He is a Fellow of the American Statistical Association, a Fellow of the Royal Statistical Society, and an Elected Member of the International Statistical Institute. He is an Associate Editor of the *Journal of Forecasting* and the *Journal of Applied Statistical Science*.

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