Exersice Sheet 7

——— **Sample Solution** ———

Task 1: Partial Correctness

(a)

$p := 2;$
$q := n - p;$
while $\neg\textbf{prime}\,(p) \lor \neg\textbf{prime}\,(q)$ **do**
  $p := p + 1;$
  $q := n - p;$
end

(b)

Let $c =$ **while** $\neg\textbf{prime}\,(p) \lor \neg\textbf{prime}\,(q)$ **do**
  $p := p + 1;$
  $q := n - p;$
end

$$\text{(seq)}\ \frac{ⓐ \qquad \text{(seq)}\ \dfrac{ⓑ \quad ⓒ}{\{p = 2\}\ q = n - p;\ c\ \{\textbf{prime}\,(p) \land \textbf{prime}\,(q) \land n = q + p\}}}{\{\textbf{true}\}\ p := 2;\ q = n - p;\ c\,\{\textbf{prime}\,(p) \land \textbf{prime}\,(q) \land n = q + p\}}$$

ⓐ

$$\text{(cons)}\ \frac{\models (\textbf{true} \Rightarrow 2 = 2) \qquad \text{(asgn)}\ \dfrac{}{\{2 = 2\}\ p := 2;\ \{p = 2\}} \qquad \models (p = 2 \Rightarrow p = 2)}{\{\textbf{true}\}\ p := 2;\ \{p = 2\}}$$

ⓑ

$$\text{(cons)}\ \frac{\models (p = 2 \Rightarrow n - p = n - p) \qquad \text{(asgn)}\ \dfrac{}{\{n - p = n - p\}\ q := n - p;\ \{q = n - p\}} \qquad \models (q = n - p \Rightarrow q = n - p)}{\{p = 2\}\ q = n - p;\ \{q = n - p\}}$$

ⓒ

$$\text{(cons)}\ \frac{\models (q = n - p \Rightarrow n = q + p) \qquad \text{(while)}\ \dfrac{ⓓ}{\{n = q + p\}\ c\ \{\textbf{prime}\,(p) \land \textbf{prime}\,(q) \land n = q + p\}} \qquad \models ((\textbf{prime}\,(p) \land \textbf{prime}\,(q) \land n = q + p) \Rightarrow (\textbf{prime}\,(p) \land \textbf{prime}\,(q) \land n = q + p))}{\{q = n - p\}\ c\ \{\textbf{prime}\,(p) \land \textbf{prime}\,(q) \land n = q + p\}}$$

ⓓ

$$\text{(cons)}\ \frac{\models (n = q + p \land (\neg\textbf{prime}\,(p) \lor \neg\textbf{prime}\,(q)) \Rightarrow p + 1 = p + 1) \qquad \text{(seq)}\ \dfrac{\text{(asgn)}\ \dfrac{}{\{p + 1 = p + 1\}\ p := p + 1\ \{p = p + 1\}} \quad ⓔ}{\{p + 1 = p + 1\}\ p := p + 1;\ q := n - p;\ \{n = q + p\}} \qquad \models (n = q + p \Rightarrow n = q + p)}{\{n = q + p \land (\neg\textbf{prime}\,(p) \lor \neg\textbf{prime}\,(q))\}\ p := p + 1;\ q := n - p;\ \{n = q + p\}}$$

ⓔ

$$\text{(cons)}\ \frac{\models (p = p + 1 \Rightarrow n - p = n - p) \qquad \text{(asgn)}\ \dfrac{}{\{n - p = n - p\}\ q := n - p\ \{q = n - p\}} \qquad \models (q = n - p \Rightarrow n = q + p)}{\{p = p + 1\}\ q := n - p\ \{n = q + p\}}$$

Task 2: Total Correctness

(a)

$$\text{(cons)}\ \frac{\models (\textbf{true} \Rightarrow \textbf{true}) \qquad \text{(while)}\ \dfrac{\text{(cons)}\ \dfrac{\models ((\textbf{true} \land x \neq 0) \Rightarrow \textbf{true}) \qquad \dfrac{}{\{\textbf{true}\}\ x := x - 1\ \{\textbf{true}\}} \qquad \models (\textbf{true} \Rightarrow \textbf{true})}{\{\textbf{true} \land x \neq 0\}\ x := x - 1\ \{\textbf{true}\}}}{\{\textbf{true}\}\ \textbf{while}\ x \neq 0\ \textbf{do}\ x := x - 1\ \textbf{end}\ \{\textbf{true} \land x = 0\}} \qquad \models ((x = 0 \land \textbf{true}) \Rightarrow \textbf{true})}{\{\textbf{true}\}\ \textbf{while}\ x \neq 0\ \textbf{do}\ x := x - 1\ \textbf{end}\ \{\textbf{true}\}}$$

(b)

$$\text{(cons)}\ \frac{\models (x > 0 \Rightarrow (\exists i \geq 0 \land x = i)) \qquad \text{(while)}\ \dfrac{\models ((i \geq 0 \land x = i + 1) \Rightarrow x \neq 0) \qquad \text{(cons)}\ \dfrac{\models ((i \geq 0 \land x = i + 1) \Rightarrow x = i + 1) \qquad \dfrac{}{\{x = i + 1\}\ x := x - 1\ \{\Downarrow x = i\}} \qquad \models (x = i \Rightarrow x = i)}{\{i \geq 0 \land x = i + 1\}\ x := x - 1\ \{\Downarrow x = i\}} \qquad (x = 0 \Rightarrow x = 0)}{\{\exists i \geq 0 \land x = i\}\ \textbf{while}\ x \neq 0\ \textbf{do}\ x := x - 1\ \textbf{end}\ \{\Downarrow x = 0\}} \qquad (x = 0 \Rightarrow \textbf{true})}{\{x > 0\}\ \textbf{while}\ x \neq 0\ \textbf{do}\ x := x - 1\ \textbf{end}\ \{\Downarrow \textbf{true}\}}$$

(c)

This statement will be disproved by contradiction.

$$\text{(cons)}\ \dfrac{\models (x<0 \Rightarrow x<0) \qquad \text{(while)}\ \dfrac{\text{(cons)}\ \dfrac{\models ((x<0 \land x \neq 0) \Rightarrow x<0) \qquad \text{(cons)}\ \dfrac{\models (x<0 \Rightarrow x<1) \qquad \dfrac{}{\{x<1\}\ x := x-1\ \{x<0\}} \qquad \models (x<0 \Rightarrow x<0)}{\{x<0\}\ x := x-1\ \{x<0\}} \qquad \models (x<0 \Rightarrow x<0)}{\{x<0 \land x \neq 0\}\ x := x-1\ \{x<0\}}}{\{x<0\}\ \textbf{while}\ x \neq 0\ \textbf{do}\ x := x-1\ \textbf{end}\ \{x<0 \land x=0\}} \qquad \models ((x=0 \land x<0) \Rightarrow \textbf{false})}{\{x<0\}\ \textbf{while}\ x \neq 0\ \textbf{do}\ x := x-1\ \textbf{end}\ \{\textbf{false}\}}$$

As this statement holds the total correctness of $\{\textbf{true}\}$ $\textbf{while}\ x \neq 0\ \textbf{do}\ x := x-1\ \textbf{end}$ $\{\Downarrow \textbf{true}\}$ is not satisfied.