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Sub : SOP [CS536] Assignment 1

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Q. 1

a]

→ $e_1 = e_2$ does not logically imply $e_1 \equiv e_2$

For example,

$$4 + 4 = 8$$

but

$$4 + 4 \not\equiv 8$$

They are not textually equal / identical, hence they cannot be syntactically equal.

$\boxed{\text{So } e_1 = e_2 \text{ does not logically imply } e_1 \equiv e_2}$

b]

→ $e_1 \neq e_2$ logically implies $e_1 \not\equiv e_2$

for example,

if $2 + 2 \neq 3$

then

$$2 + 2 \not\equiv 3$$

If two expression are not semantically equal, they cannot also be syntactically equal.

$\boxed{\text{So, } e_1 \neq e_2 \text{ logically implies } e_1 \not\equiv e_2}$

Q. 2.

a]

→ Truth table to prove $(p \vee q) \wedge q \Leftrightarrow q$

p	q	$p \vee q$	$(p \vee q) \wedge q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	F

Hence, we see that $(p \vee q) \wedge q \Leftrightarrow q$ as both the truth table values are identical

b)

→ Truth table to prove $\neg(p \leftrightarrow q) \Leftrightarrow \neg p \leftrightarrow q$

p	q	$\neg p$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg p \leftrightarrow q$
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Hence, $\neg(p \leftrightarrow q) \Leftrightarrow \neg p \leftrightarrow q$ as both truth table values are identical

c)

→ Truth table to prove $\neg p \wedge (p \vee q) \rightarrow q \Leftrightarrow T$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Hence, we can see that $\neg p \wedge (p \vee q) \rightarrow q$ is a Tautology so $\neg p \wedge (p \vee q) \rightarrow q \Leftrightarrow T$

Q. 3.

a) $(p \rightarrow q) \vee (p \rightarrow r) \Leftrightarrow p \rightarrow q \vee r$

$\rightarrow$

$p \rightarrow q \Leftrightarrow \sim p \vee q$

$(\sim p \vee q) \vee (\sim p \vee r) \Leftrightarrow RHS$ ... (Law of implication)

$\sim p \vee (q \vee r) \Leftrightarrow RHS$ ... (Law of distributive)

$\therefore \sim p \vee (q \vee r) \Leftrightarrow RHS$

$LHS \Leftrightarrow p \rightarrow q \vee r$ ... (simplifying RHS)

$LHS \Leftrightarrow \sim p \vee (q \vee r)$ ... (Law of equivalense)

$\therefore \sim p \vee (q \vee r) \Leftrightarrow \sim p \vee (q \vee r)$

$\therefore LHS \Leftrightarrow RHS$

b) $(p \vee q) \wedge \neg q \Leftrightarrow \neg (p \rightarrow q)$

$\rightarrow$

Here,

$LHS \Leftrightarrow \sim (p \rightarrow q)$ ... (Law of Implication)

$LHS \Leftrightarrow \sim (\sim p \vee q)$ ...

$LHS \Leftrightarrow p \wedge \sim q$ ... (Demorgan's law of negation)

$\therefore LHS \Leftrightarrow p \wedge \sim q$

Now, we'll replace LHS

$(p \vee q) \wedge \sim q \Leftrightarrow RHS$

$(p \wedge \sim q) \vee (q \wedge \sim q) \Leftrightarrow RHS$ (Distributive Law)

$(p \wedge \sim q) \vee \text{false} \Leftrightarrow RHS$ ... (Contradiction law)

$(p \wedge \sim q) \Leftrightarrow RHS$

Hence proved

$p \wedge \sim q \Leftrightarrow p \wedge \sim q$

$\therefore LHS \Leftrightarrow RHS$

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c) $(p \to q) \wedge (\neg p \to q) \Leftrightarrow q$

$\to$

$(p \to q) \wedge (\neg p \to q) \Leftrightarrow q$

$(\sim p \vee q) \wedge (p \vee q) \Leftrightarrow q$ ... (law of implication)

$(\sim p \wedge p) \vee (\sim p \wedge q) \vee (q \wedge p) \vee (q \wedge q)$

... (Law of distributive)

false $\vee (\sim p \wedge q) \vee (q \wedge p) \vee q$

... (law of contradition & simplify the equation)

$(\sim p \wedge q) \vee (q \wedge p) \vee q$ ... (Law of Identity)

$(q \wedge \sim p) \vee (q \wedge p) \vee q$ ... (Law of commutative)

$q \wedge (\sim p \vee p) \vee q$ ... (Distributive Law)

$q \wedge$ True $\vee q$

... (Law of Identity)

$q \vee q$

$\therefore q \Leftrightarrow q$

$\therefore$ Hence proved LHS $\Leftrightarrow$ RHS

Q. 4.

a) $\neg (p \wedge q) \wedge p \Rightarrow \neg q$

$\to$

Here,

$(\sim p \vee \sim q) \wedge p$ ... (Demorgans law)

$(\sim p \wedge p) \vee (\sim q \wedge p)$ ... (Distributive law)

False $\vee (\sim q \wedge p)$ ... (Contradiction law)

$(\sim q \wedge p)$ ... (Identity law)

$\sim q$ ... (Conjuction elimination)

Hence

$\boxed{\neg (p \wedge q) \wedge p \Rightarrow \neg q}$

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b) $p \wedge q \vee q \wedge r \Rightarrow p \vee q \vee r$

→

Here,

$(p \wedge q) \vee (q \wedge r) \Rightarrow p \vee q \vee r$

$(p \vee (q \wedge r)) \vee q$ ... (Law of associativity)

$(p \vee q) \vee r \vee q$ ... (Law of distributivity)

$(p \vee q) \vee r$ ... (Idempotent law $\because q \vee q = q$)

$p \vee q \vee r$ ... (Law of associativity)

$\boxed{\therefore (p \wedge q) \vee (q \wedge r) \Rightarrow p \vee q \vee r}$

c) $(p \rightarrow q) \wedge (\neg p \rightarrow r) \Rightarrow q \vee r$

→

Here,

$(p \rightarrow q) \wedge (\neg p \rightarrow r) \Rightarrow q \vee r$

$((p \rightarrow q) \wedge (\sim p \rightarrow r)) \wedge (p \vee \sim p)$

.. (Conjuction LEM (Law of Excluded middle))

$((\sim p \vee q) \wedge (p \vee r)) \wedge (p \vee \sim p)$

... (Implication & double negation)

$(\sim p \vee q) \wedge (p \vee r) \wedge \text{True}$ $[p \vee \sim p \equiv T]$

$(\sim p \vee q) \wedge (p \vee r)$ $[a \wedge T \equiv a$ .. law of identity$]$

$(\sim p \wedge (p \vee r)) \vee (q \wedge (p \vee r)$

.. (Distributive law)

$(\text{False} \vee (\sim p \wedge r)) \vee ((q \wedge p) \vee (q \wedge r))$

.. (Law of contradiction)

$(\sim p \wedge r) \vee (q \wedge p) \vee (q \wedge r)$

... (Identity law)

$r \vee q \vee (q \wedge r)$ -- (law of conjuction elimination)

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$r \vee q$ ... (law of absorption)

$\therefore q \vee r$ ... (law of commutative)

Hence,

$$\boxed{(p \rightarrow q) \wedge (\neg p \rightarrow r) \Rightarrow q \vee r}$$

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Q. 5.

→ In general, there can be only eight different states that are possible containing only p, q, r which are proper for $p \longleftrightarrow q \longleftrightarrow r$

But, there are only four states that satisfy $p \longleftrightarrow q \longleftrightarrow r$ which are as follows.

i) For state $\sigma = \{p = T, q = T, r = T\}$:

Here,

$p = T$
$q = T$
$r = T$

since,

$\sigma(q \longleftrightarrow r) = \text{True}$

and

$\sigma(p) \longleftrightarrow \text{True} = \text{True}$

Therefore,

It satisfies $p \longleftrightarrow q \longleftrightarrow r$

ii) For state $\sigma = \{p = F, q = F, r = T\}$:

Here,

$p = F$
$q = F$
$r = T$

since,

$\sigma(q \longleftrightarrow r) = \text{False}$

and

$\sigma(p) \longleftrightarrow F = \text{True}$

Therefore,

It satisfies $p \longleftrightarrow q \longleftrightarrow r$

iii) For state $\sigma = \{p = T, q = F, r = F\}$

Here,

$p = T$
$q = F$
$r = F$

since,

$\sigma(q \leftrightarrow r) = \text{True}$

and

$\sigma(p) \leftrightarrow T = \text{True}$

Therefore,

$p \leftrightarrow q \leftrightarrow r$ is satisfied

iv) For state $\sigma = \{p = F, q = T, r = F\}$

Here,

$p = F$
$q = T$
$r = F$

since,

$\sigma(q \leftrightarrow r) = \text{False}$

and

$\sigma(p) \leftrightarrow F = \text{True}$

Therefore,

It satisfies $p \leftrightarrow q \leftrightarrow r$

→ Truth table for $p \longleftrightarrow q \longleftrightarrow r$

p	q	r	$q \longleftrightarrow r$	$p \longleftrightarrow q \longleftrightarrow r$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	T	F
F	T	F	F	T
F	F	T	F	T
F	F	F	T	F

Q. 6.

a) $\{b = 5, i = 0, x = 6\}$ is proper for predicate $x > b[i]$
→

The above statement is False because $b = 5$ is a single integer not an array.
To evaluate $x > b[i]$ b needs to be an array. Therefore indexing b at i is not a proper form.

∴ The statement is False.

b) $\{x = 4, y = -1\}$ is proper for expression $x / sqrt(y)$
→

The above statement is True for the expression $\frac{x}{\sqrt{y}}$ because there would be runtime error due to square root of negative number i.e. $y = -1$.

Thus, the statement is valid & true in terms of evaluation even though result will cause an error during the execution of expression.

∴ The statement is True.

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c) $[x = 5, y = 2] \models T$

→ The above statement is True because in the above expression T represents a Tautology state which is always true regardless of the state of the other different variables.

$\therefore$ The statement is True.

d) Let $\sigma = \{p = T, b = (2, 0, 4)\}$, then $\sigma \models p \leftrightarrow b[b[1]] = 2$

→ The above statement is True because,

$$p = T$$
$$b[1] = 0 \quad \&$$
$$b[b[1]] = b[0]$$
$$\therefore b[0] = b[0]$$

$\therefore$ The statement is True

e) If $a \equiv b$, then if $x \geq 0$ then $b[0]$ else $a[1][3]$ fi is not a legal expression

→

The above statement is True because $a \equiv b$ means a & b are equivalent

Here, $a \equiv b$ is in if clause

$b[0]$ is in then clause

& $a[1][3]$ is in else clause

So, if $a \equiv b$ i.e. a & b are identical it means it has the same structure.

Therefore, $b[0]$ & $\{a[1][3]\}$ are within the bounds.

$\therefore$ The statement is True

Q. 7.

a) $\neg A x \geq 1 \cdot x^2 > x \iff \exists x . x \geq 1 \wedge x^2 \leq x$

$\rightarrow$ Here,

$LHS = \sim \forall x \geq 1 \cdot x^2 > x$

By negating a universal quantifier it becomes existential quantifier so we get

$\therefore \exists x \geq 1 \cdot \sim (x^2 > x)$

$\therefore \exists x \geq 1 \cdot x^2 \leq x$

... (negation of inequality)

$\therefore \exists x . x \geq 1 \wedge x^2 \leq x$

... (Combining existential quantifier with conjuction)

$\therefore$ Hence proved

$\neg A x \geq 1 \cdot x^2 > x \iff \exists x . x \geq 1 \wedge x^2 \leq x$

b) $\neg \exists x . \exists y . x > y \wedge x < y \iff \forall x . \forall y . x \leq y \vee x \geq y$

$\rightarrow$

Here,

$\cancel{LHS = \sim \exists x . \exists y . x > y \geq x}$

$LHS = \sim \exists x . \exists y . x > y \wedge x < y$

$= \forall x . \forall y \sim (x > y \wedge x < y)$

... (Negation of existential quantifier)

$= \forall x . \forall y . x \leq y \vee x \geq y$

... (Demorgan's law & Negation of inequalities)

$\therefore$ Hence proved

$\neg \exists x . \exists y . x > y \wedge x < y \iff \forall x . \forall y . x \leq y \vee x \geq y$

c) $\neg((\exists x . \exists y . Q(x,y)) \wedge \forall x . \forall y . Q(y,x)) \iff (\forall x . \forall y . \neg Q(x,y)) \vee \exists x . \exists y . \neg Q(y,x)$ Here, $Q(x,y)$ is a predicate function.

$\rightarrow$

Here,

$Q(x,y)$ is a predicate function

$\therefore$ LHS : $\sim((\exists x . \exists y . Q(x,y)) \wedge \forall x . \forall y . Q(y,x))$

$\therefore \sim(\exists x . \exists y . Q(x,y)) \vee \sim(\forall x . \forall y . Q(y,x))$

... by Demorgan's law

$\therefore [\forall x . \forall y \sim Q(x,y)] \vee [\exists x . \exists y . \sim Q(y,x)]$

... by negating the quantifiers

$\therefore$ Hence proved :

$\neg((\exists x . \exists y \; Q(x,y)) \wedge \forall x . \forall y . Q(y,x)) \iff (\forall x . \forall y . \neg Q(x,y)) \vee \exists x . \exists y . \neg Q(y,x)$

Q. 8.

a) $\rightarrow$ $isGreater(b, m, x) \equiv 0 < m \leq size(b) \wedge \forall i : 0 \leq i < m \rightarrow x > b[i]$

b) $\rightarrow$ $hasGreater(a, b) \equiv \forall j \; \cancel{\exists j} \; \exists i \; ((0 \leq i < size(b)) \wedge (0 \leq j < size\ b)) \rightarrow (b[j] > a[i])$

c) $\rightarrow$ $Extends(a, b) \equiv [size[a] \leq size[b]] \wedge \forall i : [0 \leq i < size[a]] \rightarrow [a[i] = b[i]]$

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Q. 9.

→

		$\sigma[u \mapsto \alpha][v \mapsto \beta] = \sigma[v \mapsto \beta][u \mapsto \alpha]$?	$\sigma[u \mapsto \alpha][v \mapsto \beta] \equiv \sigma[v \mapsto \beta][u \mapsto \alpha]$?
$u \equiv v$	$\alpha = \beta$	Both the statements are syntactically equal ∴ Yes	In the both L.H.S & R.H.S side we are updating the same variable with same value. ∴ Yes
$u \equiv v$	$\alpha \neq \beta$	Here, on the R.H.S., $u \equiv v$ is bind with α & L.H.S. $u \equiv v$ is bind with β. ∴ No	In the above statements they will be symantically different. ∴ No
$u \not\equiv v$	$\alpha = \beta$	Here, $\alpha = \beta$ i.e. α & β both are equal so we can say the u & v both are binded with α ∴ Yes	Here, both u & v are different variables as $u \not\equiv v$ so they are not identical & will have different procedures ∴ No
$u \not\equiv v$	$\alpha \neq \beta$	Here, on both sides i.e. on L.H.S & R.H.S. u is bind with α & v is binded with β. ∴ Yes	Here, u & v both are different variables as $u \not\equiv v$ so they will have different procedures ∴ No

Q. 10. Consider $\sigma = \{x = 2, y = 5\}$

a] Find state $\sigma[x \mapsto \sigma(y)][y \mapsto \sigma(x)]$

→ Here,

Initial state $\sigma = \{x = 2, y = 5\}$

So,

$\sigma(x) = 2$

$\sigma(y) = 5$

step i

substitute value of x with $\sigma(y)$ i.e. 5

$\therefore$ ~~$\sigma\{x$~~ $\sigma = \{x = 5, y = 5\}$

step ii

substitute value of y with $\sigma(x)$ i.e. 2

$\therefore \sigma = \{x = 5, y = 2\}$

Therefore,

$\sigma[x \mapsto 5][y \mapsto 2] = \{x = 5, y = 2\}$

$\boxed{\therefore \sigma[x \mapsto \sigma(y)][y \mapsto \sigma(x)] = \{x = 5, y = 2\}}$

b] Let $\tau = \sigma[x \mapsto 3]$, and $y = \tau[y \mapsto \tau(x) * 4]$ Find y

→ Here,

Initial state $\sigma = \{x = 2, y = 5\}$

So,

step i

substitute & replace value of x with 3

$\therefore \sigma = \{x = 3, y = 5\}$

step ii

Find τ

After replacing value of x with 3

$\tau = \sigma[x \mapsto 3] = \{x = 3, y = 5\}$

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step iii :

Find y :

In τ, $x = 3$ & we calculate y as follows

$\therefore y = \tau(x) * 4$

$= 3 * 4$

$\therefore y = 12$

Therefore,

after substitution $\{x = 3, y = 12\}$

$\therefore y = \tau[y \mapsto \tau(x) * 4] = \{x = 3, y = 12\}$

$$\boxed{\therefore y = \tau[y \mapsto 12] = \{x = 3, y = 12\}}$$