ELEMENTARY NUMBER THEORY

ASSIGNMENTS - LECTURE 1

Letters $a, b, c, \ldots, m, n$ denote integers.

(1) The triple (a, b, c) is said to be a *pythagorean triple* if $a^2 + b^2 = c^2$. If (a, b, c) is a pythagorean triple, show that $60|abc$.
(2) Show that
$$\sum_{k=1}^{n} \frac{1}{k}$$
cannot be an integer for $n > 1$.
(3) If $2^n - 1$ is prime, show that n is prime.
(4) If
$$\frac{1}{a} - \frac{1}{b} = \frac{1}{c}$$
and $h := (a, b, c)$, show that both $abch$ and $h(b - a)$ are perfect squares.
(5) If n is an even number, is possible to write 1 as the sum of the reciprocals of n odd numbers?
(6) Prove that there are infinitely many primes p such that $p - 1$ is multiple of 4.
(7) Let p be a prime number. Given distinct integers m and n, there is an unique $t = t(m, n)$ such that $m - n = p^t k$ where k is an integer not divisible by p. Define a function $d\colon \mathbf{Z} \times \mathbf{Z} \to \mathbf{R}$ by the correspondence $d(m, n) = 0$ for $m = n$ and $d(m, n) = p^{-t}$ for $m \neq n$. Prove that $(\mathbf{Z}, d)$ is a metric space.

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