

TIME SERIES DFPHASE I

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Summary. Modify the `mphase1` procedure to allow for nonparametric testing of the stability of a time series. Since we assume the goal to be Phase II prospective monitoring, the model is supposed not to contain a stochastic trend. Otherwise, the process would not be stationary by assumption and thus Phase II prospective monitoring of the mean would be useless. We want to estimate location shifts in the Phase I sample, that is, we want to test the hypothesis in Capizzi and Masarotto (2017),

$$\begin{cases} H_0 : \text{all the } \beta_{\text{step}} \text{ and } \beta_{\text{isolated}} \text{ are zero} \\ H_1 : \text{at least one } \beta_{\text{step}} \text{ or } \beta_{\text{isolated}} \text{ is nonzero.} \end{cases}$$

Idea. Substitute the permutation p -value calculation with a procedure based on bootstrap for time series. Specifically, we could consider the following approaches:

› (Circular) Block bootstrap:

1. select block length b ;
2. resample blocks $\mathcal{B} = \{(x_k, x_{k+1}, \dots, x_{(k+b) \bmod m})\}$ to construct the bootstrap time series x_1^*, \dots, x_m^* , where $k \stackrel{\text{iid}}{\sim} \text{DiscreteUniform}(1, \dots, m)$.
3. **Problem:** the resulting resampled stochastic process is not guaranteed to be stationary even when x_1, \dots, x_m is stationary. Thus we possibly include non-stationary samples in the p -value calculation.

› AR-sieve bootstrap:

1. select order p of the (V)AR stationary stochastic process;
2. fit the model and generate pseudo-observations using the estimated model;
3. fit the forward search and calculate the p -values.

› (Circular) Stationary bootstrap:

1. select expected block length b ;
2. resample blocks $\mathcal{B} = \{(x_k, x_{k+1}, \dots, x_{(k+b) \bmod m})\}$ to construct the bootstrap time series x_1^*, \dots, x_m^* , where $k \stackrel{\text{iid}}{\sim} \text{DiscreteUniform}(1, \dots, m)$ and $b \stackrel{\text{iid}}{\sim} \text{Geom}(1/b)$.
3. fit the forward search and calculate the p -values.

REFERENCES

Capizzi, G. and Masarotto, G. (2017). “Phase I Distribution-Free Analysis of Multivariate Data”. In: *Technometrics* 59.4, 484–495.