CS525: Advanced Database Organization

Notes 5: Indexing and Hashing
Part II: Tree Indexes - B$^+$-Tree

Gerald Balekaki

Department of Computer Science

Illinois Institute of Technology

gbalekaki@iit.edu

June 12st 2023

Slides: adapted from courses taught by Hector Garcia-Molina, Stanford, Andy Pavlo, Carnegie Mellon University Elke A. Rundensteiner, Worcester Polytechnic Institute, Shun Yan Cheung, Emory University, Marc H. Scholl, University of Konstanz, Principles of Database Management , Ellen Munthe-Kaas, Universitetet i Oslo, & Joy Arulraj, Georgia Teck

Today's Agenda

- Conventional indexes
  - Basic Ideas: sparse, dense, multi-level ...
  - Duplicate Keys
  - Deletion/Insertion
  - Secondary indexes
- B+-Trees (Today)
  - B+-Tree Overview
  - Design Decisions
  - Optimizations
- Hash Tables (Next)

Database Indexes

- There are a number of different data structures one can use inside of a database system for purposes such as internal meta-data, core data storage, temporary data structures, or table indexes.
- A data structure that improves the speed of data retrieval operations on a table at the cost of additional writes and storage space.
- Indexes are used to quickly locate data without having to search every row in a table every time a table is accessed.

Data Structures

- Two types of data structures:
  - Order Preserving Indexes
    - A tree-like structure that maintains keys in some sorted order.
    - Supports all possible predicates with *O(log n)* searches[1].
    - Example: `AGE > 20 AND AGE < 30`
  - Hashing Indexes
    - An associative array that maps a hash of the key to a particular record.
    - Only supports equality predicates with *O(1)* searches.
    - Example: `AGE = 20`

[1] *Given n nodes in a tree, how many iterations needed, on average, to reach any node in the tree.*

Multilevel Indexes Revisited

- Creating index-to-an-index results in multilevel indexes
- Multilevel indexes useful for speeding up data access if lowest level index becomes too large
- Index can be considered as a sequential file and building an index-to-the-index improves access
- Higher-level index is, again, a sequential file to which index can be built and so on
- Lowest level index entries may contain pointers to disk blocks or records
- Higher-level index contains as many entries as there are blocks in the immediately lower level index
- Index entry consists of search key value and reference to corresponding block in lower level index
- Index levels can be added until highest-level index fits within single disk block
- First-level index, second-level index, third-level index etc.

Multilevel Indexes Revisited

- Multilevel index can be considered as search tree, with each index level representing level in tree, each index block representing a node and each access to the index resulting in navigation towards a subtree in the tree
- Multilevel indexes may speed up data retrieval, but large multilevel indexes require a lot of maintenance in case of updates

B-tree Family

- Almost every modern DBMS that supports order-preserving indexes uses a B+-Tree.
- There is a specific data structure called a B-Tree, but then people also use the term to generally refer to a class of data structures.
  - B-Tree (1971)
  - B+-Tree (1973)
  - B*-Tree (1977)
  - B^link-Tree (1981) [2]
- The primary difference between the original B-Tree and the B+-Tree is that B-Trees store keys and values in all nodes, while B+-Trees store values only in leaf nodes
- Modern B+-Tree implementations combine features from other B-Tree variants, such as the sibling pointers used in the B^link-Tree.

[2] Efficient locking for concurrent operations on B-trees

B$^+$-Tree Structure

- B$^+$-Tree[3]: a dynamic multi-level index
- B$^+$-Tree is a self-balancing tree data structure[4] that keeps data sorted and allows searches, sequential access, insertion, and deletions in $O(\log n)$.
  - Generalization of a binary search tree in that a node can have more than two children.
  - Optimized for systems that read and write large blocks of data.
- It is perfectly balanced (i.e., every leaf node is at the same depth from the root node in tree).

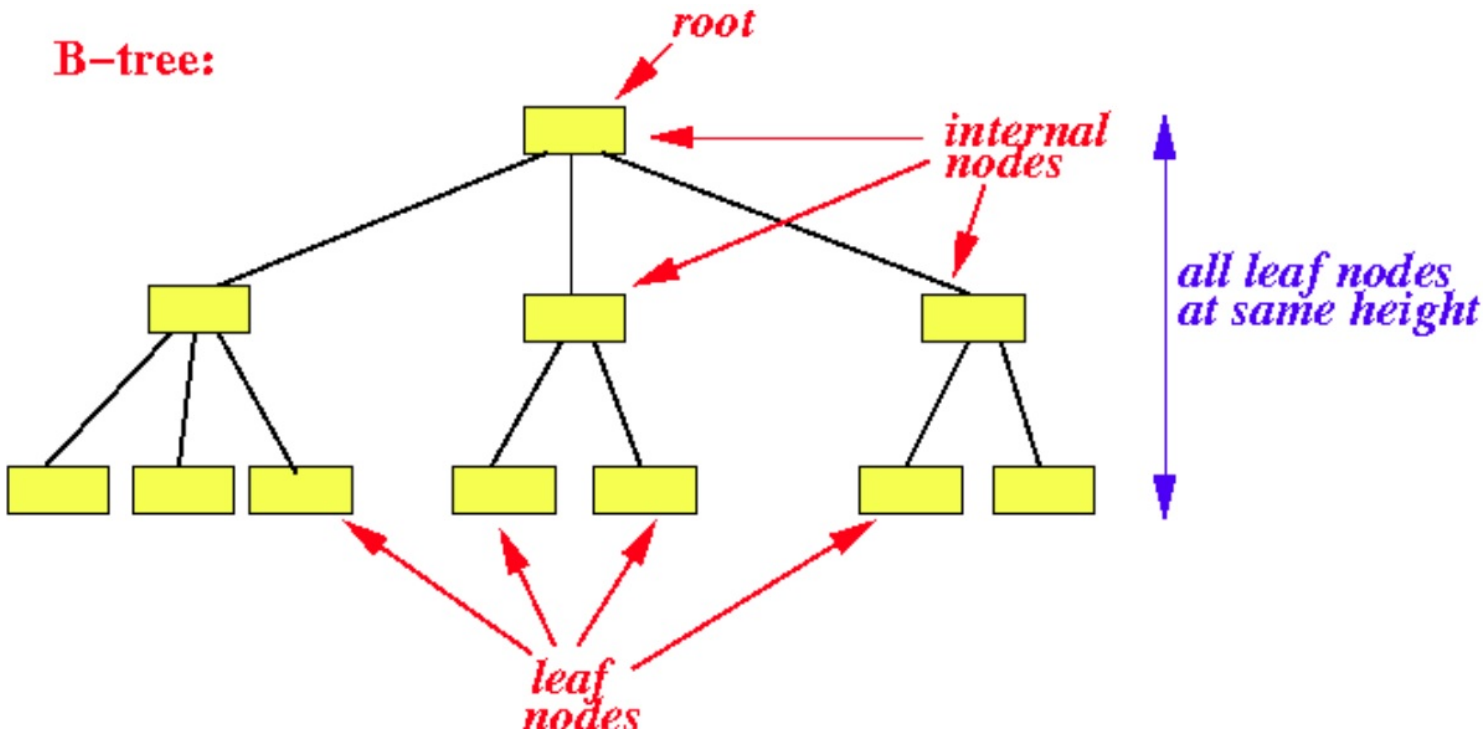
[3] *Ubiquitous B-Tree by Douglas Comer.*

[4] *Self-Balancing Binary Search trees defined as trees that automatically keeps height (maximal number of levels below the root) as small as possible when insertion and deletion operations are performed on tree*

B+tree Nodes

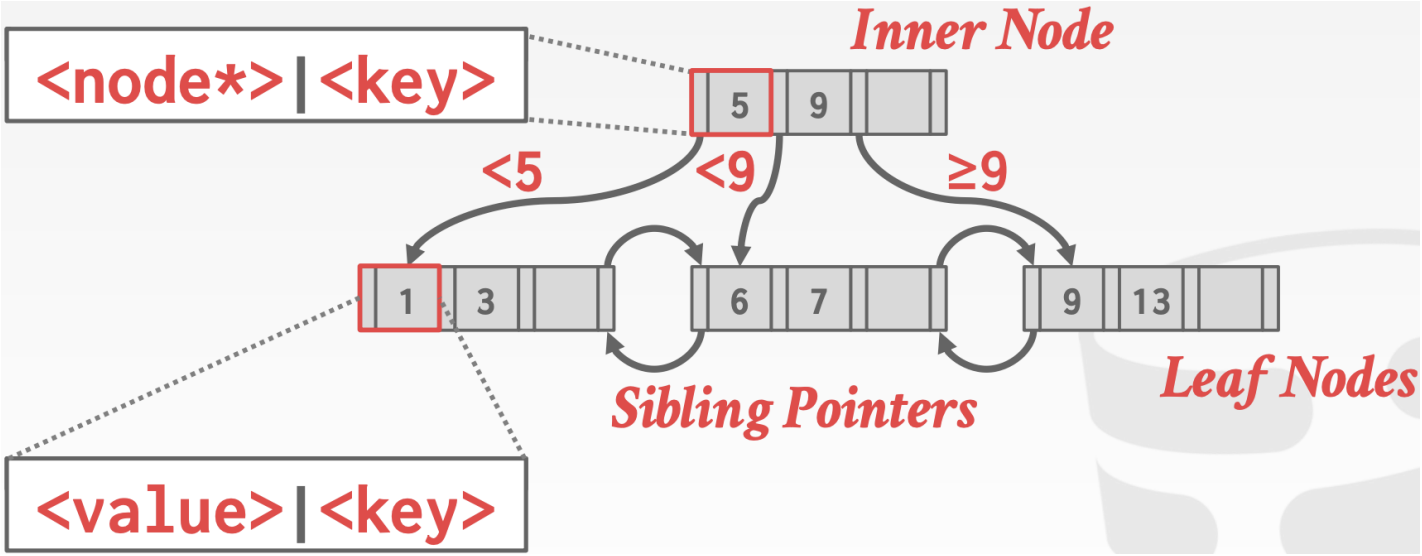
- Every node in a B+-Tree is comprised of an array of *key/value* pairs:
  - The arrays are (usually) kept in sorted key order.
  - The *keys* are derived from the attributes(s) that the index is based on.
  - The *values* will differ based on whether the node is classified as inner nodes or leaf nodes.
  - The value array for inner nodes will contain pointers to other nodes.

B$^+$tree Nodes



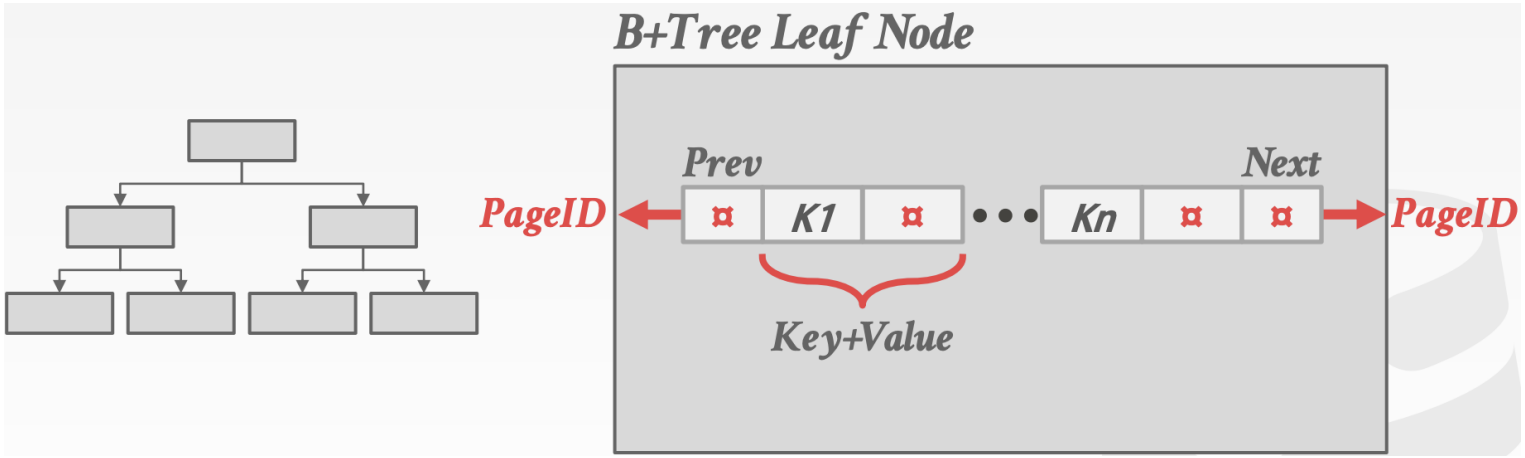
- Each node is stored in one block on disk
- `Root node`: the node at the "top" of the tree
- `Inner/Internal/intermediate node`: a node that has one or more child node(s)
- `Leaf node`: a node that does not have any children nodes

B+-Tree Diagram: Example: $n = 3$

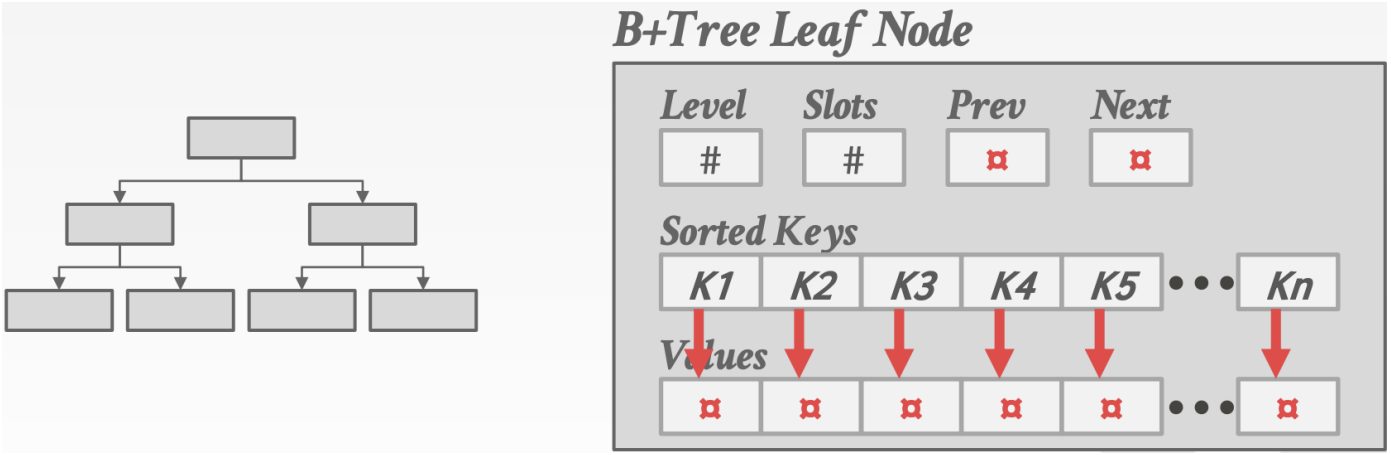


- The value array for `inner nodes` will contain pointers to other nodes.

B$^+$-Tree Leaf Nodes



B$^+$-Tree Leaf Nodes



Leaf Node Values

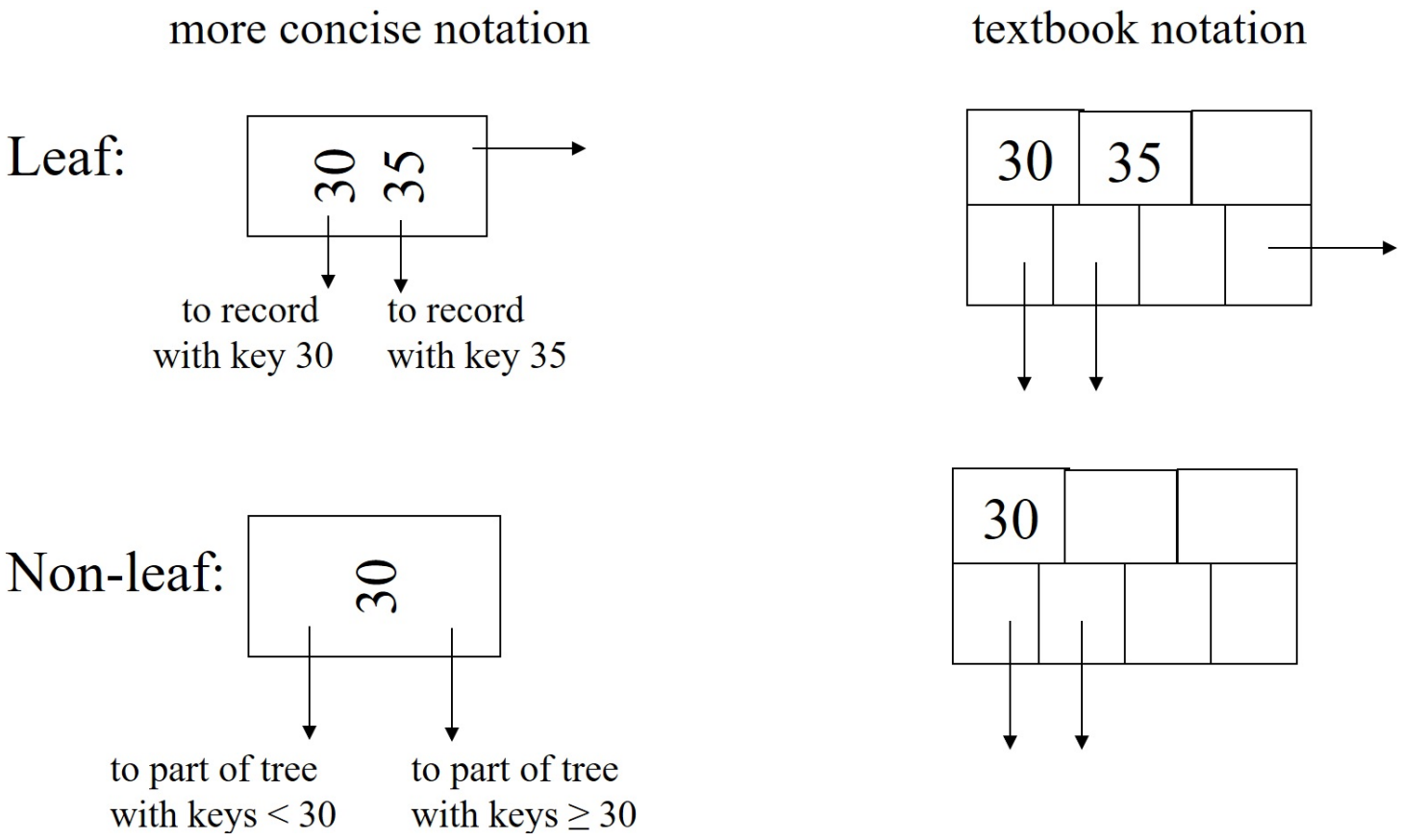
- The *values* will differ based on whether the node is classified as `inner nodes` or `leaf nodes`.
- Two approaches for leaf node values
  - Approach 1: Record IDs
    - A pointer to the location of the tuple that the index entry corresponds to.
    - Used in `PostgreSQL, MS SQL Server, IBM DB2, Oracle`
  - Approach 2: Tuple Data
    - The actual contents of the tuple is stored in the leaf node
    - Only works for primary key indexes. Secondary indexes have to store the **record id** as their values.
    - Used in `MS SQL Server, MySQL, Oracle`

Main Challenge in $\mathtt{B^+Trees}$

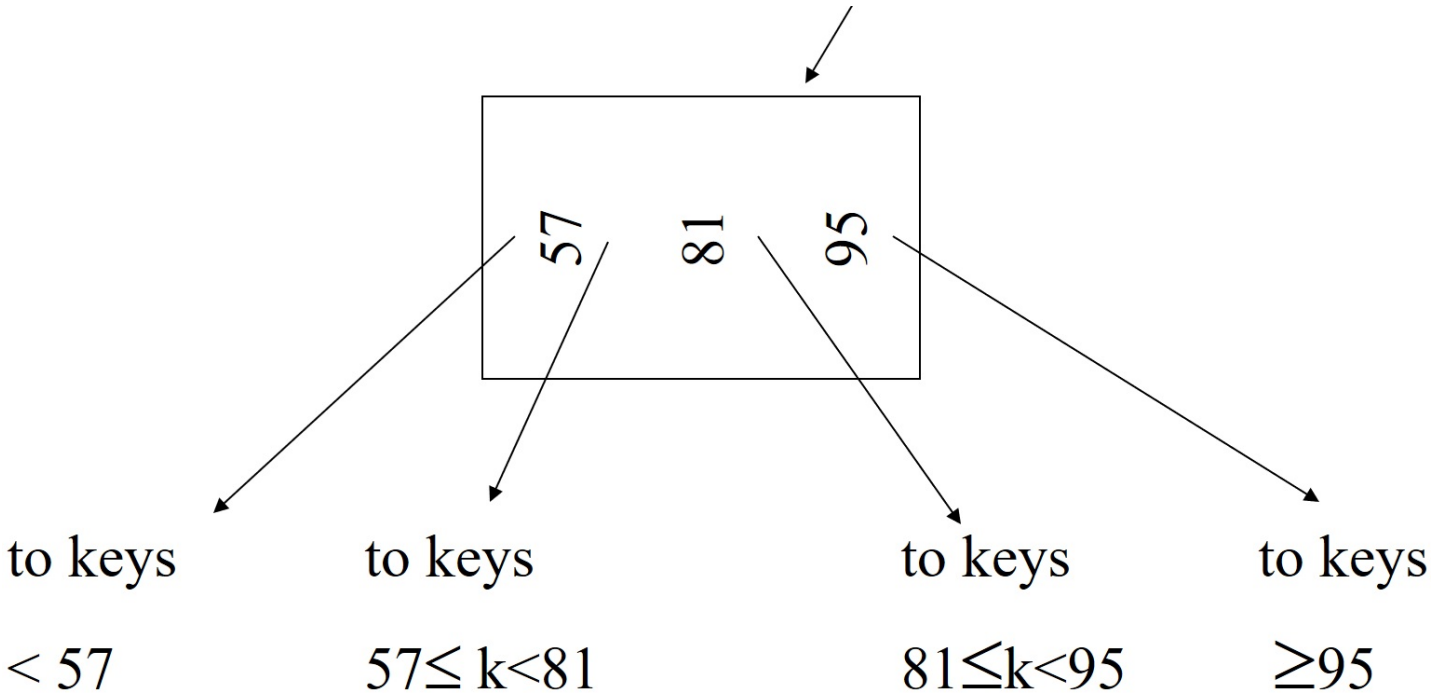
- To ensure good search cost, the tree should be kept of **minimum and uniform height**. Keep the tree:
  - full (packed), and
  - balanced (almost uniform height).
- This can be difficult – in face of insertions and deletions.
- **Solution**: Keep the tree "semi-full" (i.e., each tree node half-full)
  - Makes it easy to keep the tree balanced and semi-optimal.

Example B$^+$-Tree nodes with $n = 3$

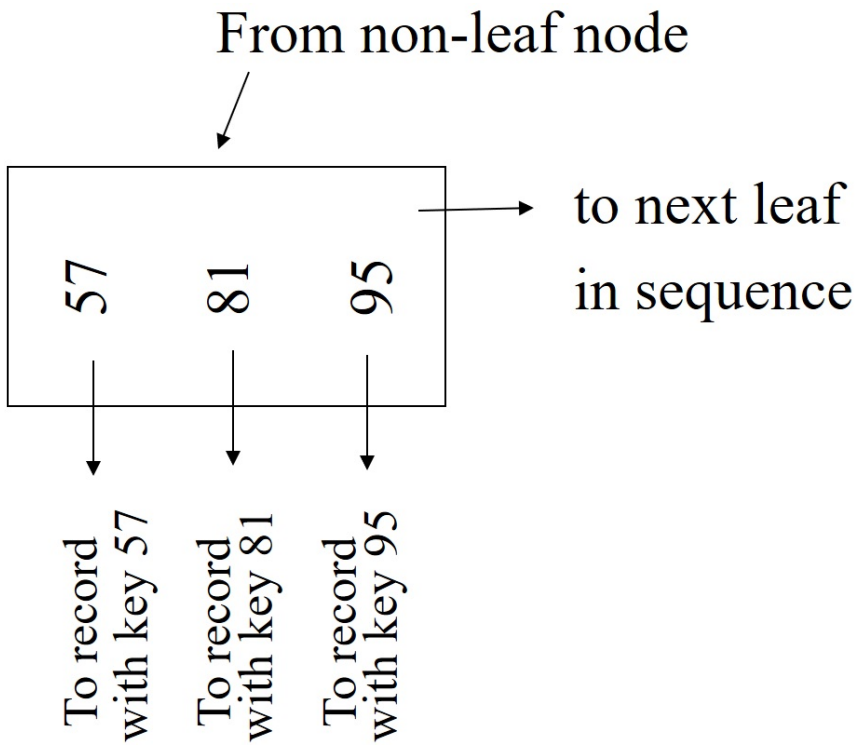
- Each internal node is stored in one block on disk
- and contains at most *n keys* and *(n+1) pointers*



Sample non-leaf

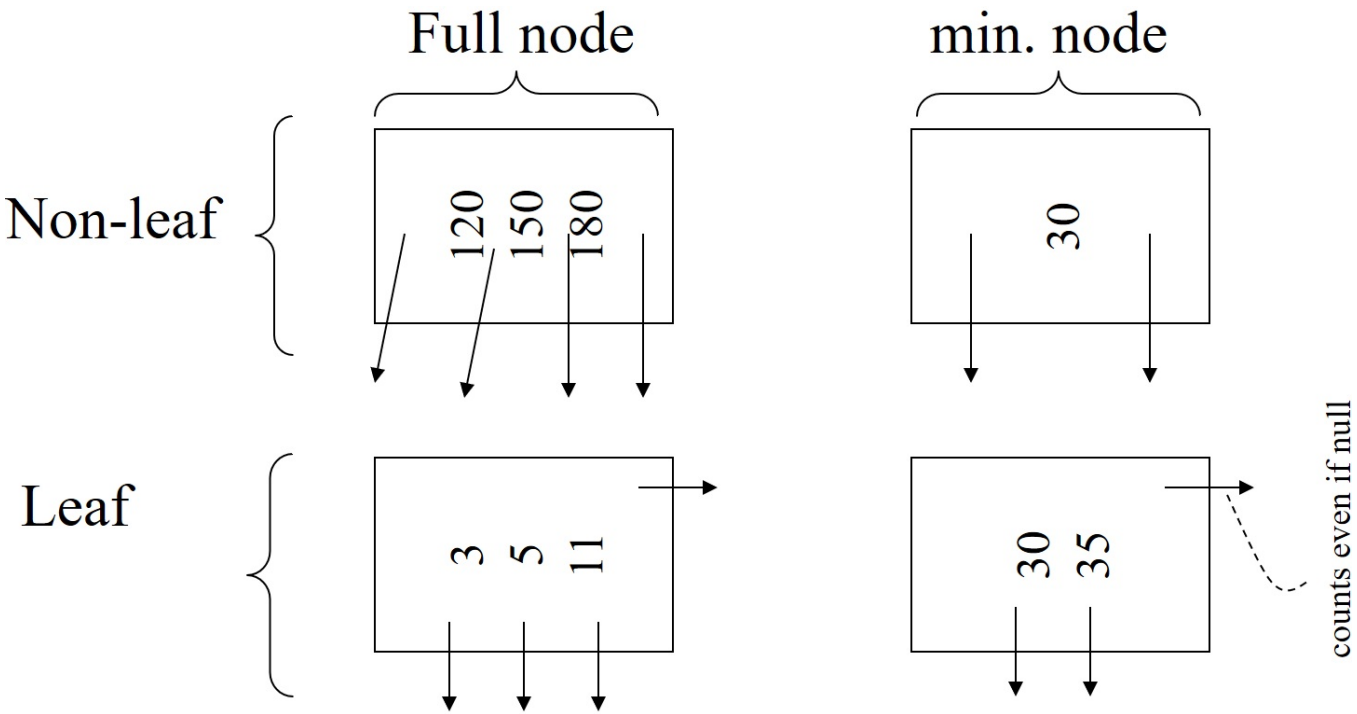


Sample leaf node



The last pointer, *sibling pointer*, points to the next leaf node (a disk block) in the B$^+$-Tree

$n = 3$



- Each internal node is stored in one block on disk and contains at most *n keys* and *n+1 pointers* (non-null children)

$$Size\ of\ nodes: \begin{cases} n+1\ pointers \\ \\ n\ keys \end{cases} \qquad (fixed)$$

B$^+$tree Nodes

- Each `node` in the tree is a `block`, which contains `search keys` and `pointers`
- Parameter n, which is largest value so that $n+1$ pointers and n keys fit in one `block`

Example (1)

If block size is 4096 bytes, keys be integers (4 bytes each), and pointers be 8 bytes each, then $n = 340$.

Don't want nodes to be too empty

- Use a "Fill Factor" to control the growth and the shrinkage. A 50% fill factor would be the minimum for `B+-Tree`
- Use at least (to ensure a balanced tree)
  - `Non-leaf`: $\lceil \frac{n+1}{2} \rceil$ pointers (to nodes)
    - Except `root`: required at least 2 be used
  - `Leaf`: $\lfloor \frac{n+1}{2} \rfloor$ pointers (to data)

Number of pointers/keys for B$^+$-Tree

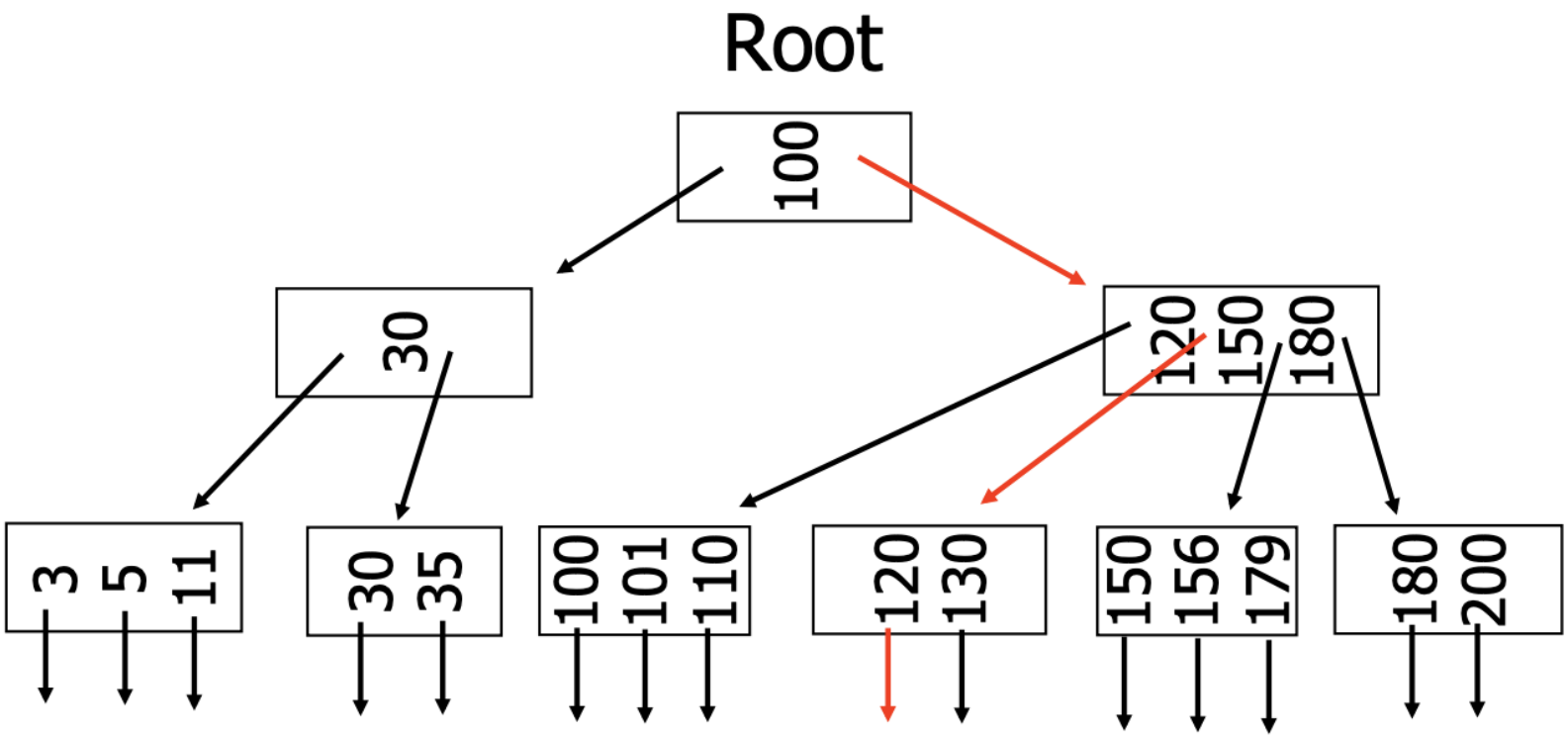
	Max ptrs	**Max keys**	**Min ptrs**	**Min keys**
Non-leaf (non-root)	$n+1$	n	$\lceil \frac{n+1}{2} \rceil$	$\lceil \frac{n+1}{2} \rceil - 1$
Leaf (non-root)	$n+1$	n	$\lfloor \frac{n+1}{2} \rfloor + 1$	$\lfloor \frac{n+1}{2} \rfloor$
Root	$n+1$	n	2*	1

*When there is only one record in the B$^+$-Tree, min pointers in the root is 1 (the other pointers are null)

Search Algorithm

- Search for `key k`
- Start from root until leaf is reached
- For current node find i so that
  - `Key[i]` $<$ `k` $\leq$ `Key[i + 1]`
  - Follow $(i+1)^{th}$ pointer
- If current node is leaf, return pointer to record or fail (no such record in tree)

Search Example: k=120, n=3



Insert into B$^+$-Tree

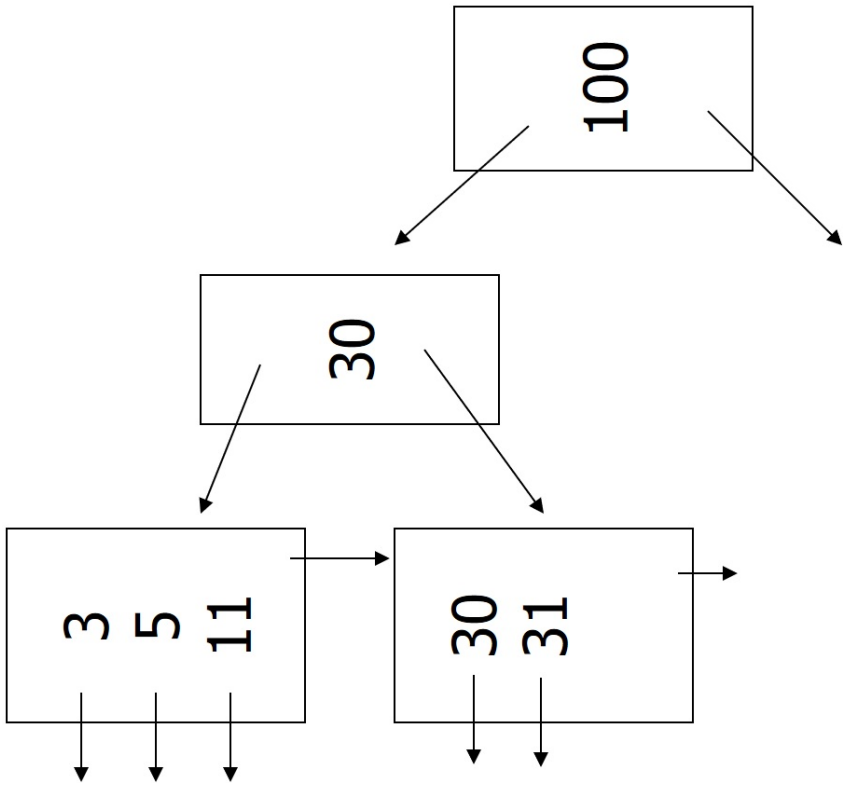
a) simple case
   - space available in `leaf`

b) `leaf` overflow

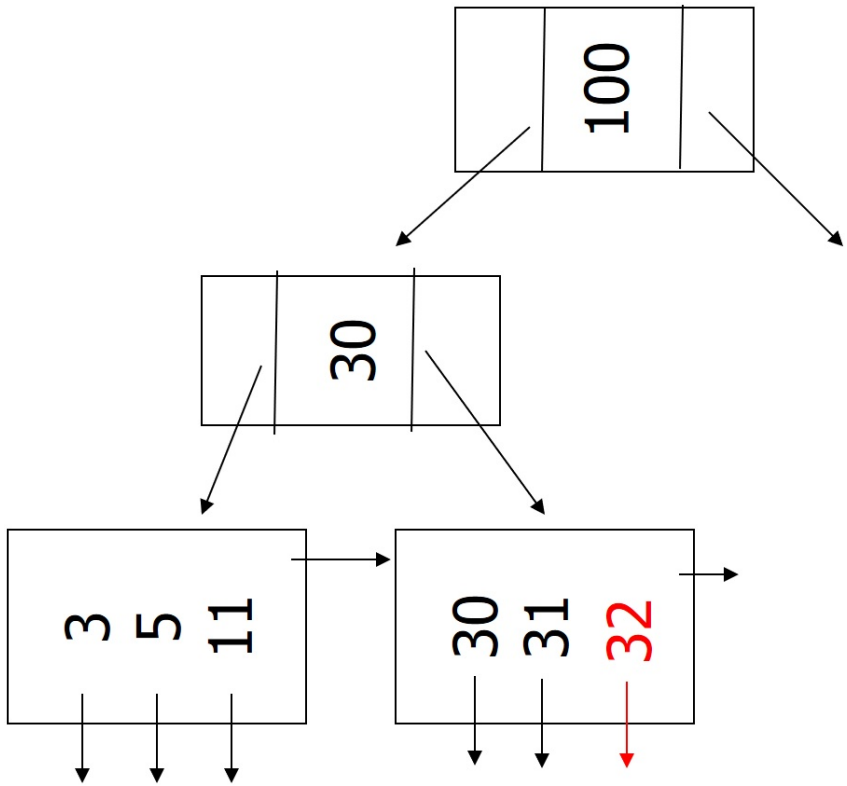
c) `non-leaf` overflow

d) new `root`

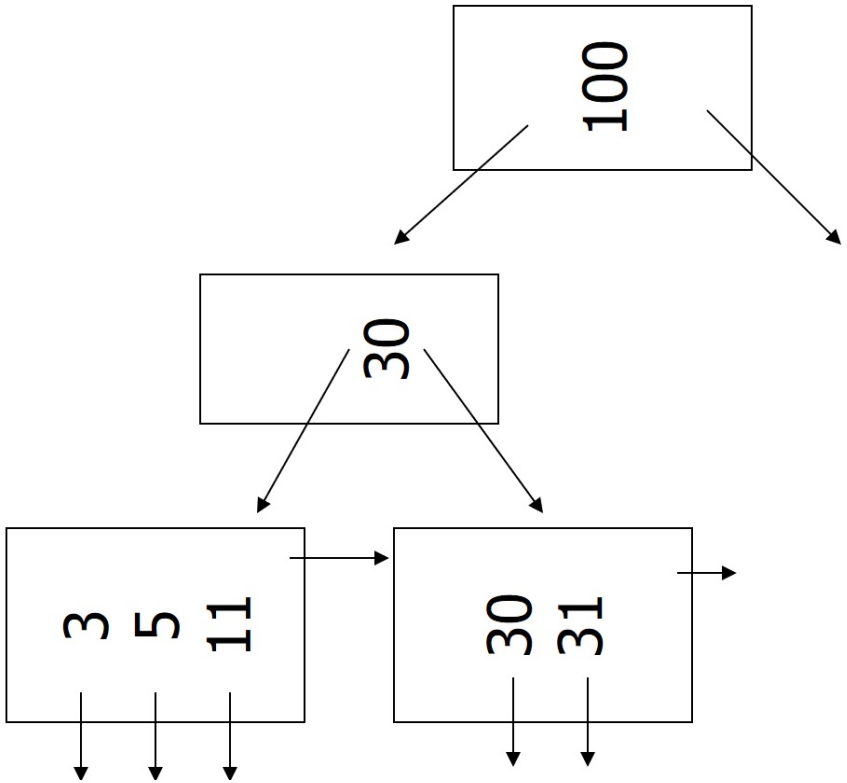
a) Insert key $= 32$, $n = 3$



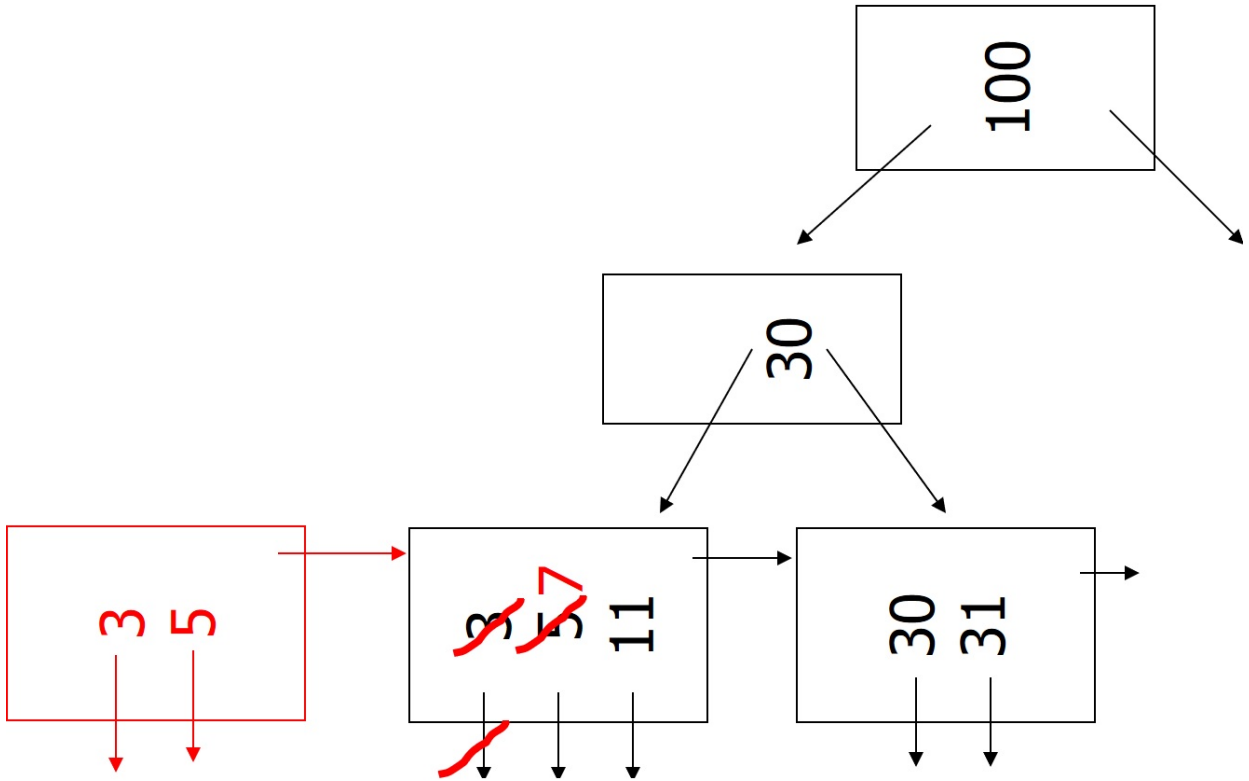
a) Insert key $= 32$, $n = 3$



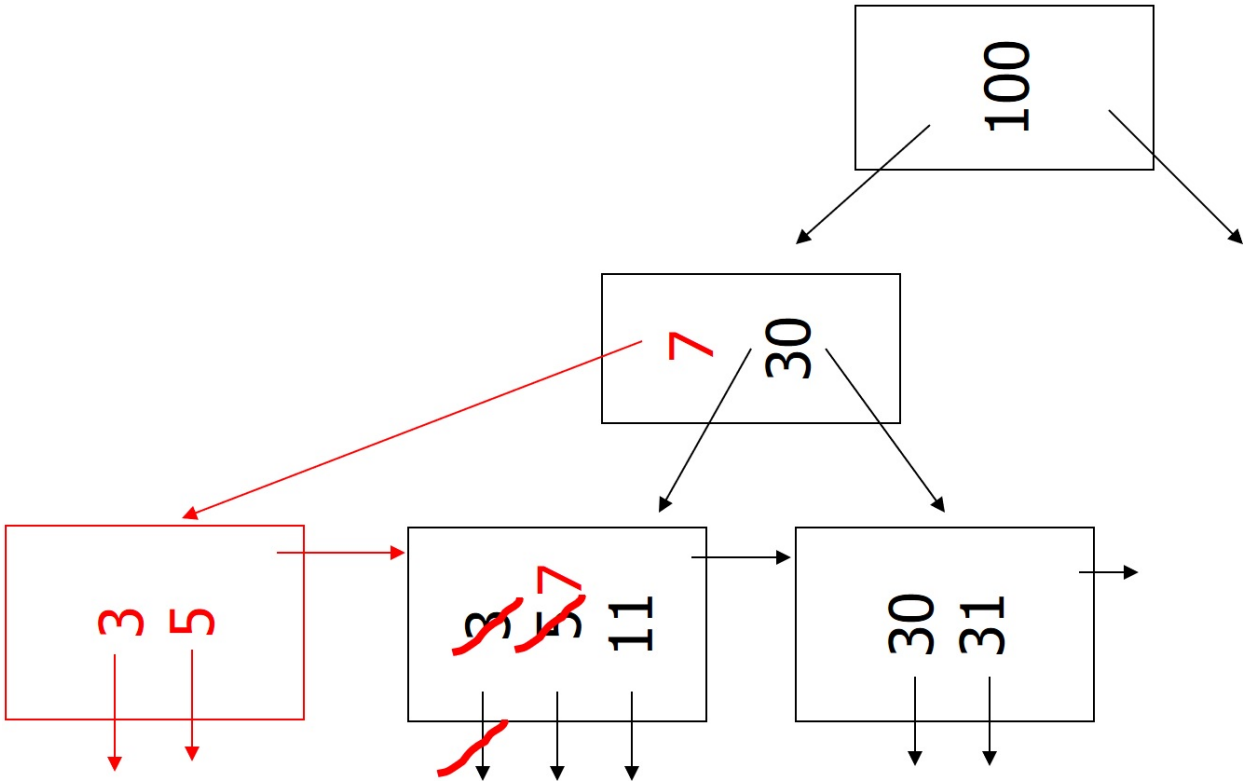
b) Insert key $= 7$, $n = 3$



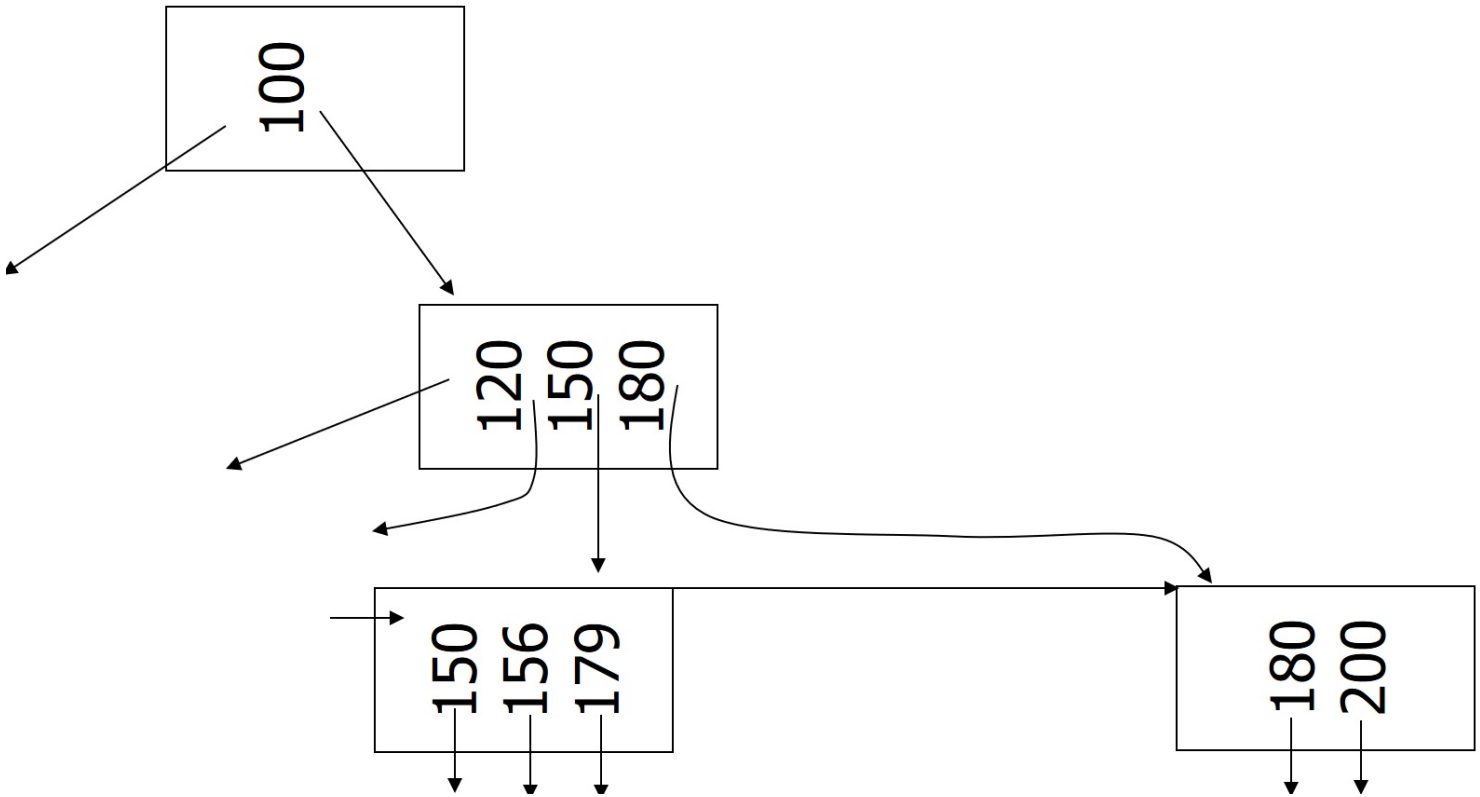
b) Insert key $= 7$, $n = 3$



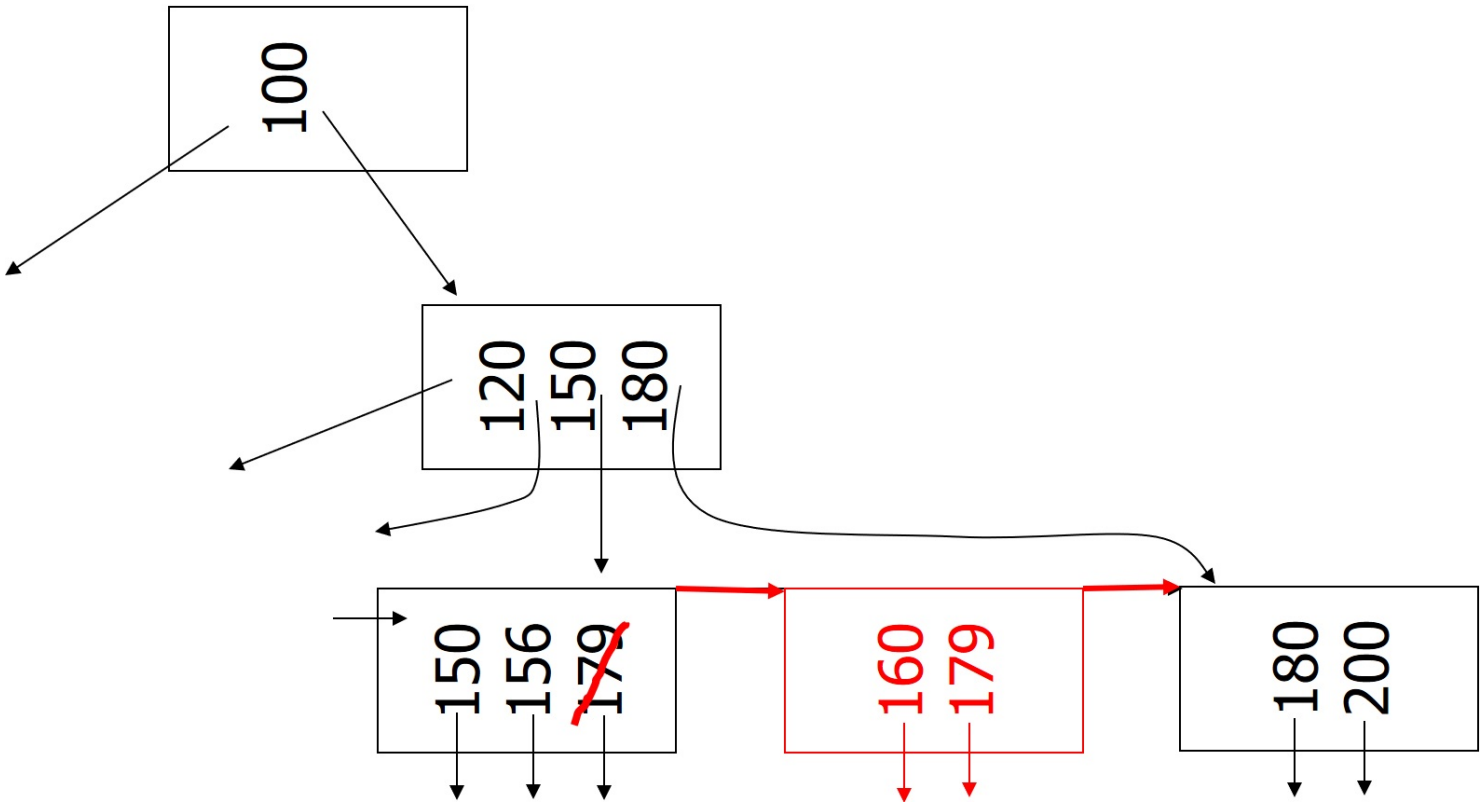
b) Insert key $= 7$, $n = 3$



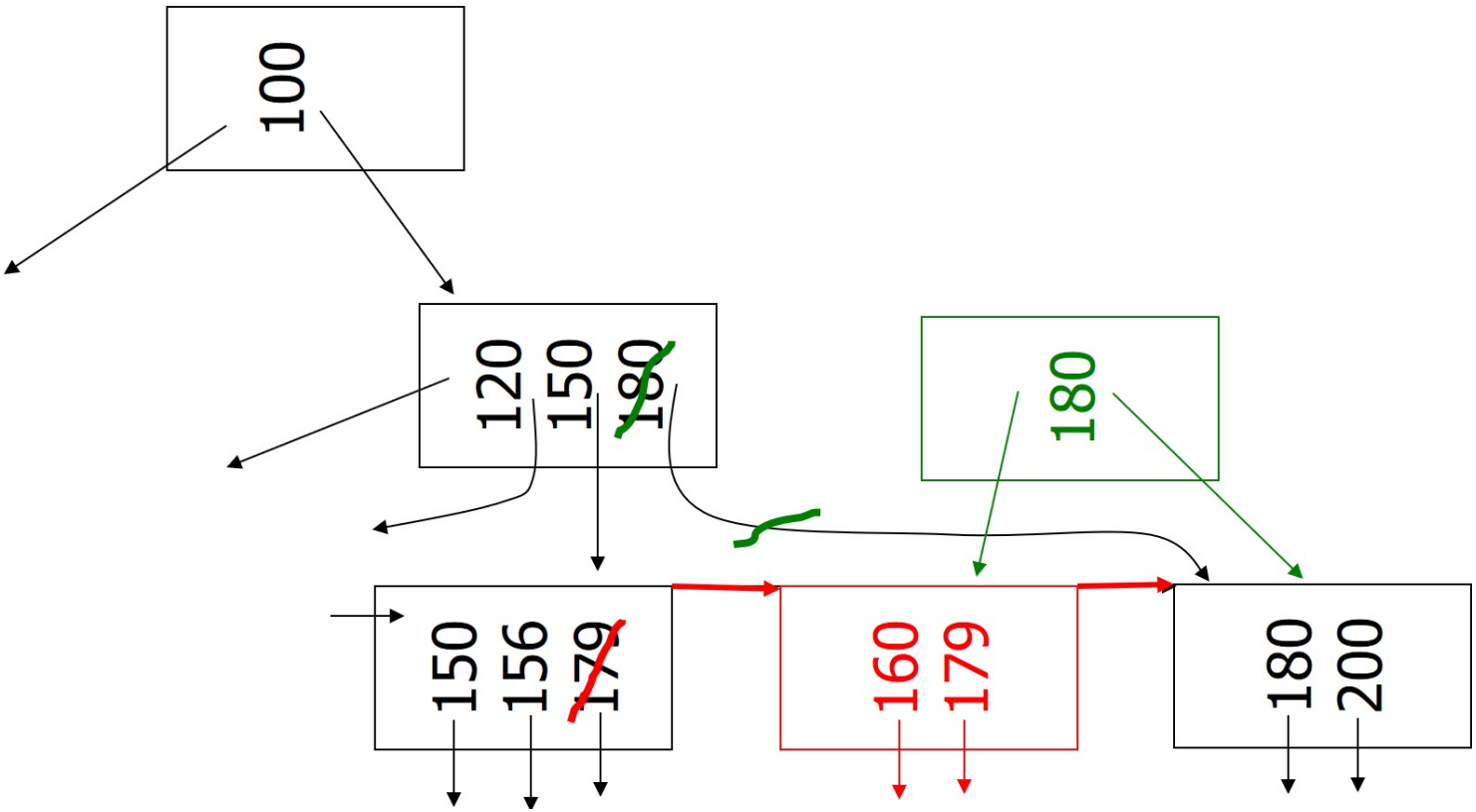
c) Insert key = 160, $n = 3$



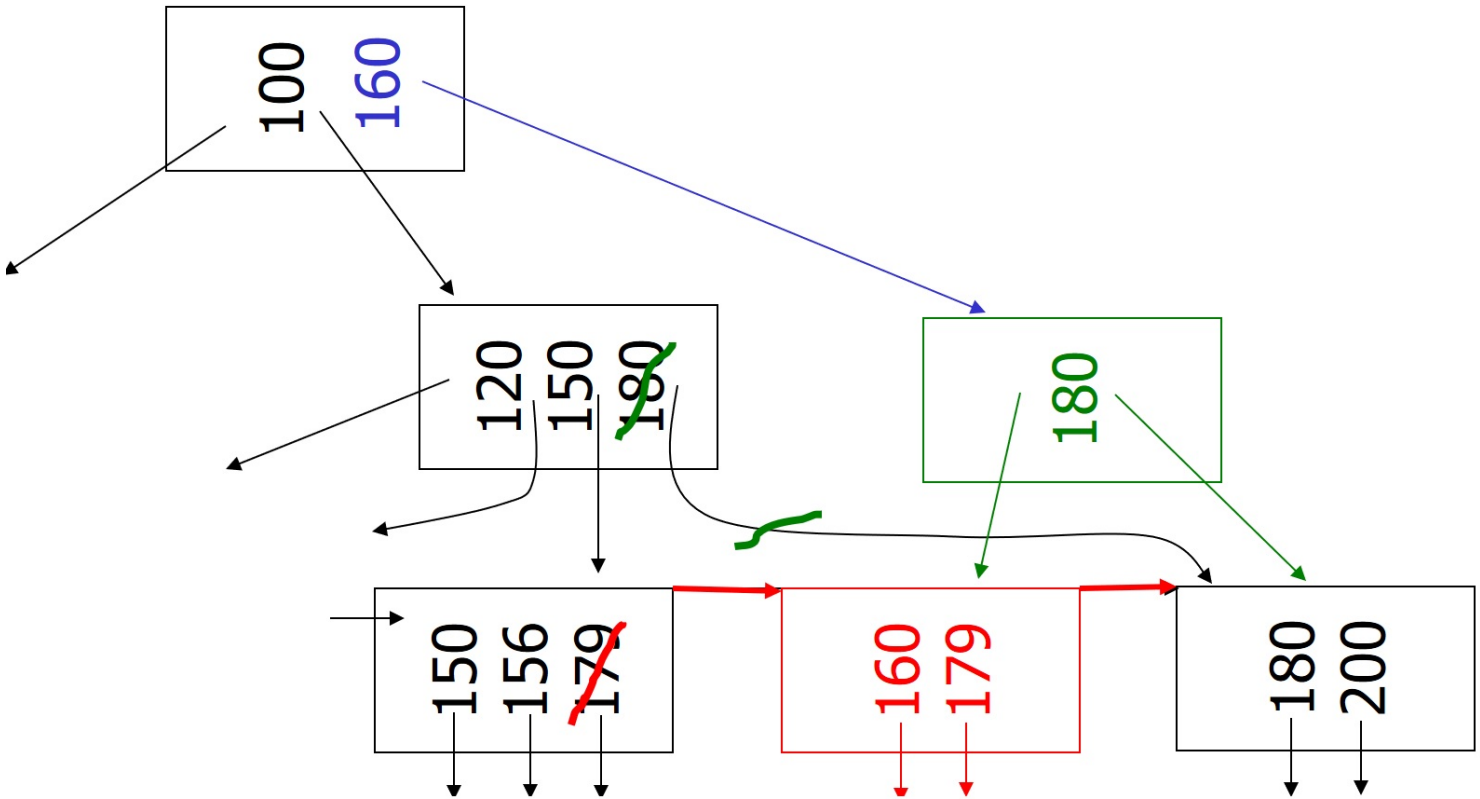
c) Insert key = 160, $n = 3$



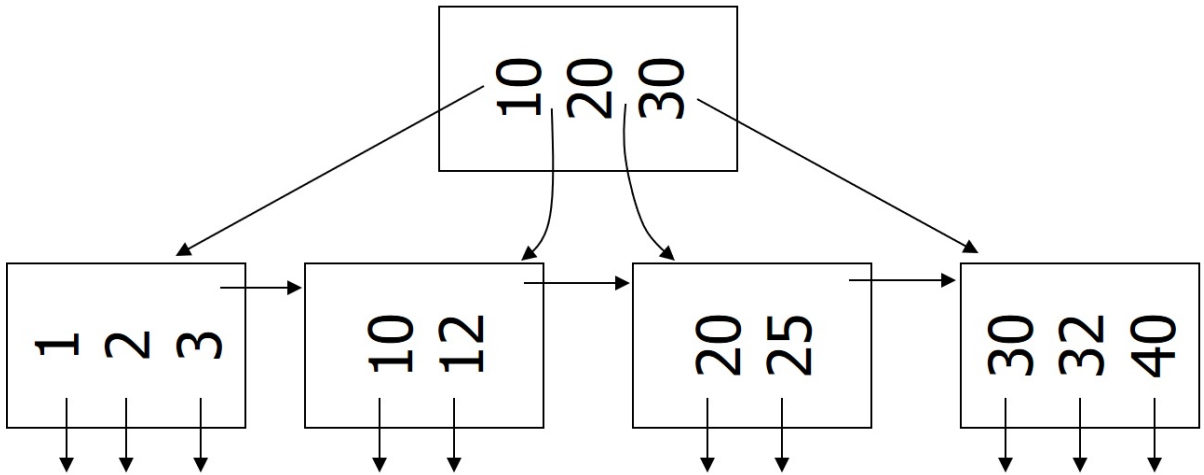
c) Insert key = 160, $n = 3$



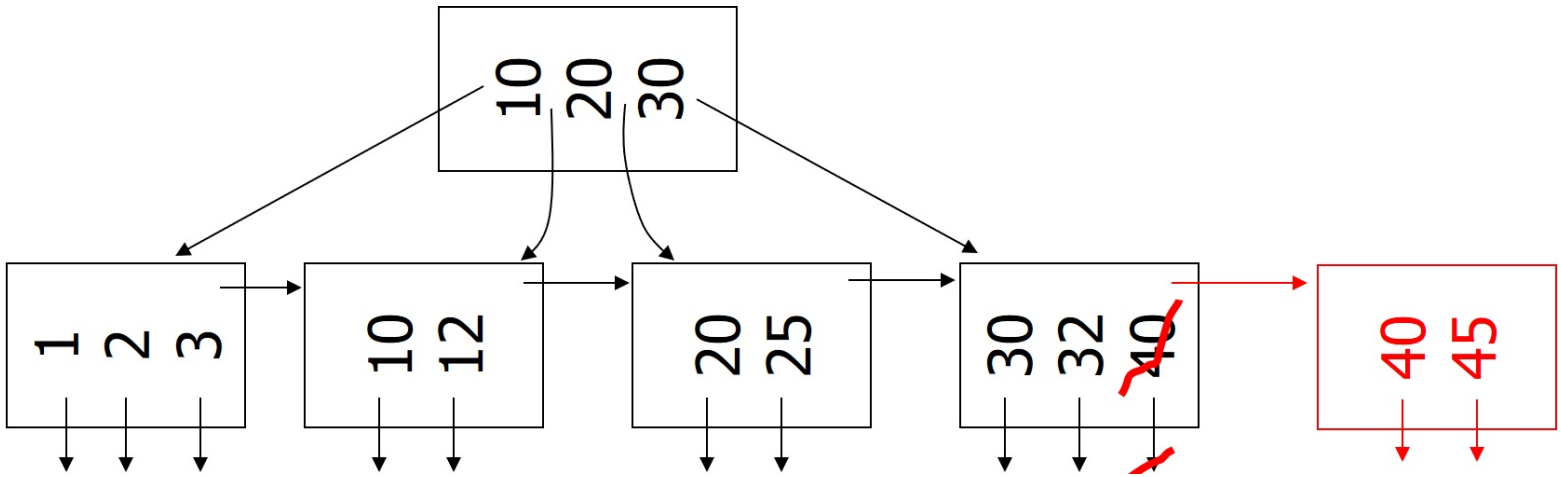
c) Insert key = 160, $n = 3$



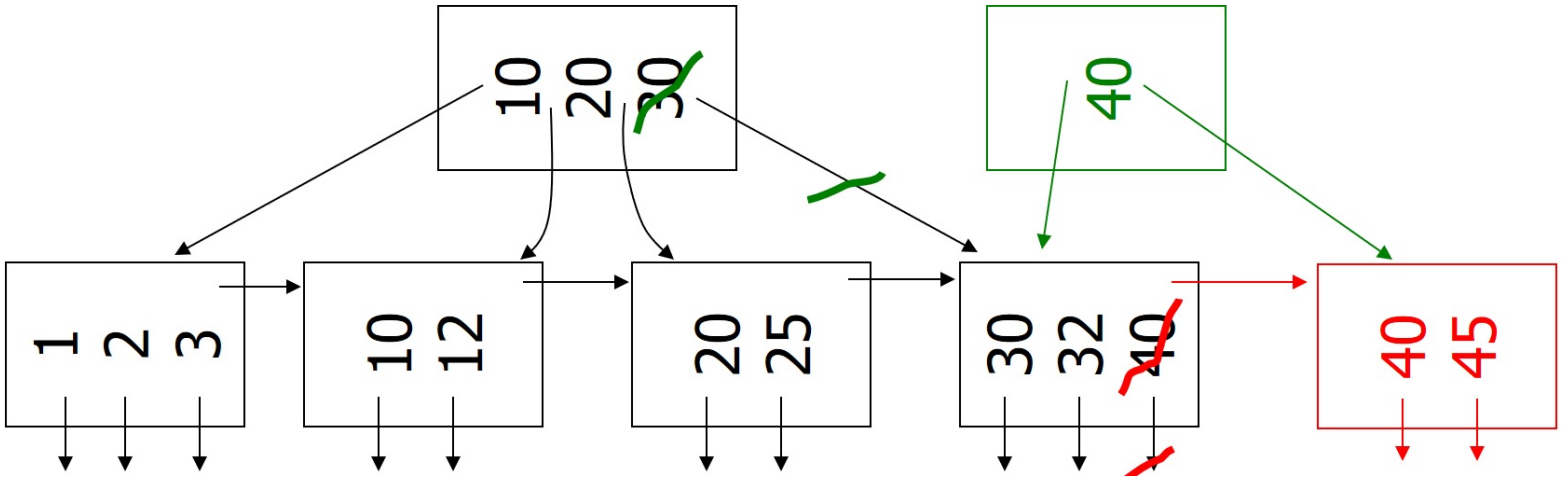
d) New `root`, insert 45, $n = 3$



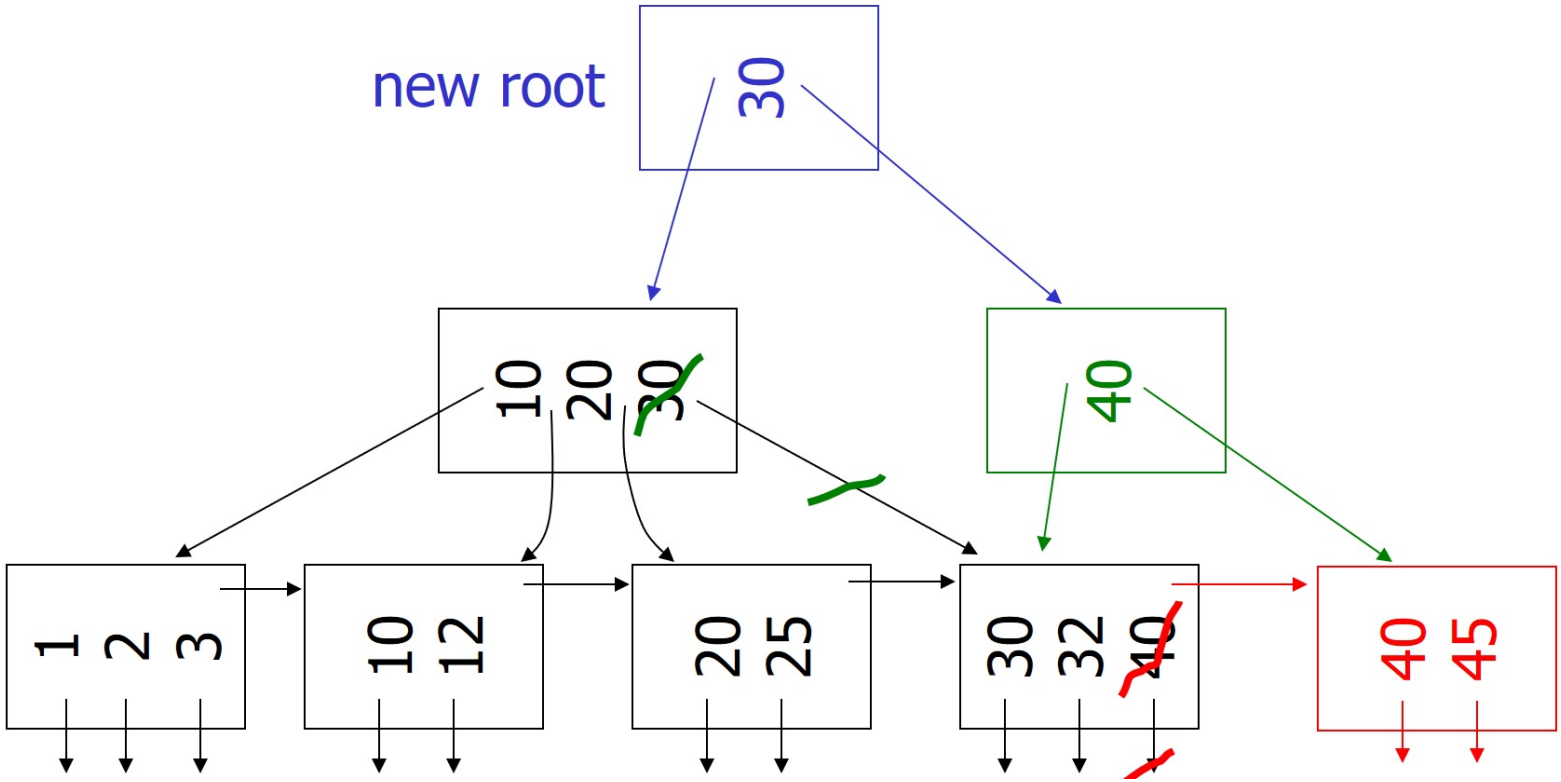
d) New `root`, insert 45, $n = 3$



d) New `root`, insert 45, $n = 3$



d) New `root`, insert 45, $n = 3$



Insertion Algorithm

To Insert Record with `key k`

1. Find correct `leaf node L` for `k`
2. Add new entry into `L` in sorted order:
   - If `L` has enough space, the operation **done**.
   - Otherwise
     - Split `L` into two nodes `L` and L_1.
     - Redistribute entries evenly and **copy up** middle key (new leaf's smallest key)
     - Insert index entry pointing to L_1 into parent of `L`
3. To split an `inner node`, redistribute entries evenly, but **push up** the middle key.

Deletion from `B+-Tree`

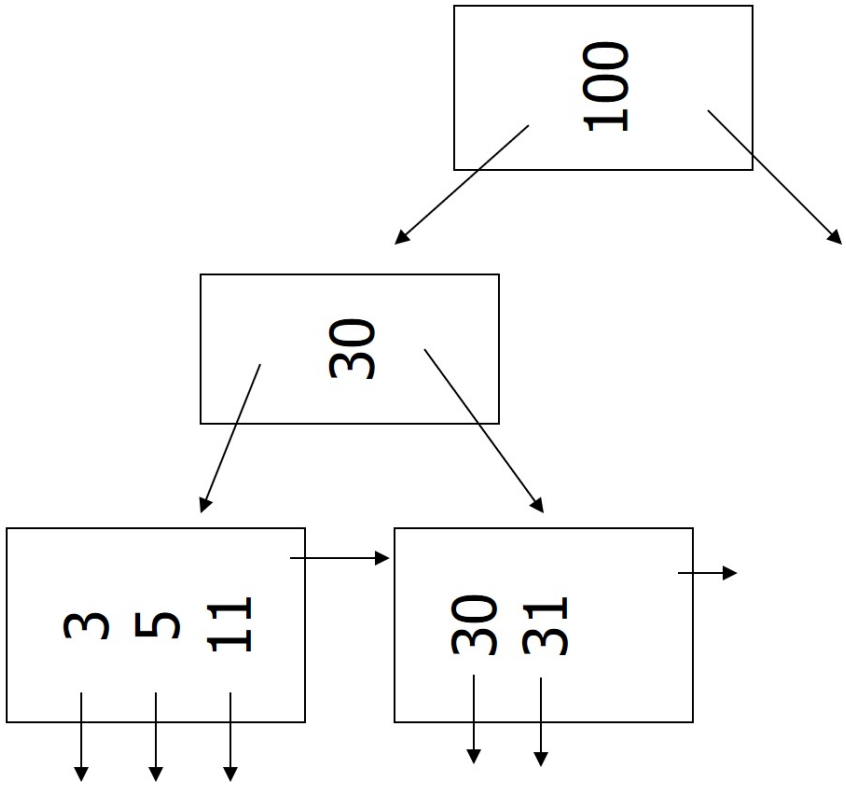
a) Simple case

b) Coalesce with neighbor (sibling)

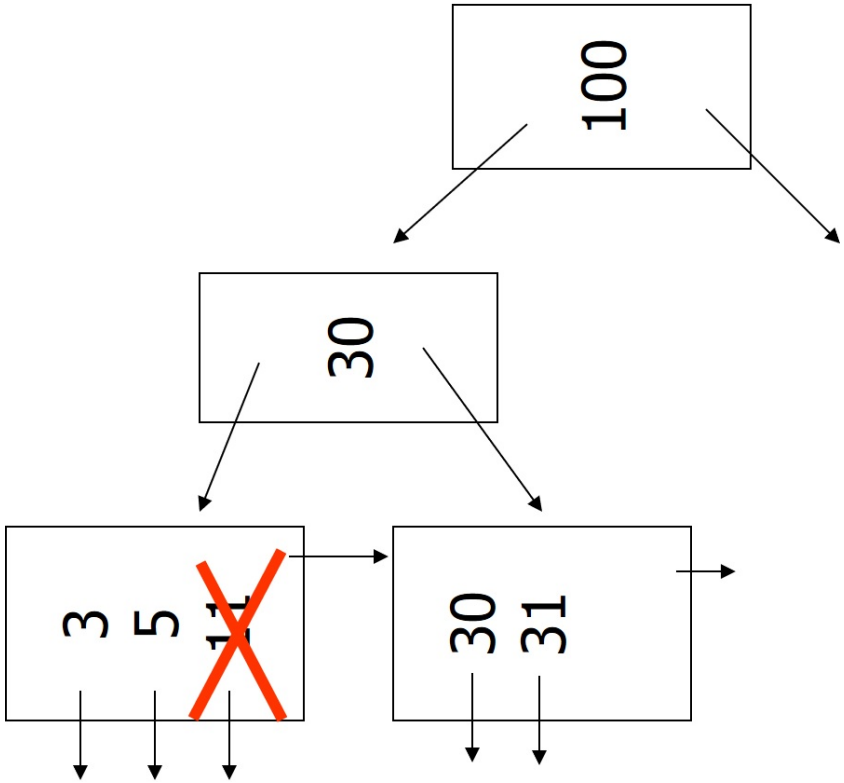
c) Re-distribute keys

d) Cases b) or c) at `non-leaf`

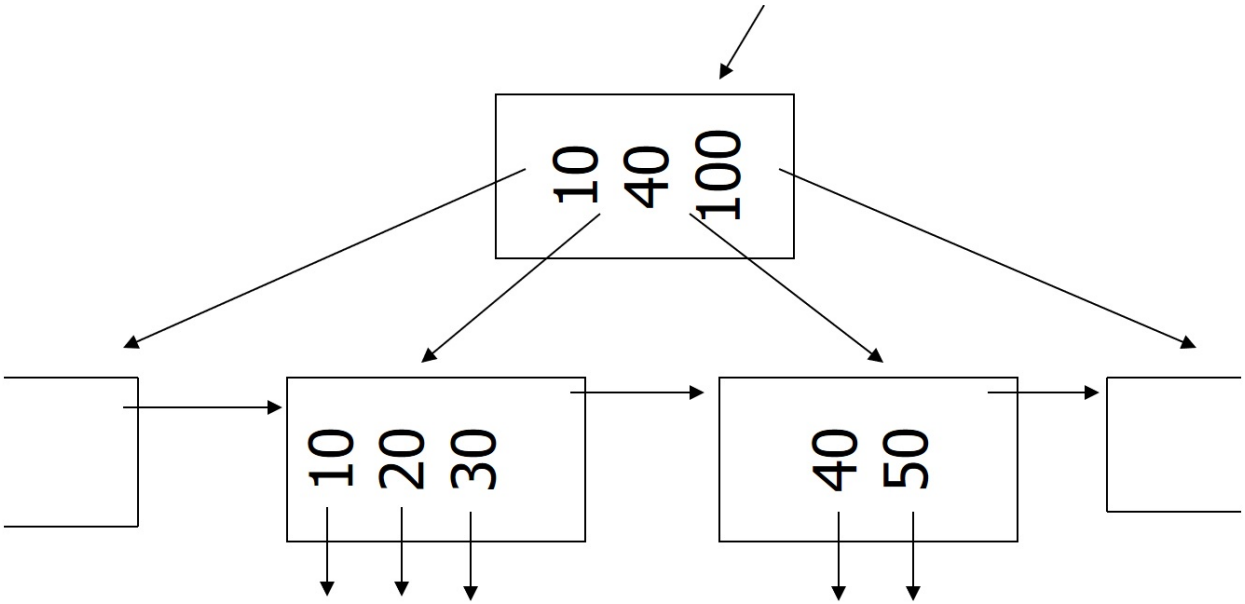
a) Delete key = 11, $n = 3$



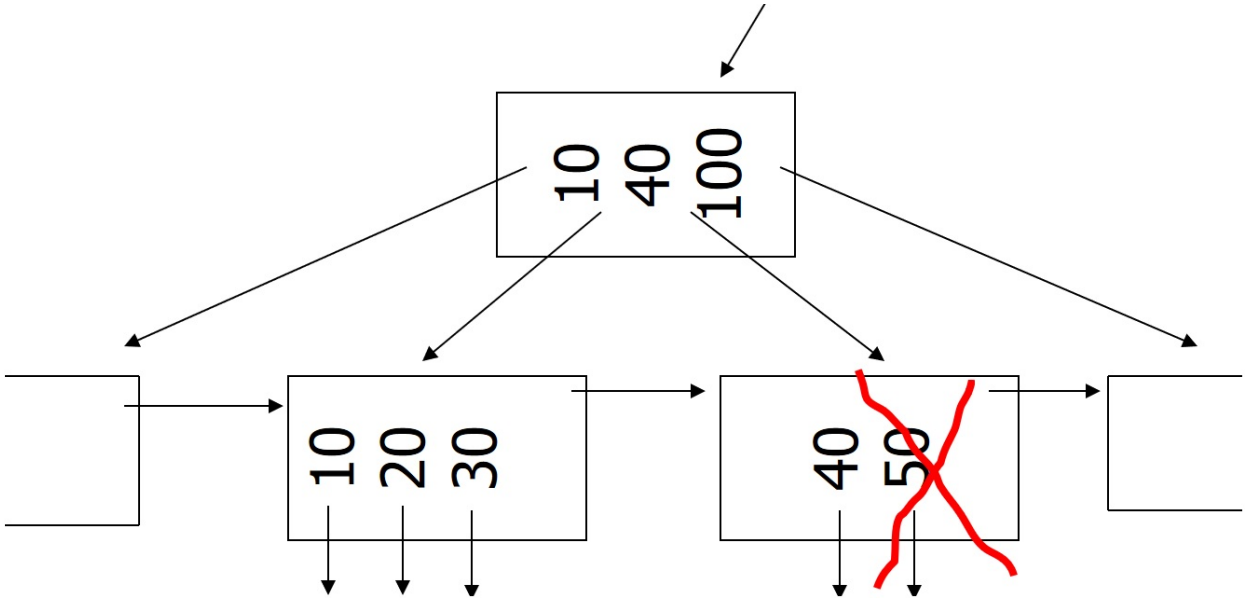
a) Delete key = 11, $n = 3$



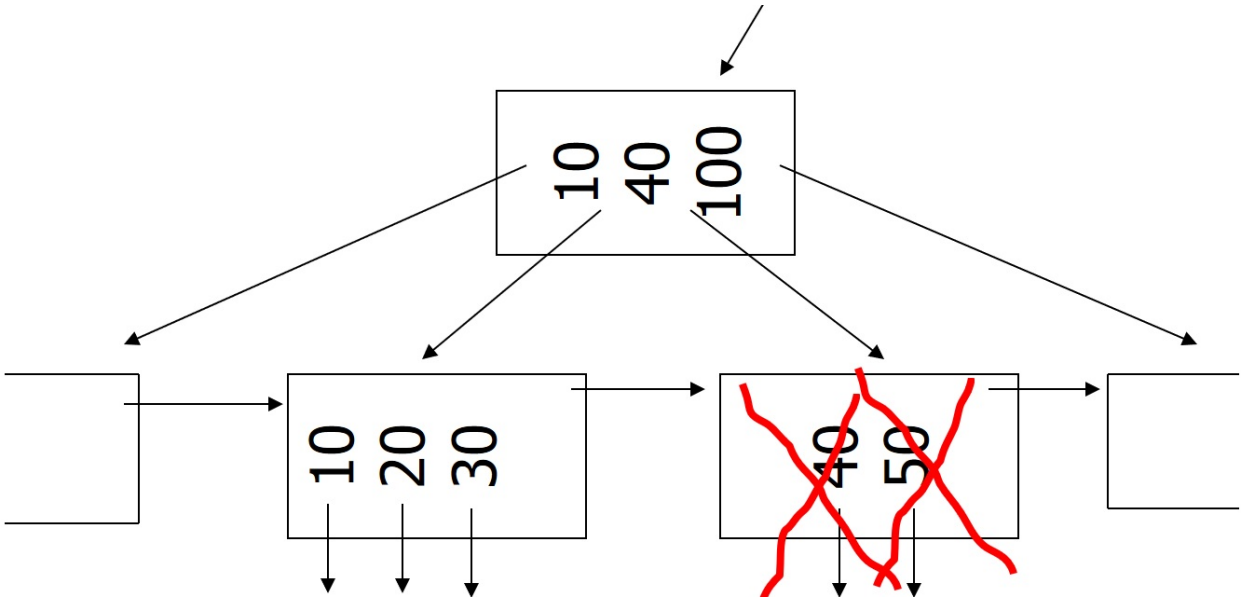
b) Coalesce with sibling: Delete 50, $n = 4$



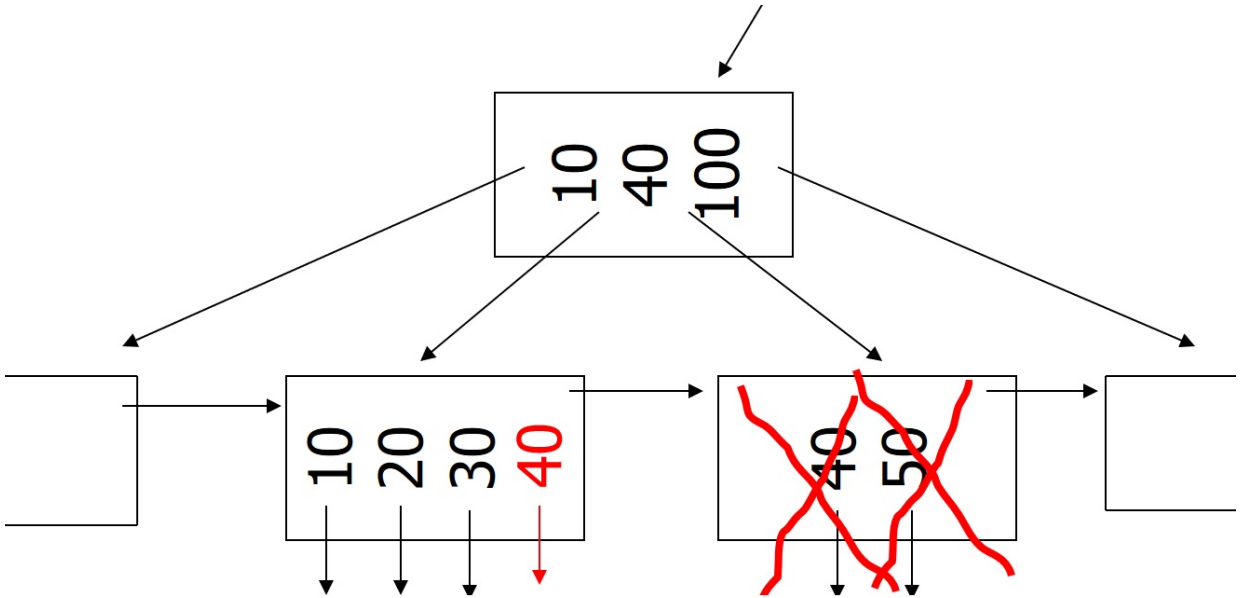
b) Coalesce with sibling: Delete 50, $n = 4$



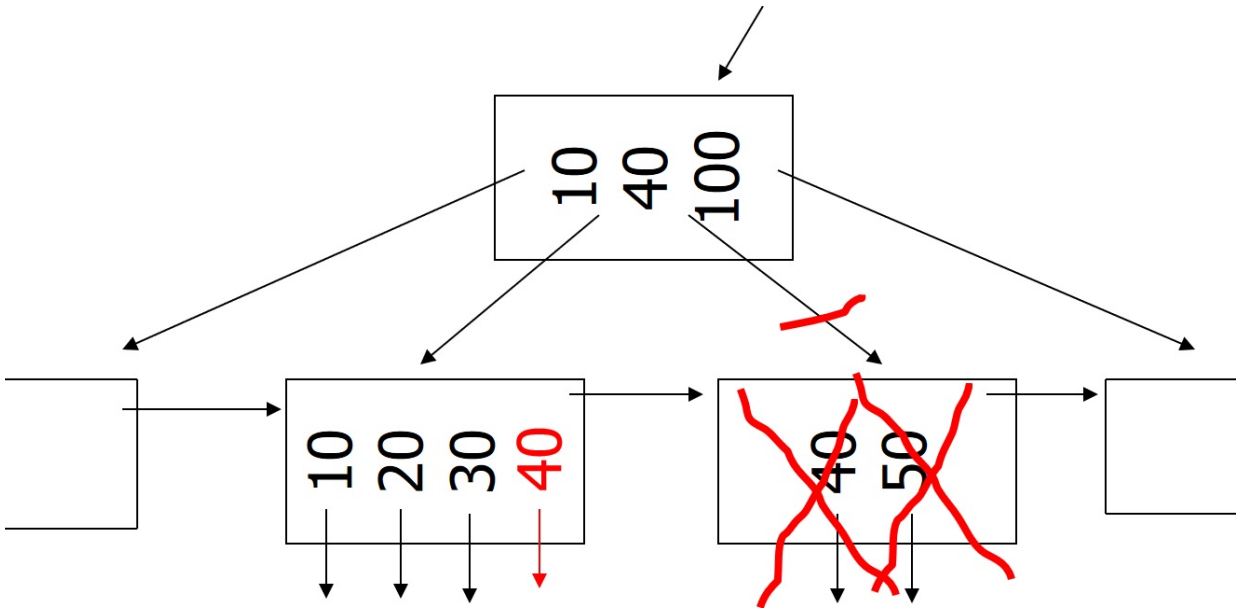
b) Coalesce with sibling: Delete 50, $n = 4$



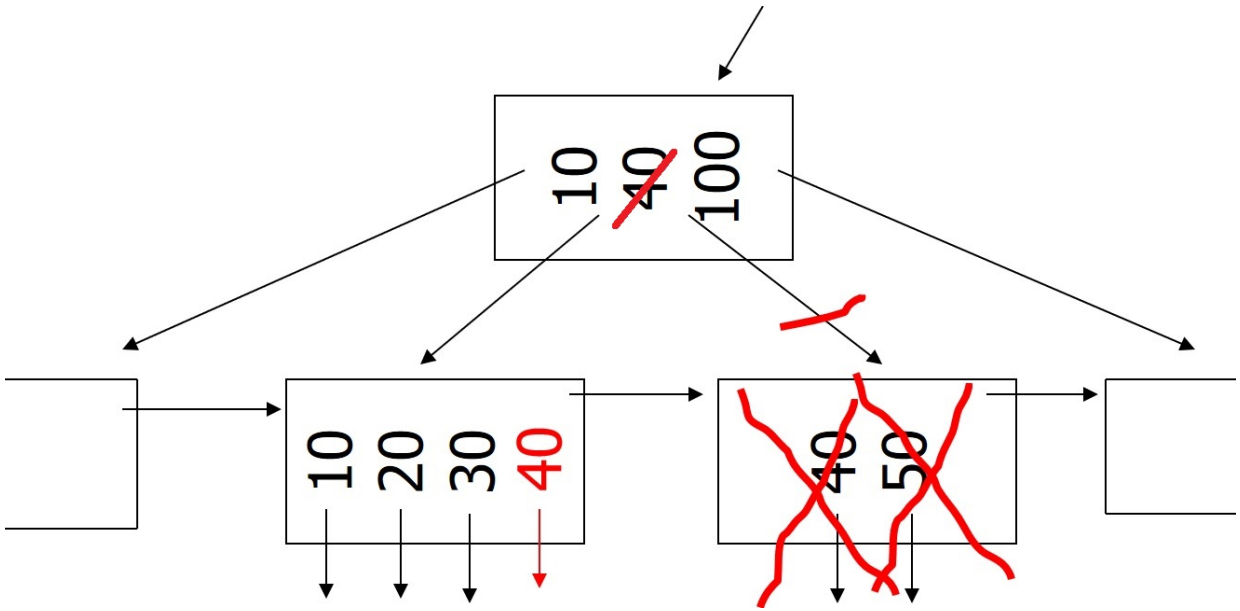
b) Coalesce with sibling: Delete 50, $n = 4$



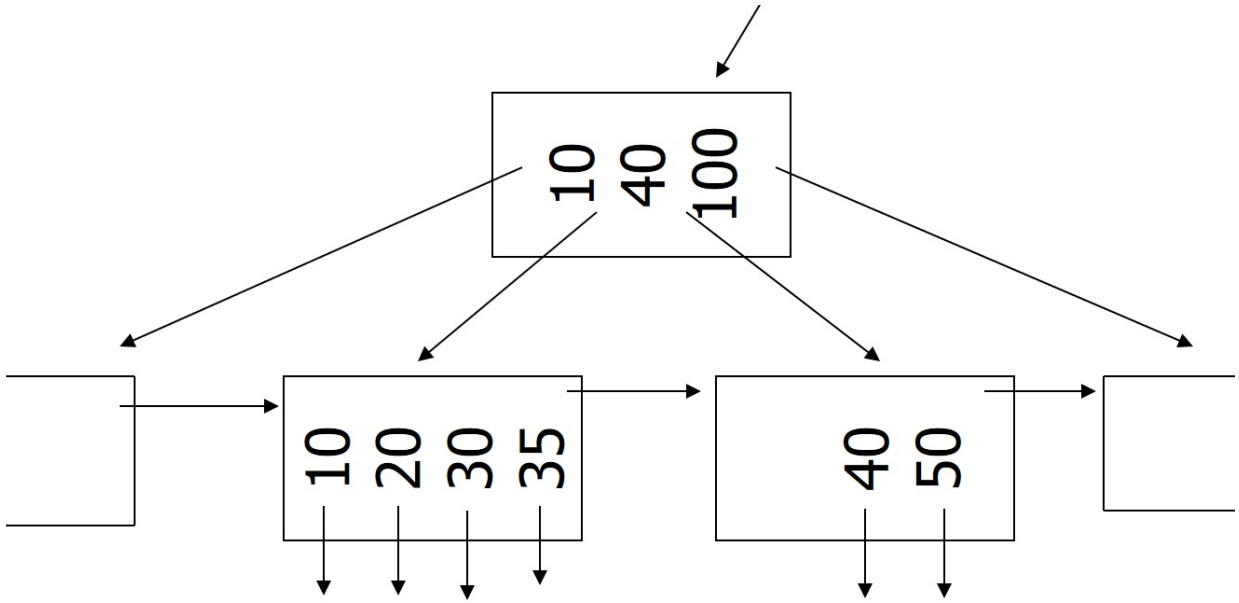
b) Coalesce with sibling: Delete 50, $n = 4$



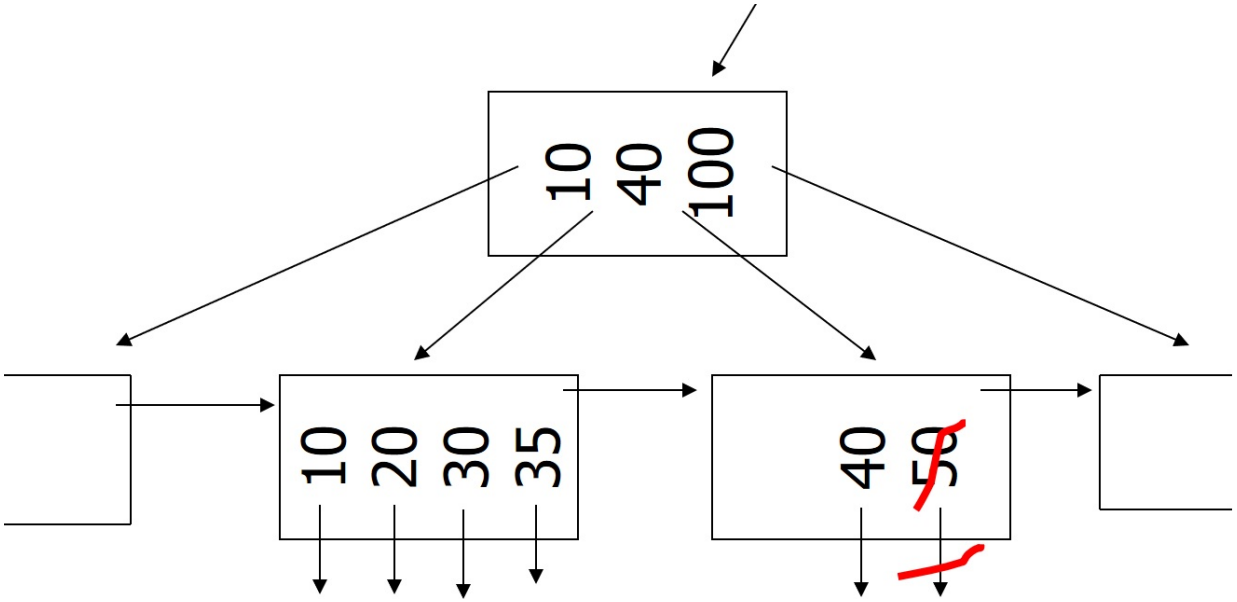
b) Coalesce with sibling: Delete 50, $n = 4$



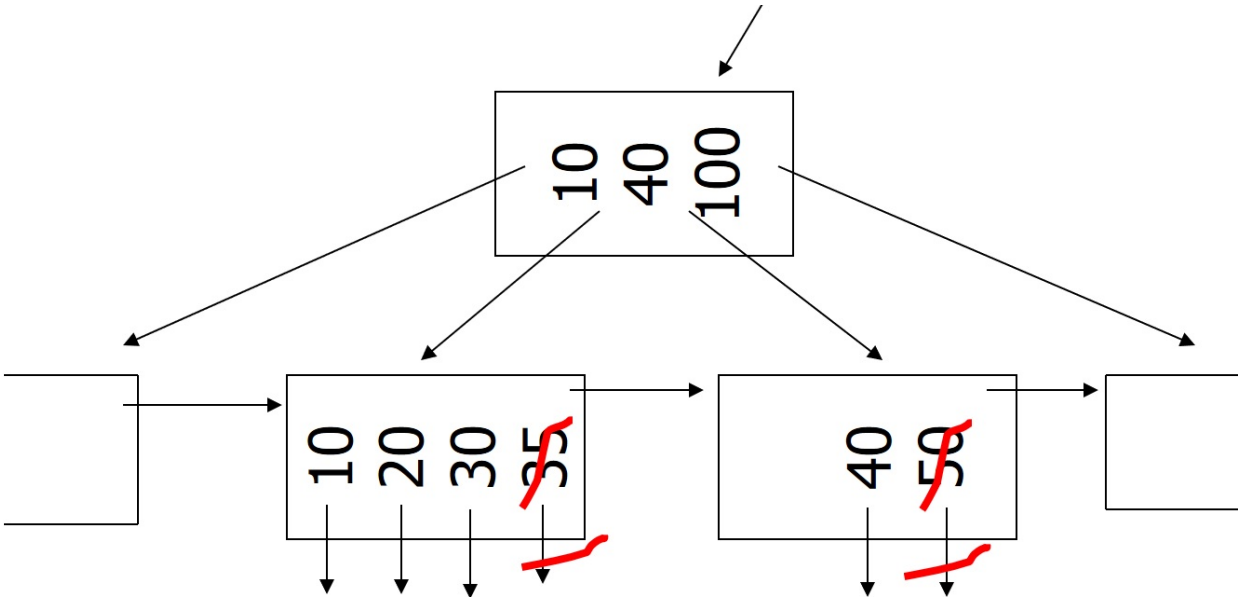
c) Redistribute keys: Delete 50, $n = 4$



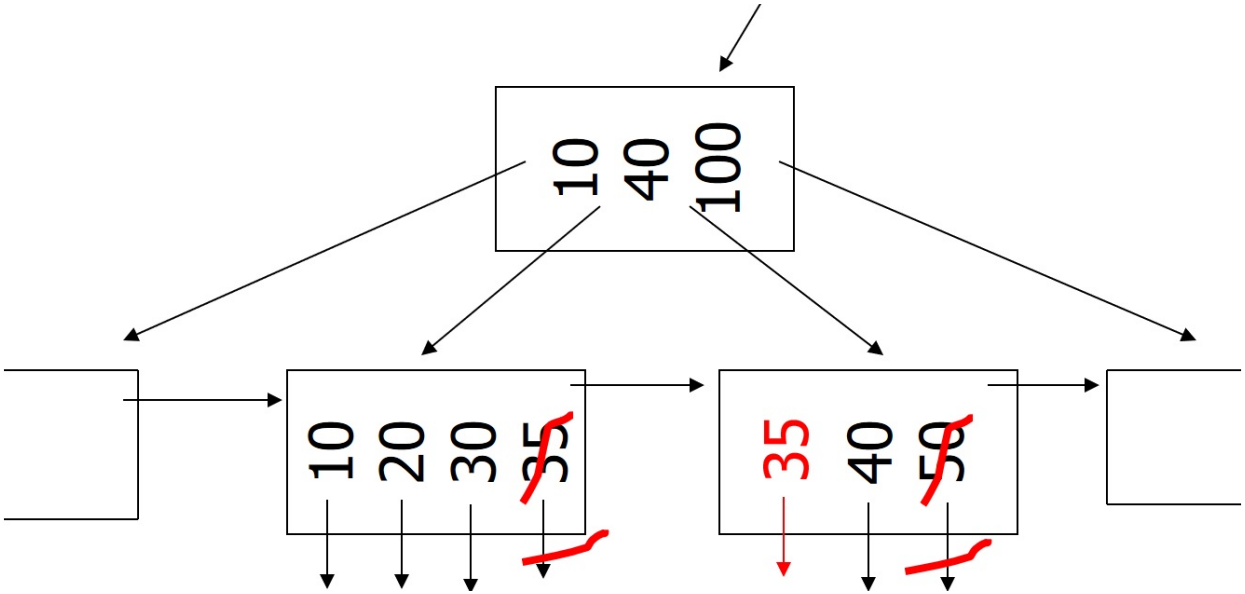
c) Redistribute keys: Delete 50, $n = 4$



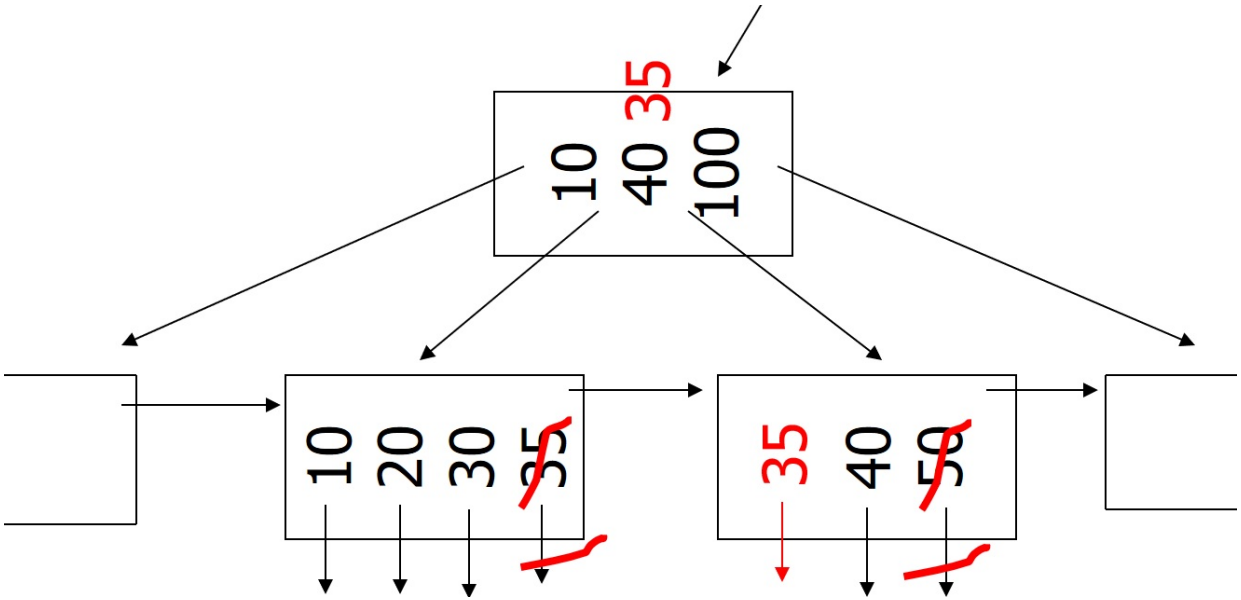
c) Redistribute keys: Delete 50, $n = 4$



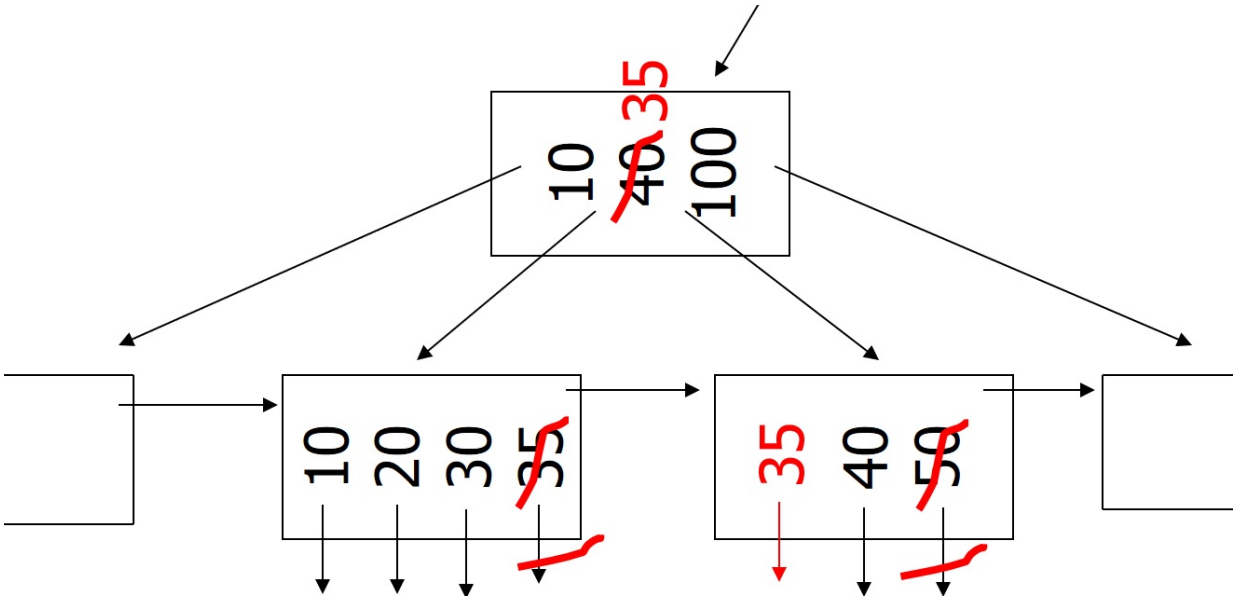
c) Redistribute keys: Delete 50, $n = 4$



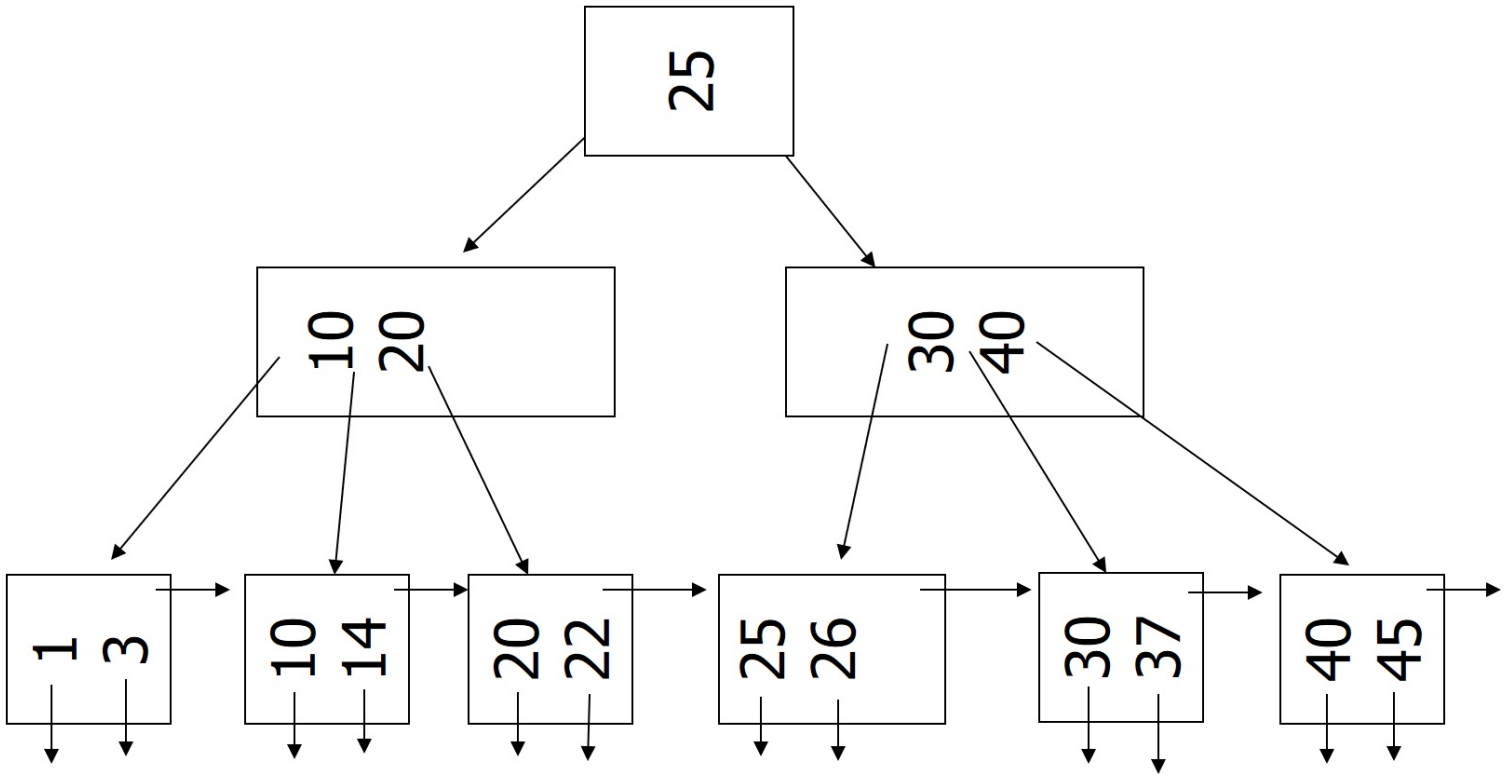
c) Redistribute keys: Delete 50, $n = 4$



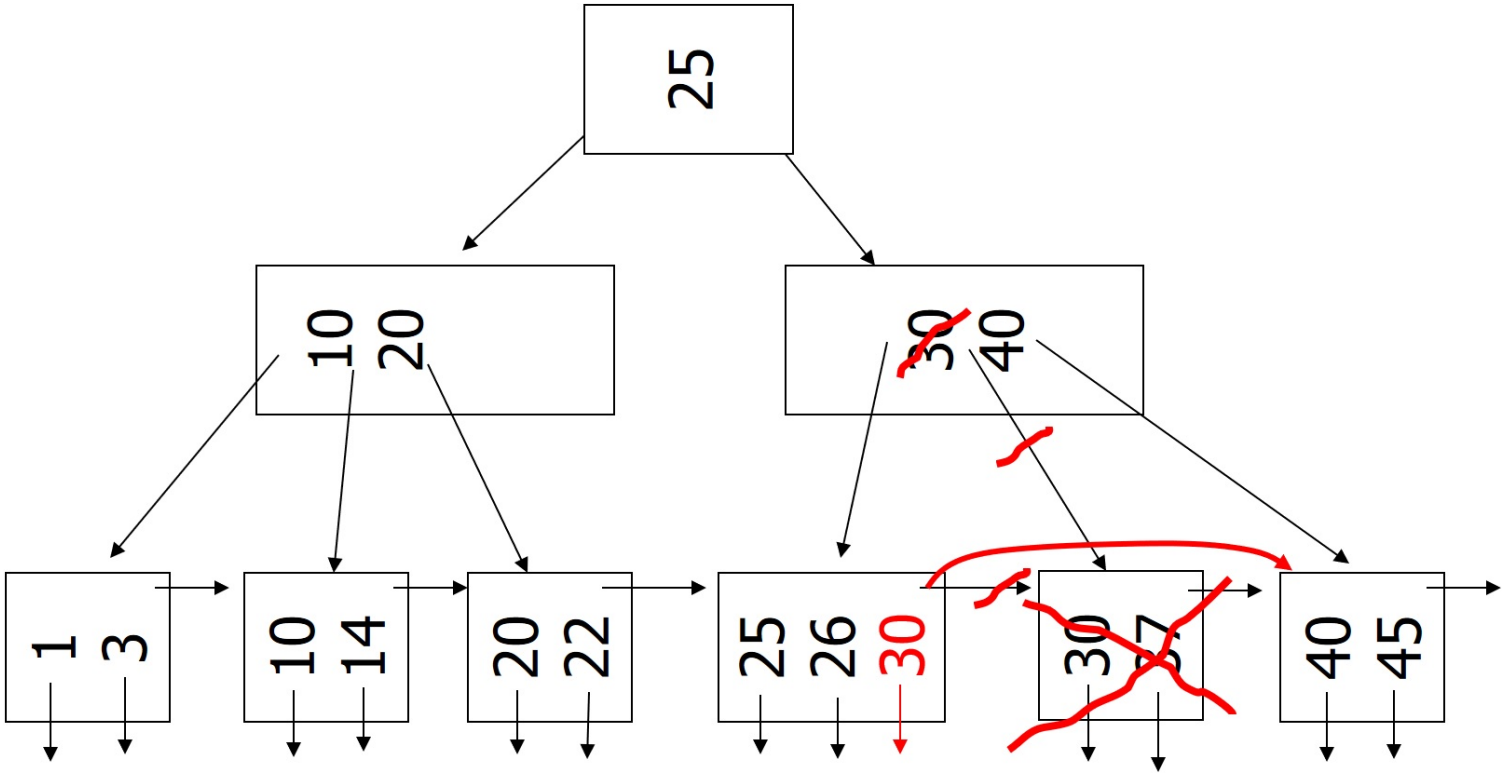
c) Redistribute keys: Delete 50, $n = 4$



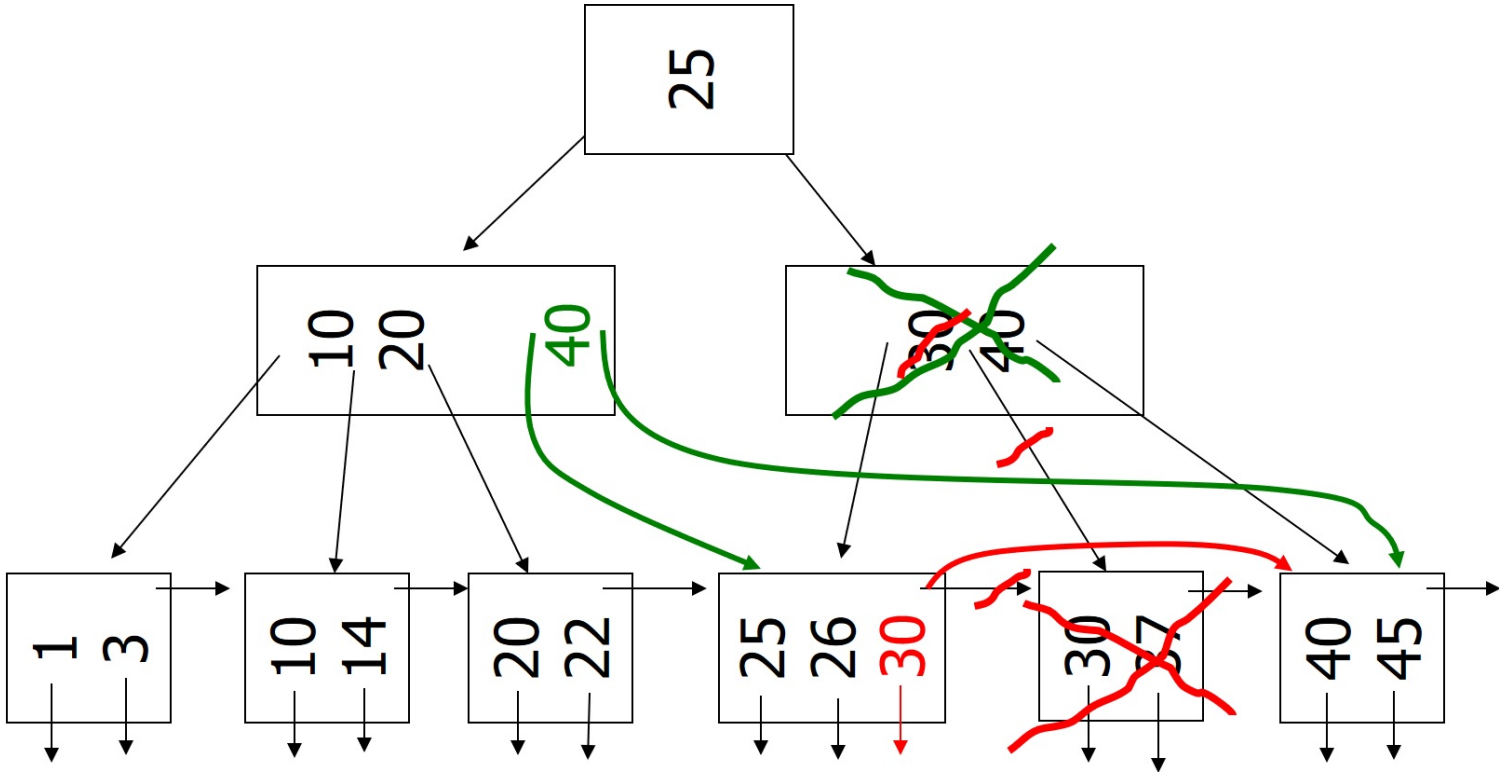
d) Non-leaf Coalesce: Delete 37, $n = 4$



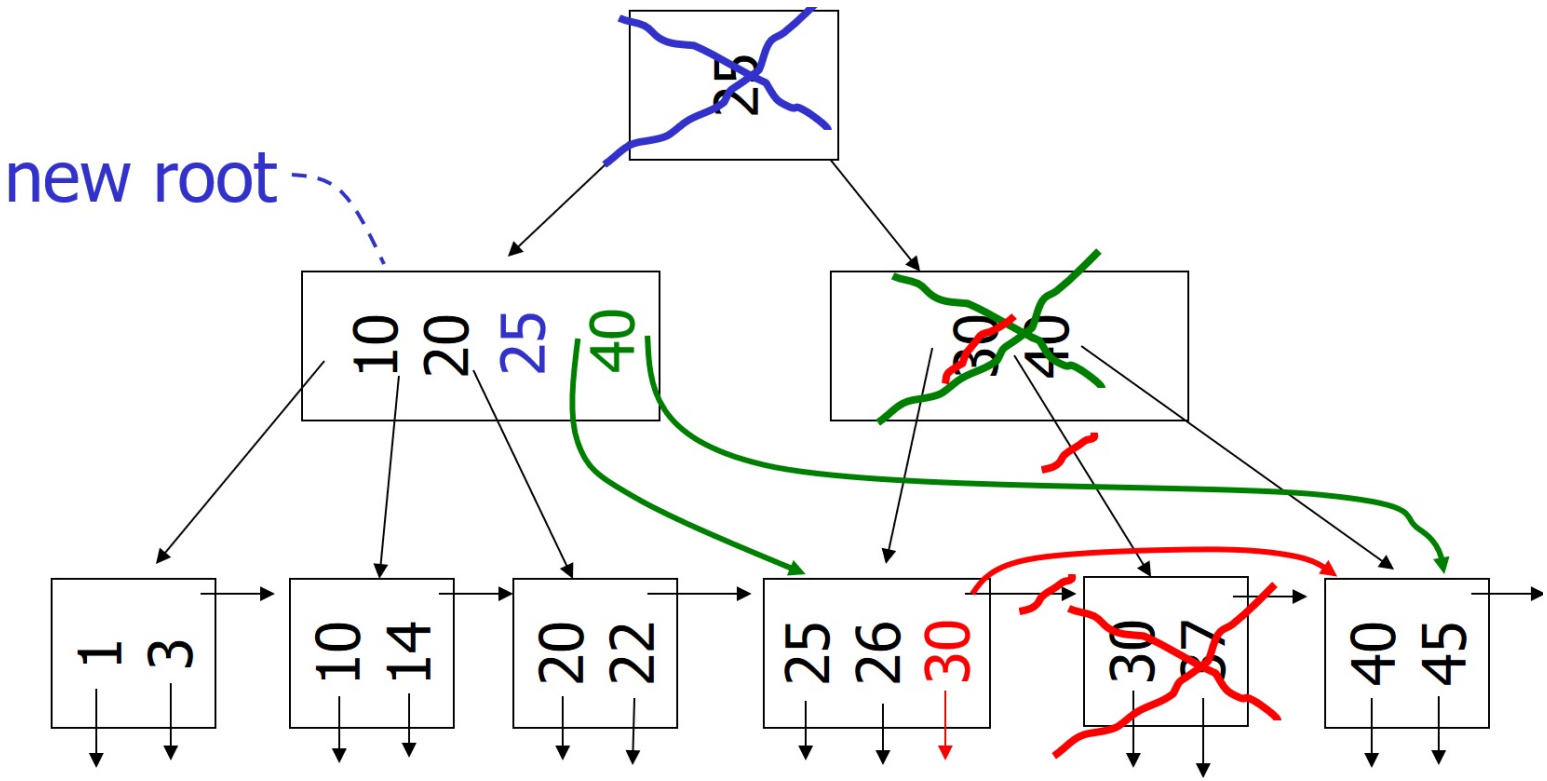
d) Non-leaf Coalesce: Delete 37, $n = 4$



d) Non-leaf Coalesce: Delete 37, $n = 4$



d) Non-leaf Coalesce: Delete 37, $n = 4$



Deletion Algorithm

To Delete record with `key k`

1. Find correct `leaf L`.
2. Remove the entry:
   - If `L` is at least half full, the operation is **done**.
   - Otherwise
     - You can try to redistribute, borrowing from sibling.
     - If redistribution fails, merge `L` and sibling.
3. If merge occurred, you must delete entry in parent pointing to `L`.

- Note
  - *B^+-Tree removes a key from a leaf node but not necessary from the non-leaf/inner nodes.*

B^+-Tree deletions in practice

- Often, coalescing is not implemented
  - Too hard and not worth it!
  - Assumption: nodes will fill up in time again

Splitting or Merging nodes

- When splitting or merging nodes follow these conventions:
  - **Leaf Split**: In case a leaf node needs to be split during insertion and n is even, the left node should get the extra key. E.g, if $n = 2$ and we insert a key 4 into a node [1,5], then the resulting nodes should be [1,4] and [5]. For odd values of n we can always evenly split the keys between the two nodes. In both cases the value inserted into the parent is the smallest value of the right node.
  - **Non-Leaf Split**: In case a non-leaf node needs to be split and n is odd, we cannot split the node evenly (one of the new nodes will have one more key). In this case the "middle" value inserted into the parent should be taken from the right node. E.g., if $n = 3$ and we have to split a non-leaf node [1,3,4,5], the resulting nodes would be [1,3] and [5]. The value inserted into the parent would be 4
  - **Node Underflow**: In case of a node underflow you should first try to redistribute values from a sibling and only if this fails merge the node with one of its siblings. Both approaches should prefer the left sibling. E.g., if we can borrow values from both the left and right sibling, you should borrow from the left one.

Comparison: B$^+$-Tree vs. indexed sequential file

- Ref #1: Held & Stonebraker, "B-Trees Re-examined", CACM, Feb. 1978
- Ref #2: M. Stonebraker, "Retrospection on a database system", TODS, June 1980

Comparison: B$^+$-Tree vs. indexed sequential file

B$^+$tree

- Consumes more space, so lookup slower
- Each insert/delete potentially restructures
- Build-in restructuring
- Predictable performance

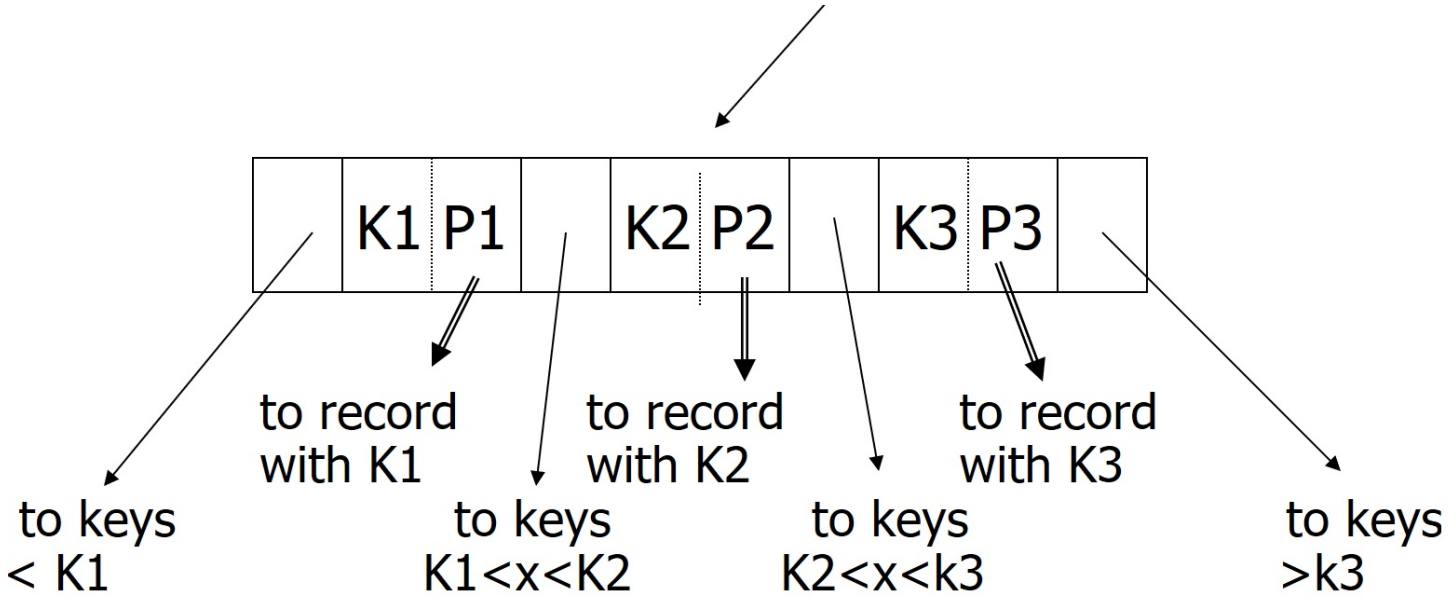
indexed seq. file

- Less space, so lookup faster
- Inserts managed by overflow area
- Requires temporary restructuring
- Unpredictable performance

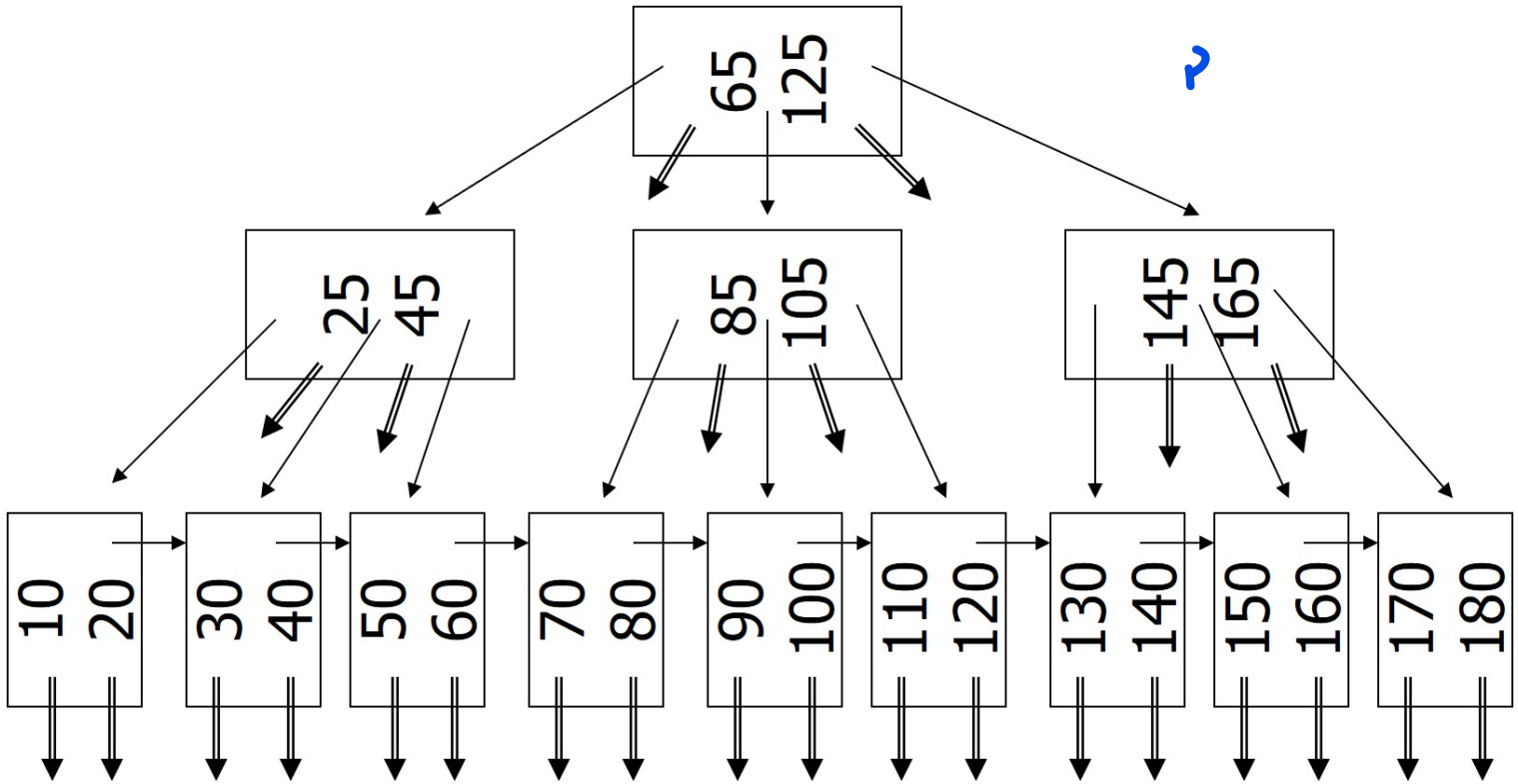
Variation on B$^+$-Tree: B-tree (no +)

- Idea:
  - Avoid duplicate keys
    - More space-efficient, since each key only appears once in the tree
  - Have record pointers in non-leaf nodes
  - In a B-tree, all search keys and corresponding data pointers are represented somewhere in the tree, but in a B$^+$-tree, all data pointers appear in the leaves (sorted from left to right)
    - In B$^+$-tree, inner nodes only guide the search process.
  - B-tree is more expensive to update when there are multiple threads.

Variation on B$^+$-Tree: B-tree (no +)



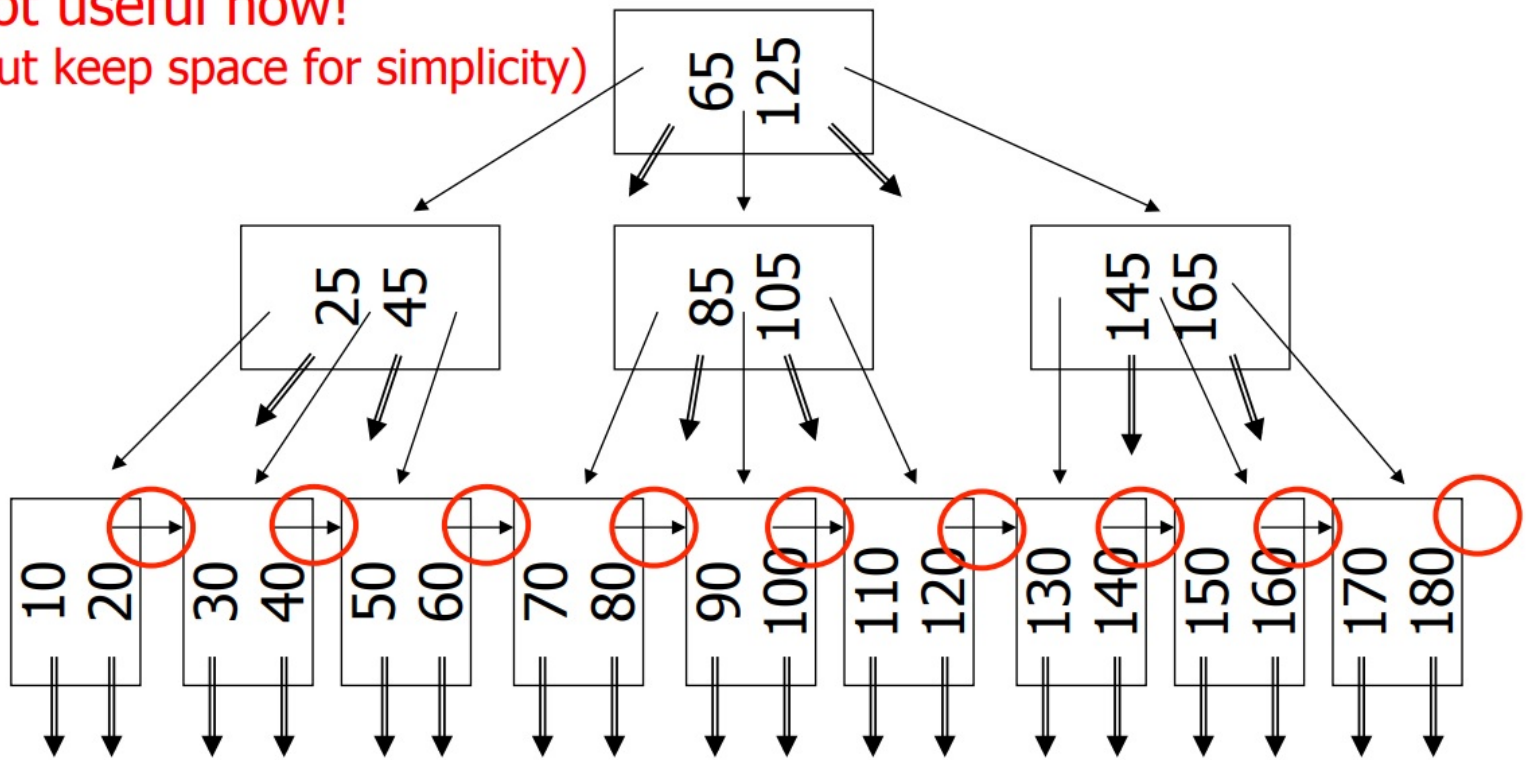
B-tree example: $n = 2$



- The leaf nodes in a B-tree is not linked
- i.e., the last record pointer in a leaf node is not used

B-tree example: $n = 2$

- sequence pointers not useful now!
  (but keep space for simplicity)



B-tree

- B-trees have faster lookup than B^+-trees
- in B-tree, non-leaf & leaf different sizes
- in B-tree, deletion more complicated

  $\Rightarrow$ B^+trees preferred

But, note:

- If `blocks` are fixed size (due to disk and buffering restrictions)
- Then `lookup` for `B+-Tree` is **actually better**

Example

- Consider a `DBMS` that has the following characteristics:
  - Pointers 4 bytes
  - Keys 4 bytes
  - Blocks 100 bytes
- Compute the maximum number of records we can index with:
  a) 2-level `B-tree`
  b) 2-level `B+-Tree`

B-tree

- Find largest integer value of n such that $4n + 4(2n + 1) \leq 100$
  $n = 8$
- `Root` has 8 keys + 8 record pointers +9 children pointers
  $= 8 \times 4 + 8 \times 4 + 9 \times 4 = 100 bytes$
- Each of 9 children: 12 records pointers (+12 keys)
  $= 12 \times (4 + 4) + 4 = 100$ bytes
- 2-level `B-tree`, maximum # of records $= 12 \times 9 + 8 = 116$

B$^+$-Tree

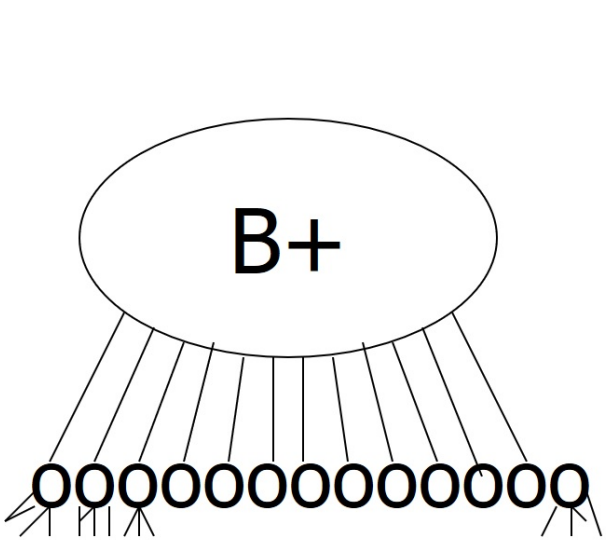
- Find largest integer value of n such that $4n + 4(n + 1) < 100$

  $n = 12$
- `Root` has 12 keys + 13 children pointers

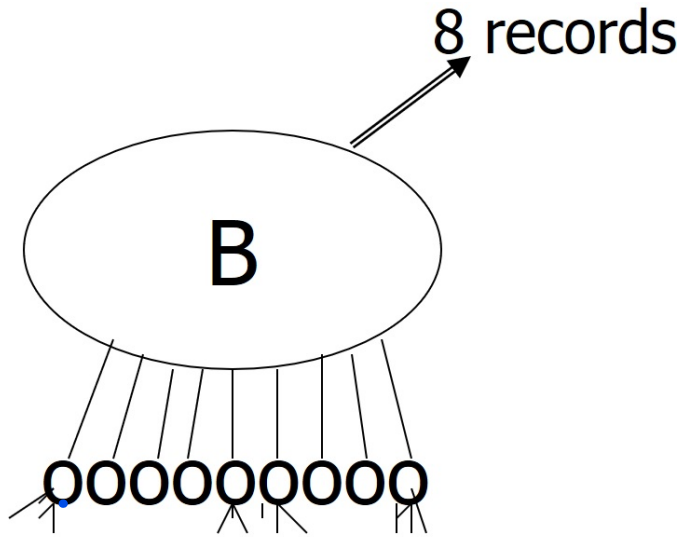
  $= 12 \times 4 + 13 \times 4 = 100 bytes$
- Each of 13 children: 12 records pointers (+12 keys)

  $= 12 \times (4 + 4) + 4 = 100$ bytes
- 2-level `B+-Tree`, maximum # of records $= 13 \times 12 = 156$

So,



156 records



108 records

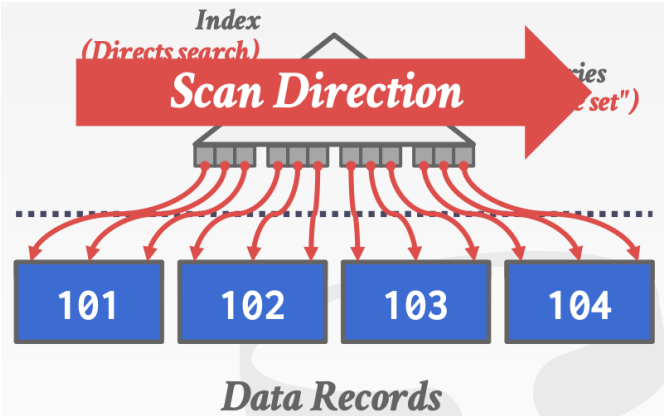
Total = 116

Data Organization

- A table can be stored in two ways:
  - Heap-organized storage: Organizing rows in no particular order.
  - Index-organized storage: Organizing rows in primary key order.
- Types of indexes:
  - Clustered index: Organizing rows in a primary key order.
  - Unclustered index: Organizing rows in a secondary key order.
- The table is stored in the sort order specified by the primary key.
  - Can be either heap- or index-organized storage.
- Some DBMSs always use a clustered index.
  - If a table doesn't contain a primary key, the DBMS will automatically make a hidden row id primary key.
- Other DBMSs cannot use them at all.

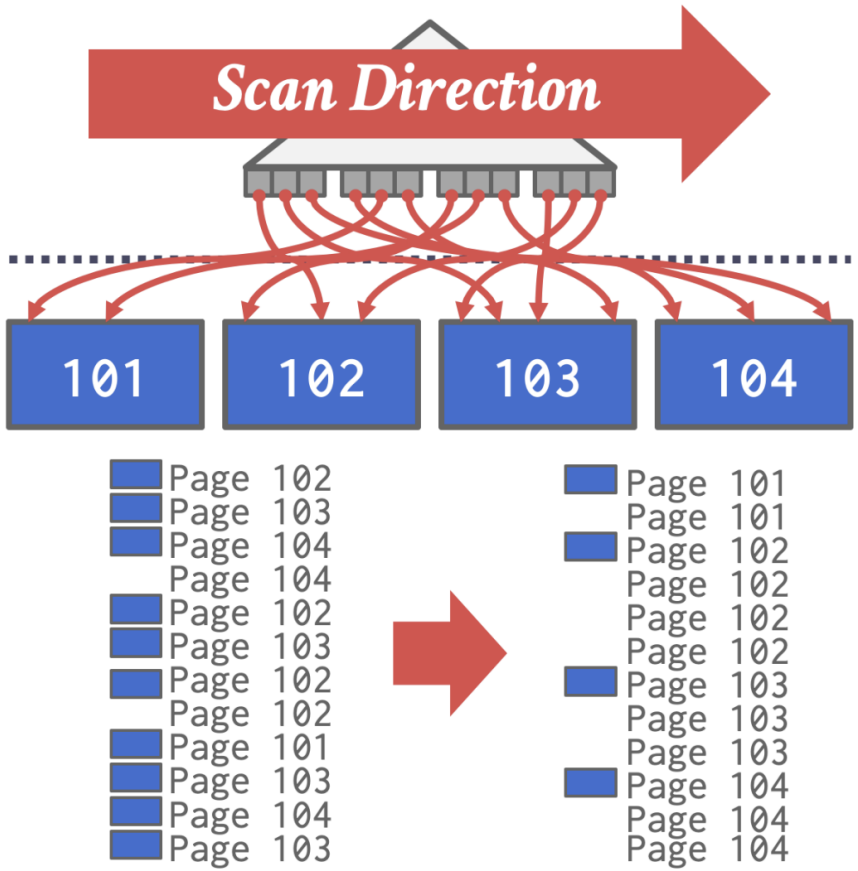
Clustered Index

- Tuples are kept sorted on disk using the order specified by primary key.
- If the query accesses tuples using the clustering index's attributes, then the DBMS can jump directly to the pages that it needs.
- Traverse to the left-most leaf page, and then retrieve tuples from all leaf pages.



Index Scan Page Sorting

- Retrieving tuples in the order that appear in an unclustered index is inefficient.
- The DBMS can first figure out all the tuples that it needs and then sort them based on their page id.



Filtering Tuples/Selection Conditions

- The DBMS can use a `B+-Tree` index if the query/filter provides any of the attributes of the search key.
- Example: Index on `<a,b,c>`
  - Supported: `(a=5 AND b=3)`
  - Supported: `(b=3)`.
- Not all DBMSs support this.
- For hash index, we must have **all attributes** in search key.

B^+-Tree Design Choices/Decisions [2]

- Node Size
- Merge Threshold
- Variable Length keys
- Non-Unique Indexes
- Intra-Node Search

[2] Modern B-Tree Techniques By Goetz Graefe
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.219.7269&rep=rep1&type=pdf

B+-Tree: Node Size

- The optimal node size for a B+-Tree depends on the speed of the disk.
- The idea is to amortize the cost of reading a node from disk into memory over as many key/value pairs as possible.
- The slower the disk, then the larger the optimal node size for a B+-Tree[3].
  - HDD: ~ 1 MB
  - SSD: ~ 10KB
  - In-Memory: ~ 512B
- Optimal sizes can vary depending on the workload
  - Some workloads may be more scan-heavy versus having more single key look-ups.
    → Leaf Node Scans (OLAP) vs. Root-to-Leaf Traversals (OLTP)
    - *as point queries would prefer as small a page as possible to reduce the amount of unnecessary extra info loaded, while a large sequential scan might prefer large pages to reduce the number of fetches it needs to do.*

[3] *For example, nodes stored on hard drives are usually on the order of megabytes in size to reduce the number of seeks needed to find data and amortize the expensive disk read over a large chunk of data, while in-memory databases may use page sizes as small as 512 bytes in order to fit the entire page into the CPU cache as well as to decrease data fragmentation.*

B+-Tree: Merge Threshold

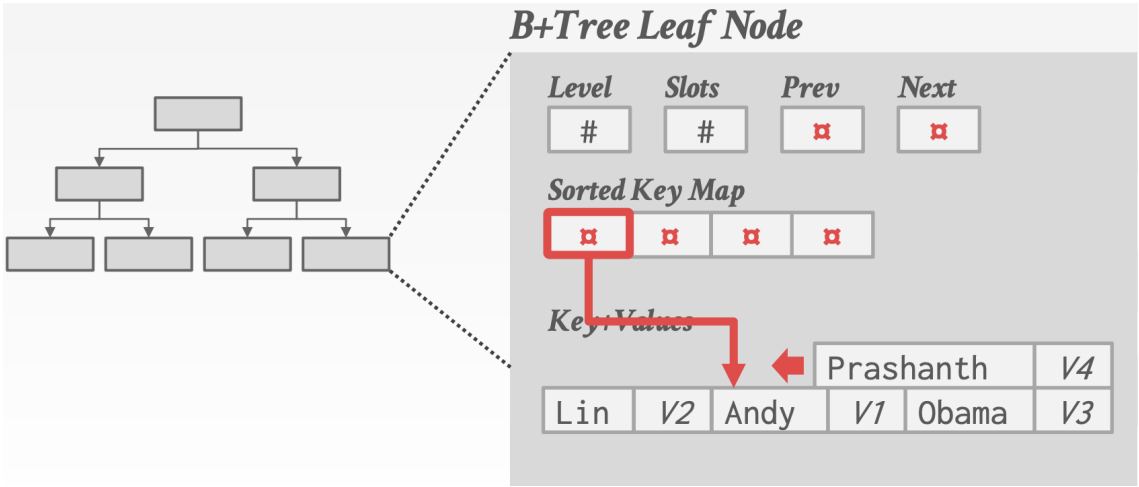
- While B+-Trees have a rule about merging underflowed nodes after a delete, sometimes it may be beneficial to temporarily violate the rule to reduce the number of deletion operation
- Some DBMS do not always merge when it's half full.
- Delaying a merge operation may reduce the amount of reorganization.
- It may be better for the DBMS to let **underflows** to occur and then periodically **rebuild** the entire tree to rebalance it.

B+-Tree: Variable Length keys

- **Approach 1: Pointers**
  - Store keys as pointers to the tuple's attribute (very rarely used).
- **Approach 2: Variable-length Nodes**
  - We could also still store the keys like normal and allow for variable length nodes.
  - The size of each node in the `B+-Tree` can vary, but requires careful memory management. This approach is also rare.
- **Approach 3: Padding**
  - Instead of varying the key size, we could set each key's size to the size of the maximum key and pad out all the shorter keys.
  - In most cases this is a massive waste of memory, so you don't see this used by anyone either.
- **Approach 4: Key Map/Indirection**
  - Embed an array of pointers that map to the `key+value` list within the node.
    - Each pointer will be an offset to a key inside the node itself
  - This is similar to slotted pages discussed before. This is the most common approach.

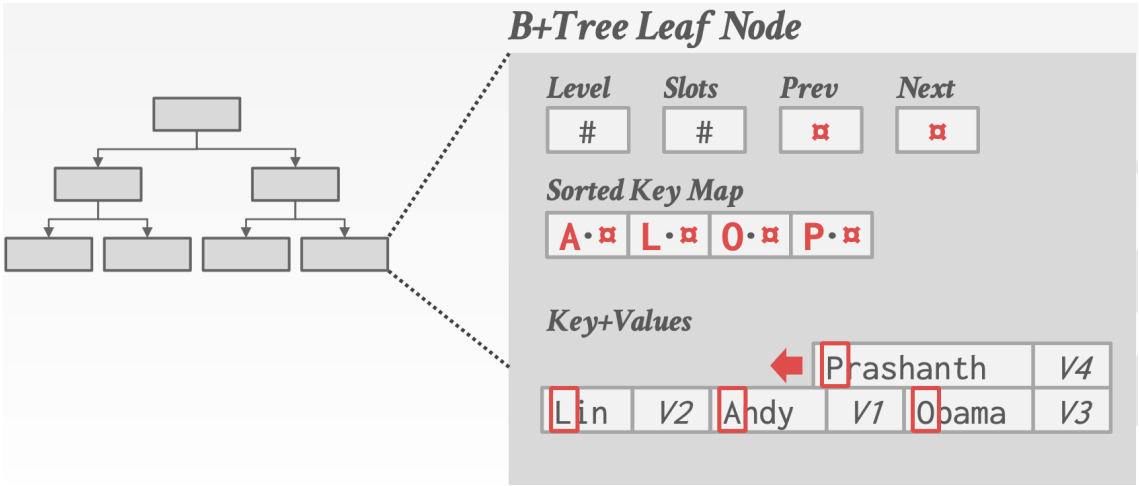
Key Map/Indirection

- Replace the keys with an index to the key-value pair in a separate dictionary.
  - Each pointer will be an offset to a key inside the node itself
  - The sorted key map is sorted based on the value of the keys



Key Map/Indirection

- Due to the small size of the dictionary index value, there is enough space to place a prefix of each key alongside the index,
- Potentially allowing some index searching and leaf scanning to not even have to chase the pointer
  - (if the prefix is at all different from the search key)

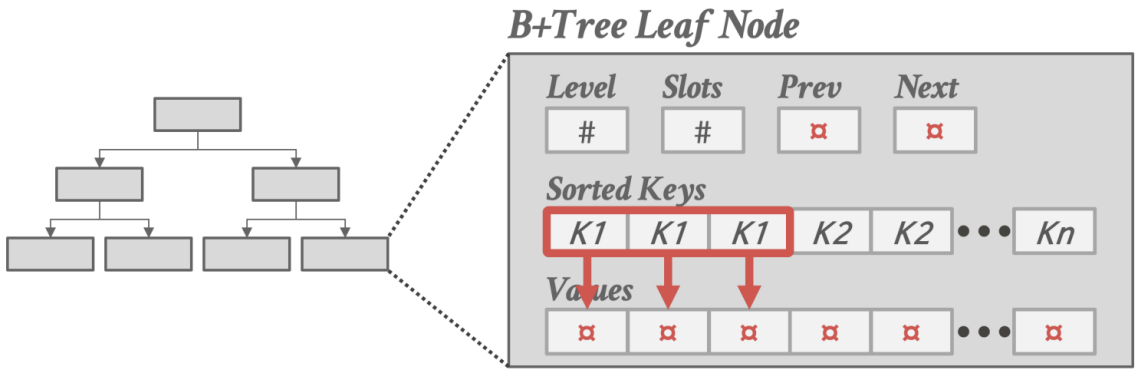


B+-Tree: Non-Unique Indexes

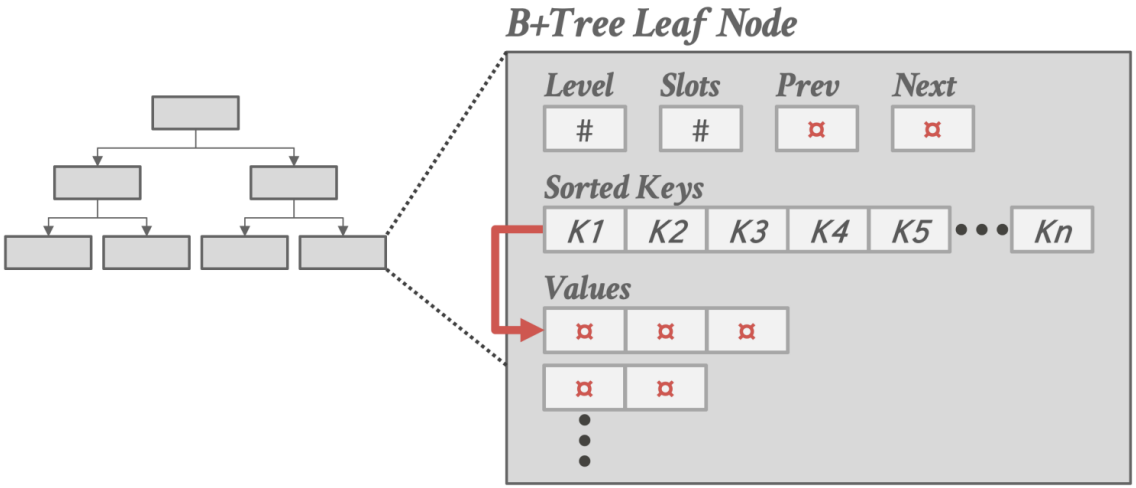
B+-Trees can deal with non-unique indexes by duplicating keys or storing value lists:

- **Approach 1: Duplicate Keys**
  - Use the same leaf node layout but store duplicate keys multiple times. (Most common)
- **Approach 2: Value Lists**
  - Store each key only once and maintain a linked list of unique values.

Non-Unique Indexes: Duplicate Keys



Non-Unique Indexes: Value Lists



Duplicate Keys

- There are two approaches to duplicate keys in a `B+-Tree`:
  - The first approach is to append record IDs as part of the key. Since each tuple's record ID is unique, this will ensure that all the keys are identifiable.
  - The second approach is to allow leaf nodes to spill into overflow nodes that contain the duplicate keys.
    - Although no redundant information is stored, this approach is more complex to maintain and modify.

B$^+$-Tree: Intra-Node Search

- **Approach 1: Linear**
  - Scan the key/value entries in the node from beginning to end. Stop when you find the key that you are looking for.
  - Use SIMD[4] to vectorize comparisons,
  - This does not require the key/value entries to be pre-sorted.
- **Approach 2: Binary**
  - Jump to the middle key, and then pivot left/right depending on whether that middle key is less than or greater than the search key.
  - This requires the key/value entries to be pre-sorted.
- **Approach 3: Interpolation**
  - Approximate the starting location of the search key based on the known low/high key values in the node. Then perform linear scan from that location.
  - This requires the key/value entries to be pre-sorted.

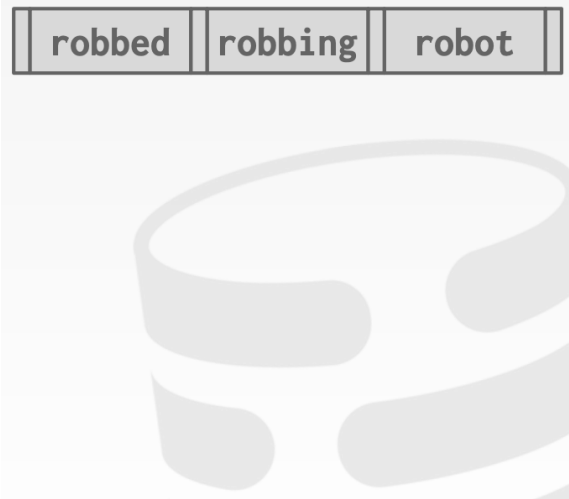
[4] *Single instruction, multiple data (SIMD): computational technique for processing a number of data values (generally a power of two) using a single instruction, with the data for the operands packed into special wide registers. One instruction can therefore do the work of many separate instructions*

B$^+$-Tree Optimizations

- Prefix Compression
- Suffix Truncation
- Bulk Insert
- Pointer Swizzling

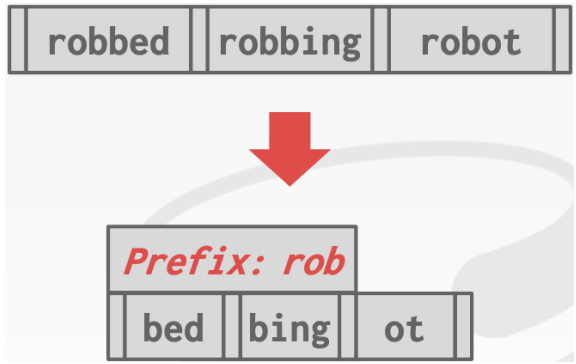
Prefix Compression

- Sorted keys in the same leaf node are likely to have the same prefix.
- Instead of storing the entire key each time, extract common prefix and store only unique suffix for each key.
  - Many variations.



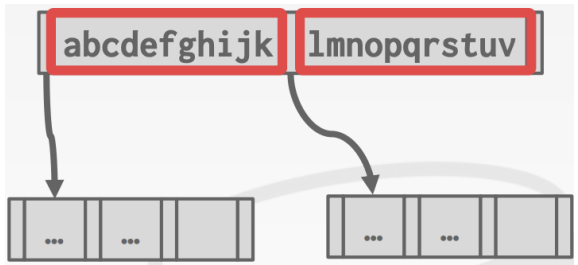
Prefix Compression

- Sorted keys in the same leaf node are likely to have the same prefix.
- Instead of storing the entire key each time, extract common prefix and store only unique suffix for each key.
  - Many variations.



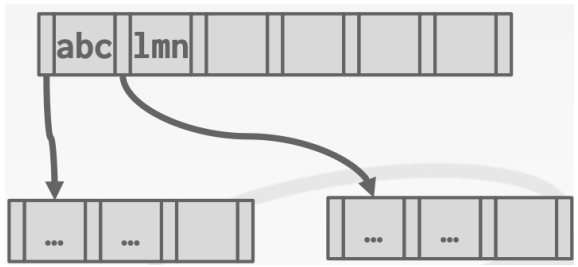
Suffix Truncation

- The keys in the inner nodes are only used to "direct traffic".
  - We don't actually need the entire key.
- Store a minimum prefix that is needed to correctly route probes into the index.



Suffix Truncation

- The keys in the inner nodes are only used to "direct traffic".
  - We don't actually need the entire key.
- Store a minimum prefix that is needed to correctly route probes into the index.

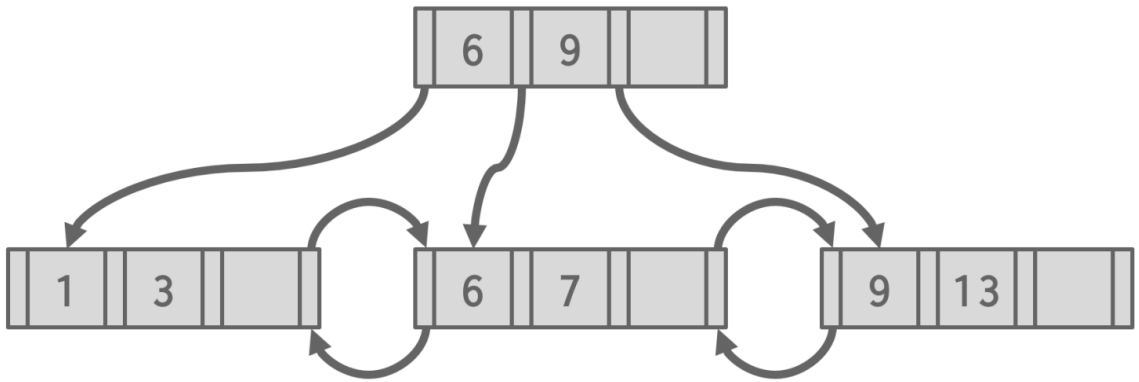


Bulk Inserts

- The fastest/best way to build a `B+-Tree` is to first sort the keys and then build the index from the bottom up.
- This will be faster than inserting one-by-one since there are no splits or merges.

Bulk Inserts

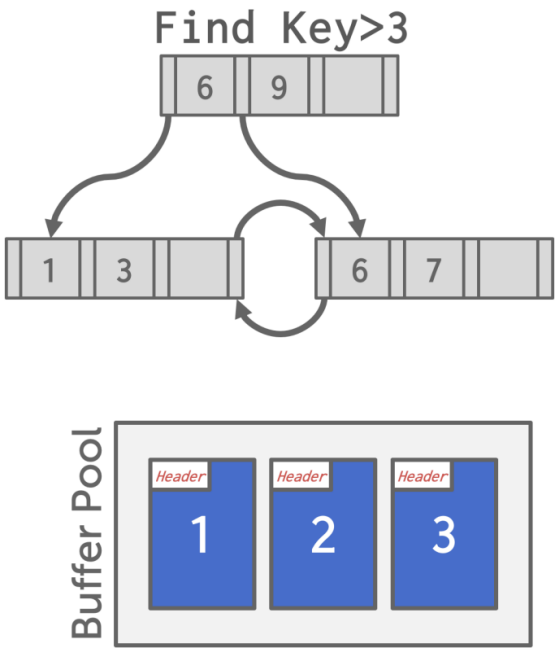
```
        Keys:  3, 7, 9, 13, 6, 1
Sorted Keys:  1, 3, 6, 7, 9, 13
```

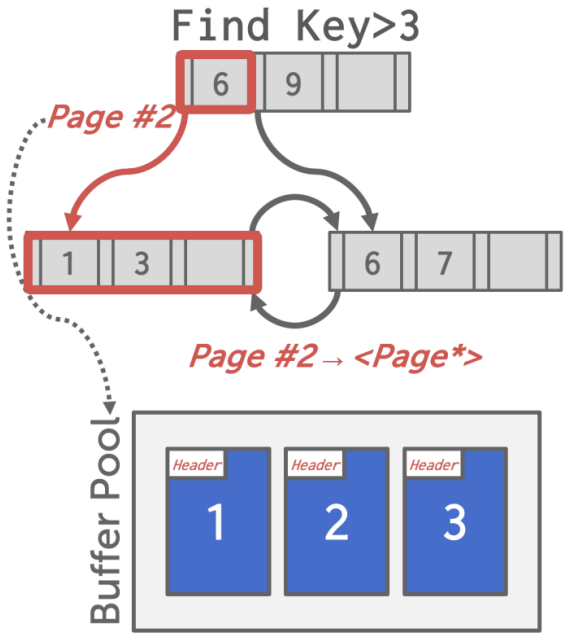


Pointer Swizzling

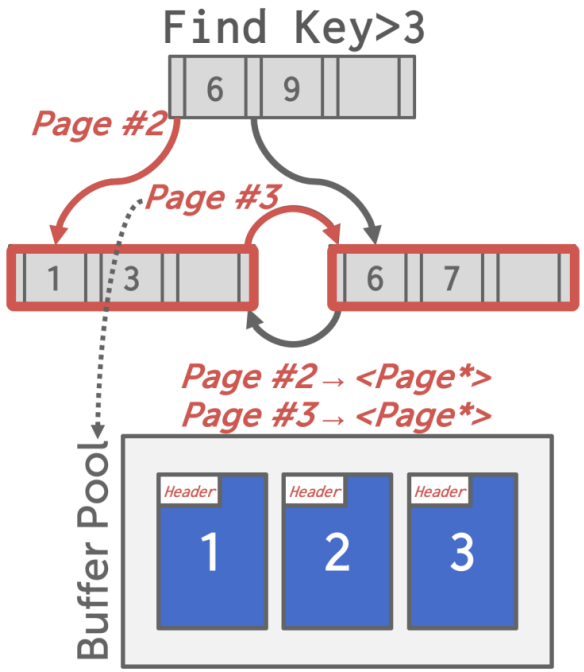
- Nodes use page ids to reference other nodes in the index.
- The DBMS must get the memory location from the page table during traversal.
- If a page is pinned in the buffer pool, then we can store raw pointers instead of page ids.
- This avoids address lookups from the page table.

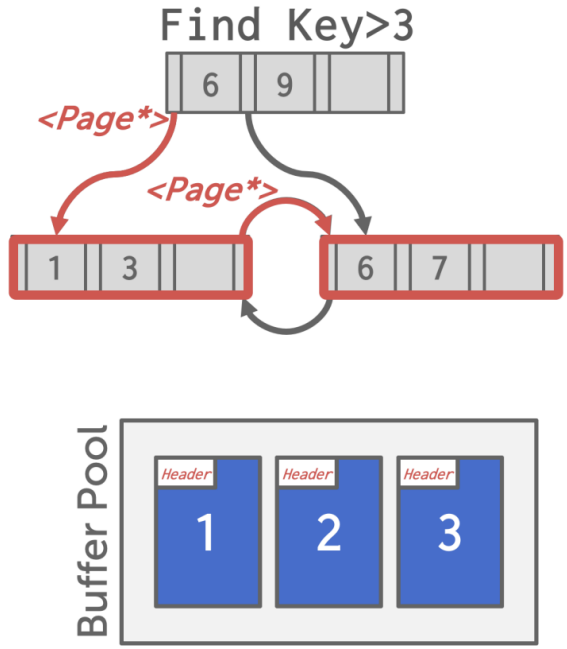
Pointer Swizzling





Pointer Swizzling





Conclusion

- The venerable `B+-Tree` is always a good choice for your DBMS.

Reading

- Chapter 4: Index Structures (uploaded on course Blackboard)
- Database System Concepts: Chapter 11 (6th Edition)/ Chapter 14 (7th Edition)
- Modern B-Tree Techniques, By Goetz Graefe

Next

- Hash Tables