

Magnetic flux density

$$\partial_t B = -\nabla \times E, \quad \nabla \cdot B = 0 \quad (1)$$

Magnetic field strength and Ohm's Law

$$\nabla \times H = J = \sigma(E + u \times B) \quad (2)$$

For a magnetic material

$$E = -u \times B + \frac{1}{\sigma} \nabla \times H, \quad H = \frac{1}{\mu} B \quad (3)$$

Therefore, the Induction equation

$$\partial_t B = \nabla \times \left( u \times B - \frac{1}{\sigma} \nabla \times \frac{B}{\mu} \right) \quad (4)$$

We can use a vector potential

$$B = \nabla \times A, \quad \partial_t A + \nabla \phi = u \times B - \frac{1}{\sigma} \nabla \times \frac{B}{\mu} \quad (5)$$

In the Column gauge  $\nabla \cdot A = 0$ .

$$\text{In 2D} \quad B = \nabla \times (A \hat{z}), \quad \nabla \times \frac{\nabla \times (A \hat{z})}{\mu} = -\nabla \cdot \left( \frac{1}{\mu} \nabla A \right) \hat{z} \quad (6)$$

$$\partial_t A + u \cdot \nabla A = \sigma^{-1} \nabla \cdot (\mu^{-1} \nabla A) \quad (7)$$

The Lorenz force

$$\mathcal{F} = J \times B = (\nabla \times H) \times B \quad (8)$$

The Maxwell stress

$$\Sigma \equiv \frac{1}{\mu} \left[ B B - \frac{|B|^2}{2} I \right] \quad (9)$$

$$\mathcal{F} = \nabla \cdot \Sigma + \frac{|B|^2}{2\mu} \nabla \log \mu \quad (10)$$