Optimal Group Orderings for Bias Bounties

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Introduction

The Bias Bounty competition held in class generated a large sum of data from independent teams each trying to solve the same binary classification task. While this is not the implementation which Bias Bounties were originally intended for, there are interesting methods which this version can be adapted into a competition which yields a similar final product indistinguishable from the Bayes Optimal Classifier.

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Use Cases:

Given a Kaggle style competition where teams submit their best Pointer Decision List (PDL), construct a new PDL which has optimal (lowest) error over the test set using all subgroup-hypothesis pairs submitted from any group.

Motivation

Let $X = [x_1, x_2, x_3, x_4]$ and $Y = [1, 1, 1, 1]$. Suppose there are three subgroup-hypothesis pairs

$$g_1(X) = [1, 1, 1, 1],\ g_2(X) = [1, 1, 1, 1],\ g_3(X) = [1, 1, 0, 0]$$
$$h_1(X) = [1, 1, 1, 0],\ h_2(X) = [0, 0, 1, 1],\ h_3(X) = [1, 1, 0, 0].$$

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$\sigma' = (3, 2, 1),\ f_{\sigma'}(X) = 0$

Coalition Game Theory

- A **coalition game** (N, v) consists of a set of players $N = \{1, \ldots, n\}$ and a **characteristic function** $v : 2^n \to \mathbb{R}$ which maps subsets (or coalitions) of players S to real valued payoffs written as $v(S)$.

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Example:

$$v(S) = \begin{cases} 1 & \text{if } S = N \\ 0 & \text{otherwise.} \end{cases}$$

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Fair distribution of the reward is $1/n$ for each player in the case where $S = N$.

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Should each player receive an equal piece of the pie in a game?

Payoff Functions

Given a coalition game (N, v), we define a payoff function $\psi : \mathbb{N} \times \mathbb{R}^{2^n} \to \mathbb{R}^n$ which returns each player's marginal payoff. We denote player i's payoff by $\psi_i(N, v)$.

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How do we ensure that a payoff function $\psi_i(N, v)$ is 'fair' for each player *i*?

Axiomatic Fairness

1. **Axiom 1** (Symmetry) For any subset S not containing players i and j, if $v(S \cup \{i\}) = v(S \cup \{j\})$, then i, j are said to be equivalent. For any v, if i and j are equivalent players, then $\psi_i(N, v) = \psi_j(N, v)$.

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2. **Axiom 2** (Null Player) A player i is said to be null if for any subset S not containing i, $v(S \cup \{i\}) - v(S) = v(\{i\})$. For any v, if i is a null player, then $\psi_i(N, v) = v(\{i\})$.

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3. **Axiom 3** (Additivity) Given two characteristic functions v_1, v_2 over the same players in different coalition games, then for any player i, we have $\psi_i(N, v_1 + v_2) = \psi_i(N, v_1) + \psi_i(N, v_2)$ where the game $(N, v_1 + v_2)$ is defined by $(v_1 + v_2)(S) = v_1(S) + v_2(S)$ for any subset S.

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Shapley Value

Theorem (Shapley Value)

There exists a unique function ψ which satisfies Axioms 1-3, for games with finite coalitions; it is given by the formula

$$\psi_i(N, v) = \frac{1}{|N|!} \sum_{S \subseteq N} |S|!(|N| - |S| - 1)! \left(v(S) - v(S - \{i\}) \right).$$

Connection to Bias Bounties

Suppose we have a universe of subgroup-hypothesis pairs $U = \{(g_1, h_1), \ldots, (g_n, h_n)\}$. Let Σ be the permutation set of all orderings of pairs in U. Then, a permutation $\sigma \in \Sigma$ is an order of the n pairs and is an ordered subset of U.

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Define the coalition game (N, v) where $N = U$ and $v(\sigma)$ is the error on the PDL where pairs are iteratively introduced to an updater function in order defined by σ. Let $v(\sigma^{-i})$ be the error on the PDL defined by permutation σ where pair i is not introduced.

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The Shapley Value of player (pair) i is semantically equivalent to

$$\psi_i(N, v) = \sum_{\sigma \in \Sigma} \left(v(\sigma) - v(\sigma^{-i}) \right)$$

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Implementation

- Data set: California, 2018, Income Task, same features as Bias Bounties class project, 30,000 instances.
- 10 subgroup-hypothesis pairs which are accepted in order 1-10.
- Code working in parallel on 30 series graphics card.

Experiment 1

1. Using pairs 1-8, create the permutation list of all orderings using the 8 pairs.
2. Construct every PDL created from a permutation in the permutation list and store its overall error on the test set.
3. Output an optimal ordering with the lowest error on the test set as well as a worst ordering with highest error on the test set.

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Best error: 0.15383333333333338
Best Order: (1, 2, 3, 4, 5, 6, 7, 8)
Worst error: 0.15733333333333333
Worst Order: (1, 2, 3, 5, 8, 4, 6, 7)
Total time (seconds): 78542.59 $\approx$ 22 hours

Experiment 2

1. Using pairs 1-8, create the permutation list of all orderings using the 8 pairs.
2. Randomly select permutation and compute the Shapley contribution of each pair in the specific permutation
3. Loop process until termination or complete necessary number of trials
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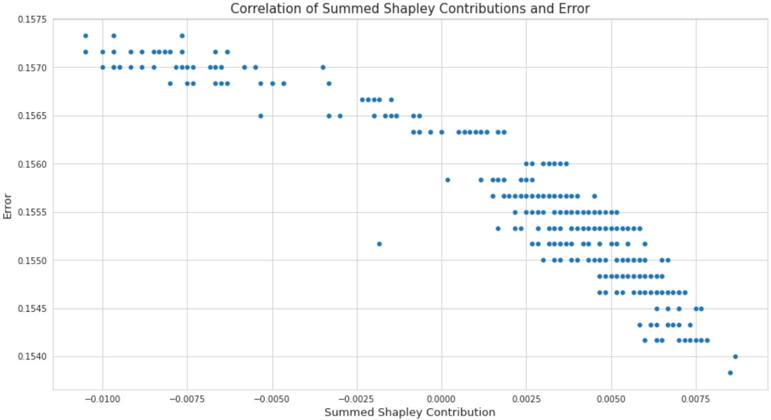
Computed approximately 1,800 Shapley contribution values over the course of 24 hours.

Results (Main)



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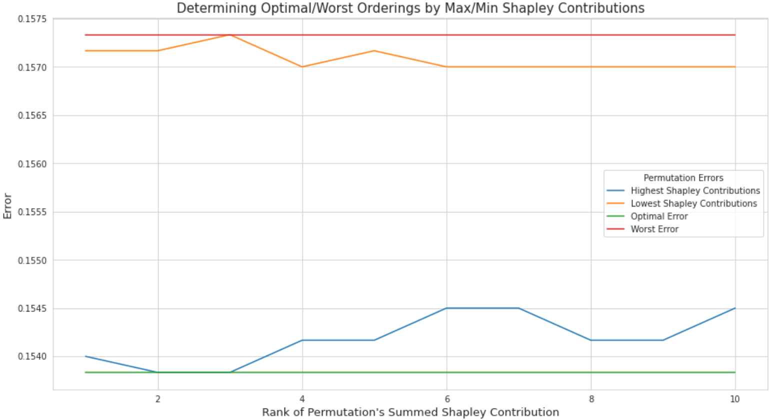

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Future Plans

- Using average Shapley values to solve the max sum optimization problem for Shapley contributions by permutation ordering.
- Bounding the gap between optimal and worst case error given a universe of subgroup-hypothesis pairs and their expected Shapley values.