COMP 202. Introduction to Electronics

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Minterm/Maxterm Expansion

Find the minterm & maxterm expansion of $f(a,b,c) = \bar{a}b + b\bar{c}$

$f = \bar{a}b(c + \bar{c}) + (\bar{a} + a)b\bar{c}$

$f = \bar{a}bc + \bar{a}b\bar{c} + \bar{a}b\bar{c} + ab\bar{c}$

since $X + X = X$

$f = \bar{a}bc + \bar{a}b\bar{c} + ab\bar{c}$

In decimal notation this is the same as, in minterm (SOP)

$f = \bar{a}bc\ (011) + \bar{a}b\bar{c}\ (010) + ab\bar{c}\ (110) = \sum m(2,3,6)$

The maxterm (POS) equivalent is

$f = \prod M(0,1,4,5,7)$

Exercises

Find the minterm/maxterm expansion of $f(a, b, c, d) = \bar{a}(\bar{b} + d) + ac\bar{d}$.

Find a minimum sum-of-products expression for $F(a, b, c) = \sum m(0, 1, 2, 5, 6, 7)$

- using algebraic simplification
- using Karnaugh map

Karnaugh Maps & Minterms

If a function is given in algebraic form, it is unnecessary to expand it to *minterm* form before plotting it on a map.

If the algebraic expression is converted to sum-of-products form, then each product term can be plotted directly as a group of 1's on the map.

For example, given, $f(a, b, c) = ab\bar{c} + \bar{b}c + \bar{a}$

We then fill a three variable Karnaugh Map as follows:

The term $ab\bar{c}$ is 1 when a = 1 and bc = 10, so we place a 1 in the square which corresponds to the a = 1 row and the bc = 10 column of the map.

The term $\bar{b}c$ is 1 when bc = 01, so we place 1's in both squares of the bc = 01 column of the map.

The term $\bar{a}$ is 1 when $a = 0$, so we place 1's in all the squares of the a = 0 row of the map.

Example

		bc			
		00	01	11	10
a	0	1	1	1	1
	1	0	1	0	1

Example

Plot the following function, $f(a, b, c, d) = acd + \bar{a}b + \bar{d}$, on a Karnaugh map

Solution:

The first term is 1 when $a = c = d = 1$, so we place 1's in the two squares which are in the $a = 1$ row and $cd = 11$ column.

The term $\bar{a}b$ is 1 when $ab = 01$, so we place four 1's in the $ab = 01$ row.

Finally, $\bar{d}$ is 1 when $d = 0$, so we place eight 1's in the two rows for which $d = 0$. (Duplicate 1's are not plotted because $1 + 1 = 1$.)

Example

ab \ cd	00	01	11	10
00	1			1
01	1	1	1	1
11	1		1	1
10	1		1	1

Exercise

Determine the minimum sum of products and minimum product of sums for the following

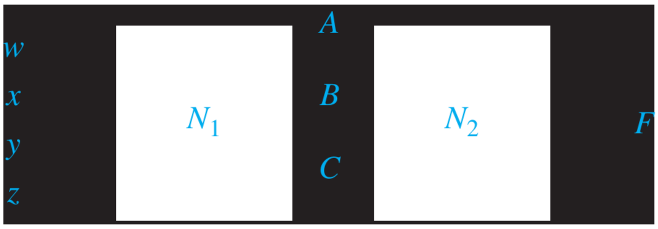
(a) $f = \bar{b}\bar{c}\bar{d} + bcd + ac\bar{d} + \bar{a}\bar{b}c + \bar{a}b\bar{c}d$

(b) $f = \bar{x}\bar{z} + wyz + \bar{w}\bar{y}\bar{z} + \bar{x}y$

Incompletely Specified Functions

A large digital system is usually divided into many subcircuits.

Consider the following example in which the output of circuit N_1 drives the input of circuit N_2.



Let us assume that the output of N_1 does not generate all possible combinations of values for A, B, and C. In particular, we will assume that there are no combinations of values for w, x, y, and z which cause A, B, and C to assume values of 001 or 110.

Hence, when we design N_2, it is not necessary to specify values of F for $ABC = 001$ or 110 because these combinations of values can never occur as inputs to N_2.

Incompletely Specified Functions

For example, F might be specified by Table below:

A B C	F
0 0 0	1
0 0 1	X
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	X
1 1 1	1

The X's in the table indicate that we don't care whether the value of 0 or 1 is assigned to F for the combinations $ABC = 001$ or 110.

In this example, we *don't care* what the value of F is because these input combinations never occur anyway.

The function F is then *incompletely specified*.

Incompletely Specified Functions

The minterms $\overline{A}\,\overline{B}C$ and $AB\overline{C}$ are referred to as don't-care minterms, since we don't care whether they are present in the function or not.

When we realize the function, we must specify values for the don't-cares.

It is desirable to choose values which will help simplify the function.

If we assign the value 0 to both X's, then

$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC + ABC = \overline{A}\,\overline{B}\,\overline{C} + BC$

If we assign 1 to the first X and 0 to the second, then

$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}BC + ABC = \overline{A}\overline{B} + BC$

If we assign 1 to both X's, then

$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}BC + AB\overline{C} + ABC = \overline{A}\,\overline{B} + BC + AB$

The <u>second choice</u> of values leads to the simplest solution.

Slide Correction

I had incorrectly agreed earlier that $\overline{A}\,\overline{B} + BC + AB$ is that same as BC but I did the logic and it's not.

This is easily verifiable with a Karnaugh map as below:

		B	
		0	1
A	0	1	
	1		1

Thus, $\overline{A}\,\overline{B} + AB$ is NOT 0. DO take note.

Incompletely Specified Functions

Note that way in which incompletely specified functions can arise, and there are many other ways. In the preceding example, don't-cares were present because certain combinations of circuit inputs did not occur.

In other cases, all input combinations may occur, but the circuit output is used in such a way that we do not care whether it is 0 or 1 for certain input combinations.

Incompletely Specified Functions

When writing the minterm expansion for an incompletely specified function, *m* is often used to denote the required minterms and *d* to denote the don't-care minterms.

Using this notation, the minterm expansion for Table above is

$F = \sum m(0,3,7) + \sum d(1,6)$

Incompletely Specified Functions

The Karnaugh map method is easily extended to functions with don't-care terms.

The required minterms are indicated by 1's on the map, and the don't-care minterms are indicated by *X*'s.

When choosing terms to form the minimum sum of products, all the 1's must be covered, but the *X*'s are only used if they will simplify the resulting expression.

Other Uses of Karnaugh Maps

Many operations that can be performed using a truth table or algebraically can be done using a Karnaugh map.

A map conveys the same information as a truth table—it is just arranged in a different format.

If we have a function *F* on a map, we can read off the minterm and maxterm expansions for F and for F'.

If the minterm is $f = \sum m(0, 2, 3, 4, 8, 10, 11, 15)$, and because each 0 corresponds to a maxterm, the maxterm expansion of *f* is $f = \prod M(1, 5, 6, 7, 9, 12, 13, 14)$

We can prove that two functions are equal by plotting them on maps and showing that they have the same Karnaugh map.

When simplifying a function algebraically, the Karnaugh map can be used as a guide in determining what steps to take.