COMP 202. Introduction to Electronics

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Theorem 8: DeMorgan's Theorem

(a) $\overline{a+b} = \overline{a} \cdot \overline{b}$

(b) $\overline{a \cdot b} = \overline{a} + \overline{b}$

Let's prove part(a)

Theorem 8: DeMorgan's Theorem

If $X = a + b$, then $\overline{X} = \overline{(a + b)}$. By Postulate 6, $X \cdot \overline{X} = 0$ and $X + \overline{X} = 1$. If $X \cdot Y = 0$ and $X + Y = 1$, then $Y = \overline{X}$ because the complement of X is unique. Therefore, we let $Y = \overline{a}\overline{b}$ and test $X \cdot Y$ and $X + Y$:

Theorem 8: DeMorgan's Theorem

$$
\begin{aligned}
X \cdot Y &= (a + b)(\overline{a}\overline{b}) \\
&= (\overline{a}\overline{b})(a + b) && [P3(b)] \\
&= (\overline{a}\overline{b})a + (\overline{a}\overline{b})b && [P5(b)] \\
&= a(\overline{a}\overline{b}) + (\overline{a}\overline{b})b && [P3(b)] \\
&= (a\overline{a})\overline{b} + \overline{a}(\overline{b}b) && [P4(b)] \\
&= 0 \cdot \overline{b} + \overline{a}(b \cdot \overline{b}) && [P6(b), P3(b)] \\
&= \overline{b} \cdot 0 + \overline{a} \cdot 0 && [P3(b), P6(b)] \\
&= 0 + 0 && [T2(b)] \\
&= 0 && [P2(a)]
\end{aligned}
$$

Theorem 8: DeMorgan's Theorem

$$
\begin{aligned}
X + Y &= (a + b) + \bar{a}\bar{b} \\
&= (b + a) + \bar{a}\bar{b} && [P3(a)] \\
&= b + (a + \bar{a}\bar{b}) && [P4(a)] \\
&= b + (a + \bar{b}) && [T5(a)] \\
&= (a + \bar{b}) + b && [P3(a)] \\
&= a + (\bar{b} + b) && [P4(a)] \\
&= a + (b + \bar{b}) && [P3(a)] \\
&= a + 1 && [P6(a)] \\
&= 1 && [T2(a)]
\end{aligned}
$$

Therefore, by the uniqueness of $\overline{X}$,$Y = \overline{X}$, and therefore $\bar{a}\bar{b} = \overline{a + b}$

Theorem 8: DeMorgan's Theorem

Theorem 8 may be generalised as follows.

(a) $\overline{a + b + \ldots + z} = \bar{a} \cdot \bar{b} \ldots \cdot \bar{z}$

(b) $\overline{ab \ldots z} = \bar{a} + \bar{b} + \ldots + \bar{z}$

Note:

The rule to follow when complementing an expression is to use relation (a) or (b), replacing each $+$ (OR) operator with an $\cdot$ (AND) operator, and vice versa, and replacing each variable with its complement.

In applying DeMorgan's theorem, operator precedence must be observed: $\cdot$ takes precedence over $+$. The following example illustrates this important point.

Example 1

Q: Complement the expression $a + bc$
Solution:

$$\begin{aligned}
\overline{a + b \cdot c} &= \overline{a + (b \cdot c)} \\
&= \overline{a} \cdot \overline{(b \cdot c)} \\
&= \overline{a} \cdot (\overline{b} + \overline{c}) \\
&= \overline{a}\overline{b} + \overline{a}\,\overline{c}
\end{aligned}$$

Notice how: $\overline{a + b \cdot c} \;\neq\; \overline{a} \cdot \overline{b} + \overline{c}$

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Example 2

$$\begin{aligned}
\overline{X + \overline{Y}} &= \overline{X} \cdot \overline{\overline{Y}} && [T8(a)] \\
&= \overline{X} \cdot Y && [T3]
\end{aligned}$$

Example 3

Q: Complement the expression $a(b+z(x+\overline{a}))$, and simplify the result so that the only complemented terms are individual variables.
Solution:

$$\begin{aligned}
\overline{a(b+z(x+\overline{a}))} &= \overline{a} + \overline{(b+z(x+\overline{a}))} \\
&= \overline{a} + \overline{b}\ \overline{(z(x+\overline{a}))} \\
&= \overline{a} + \overline{b}(\overline{z} + \overline{(x+\overline{a})}) \\
&= \overline{a} + \overline{b}(\overline{z} + \overline{x} \cdot \overline{\overline{a}}) \\
&= \overline{a} + \overline{b}(\overline{z} + \overline{x}a)
\end{aligned}$$

Example 4

Repeat Example 3 for the expression $a(b+c)+\overline{a}b$

Theorem 9: Consensus

(a) $ab + \overline{a}c + bc = ab + \overline{a}c$.

(b) $(a + b)(\overline{a} + c)(b + c) = (a + b)(\overline{a} + c)$.

Proof.

$$\begin{aligned} ab + \overline{a}c + bc &= ab + \overline{a}c + 1 \cdot bc \\ &= ab + \overline{a}c + (a + \overline{a})bc \\ &= ab + \overline{a}c + abc + \overline{a}bc \\ &= (ab + abc) + (\overline{a}c + \overline{a}cb) \\ &= ab + \overline{a}c \end{aligned}$$

The key to using this theorem is to find an element and its complement, note the associated terms, and eliminate the included term (the 'consensus' term), which is composed of the associated terms.

Examples

$$AB + \overline{A}CD + BCD = AB + \overline{A}CD$$
$$(a + \overline{b})(\overline{a} + c)(\overline{b} + c) = (a + \overline{b}(\overline{a} + c)$$

$$\begin{aligned} ABC + \overline{A}D + \overline{B}D + CD &= ABC + (\overline{A} + \overline{B})D + CD \\ &= ABC + \overline{AB}D + CD \\ &= ABC + \overline{AB}D \\ &= ABC + (\overline{A} + \overline{B})D \\ &= ABC + \overline{A}D + \overline{B}D \end{aligned}$$

In each of the preceding examples, an element or expression and its complement offer the key to reducing the expression.

Positive Versus Negative Logic

The AND and OR logic functions are realised by AND and OR gates, respectively, if the *positive logic* conventions is used for all gate inputs and outputs; i.e. if the signals connected to the gate inputs and outputs are all *active high*.

When the signals connected to the gate inputs and output are all active low, the roles of these gates are reversed.

Thus in negative logic convention, 1 is represented by a low voltage and 0 by a high voltage.

Positive Versus Negative Logic

This may be verified by applying *involution* (Theorem 3) and DeMorgan's theorem to the expression for the logical AND function as follows:

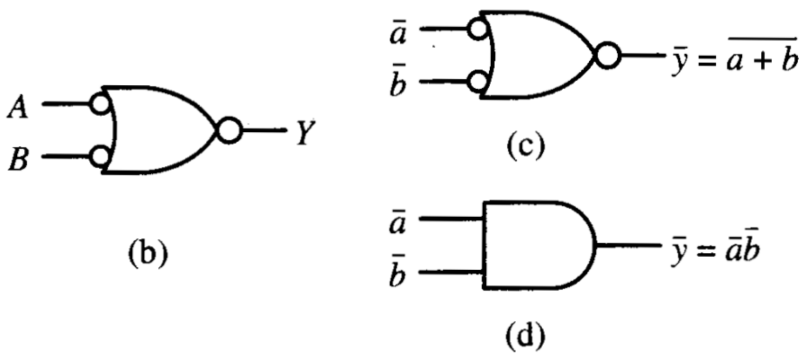
$$\begin{aligned} y &= a \cdot b \\ &= \overline{\overline{a \cdot b}} \\ &= \overline{\overline{a} + \overline{b}} \\ &= \bar{f}_{OR}(\overline{a}, \overline{b}) \end{aligned}$$

The equation indicates that an AND gate symbol can be drawn as an OR function with active-low inputs and output.

Symbols for AND Negative Logic

A	B	Y
1	1	1
1	0	1
0	1	1
0	0	0

(a)



AND gate usage in a negative logic system. (a) AND gate truth table ($L = 1, H = 0$. (b) Alternative AND gate symbol (negative logic). (c) Preferred usage. (d) Improper usage.
Note that the alternative form, shown in (d) is not incorrect but is more difficult to analyze, and should therefore be avoided when negative logic is being used.

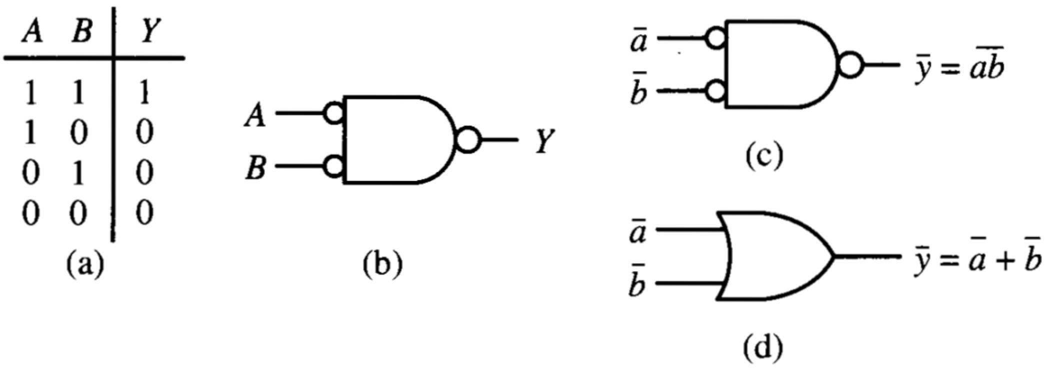
OR Negative Logic

Similarly, *involution* and DeMorgan's Theorem can be applied to show the OR negative logic as follows:

$$\begin{aligned} y &= a + b \\ &= \overline{\overline{a + b}} \\ &= \overline{\bar{a} \cdot \bar{b}} \\ &= \bar{f}_{AND}(\bar{a}, \bar{b}) \end{aligned}$$

The equation indicates that an OR gate symbol can be drawn as an AND function with active-low inputs and output.

Symbols for OR Negative Logic



OR gate usage in a negative logic system. (a) OR gate truth table ($L = 1, H = 0$). (b) Alternate OR gate symbol (negative logic). (c) Preferred usage. (d) Improper usage.

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Example 2

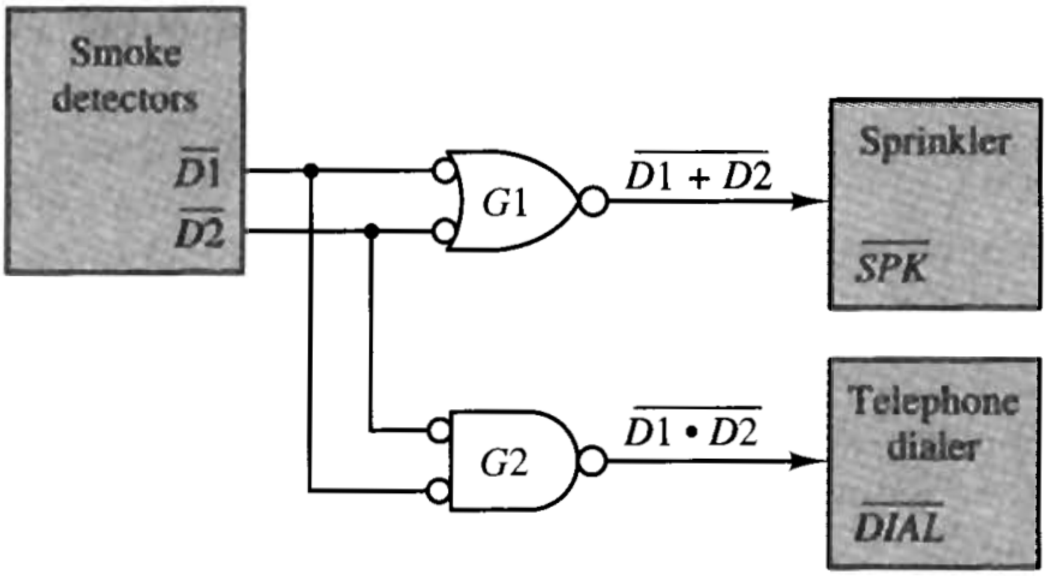
Solution: The logic equations for the sprinkler and telephone dialer are derived by determining the conditions that should activate each device. The sprinkler is to be activated whenever either smoke detector output is asserted. The desired operation is $SPK = D1 + D2$. Since these signals are only available in active-low form, we write

$$\overline{SPK} = \overline{D1 + D2}$$

Likewise, the dialer is to be activated whenever both smoke detector outputs are asserted; thus, $DIAL = D1 \cdot D2$. Since these signals are only available in active-low form, we write

$$\overline{DIAL} = \overline{D1 \cdot D2}$$

Example 2



Smoke alarm system, illustrating negative logic.

Applications: 1

A burglar alarm for a bank is designed so that it senses four input signal lines. Line A is from the secret control switch, line B is from a pressure sensor under a steel safe in a locked closet, line C is from a battery-powered clock, and line D is connected to a switch on the locked closet door. The following conditions produce a logic 1 voltage on each line:

A: The control switch is closed.

B: The safe is in its normal position in the closet.

C: The clock is between 10:00 and 14:00 hours.

D: The closet door is closed.

Write the equations of the control logic for the burglar alarm that produces a logic 1 (rings a bell) when the safe is moved and the control switch is closed, or when the closet is opened after banking hours, or when the closet is opened with the control switch open.

Solution

The statement “when the safe is moved and the control switch is closed” is represented by $A\overline{B}$. “When the closet is opened after banking hours” is represented by $\overline{C}\,\overline{D}$. “When the closet is opened with the control switch open” is represented by $\overline{A}\,\overline{D}$. Therefore, the logic equation for the burglar alarm is

$$f(A,B,C,D) = A\overline{B} + \overline{C}\,\overline{D} + \overline{A}\,\overline{D}$$

Applications: 2

John and Jane Doe have two children, Joe and Sue. When eating out they will go to a restaurant that serves only hamburgers or one that serves only chicken. Before going out, the family votes to decide on the restaurant. The majority wins, except when Mom and Dad agree, and in that case they win. Any other tie votes produce a trip to the chicken restaurant. We wish to design a logic circuit that will automatically select the restaurant when everyone votes.

Solution

If we let a 1 represent a vote cast for hamburgers and a 0 represent a vote cast for chicken, the truth table for the voting circuit is given below:

John A	Jane B	Joe C	Sue D	Vote for Hamburger f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

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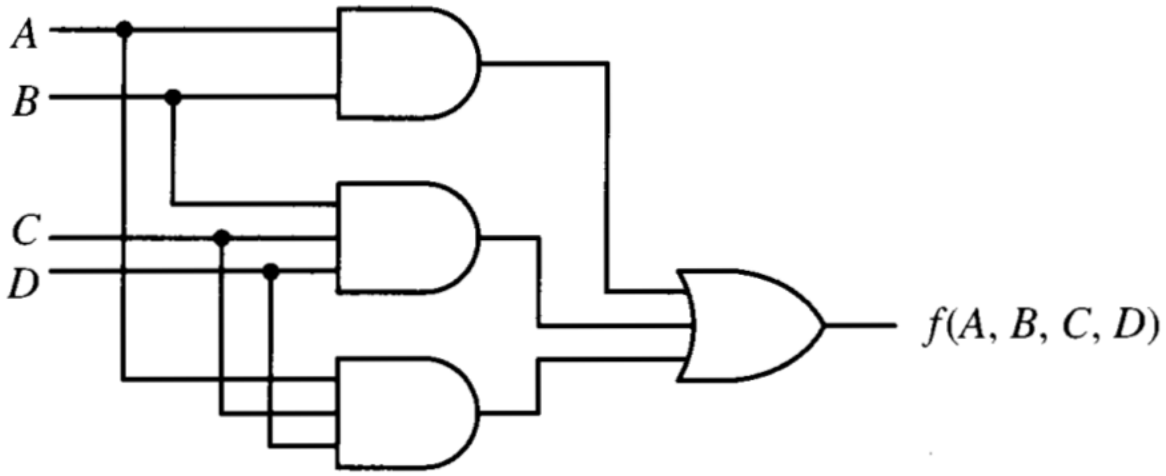
Solution

$$\begin{aligned}
f(A,B,C,D) &= \overline{A}BCD + A\overline{B}CD + AB\overline{C}\,\overline{D} + AB\overline{C}D + \\
&\quad ABC\overline{D} + ABCD \\
&= \overline{A}BCD + A\overline{B}CD + AB \\
&= AB + ACD + \overline{A}BCD \\
&= AB + ACD + BCD
\end{aligned}$$

Solution



Question

A burglar alarm is designed so that it senses four input signal lines. Line *A* is from the secret control switch, line *B* is from a pressure sensor under a steel safe in a locked closet, line *C* is from a battery-powered clock, and line D is connected to a switch on the locked closet door. The following conditions produce a logic 1 voltage on each line.

A: The control switch is closed.

B: The sage is in its normal position in the closet.

C: The clock is between 1000 and 1400 hours.

D: The closet door is closed.

Write the switching expression for the burglar alarm that produces a logic 1 (rings a bell) when the sage is moved and the control switch is closed, or when the closet is opened after banking hours, or when the closet is opened with the control switch open.

Question

A long hallway has three doors, one at each end and one in the middle. A switch is located at each door to operate the incandescent lights along the hallway. Label the switches A, B, and C. Design a logic network to control the lights.

Question

Joe, Jack, and Jim get together once a week to either go to a movie or go bowling. To decide what to do, they vote and a simple majority wins. Assuming a vote for the movie is represented as a 1, design a logic circuit that automatically computes the decision.