



(2.3.8) Amortized Analysis & Dynamic Array



-> Using an Amortized analysis, we can show that the average Cost of an operation is small, even though a single operation within the sequence might be expensive.

Suppose you commute by bus and have 2 options ① Single ticket - Rs 50 per ride ② Monthly travel pass - Rs 1000, unlimited side

17 you ride bus once a month-Rs 1000

-> Example:

If you ride bus 50 times a month > [Rs 1000, 0, 0, 0, 0, 0 --- 0] Cost per ride = Rs 1000/50 = Rs 20 -> Cheaper per ride.

-> Amortized analysis is implicitly used in cost analysis of many Algorithms.



Dynamic Array

- -> A dynamic array is an array that:

 ① Automatically resizes itself when it becomes full.
 - (2) Allows O(i) random access like a normal array.
 - 3 Grows in Amortized O(1) time per insertion.
 - * How it works?
- we start with empty dynamic array of size 0.
- -> Let's define
 - 10 when the array is full, we double its size.
 - 2 Insert n element one by one
- -> Each insert works like this
 - (1) If space is available \rightarrow insert in O(1) time
 - 2 If full create a new array of double size and

Ly Copy all old elements Ly Insert the new element.



* Amostized Time Complexity Analysis

Resize no.	New Capacity	Copy Cost
N. Committee	1	0
2	2	1
3	4	2
4	8	Ч
5	16	8
K	(K-1)	2 ^(k-2)

Total Cost of n însextion is

D Each insextion takes 1 unit

2 Plus Cost of Copying during resizes

$$\rightarrow$$
 Let's say $2^{(k-1)} = m$
 $k-1 = \log m$

K = 1+ log m

Total Cost of $n = m + 1 + 2 + 4 + - \cdots + \frac{m}{2}$ Insertion Simple Resize

Insert



Ly Geometric
$$\Rightarrow 2^0 + 2^1 + 2^2 + \dots = n/2$$

Series $\Rightarrow 3^0 + 2^1 + 2^2 + \dots = n/2$
 $\Rightarrow 5n = \frac{\alpha(r^{n-1} - 1)}{r-1} = \frac{1 \cdot (2^{\log n} - 1)}{2 - 1} = (n-1)$

T(n) = O(n)

L) Amortized Cost per Insertion =
$$\frac{T(n)}{n} = \frac{2n-1}{n} \approx 2 = O(1)$$

-> Dynamic Array functionality in Python can be achieved using List data structures.

T(n) = m + (n-1) = 2n-1

List in Python

-> Python List is a dynamic array



- -> Not a linked list uses contiguous memory.
- -> Elements are stored as references to Python objects
- -> Performance Summary.

Operation	Time Complexity	Notes
append()	Amortized - O(1)	occasional o(n) when resized
insert (i, x)	0(n)	Element must be shifted
pop()	0(1)	Last elements
pob(i)	0(n)	shift elements
ust[i]	0(1)	direct index access
(en (list)	O(1)	Stored in metadata



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