Learning in finite and indefinite public good games

Ivan Dedyukhin, Lior Bakalo, Xiaojie Zhang, Inigo De Juan Razquin, Sabrina Rittinger

UPF - COMP24 Summer School

June 09, 2024

Universitat Pompeu Fabra Barcelona

Presentation Outline

Outline

Introduction

- Social dilemmas are studied extensively as they represent the trade-off between individually rational and socially efficient outcomes
- Linear VCM games is an example: contributions that benefit everyone
- In life, public good games are usually played repeatedly:
  - under a shadow of future: fundraising, media donations
  - with definite horizon: announcement of the last period
- We explore the Q-learning model to explore what equilibrium can be achieved in the indefinite horizon and if conditional cooperation can be an outcome of a finite game

Public Good Game

- N subjects (4 or 10)
- Each subject decides how to allocate W tokens:
  1. private account
  2. group project
- Group project payoff to each agent is *MPCR* per token allocated (0.3 or 0.75)
- Hence if agent i allocates g_i to group account payoffs are

$$\pi_i = W - g_i + MPCR * \sum_{j=1}^{N} g_j$$

Outline

Literature

- Prisoner's dilemma: Dal Bó and Fréchette (2018); Embrey, Fréchette, and Yuksel (2018)
- VCM in the finite settings: Isaac and Walker (1988)
- VCM in the infinite settings: Lugovskyy, Puzzello, Sorensen, Walker, and Williams (2017); Tan and Wei (2014)
- Conditional cooperation: Thöni and Volk (2018)
- Learning models in public goods: Arifovic and Ledyard (2012)

Related Experiments - Finite Public Goods Game

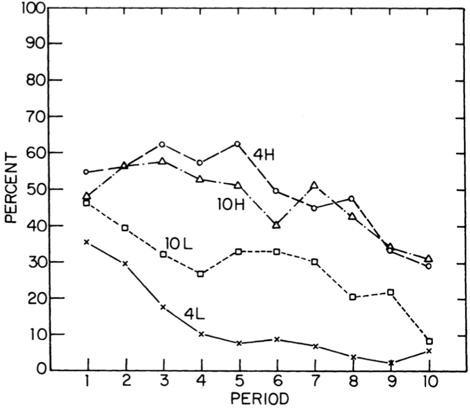


Figure: Mean Percentage of Tokens Contributed to the Public Good Isaac and Walker (1988)

Related Experiments - Finite Public Goods Game

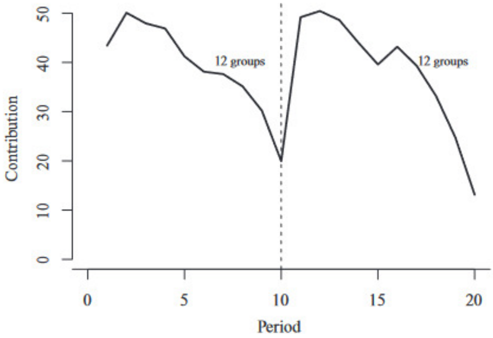


Figure: Mean Percentage of Tokens Contributed to the Public Good Tan and Wei (2014)

Related Experiments - Indefinite Public Goods Game

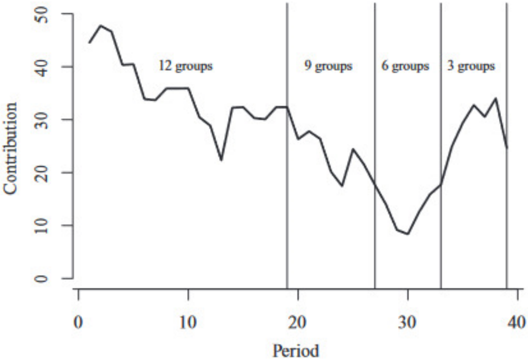
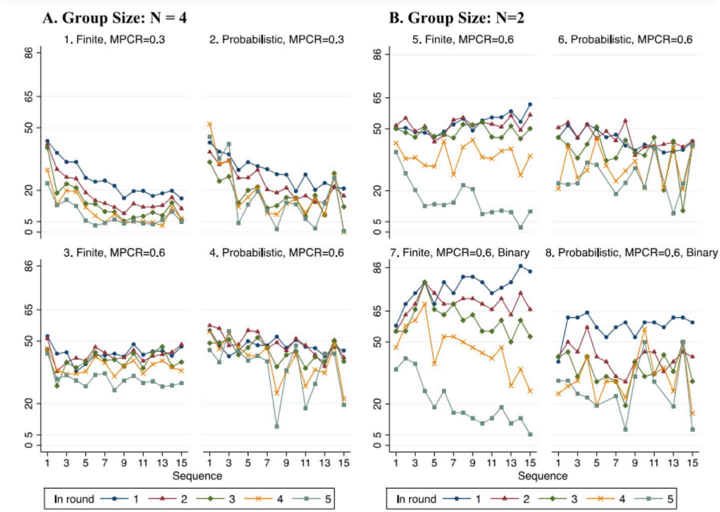


Figure: Indefinite horizon public goods game Tan and Wei (2014)

Related Experiments - Finite & Indefinite Public Goods Game



Outline

Model

- We use the Q-learning model:
    - Discrete choice of contributions out of endowment of 5
    - States are:
        - Previous round contribution of other players in **indefinite** setting
        - Previous round contribution of other players and round in **finite** setting
- We will train 2x2x2=8 different models:
    - Finite or indefinite
    - *MPCR* from {0.3, 0.75}
    - Group size from {4, 10}
- We will discretize the previous round contributions by other players in 5 bins with a step of 20pp of endowment

Learning model

- Actions Space is {0,1,2,3,4,5}
- Softmax policy with temperature $\lambda = 5$
- Learning rate $\alpha = 0.1$

1. Indefinite:
   - State space: 6 prev round contributions
   - Values $Q(a,s)$ are initialized with average possible outcomes
   - Update rule:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left[r + \delta \max_{a'} Q\left(s', a'\right)\right]$$

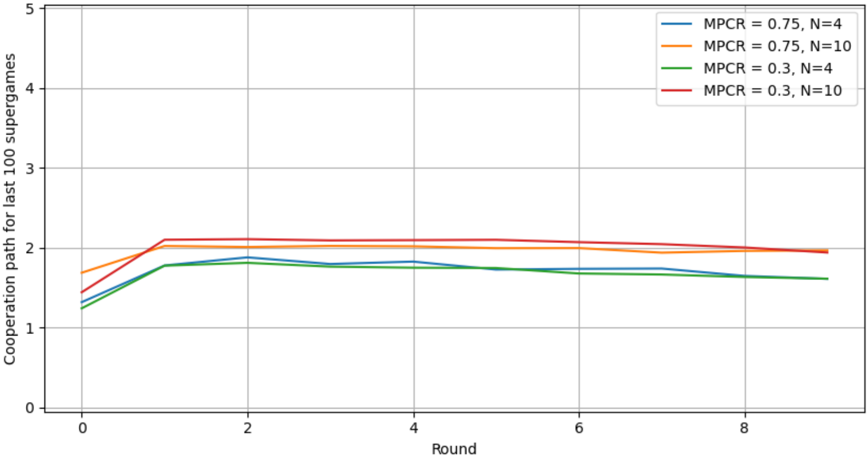
   - $\delta = 0.9$

2. Finite:
   - State space: 6 prev round contributions x 10 rounds
   - Values $Q(a,s)$ are initialized with average possible outcomes times number of remaining periods
   - Update rule:

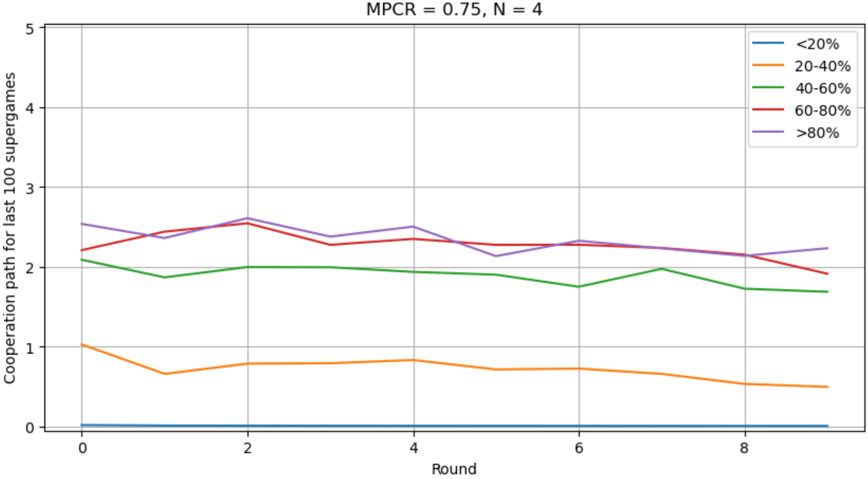
$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left[r + \max_{a'} Q\left(s', a'\right)\right]$$

Outline

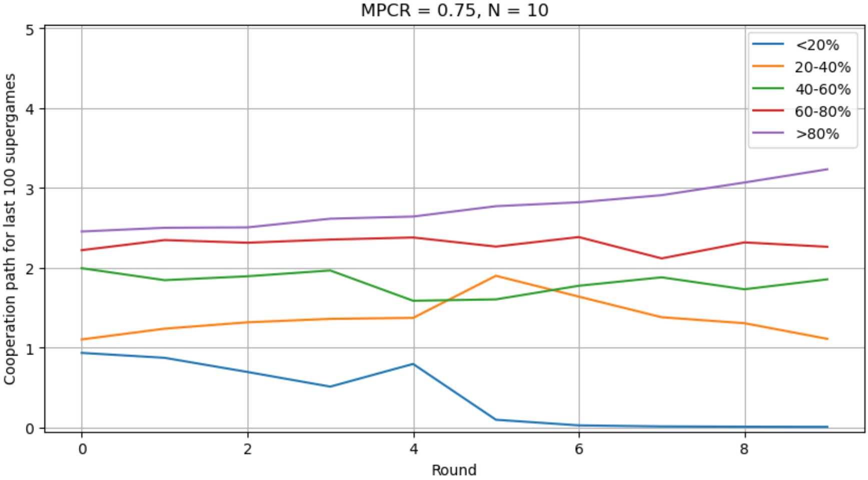
Simulation results: finite



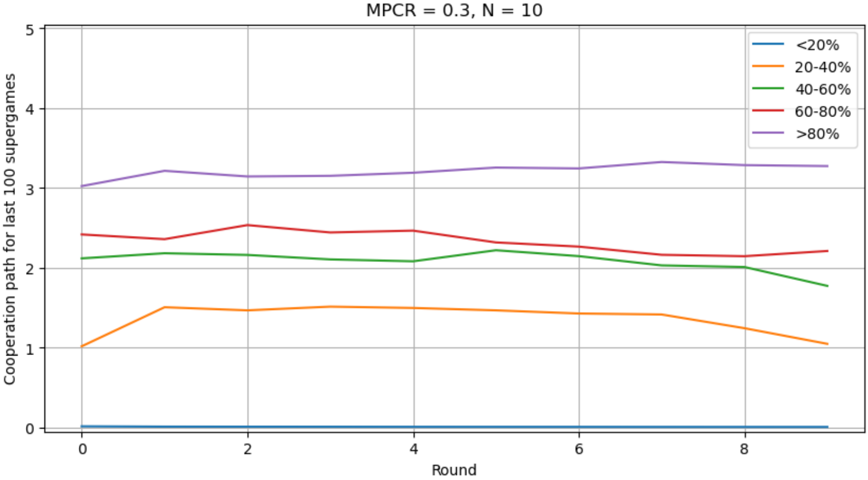
Simulation results: finite



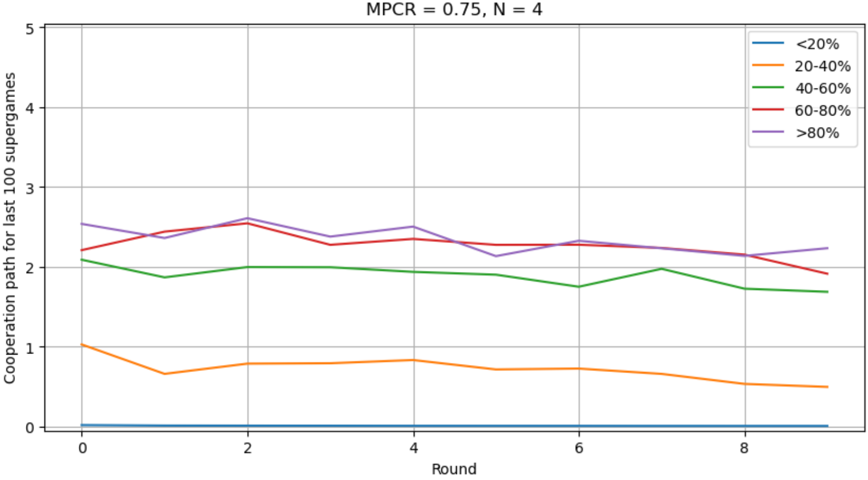
Simulation results: finite



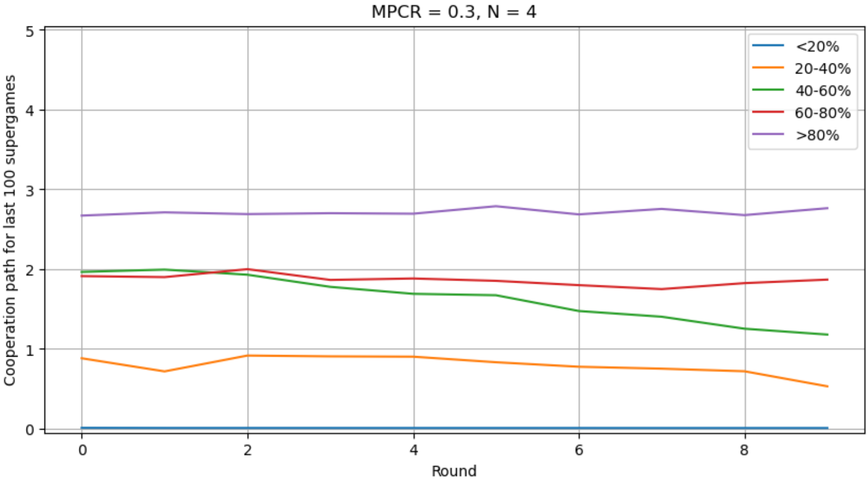
Simulation results: finite



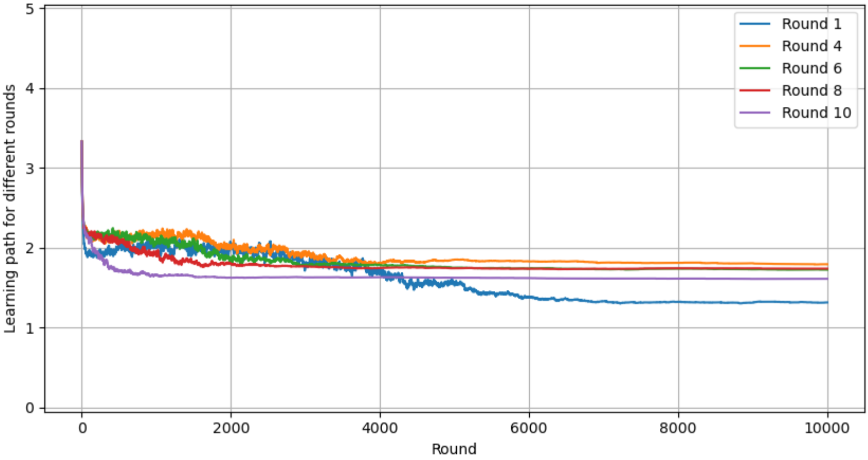
Simulation results: finite



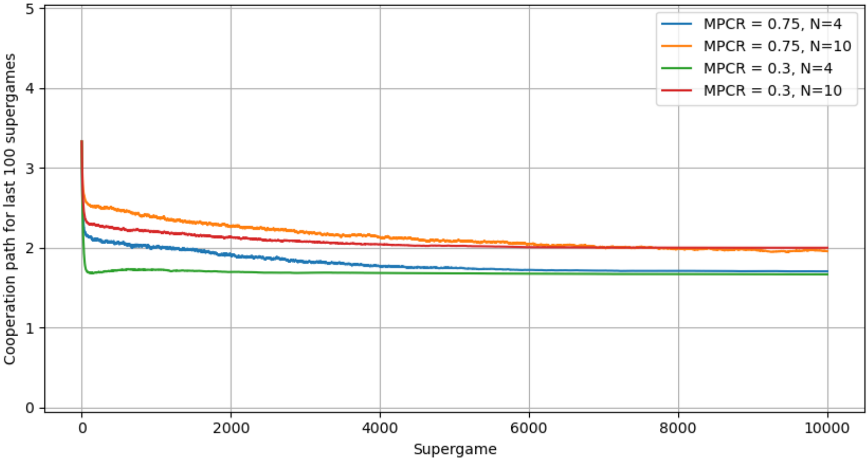
Simulation results: finite



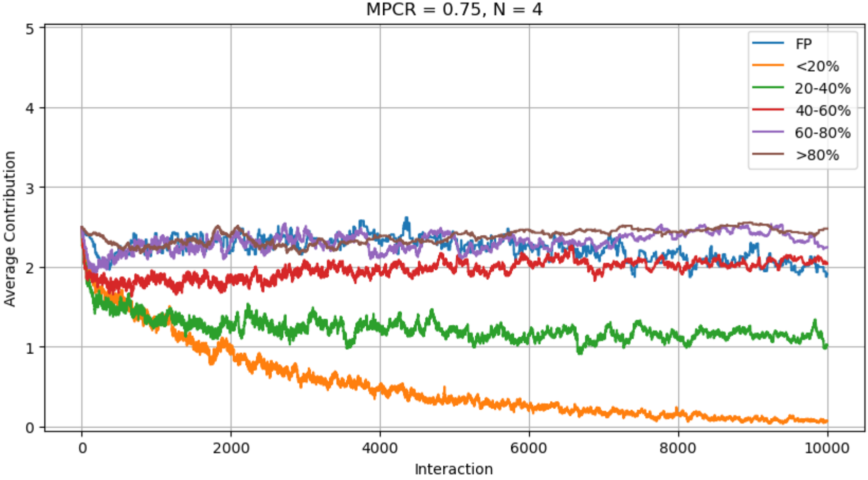
Simulation results: finite



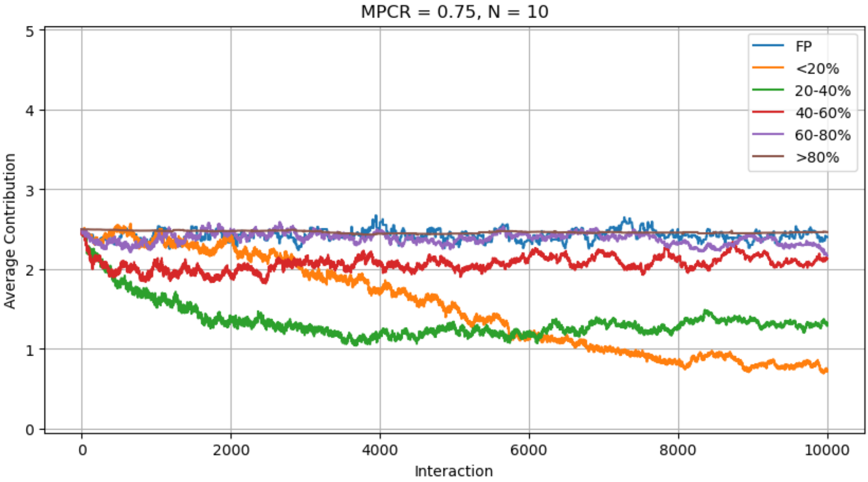
Simulation results: indefinite



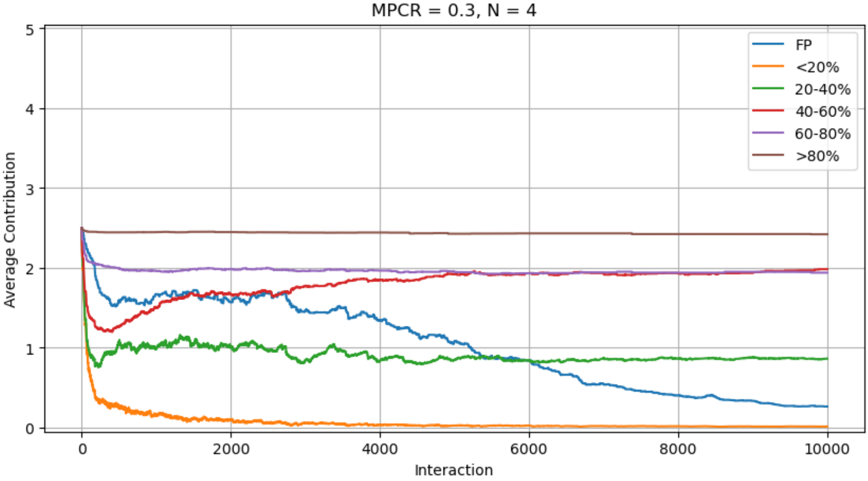
Simulation results: indefinite



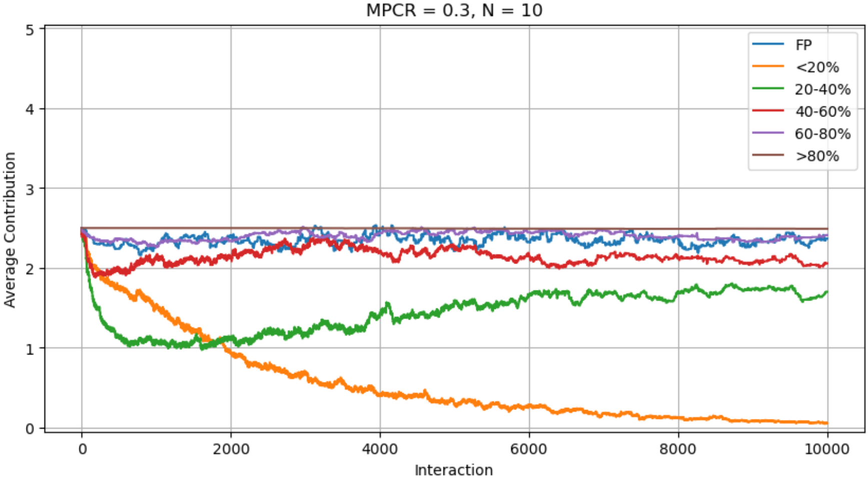
Simulation results: indefinite



Simulation results: indefinite



Simulation results: indefinite



Conclusion

- Both learning models do not follow the experimental results
- The algorithms learn to cooperate conditionally
- It would be good to unify the algorithm so it might fit both settings
- We assumed no social preferences and did not fit the data

References

Arifovic, J., & Ledyard, J. (2012). Individual evolutionary learning, other-regarding preferences, and the voluntary contributions mechanism. *Journal of Public Economics*, *96*(9-10), 808–823.

Dal Bó, P., & Fréchette, G. R. (2018). On the determinants of cooperation in infinitely repeated games: A survey. *Journal of Economic Literature*, *56*(1), 60–114.

Embrey, M., Fréchette, G. R., & Yuksel, S. (2018). Cooperation in the finitely repeated prisoner's dilemma. *The Quarterly Journal of Economics*, *133*(1), 509–551.

Isaac, R. M., & Walker, J. M. (1988). Group size effects in public goods provision: The voluntary contributions mechanism. *The Quarterly Journal of Economics*, *103*(1), 179–199. Retrieved 2024-06-08, from `http://www.jstor.org/stable/1882648`

Lugovskyy, V., Puzzello, D., Sorensen, A., Walker, J., & Williams, A. (2017). An experimental study of finitely and infinitely repeated linear public goods games. *Games and Economic Behavior*, *102*, 286–302.

Tan, L., & Wei, L. (2014). Voluntary contribution mechanism played over an infinite horizon. *Pacific Economic Review*, *19*(3), 313–331.

Thöni, C., & Volk, S. (2018). Conditional cooperation: Review and refinement. *Economics Letters*, *171*, 37–40.