Exersice Sheet 3

———— Sample Solution ———

Task 1: Operational Equivalence

Prove or disprove:

repeat c until $b \sim c$; while b do c end

The claim will be disproved using a counter example.

Lets assume $b := \mathbf{true}$ and $c := \mathbf{skip}$.

The **repeat** statement will terminate after the first iteration with $\langle \mathbf{repeat} \ c \ \mathbf{until} \ b, \ \sigma \rangle \to \sigma$ while the **while** statement will never terminate as its condition is always satisfied.

Task 2: Translation of Statements

$$\mathfrak{T}_{c}\llbracket\mathbf{repeat}\;c\;\mathbf{until}\;b\rrbracket=\mathfrak{T}_{c}\llbracket c\rrbracket;\;\mathfrak{T}_{b}\llbracket b\rrbracket;\;\mathbf{JMPF}\left(-|\mathfrak{T}_{c}\llbracket c\rrbracket|+|\mathfrak{T}_{b}\llbracket b\rrbracket|\right)$$

Task 3: loop Loops

(a)

$$\frac{\langle x>0,\ \sigma\rangle \to \mathbf{false}}{\langle \mathbf{loop}\ x\ \mathbf{begin}\ c\ \mathbf{end},\ \sigma\rangle \to \sigma}$$

$$\frac{\langle x>0,\;\sigma\rangle\rightarrow\mathbf{true}\quad \langle c,\;\sigma\rangle\rightarrow\sigma^{'}\quad \left\langle z:=x-1,\;\sigma^{'}\right\rangle\rightarrow\sigma^{''}\quad \left\langle \mathbf{loop}\;z\;\mathbf{begin}\;c\;\mathbf{end},\;\sigma^{''}\right\rangle\rightarrow\sigma^{'''}}{\left\langle \mathbf{loop}\;x\;\mathbf{begin}\;c\;\mathbf{end},\;\sigma\right\rangle\rightarrow\sigma^{'''}}$$

(b)

$$\begin{split} &\mathfrak{T}_{c}\llbracket \mathbf{loop} \ x \ \mathbf{begin} \ c \ \mathbf{end} \rrbracket = \mathbf{LOAD}(x); \ \mathbf{STO}(\xi); \\ &\mathbf{LOAD}(\xi); \ \mathbf{PUSH}(0); \ \mathbf{GT}; \ \mathbf{JMPF}(|\mathfrak{T}_{c}\llbracket c \rrbracket| + 6); \\ &\mathfrak{T}_{c}\llbracket c \rrbracket; \ \mathbf{LOAD}(\xi); \ \mathbf{PUSH}(1); \ \mathbf{SUB}; \ \mathbf{STO}(\xi); \ \mathbf{JMP}(-(|\mathfrak{T}_{c}\llbracket c \rrbracket| + 8)) \end{split}$$