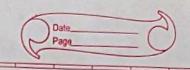
Date\_\_\_\_\_ CWID: A20545137 5-6: (5536 sop Assignment 4 a) P[y+z/x]  $P[y+z/x] = (w \cdot x \neq 0 \land z \leq Z \rightarrow f(w) > 0$ 1 yn. 7y. 0 ky < x > f (w - x) + y > f(z)) P[4+2/r] = (w \* e + 0 ^ Z < 2) -> (f (w) > 0 ^ Vx. 3y. 0 < y < x A f ( w ÷ x ) + y > f(z) Ply+z/x] = w + (y + z) + 0 x z < 2 -> f(w) > 0 1 4x. fy. 0 &y &x 1 ( w = x) + y > f(z) b) p[x+z/w] P[x+2/w] = ((x+z)·x + 0 1 2 (2 -> f(x+z)) 0 x yxo. 74. 0 < y < xo x f ((x+z) = xo) + 4 > f(2)) P[x+z/w]  $= ((x+z) \cdot x \neq 0 \land z \leq 2 \rightarrow f(x+z) > 0 \land \forall x_0 \cdot \exists y \cdot 0 \leq y \leq x_0 \land f(x+z) = x_0)$ + y > f(z)) () P[x+y/2] P[x+y/z] = (w.x \$ 0 \ (x+y) \ \ 2 \rightarrow \ f(\omega) > 0 1 4x0. 340.0 840 (No 1 f (W - 40) + 40 > + ( xx + y ))

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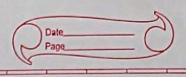
Let x & y be two different integer variables:

(x \* y) [e/x][e'/y] = (x \* y) [e'/y][e/x] al show an example in which the above conjecture works e = 4 and e' = 2 (x \* y)[4/x][2/y] = (4 \* y)[2/y]= 4 \* 2 = 8 (xxy)[2/y][4/x] = (xx2)[4/x] LHS = RHS b) Disaprove the above conjecture with a counterexample e = y and e' = x (x \* y) [ y/x] [x/y] = y \* y [x/y] = x \* x (x \* y) [x/y] [y/x] = x \* x [y/x] Therefore, this disaproves the conjecture



Prove that "If partle sp(p, s) => q"
Here p <=> w/p (s,q) implies that = {p} s {q} ... by definition of weakest liberal precondition = { w/p(5, q) } S fq } weakening the precondition let T be a state such that T = sp (p, s) By the definition of sp(p, S) there exists some of Fp such that TEM(5,0)-1. F { p } s { q } implies that m (s, v) - 1 = q Thus, if t = sp (p, s), then t = q which implies sp (p, s) => q. Hence proved. b) Disprove that "if p (=> w|p(s,q) then q => sp(p,s)"

Here p = wp(s,q) implies that = [p & s § 9] -- by definition of weakest liberal precondition = { w/p (5, q) } 5 } 9 } --- weakening the precondition By the definition of sp(p,s) we have = sp3 s sip (p,s) } Since sp(p,s) => q, it implies that a triple may or may not hold true if post condition is weakened. It contradicts the claim that sp(p, s) is stronger than q Counterexample can be if s = x := x x x and g = x < b then wip (5, q) = x x x < 1 (=> x = 0. B-t sp (x=0, x = x x x) = (x0 = 0 x x = x0 x x0) => x = 0 , which is strictly stronger than x < 1 Hence, the above statement compot be true



4 Etot [ P ] S { 5 } Explanation: 5 (=> sp (p, S) But it is possible that 3T = 1 E M(5,0) Hence, it does not satisfy triple under total correctness b) There exists some or = p such that or # {pfs {q}} False Explanation: It a state is satisfying pre-condition & s is logically equivalent to sp (p, s) can satisfy the pre-condition & may not satisfy the strongest postcondition. It can either diverge or create errors on S. i.e. if of p then M(s, o) -1 = s c) For each state of p we have that M(s, o) +s - False txplanation: If ofp then YTEM(S, o). T = I V x = S which means it either leads to pseudo-states or satisfies the strongest post condition

If m(s, \sigma) - 1 \mathbb{F}s, then \sigma \mathbb{F}p

False

Explanation:

Knowing that S terminates & that s holds upon termination

does not guarantee that the precondition p was initially true.

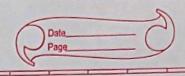
s could hold as a result of conditions or assignments within

S, even if p did not hold before execution i.e. just because

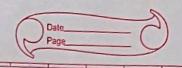
s terminates & s holds afterward does not imply that the

initial condition is p was true.

e. If \sigma \mathbb{F} \tap \tap \mathbb{F} = \mathbb{



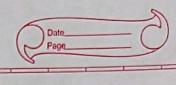
(alculate sp (x=y, if x>0 -> x:= y+1; z:=x Dx <0-> → 1 (+ z:= y fi) IF = if x>0 -> x:=y+1; z:=x 1 x <0 -> y:=x-1; z:= 4 fi sp (x = y , IF) = sp (x=y /x=x0/y=y0/x>0, x:= y+1; Z:= x) V sp (x=y ~ x= x0 ~ y = y0 ~ x <0 y:=x-1; z := y) = sp (x0 = y ^ x0 = x0 ^y = y0 ^ x0 > 0 ^ x = y+1, 2:= x) V sp (x= yo x x = xo x yo = yo ~ x < 0 ^ y = x - 1 , z = y) = (x0 = y ~ x0 = x0 ~ y = y0 ~ x0 > 0 ~ x = y+1 1 2 = 80 ) V (x = y0 1 x = x0 1 y0 = yo ~ x < 0 1 y = x - 1 1 z = y) (alcolate sp(y=x+1, y:=y+1; if x <0 then y:=-y fi) - y = x + 1 + 1 Now lets continuate conditional statement: if x < 0 then y : = - y fi 7 = - ( 8 + 2 )



for talse statement i.e. if x > 0, then y = x + 2 the strongest postcondition sp (y = xx + 1 y:= y+1

if x<0 then y:= -y fi) is therefore

(=> (x < 0 x y = -x-2) \*\* y (x >, 0 A y = x+2) -- y = f x+2 if x>,0 For tormal proof: To prove: + {p} if B then S, else Sz fi {q1 v q2} ○ {p ∧ B 4 5, {q1 } --- given @ {p ^ ¬ B } S2 { 92 } (D) Sp ~ B 3 S1 Sq 1 v 92 5 @ fp ~ ~ B } 52 { 9, ~ 92 } (B → (p ∧ B)) ∧ (¬B → (p ∧ ¬B)) <=> p Logical equivalence 0 9 (B-) (p ~ B)) ~ (-B-) (p ~ -B)) } if B then si else so fi fqi v q2 } -- By conditional rule 2 using steps 3 & @ @ Ep 4 if B then SI else Sz fi [q1 v q2 f -- By consequence rule using steps @ 2 @ : Hence proved



S = x := x + x ; y := 2 \* y step ( calculate w/p (5, x=y) for y := 2 \* y ... Before this x = 2 \* y must hold -- Before this x \* x = 2 \* y must hold : w/p (5, x=y) (=) x + x = 2 \* y step @ Proof + Ip & S [x = y ] p = w/p (5, x=y) p = [x \* x = 2 \* y) The weed to brove + (b & 2 x = 1 } il Precondition: p = (x \* x = 2 x q) ii) sequence of statements: X := X \* X y:= 2 \* y in 15 how that executing x := xe x xe preserves p: x \* x = 2 \* y the intermediate condition x = 2 \* y will hold after this step. iv) show that executing y = 2 x y establishes the postcondition x = y : After y:= 2 \* y y takes the value 2 \* g

Since xe=2 \* y held after first assignment xe=y now holds. + {x x x = 2 x y 3 5 5x = 4 4 Hence proved.

