Exersice Sheet 3

——— Sample Solution ———

Task 1: Operational Equivalence

Prove or disprove:

$$\textbf{repeat } c \textbf{ until } b \sim c;\ \textbf{while } b \textbf{ do } c \textbf{ end}$$

The claim will be disproved using a counter example.

Lets assume $b := \textbf{true}$ and $c := \textbf{skip}$.
The **repeat** statement will terminate after the first iteration with $\langle \textbf{repeat } c \textbf{ until } b,\ \sigma\rangle \to \sigma$ while the **while** statement will never terminate as its condition is always satisfied.

Task 2: Translation of Statements

$$\mathfrak{T}_c[\![\textbf{repeat } c \textbf{ until } b]\!] = \mathfrak{T}_c[\![c]\!];\ \mathfrak{T}_b[\![b]\!];\ \textbf{JMPF}\,(-|\mathfrak{T}_c[\![c]\!]| + |\mathfrak{T}_b[\![b]\!]|)$$

Task 3: loop Loops

(a)

$$\frac{\langle x > 0,\ \sigma\rangle \to \textbf{false}}{\langle \textbf{loop } x \textbf{ begin } c \textbf{ end},\ \sigma\rangle \to \sigma}$$

$$\frac{\langle x > 0,\ \sigma\rangle \to \textbf{true} \quad \langle c,\ \sigma\rangle \to \sigma' \quad \langle z := x - 1,\ \sigma'\rangle \to \sigma'' \quad \langle \textbf{loop } z \textbf{ begin } c \textbf{ end},\ \sigma''\rangle \to \sigma'''}{\langle \textbf{loop } x \textbf{ begin } c \textbf{ end},\ \sigma\rangle \to \sigma'''}$$

(b)

$$\mathfrak{T}_c[\![\textbf{loop } x \textbf{ begin } c \textbf{ end}]\!] = \textbf{LOAD}(x);\ \textbf{STO}(\xi);$$
$$\textbf{LOAD}(\xi);\ \textbf{PUSH}(0);\ \textbf{GT};\ \textbf{JMPF}(|\mathfrak{T}_c[\![c]\!]| + 6);$$
$$\mathfrak{T}_c[\![c]\!];\ \textbf{LOAD}(\xi);\ \textbf{PUSH}(1);\ \textbf{SUB};\ \textbf{STO}(\xi);\ \textbf{JMP}(-(|\mathfrak{T}_c[\![c]\!]| + 8))$$