COMP 202. Introduction to Electronics

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Combinational/Combinatorial Circuits

Karnaugh Maps

For purposes of simplification, the Karnaugh map is a convenient way of representing a Boolean function of a small number (up to four) of variables.

The map is an array of 2^n squares, representing all possible combinations of values of n binary variables.

Karnaugh Maps

The map can be used to represent any Boolean function in the following way:

> Each square corresponds to a unique product in the sum-of-products form, with a 1 value corresponding to the variable and a 0 value corresponding to the *NOT* of that variable.

Given the truth table of a Boolean function, it is an easy matter to construct the map: for each combination of values of variables that produce a result of 1 in the truth table, fill in the corresponding square of the map with 1.

To convert from a Boolean expression to a map, it is first necessary to put the expression into what is referred to as *canonical form*: each term in the expression must contain each variable.

Karnaugh Maps

So, for example, if we have,

$$F = \bar{A}B + B\bar{C} \tag{1}$$

we must first expand it into the full form of

$$F = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} \tag{2}$$

and then convert this to a map.

Here the two rows embraced by the symbol A are those in which the variable A has the value 1; the rows not embraced by the symbol A are those in which A is 0; similarly for B, C, and D.

Exercise

Using the laws of Boolean logic show that the canonical form of $F = \bar{A}B + B\bar{C}$ is $F = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}$

Combinational/Combinatorial Circuits

Karnaugh Maps

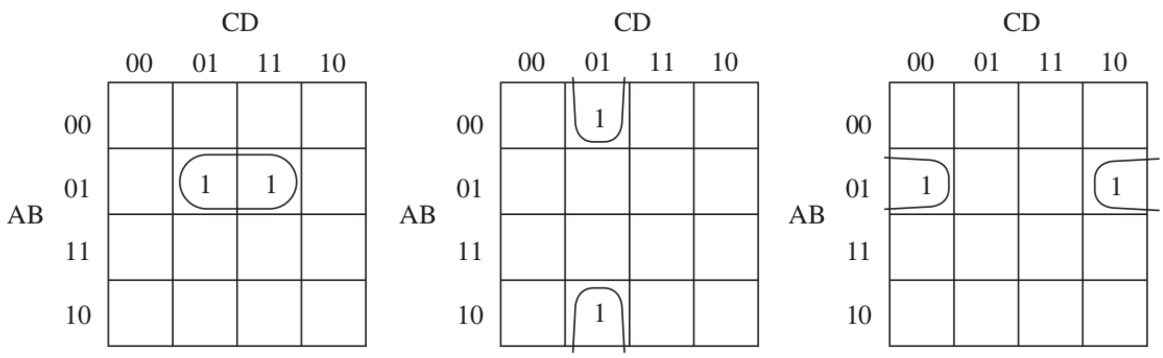
Once the map of a function is created, we can often write a simple algebraic expression for it by noting the arrangement of the 1s on the map. The principle is as follows.

> Any two squares that are adjacent differ in only one of the variables. If two adjacent squares both have an entry of one, then the corresponding product terms differ in only one variable. In such a case, the two terms can be merged by eliminating that variable.

For example, in (a) below, the two adjacent squares correspond to the two terms ABCD and ABCD. Thus, the function expressed is

$$\bar{A}B\bar{C}D + \bar{A}BCD = \bar{A}BD$$

Karnaugh Maps



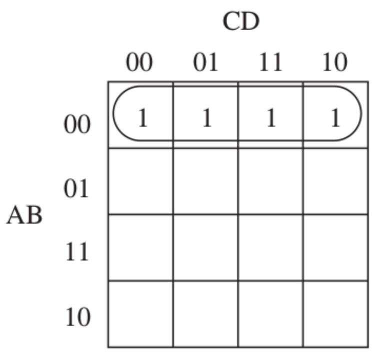
(a) $\overline{A}BD$ (b) $\overline{B}\,\overline{C}D$ (b) $\overline{A}B\overline{D}$

Karnaugh Maps

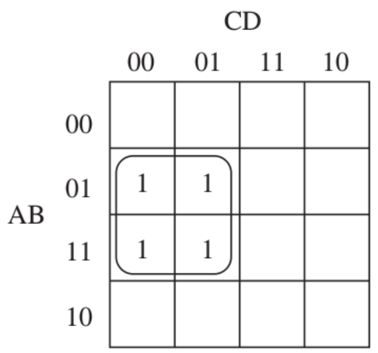
This process can be extended in several ways. First, the concept of adjacency can be extended to include wrapping around the edge of the map. Thus, the top square of a column is adjacent to the bottom square, and the leftmost square of a row is adjacent to the rightmost square. These conditions are illustrated in (b) and (c) above.

Second, we can group not just 2 squares but 2^n adjacent squares (that is, 2, 4, 8, etc.). (See examples below). Note that in this case, two of the variables can be eliminated.

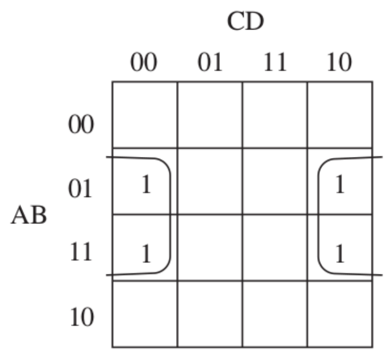
Karnaugh Maps



(d) $\overline{A}\,\overline{B}$



(e) $B\overline{C}$

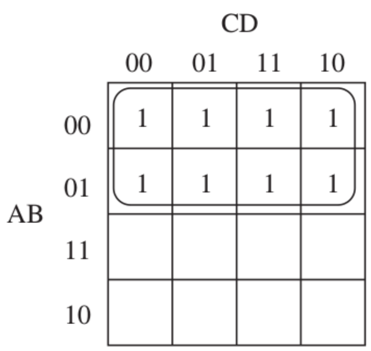


(f) $B\overline{D}$

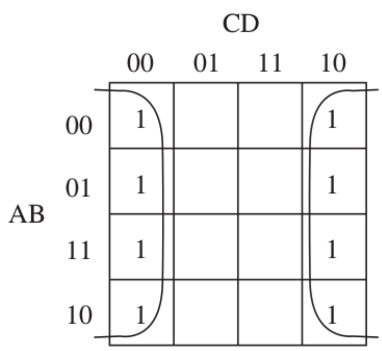
Combinational/Combinatorial Circuits

Karnaugh Maps

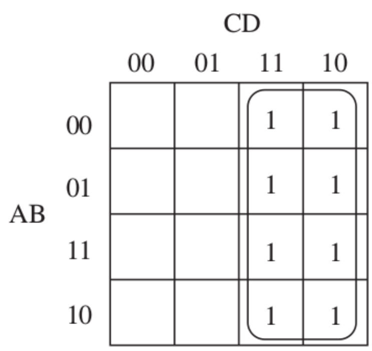
The next examples show groupings of 8 squares, which allow three variables to be eliminated.



(g) $\overline{A}$



(h) $\overline{D}$



(i) C

Karnaugh Maps

We can summarize the rules for simplification as follows:

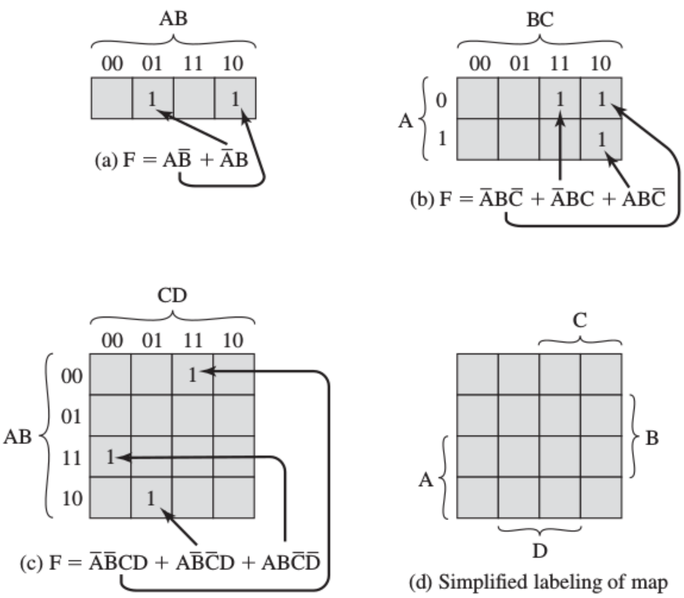
Among the marked squares (squares with a 1), find those that belong to a unique largest block of 1, 2, 4, or 8 and circle those blocks.

Select additional blocks of marked squares that are as large as possible and as few in number as possible, but include every marked square at least once. The results may not be unique in some cases. For example, if a marked square combines with exactly two other squares, and there is no fourth marked square to complete a larger group, then there is a choice to be made as two which of the two groupings to choose. When you are circling groups, you are allowed to use the same 1 value more than once.

Karnaugh Maps

Continue to draw loops around single marked squares, or pairs of adjacent marked squares, or groups of four, eight, and so on in such a way that every marked square belongs to at least one loop; then use as few of these blocks as possible to include all marked squares.

Karnaugh Maps



(a) shows the map of four squares for a function of two variables. It is essential for later purposes to list the combinations in the order 00, 01, 11, 10. Because the squares corresponding to the combinations are to be used for recording information, the combinations are customarily written above the squares. In the case of three variables, the representation is an arrangement of eight squares (b), with the values for one of the variables to the left and for the other two variables above the squares. For four variables, 16 squares are needed, with the arrangement indicated in (c). The labeling used in (d) emphasizes the relationship between variables and the rows and columns of the map. Here the two rows embraced by the symbol A are those in which the variable A has the value 1; the rows not embraced by the symbol A are those in which A is 0; similarly for B, C, and D.

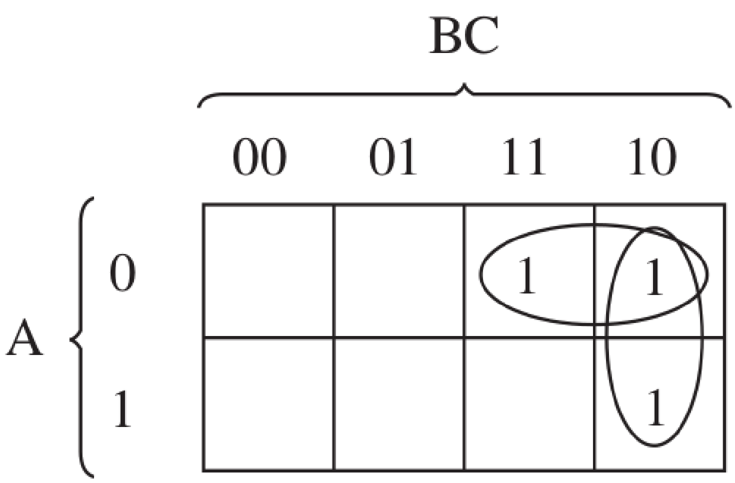
Combinational/Combinatorial Circuits

Karnaugh Maps

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Karnaugh Maps

If any isolated 1s remain after the groupings, then each of these is circled as a group of 1s



(a) $F = \overline{A}B + B\overline{C}$

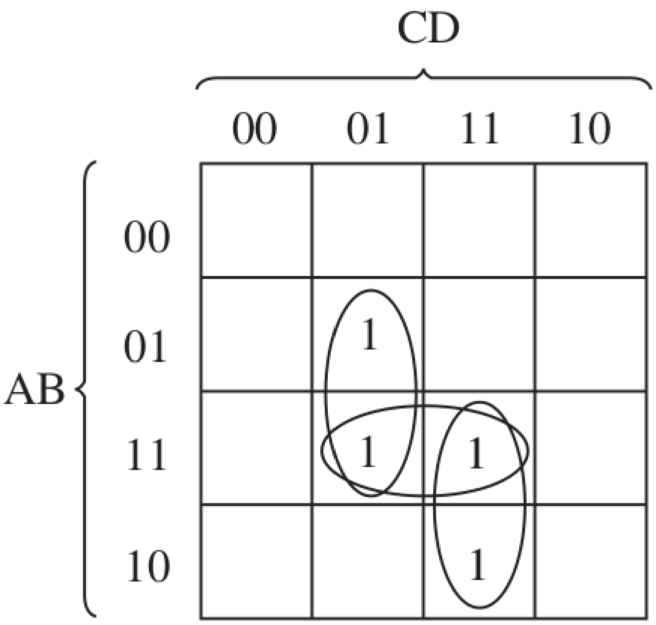
Karnaugh Maps

Finally, before going from the map to a simplified Boolean expression, any group of 1s that is completely overlapped by other groups can be eliminated.

In this case, the horizontal group is redundant and may be ignored in creating the Boolean expression.

This is shown in the figure below.

Karnaugh Maps



(b) $F = B\overline{C}D + ACD$

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Karnaugh Maps – Example 1

Given the $F = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$ we can simplify it as follows

$$
\begin{aligned}
F &= \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC \\
&= BC(\overline{A} + A) + A\overline{B}C + AB\overline{C} \\
&= B(C + A\overline{C}) + A\overline{B}C \\
&= BC + AB + A\overline{B}C \\
&= BC + A(B + \overline{B}C) \\
&= AB + BC + AC
\end{aligned}
$$

Karnaugh Maps – Example 1

By Karnaugh Maps the same result is obtained as follows:

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

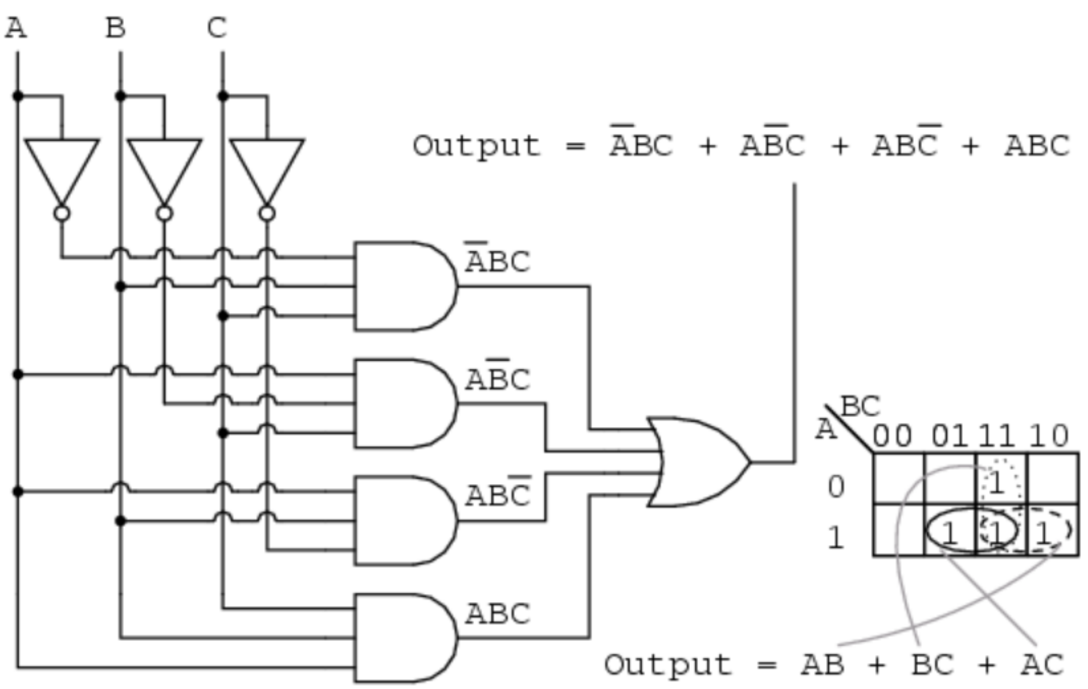
$$F = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$F = AB + BC + AC$$

The reason this simplification is necessary is shown in the figure below.

Combinational/Combinatorial Circuits

Karnaugh Maps – Example 1



Note the significant reduction in gates!

Karnaugh Maps – Example 2

A \ BC	00	01	11	10
0	1	1	0	0
1	0	0	0	0

$$
\begin{aligned}
F &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C \\
&= \bar{A}\bar{B}
\end{aligned}
$$

Karnaugh Maps – Example 3

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$$
\begin{aligned}
F &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} \\
&= \bar{A}
\end{aligned}
$$

Karnaugh Maps – Example 4

	BC	00	01	11	10
A	0	0	1	1	0
	1	0	1	1	0

$$
\begin{aligned}
F &= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC \\
&= C
\end{aligned}
$$

Combinational/Combinatorial Circuits

Karnaugh Maps – Example 5

	BC	00	01	11	10
A	0	1	1	1	1
	1	0	0	1	1

$$
\begin{aligned}
F &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C} \\
&= C
\end{aligned}
$$

Karnaugh Maps – Example 6

A \ BC	00	01	11	10
0	0	0	1	0
1	0	0	1	0

$$
\begin{aligned}
F &= \bar{A}BC + ABC \\
&= BC
\end{aligned}
$$

Karnaugh Maps – Example 7

A \ BC	00	01	11	10
0	0	0	1	1
1	0	0	1	1

$$
\begin{aligned}
F &= \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C} \\
&= B
\end{aligned}
$$

Karnaugh Maps – Example 8

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$$\begin{aligned} F &= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} \\ &= \bar{C} \end{aligned}$$

Karnaugh Maps – Example 9

A \ BC	00	01	11	10
0	1	1	1	1
1	1	0	0	1

$$\begin{aligned} F &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} \\ &= \bar{A} + \bar{C} \end{aligned}$$