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Sub : CS536 SOP Assignment 2

Q. 1

a) → $\sigma \models \exists x \in S . p$ means for this state σ & for some $\alpha \in S$, it is the case that $\sigma[x \mapsto \alpha] \models p$

b) → $\sigma \models \forall x \in S . p$ means for this state σ & for every $\alpha \in S$, it is the case that $\sigma[x \mapsto \alpha] \models p$

c) → $\sigma \not\models \exists x \in S . p$ means for this state σ & for every $\alpha \in S$, it is the case that $\sigma[x \mapsto \alpha] \not\models p$

d) → $\sigma \not\models \forall x \in S . p$ means for this state σ & for some $\alpha \in S$, it is the case that $\sigma[x \mapsto \alpha] \not\models p$

e) → $\models \exists x \in S . p$ means for every state σ, it is the case that $\sigma \models \exists x \in S . p$

f) → $\models \forall x \in S . p$ means for every state σ, it is the case that $\sigma \models \forall x \in S . p$

g) → $\not\models \exists x \in S . p$ means for some state σ, it is the case that $\sigma \not\models \exists x \in S . p$

h) → $\not\models \forall x \in S . p$ means for some state σ, it is the case that $\sigma \not\models \forall x \in S . p$

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Q. 2

a) $\{x = 2, y = 3\} \models x < 2 \rightarrow y < 3$

→ First,

substitute the value of x & y,

$x < 2 \rightarrow y < 3$

$2 < 2 \rightarrow 3 < 3$

After evaluating the truth values.

False → False

For $p \rightarrow q$, if p is false & q is false then the truth value is True

∴ The statement $\{x = 2, y = 3\} \models x < 2 \rightarrow y < 3$ is $\boxed{\text{True}}$

b) $\{b = (2, 5, 4, 8)\} \models \exists m : 0 \leq m < 4 \wedge b[m] < 2$

→ Here,

$\exists m . 0 \leq m < 4 \wedge b[m] < 2$

In the above equation, there exist an index m between 0 & 3 such that the element at index m in the sequence b is less than 2 (< 2)

So,

let's check this for each element of array b :

$b[0] = 2$, not < 2

$b[1] = 5$, not < 2

$b[2] = 4$, not < 2

$b[3] = 8$, not < 2

∴ The statement $\{b = (2, 5, 4, 8)\} \models \exists m . 0 \leq m < 4 \wedge b[m] < 2$ is $\boxed{\text{False}}$

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c) $\{x = 2, b = (2,3)\} \models \exists y . \forall 0 \le x \le 1 . b[x] = y$

→ Here,

The above statement is valid when $0 \le x \le 1$

& $b[x] = y$.

So, lets check this for above statement with above values,

when $x = 0$ $b[x] = b[0] = 2$

$x = 1$ $b[x] = b[1] = 3$

For this to be true we need a single value of y that equals both $b[0]$ & $b[1]$

No single value of y can satisfy $b[x] = y$ for both $x = 0$ & $x = 1$.

$\therefore$ The statement $\{x = 2, b = (2,3)\} \models \exists y \ \forall 0 \le x \le 1 . b[x] = y$ is $\boxed{\text{False.}}$

d) $\{x = 1, b = (5,3,6)\} \models \forall x . \forall 0 \le k < 3 . x < b[k]$

→ Here,

The above statement is valid when $0 \le k < 3$

& $x < b[k]$

First,

substitute the given values into the formula,

$\forall x . \forall 0 \le k < 3 . x < b[k]$

$\therefore \forall x . \forall 0 \le k < 3 . 1 < b[k]$

Now,

Checking for all values of k :

$k = 0$, $b[k] = b[0] = 5$, $5 > 1$, True

$k = 1$, $b[k] = b[1] = 3$, $3 > 1$, True

$k = 2$, $b[k] = b[2] = 6$, $6 > 1$, True

Even if the above statements are True, $\forall x$ is a universal identifier & the above statements should be True for all values of x

So, If $x = 5$, then $5 < 5$, False

$5 < 3$, False

$5 < 6$, True

As shown above, the condition does not hold True for all the values of x.

$\therefore$ The statement $\{x = 1 \quad b = (5,3,6)\} \models \forall x \,.\, \forall 0 \leq k < 3 \,.\, x < b[k]$ is $\boxed{\text{False.}}$

Q. 3.

a) `for (int i = 0; i < b.length; i++) { b[i] = i; }`

→

```
i := 0; while i < size(b) do b[i] := i;
    i := i + 1 od
```

b) `while (x != 1) { if (x % 2 == 0) { x = x / 2; } else { x++; } }`

→

```
while x ≠ 1 do
    if x % 2 = 0 then
        x := x / 2
    else
        x := x + 1
    fi
od
```

c) `int m = 8, p = 1, y = 1; while (++m < 20) { p = p * (y++); }`

→

```
m := 8; p := 1; y := 1; m := m + 1;
while m < 20 do
    m := m + 1;
    p := p * y;
    y := y + 1;
od
```

Q. 4.

a) $\langle$ if $x < 2$ then $x := y + 1$; $w := x + 2$ fi, $\{x = 3, y = 3, w = 4\} \rangle$

→ Here,

First evaluate if $x < 2$

Given,

$x = 3$ $\therefore$ $x < 2$ is false

Thus the branch will not get executed thus the values of x, y, w stay same.

$\therefore$ Final evaluation

$$\boxed{\langle E, \{x = 3, y = 3, w = 4\} \rangle}$$

b) $\langle$ while $x < 2$ do $x := y + 1$; $w := x + 2$ od, $\{x = 1, y = 3, w = 4\} \rangle$

→ First,

check if $x < 2$, $x = 1$, $\therefore$ $x < 2$ is True

→ $\langle x := y + 1$; $w := x + 2$; while $x < 2$ do $x := y + 1$; $w := x + 2$ od, $\{x = 1, y = 3, w = 4\} \rangle$

→ $\langle w := x + 2$; while $x < 2$ do $x := y + 1$; $w := x + 2$ od, $\{x = 4, y = 3, w = 4\} \rangle$

$\therefore$ executing $x := y + 1$

$\rightarrow$ $\langle$ while $x < 2$ do $x := y + 1$; $w := x + 2$ od,
$\{x = 4, y = 3, w = 6\}\rangle$

..... executing $w := x + 2$

Now,

check for $x = 4$, $x < 2$: $4 < 2$ is false

$\therefore$ Final Evaluation

$$\boxed{\langle E, \{x = 4, y = 3, w = 6\}\rangle}$$

c] $\langle x := y + 1 ; y := x + 1, \sigma\rangle$

$\rightarrow$ First,

execute $x := y + 1$

$\langle x := y + 1 ; y := x + 1, \sigma\rangle \longrightarrow$
$\langle y := x + 1, \sigma[x \mapsto \sigma(y) + 1]\rangle$

Next,

execute $y := x + 1$

$\langle y := x + 1, \sigma[x \mapsto \sigma(y) + 1]\rangle \longrightarrow$
$\langle E, \sigma[x \mapsto \sigma(y) + 1][y \mapsto \sigma(y) + 1 + 1]\rangle$

$\therefore$ Final Evaluation :

$$\boxed{\langle E, \sigma[x \mapsto \sigma(y) + 1][y \mapsto \sigma(y) + 2]\rangle}$$

Q. 5.

a) $\langle W, \sigma_1 \rangle$ where $\sigma_1 \vDash y < x \leq 0$

$\rightarrow$

Here,

$\langle W, \sigma_1 \rangle$

Because of $\sigma(x) > \sigma(y)$, we get,

$\rightarrow \langle S ; W, \sigma \rangle$

Now,

because $\sigma(x) \leq 0$, we will get,

$\rightarrow \langle y := -2 * x ; W, \sigma \rangle$

$\rightarrow \langle W, \sigma[y \mapsto -2 * \sigma(x)] \rangle$

Now,

because $\sigma(x) \leq 0$ & $\sigma[y \mapsto -2 * \sigma(x)](y) \geq 0$

We will get,

$\boxed{\rightarrow \langle E, \sigma[y \mapsto -2 * \sigma(x)] \rangle}$

b) $\langle W, \sigma_2 \rangle$ where $\sigma_2 \vDash x > 0 \wedge y \leq 0$

$\rightarrow$

Here, need to evaluate,

$\langle W, \sigma_2 \rangle$

$\rightarrow^* \langle W, \sigma_2[x \mapsto \sigma_2(x) + 1] \rangle$

$\rightarrow^* \langle W, \sigma_2[x \mapsto \sigma_2(x) + 2] \rangle$

Now,

because x binds with a larger value in each of the iteration, we will get,

$\boxed{\rightarrow^* \langle E, \bot_d \rangle}$

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Q. 6.

a) Calculate $M(S, \tau)$, Here τ is a state with x & y

→ Here,

$W \equiv$ while $x < 3$ do S od

$S \equiv x := x + 1 ; y := y * x$

Calculate

$M(S, \tau)$

Now,

lets assume initial values of x & y in τ as α & β respectively

$\therefore x := x + 1$

$x := \alpha + 1$

and,

$y := \cancel{\beta} y * x$

$y := \beta * (\alpha + 1)$

$\therefore M(S, \tau) = \{\tau[x \mapsto \alpha + 1][y \mapsto \beta * [\alpha + 1]]\}$

$\therefore M(S, \{x = \alpha, y = \beta\}) = \{\{x = \alpha + 1, y = \beta * (\alpha + 1)\}\}$

Where, α, β are assumed to be initial values states in τ.

$\boxed{\therefore M(S, \tau) = \{x = \alpha + 1, y = \beta * (\alpha + 1)\}}$

b) Calculate $M(W, \sigma)$, where $\sigma(x) = 4$ & $\sigma(y) = 1$

→ Here,

Initial state,

$\sigma(x) = 4, \sigma(y) = 1$

While loop won't execute since $x < 3 : 4 < 3$ is False.

Therefore, no changes are made to x or y.

$\therefore$ Final Evaluation:

$$\boxed{\begin{aligned} M(W, \sigma) &= M(W, \{x = 4, y = 1\}) \\ &= \{x = 4, y = 1\} \end{aligned}}$$

c) Calculate $M(W, \sigma)$, where $\sigma \models x = 1 \wedge y = 1$

$\rightarrow$ Here,

$$\begin{aligned} M(W, \sigma) &= M(W, \{x = 1, y = 1\}) \\ &= M(W, M(S, \{x = 1, y = 1\})) \\ &= M(W, M(x := x + 1; y := y + x, \\ &\qquad \{x = 1, y = 1\})) \\ &= M(W, M(y := y + x, \{x = 2, y = 1\})) \\ &= M(W, \{x = 2, y = 2\}) \\ &= M(W, M(S, \{x = 2, y = 2\})) \\ &= M(W, \{x = 3, y = 6\}) \\ &= \{x = 3, y = 6\} \end{aligned}$$

$$\boxed{\therefore M(W, \sigma) = \{x = 3, y = 6\}}$$

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Q. 7

a)

→ Need to calculate $M(S, \sigma)$ where $\sigma(x) = -2$ & $\sigma(y) = -1$

$$\therefore M(S, \sigma) = M\left(x := \frac{y}{x}, \sigma\right)$$

$$= M\left(E, \sigma\left[x \mapsto \sigma\left(\frac{y}{x}\right)\right]\right)$$

$$= \{\sigma[x \mapsto 0]\}$$

b)

→ Need to calculate $M(W, \sigma)$ where $\sigma = \{x = 2, y = 2, b = (0, 1, 2)\}$

$$\therefore M(W, \sigma) = M(W, \{x = 2, y = 2, b = (0, 1, 2)\})$$

$$= M(W, \{x = 1, y = 2, b = (0, 1, 2)\})$$

$$= M(W, \{x = 2, y = 2, b = (0, 1, 2)\})$$

$$= \{\perp_d\}$$

$\therefore$ As we can see the same configuration is appearing again & again Hence proved.

c)

→ Need to calculate $M(W, \sigma)$ where $\sigma = \{x = 8, y = 2, b = (4, 2, 0)\}$

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$\therefore M(W, \sigma) = M(W, \{x=8, y=2, b=(4,2,0)\})$

$= M(W, \{x=7, y=0, b=(4,2,0)\})$

$= M(W, \{x=6, y=4, b=(4,2,0)\})$

$= \{\perp e\}$

$\because$ Array index out of bounds

d) → Unfortunately, there is no such state σ such that $M(W, \sigma) = \{\perp e\}$ because of the "division by zero" error.

The only division operation $\frac{y}{x}$ appears in the if condition of statement S, & we evaluate $\frac{y}{x}$ immediately after we enters an iteration of W. So, whenever we evaluate $\frac{y}{x}$, we must need to have the evaluation of $x > 0$ to be equals to true.

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Q. 8.

→ Given,

$S \equiv x := sqrt(x) / b[y]$

& let $\sigma = \{b = (3, 0, -2, 4), x = \alpha, y = \beta\}$

Here,

α & β are two named integer constants.

We need to find all possible states σ such that

$M(S, \sigma) = \{\perp_e\}$

① σ with $\sigma(x) < 0$ and $\sigma(y)$ is any arbitary integer.

[This error is due to the square root of the negative number]

② σ with $\sigma(x) \geq 0$ & $\sigma(y) \geq 4$

[Array index out of bounds]

③ $\sigma(x) \geq 0$ and $\sigma(y) < 0$

[Array index out of bounds]

④ $\sigma(x) \geq 0$ and $\sigma(y) = 1$

[Division by zero error]

& In terms of α and β :

① $\alpha < 0$ & β = any arbitary integer

② $\alpha \geq 0$ & $\beta < 0$

③ $\alpha \geq 0$ & $\beta \geq 4$

④ $\alpha \geq 0$ & $\beta = 1$

This are all the states that result in error state $\{\perp_e\}$

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Q. 9.

a) $\perp \models T$

→ False

Explanation : The truth T is universally true in all of the states. And $\perp$ represents an error state, which indicate that the reputation does not yeild a valid result. Error state cannot satisfy truth T. Since $\perp$ is an error state, it does not satisfy Truth.

b) $\perp \nvDash F$

→ True

Explanation : $\perp$ represents an error state. F is universally false, & no state satisties F. As $\perp$ represents an error state & it cannot satisfy True as well as it cannot satisfy False.

∴ The above statement is True

c) If $\sigma(p) \neq \bot$, then $\sigma \nvDash p$

→ $\boxed{\text{False}}$

Explanation: If $\sigma(p) \neq \bot$, it means that no error occurred during the evaluation of p. There can be two states either $\sigma \vDash p$ or $\sigma \vDash \sim p$ is true. $\sigma \vDash p$ means that p evaluates to true in state σ.

$\sigma \vDash \sim p$ means that p evaluates to false in state σ. The statement given is false because $\sigma(p) \neq \bot$ allows for possibility that p evaluates to false i.e. the next statement should be $\sigma \nvDash p$.

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d) If $\sigma(p) = \bot$, then $\nvDash \neg p$

→ $\boxed{\text{True}}$

Explanation : When $\sigma(p) = \bot$ then the state σ cannot satisfy either p or $\neg p$. Because it is in a state of undefined evaluation for that given predicate. Hence, the statement is True.

e) If $\vDash p$, then $\neg \exists \sigma . \sigma(p) = \bot$

→ $\boxed{\text{True}}$

Explanation : When $\sigma(p) = \bot$ then it will indicate that evaluating p in state σ results in value that indicate an error or undefined behaviours. Since p is valid in all well formed states, these cannot be any well-formed state where p evaluates to the $\bot$. Hence, the statement is True.

Q. 10.

a) Let $\Sigma_0 \subset \Sigma$ & $\Sigma_0 \vDash p$, also let $\tau \vDash p$; then $\Sigma_0 \cup \{\tau\} \vDash p$

→ $\boxed{\text{True}}$

Explanation : The state $\Sigma_0 \vDash p$ implies that every state in the Σ_0 satisfies p & $\tau \vDash p$ means τ also satisfies p. Therefore adding τ to Σ_0 should not change the satisfication factor of p so every state in $\Sigma_0 \cup \{\tau\}$ will still satisfy p. Hence, the statement is True.

b) $\emptyset \vDash p$ & $\emptyset \vDash \neg p$ ($\emptyset$ represents an empty collection of states)

→ $\boxed{\text{True}}$

Explanation : The empty collection of states (i.e. $\emptyset$) satisfies any prediction. This is because there are no

counter parts in $\emptyset$ to refuse the prediction satisfaction.
Therefore, both $\emptyset \models p$ & $\emptyset \not\models p$ are True.

c) Let $\tau \in \Sigma$, then $\tau \models p$ or $\tau \models \neg p$

→ $\boxed{\text{False}}$

Explanation for the well formed states τ it is possible that $\tau(p) = \perp$, meaning that the evaluation of p in the state of τ results in an undefined value of errors.

If $\tau(p) = \perp$, then neither $\tau \models p$ nor the $\tau \models \neg p$ are true. Therefore the above statement is false.

d) Let $\Sigma_0 \subset \Sigma$ then $\Sigma_0 \models p$ or $\Sigma_0 \models \neg p$

→ $\boxed{\text{False}}$

Explanation: It is possible that some of the states in Σ_0 might satisfy p while some of the states in Σ_0 might satisfy $\neg p$. So, therefore, neither $\Sigma_0 \models p$ nor $\Sigma_0 \models \neg p$ would be True case.

e) Let $\sigma_1 \models p_1$ & $\sigma_2 \models p_2$, then $[\sigma_1, \sigma_2] \models p_1 \vee p_2$

→ $\boxed{\text{True}}$

Explanation: If $\sigma_1 \models p_1$, then $\sigma_1 \models p_1 \vee p_2$ because p_1 implies $p_1 \vee p_2$ as shown above. Similarly we can say that if $\sigma_2 \models p_2$, then $\sigma_2 \models p_1 \vee p_2$. Therefore, $p_1 \vee p_2$ is True in both the states so $\{\sigma_1, \sigma_2\} \models p_1 \vee p_2$. Therefore the above statement is True.

Additional assumption:

i) $\sigma 1$ can evaluate $p2$ (but we don't know if $\sigma 1 \models p2$ or $\sigma 1 \not\models p2$)

ii) $\sigma 2$ can evaluate $p1$ (but we don't know if $\sigma 2 \models p1$ or $\sigma 2 \not\models p1$)

Even if $\sigma 1 \not\models p2$ & $\sigma 2 \not\models p1$, it doesn't affect the truth of the given statement. We will still have at least one state satisfying each part of the disjunction.

∴ Therefore, the above statement is True.