

Exersice Sheet 2

———— Sample Solution ————

Task 1:

Task 2: Operational Semantics of other Statements

For $c \in Cmd$, $\sigma, \sigma', \sigma'' \in \Sigma$ and $b \in BExp$ the **repeat until relation** $\langle \text{repeat } c \text{ until } b, \sigma \rangle \rightarrow \sigma''$ is defined by:

$$\frac{\langle c, \sigma \rangle \rightarrow \sigma'' \quad \langle b, \sigma'' \rangle \rightarrow \mathbf{true}}{\langle \text{repeat } c \text{ until } b, \sigma \rangle \rightarrow \sigma''} \text{ (repeat-true)}$$

$$\frac{\langle c, \sigma \rangle \rightarrow \sigma' \quad \langle b, \sigma' \rangle \rightarrow \mathbf{false} \quad \langle \text{repeat } c \text{ until } b, \sigma' \rangle \rightarrow \sigma''}{\langle \text{repeat } c \text{ until } b, \sigma \rangle \rightarrow \sigma''} \text{ (repeat-false)}$$

Task 3: Termination

Prove that $\langle \mathbf{while } b \text{ do } c \text{ end}, \sigma \rangle \rightarrow \sigma'$ implies that $\langle b, \sigma' \rangle \rightarrow \mathbf{false}$.
This will be proven by induction on the height h of derivation trees.

Induction Base: ($h=1$)

If the derivation tree has height 1 only one derivation is possible, namely

$$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{false}}{\langle \mathbf{while } b \text{ do } c \text{ end}, \sigma \rangle \rightarrow \sigma'} \text{ (while-false)}$$

Since this rule is unambiguous the induction base holds trivially.

Induction Hypothesis:

$$\langle \mathbf{while } b \text{ do } c \text{ end}, \sigma \rangle \rightarrow \sigma' \text{ implies } \langle b, \sigma' \rangle \rightarrow \mathbf{false}$$

holds for all derivations of an arbitrary, but fixed height h and for all states σ, σ' .

Induction Step: ($h \mapsto h + 1$)

For all derivations of height $h + 1$ ($h \geq 1$), we have

$$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{true} \quad \langle c, \sigma \rangle \rightarrow \sigma' \quad \frac{\dots \text{ (derivation tree of height } h)}{\langle \mathbf{while } b \text{ do } c \text{ end}, \sigma' \rangle \rightarrow \sigma''}}{\langle \mathbf{while } b \text{ do } c \text{ end}, \sigma \rangle \rightarrow \sigma''}$$

By Induction Hypothesis $\langle \mathbf{while } b \text{ do } c \text{ end}, \sigma' \rangle \rightarrow \sigma''$ implies $\langle b, \sigma' \rangle \rightarrow \mathbf{false}$.

Due to the propagating characteristics of the derivation trees we also know that $\langle \mathbf{while } b \text{ do } c \text{ end}, \sigma \rangle \rightarrow \sigma''$ implies $\langle b, \sigma'' \rangle \rightarrow \mathbf{false}$. ■

Task 4: