UNIVERSITY OF BIRMINGHAM

CNCR

Control Theory

Mind, **B**rain, and **M**odels 2022/23

Motivation

- All systems, living and mechanical, are both information and feedback control systems - Wiener
- **What** is control theory?

"Control theory is an interdisciplinary branch of engineering and mathematics that deals with the behavior of dynamical systems with inputs. […] The usual objective of a control theory is to calculate solutions for the proper corrective action from the controller that result in system stability, that is, the system will hold the set point and not oscillate around it." [1]

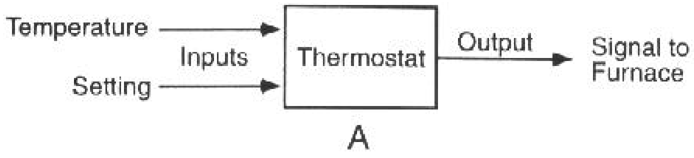
- **Why** control engineering in MBM?
  1. Common modeling language for cognitive scientists and engineers/computer scientists
  2. Useful tools for implementing experiments etc.

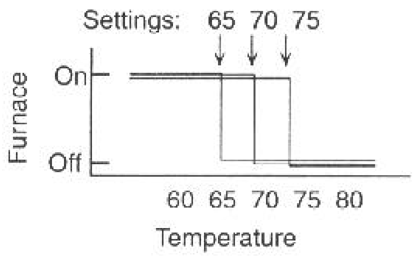
[1] http://en.wikipedia.org/wiki/Control_theory

Motivation

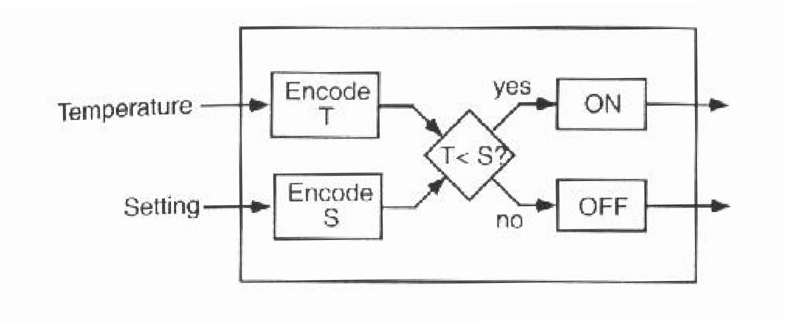
Example: models of a thermostat

According to Marr (1982)



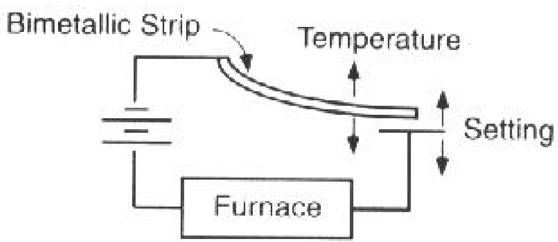


The **computational** level of description of an information processing system is the mathematical description of the mapping between the input to the system and its output

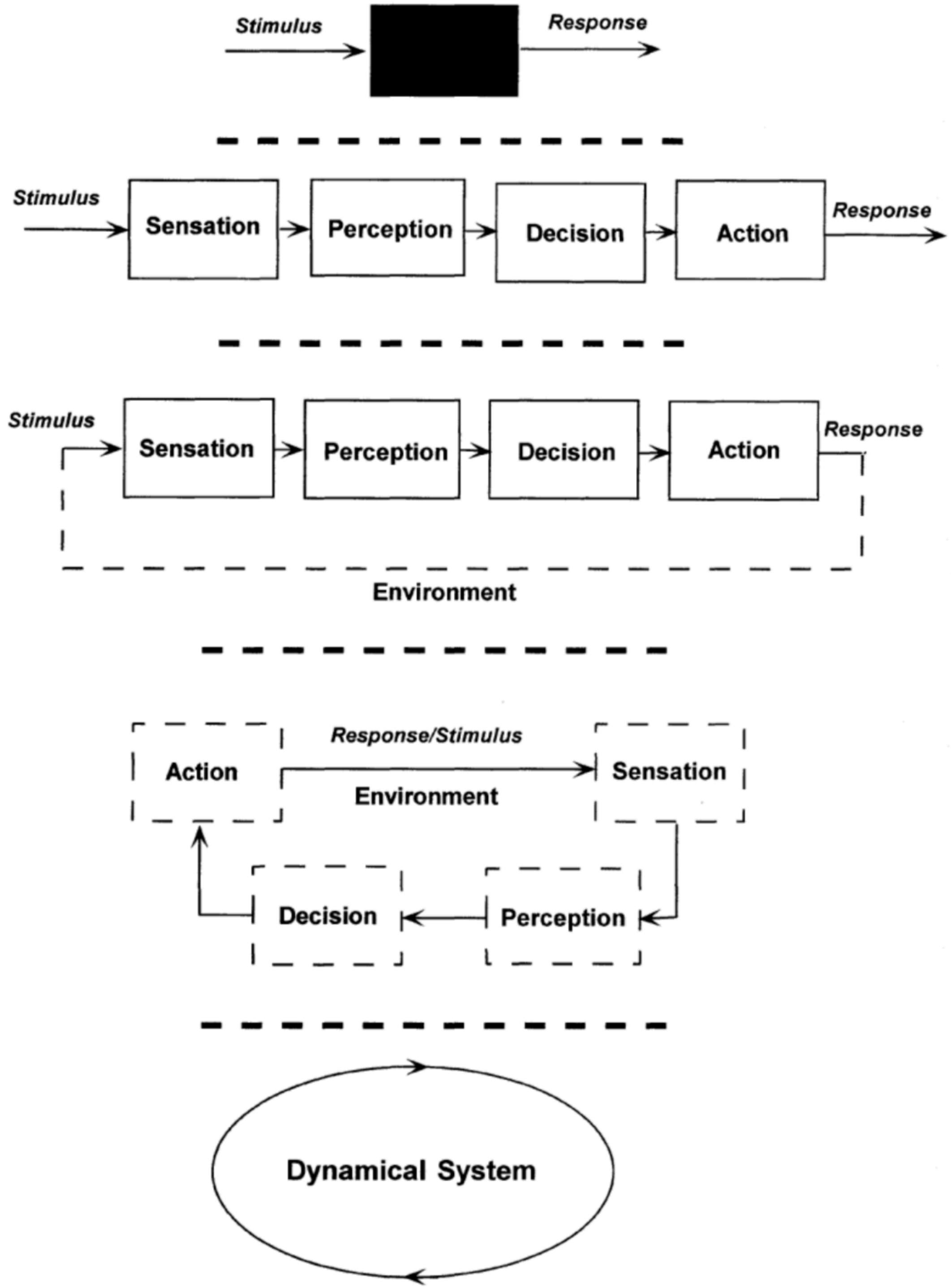


The **representation** and **algorithm** level of description of an information processing system specifies:

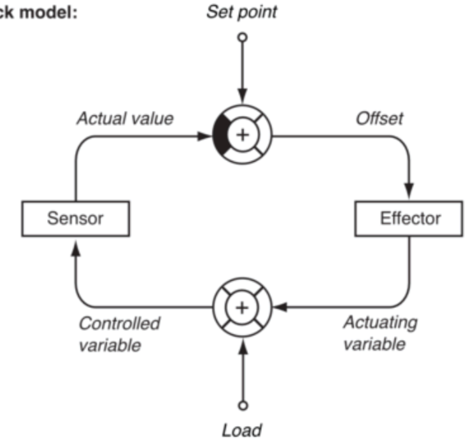
- the representation for the input and for the output
- the algorithm that transforms the input into the output



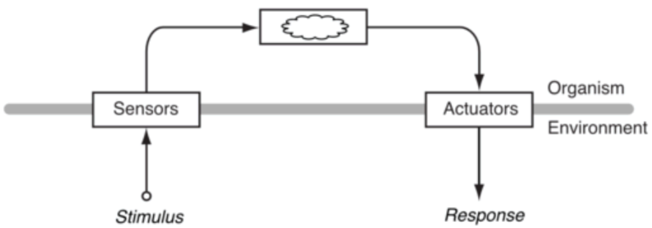
The **implementation** level of description of an information processing system specifies how an algorithm is embodied as a physical process in a physical system


Stimulus
Response
Stimulus
Sensation
Perception
Decision
Action
Response
Stimulus
Sensation
Perception
Decision
Action
Response
Environment
Response/Stimulus
Action
Sensation
Environment
Decision
Perception
Dynamical System

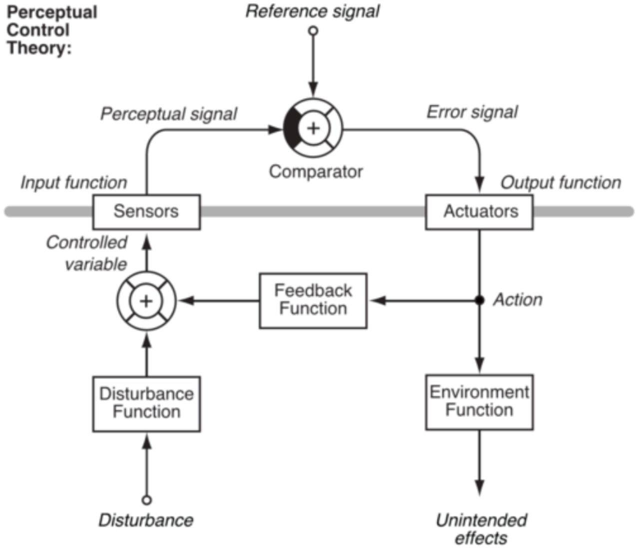
General feedback model:



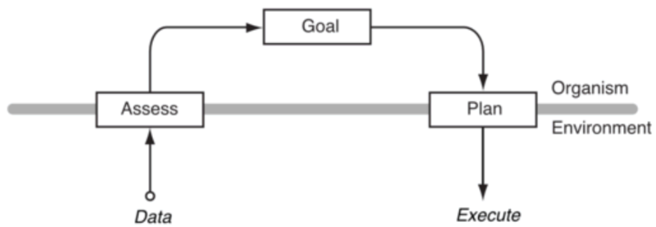
Behaviourism:



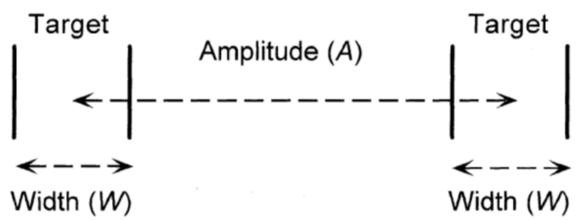
Perceptual Control Theory:

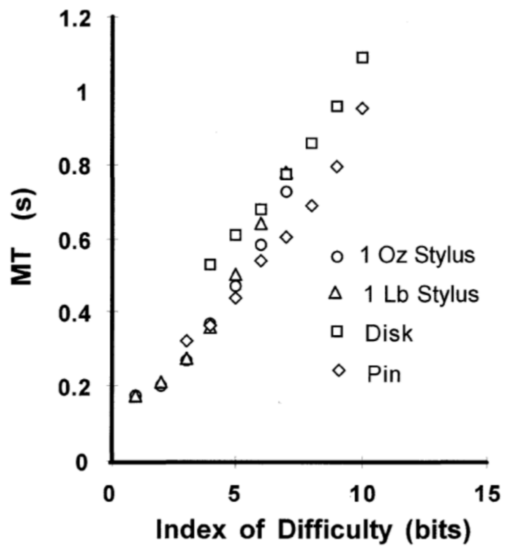


Cognitive Psychology:

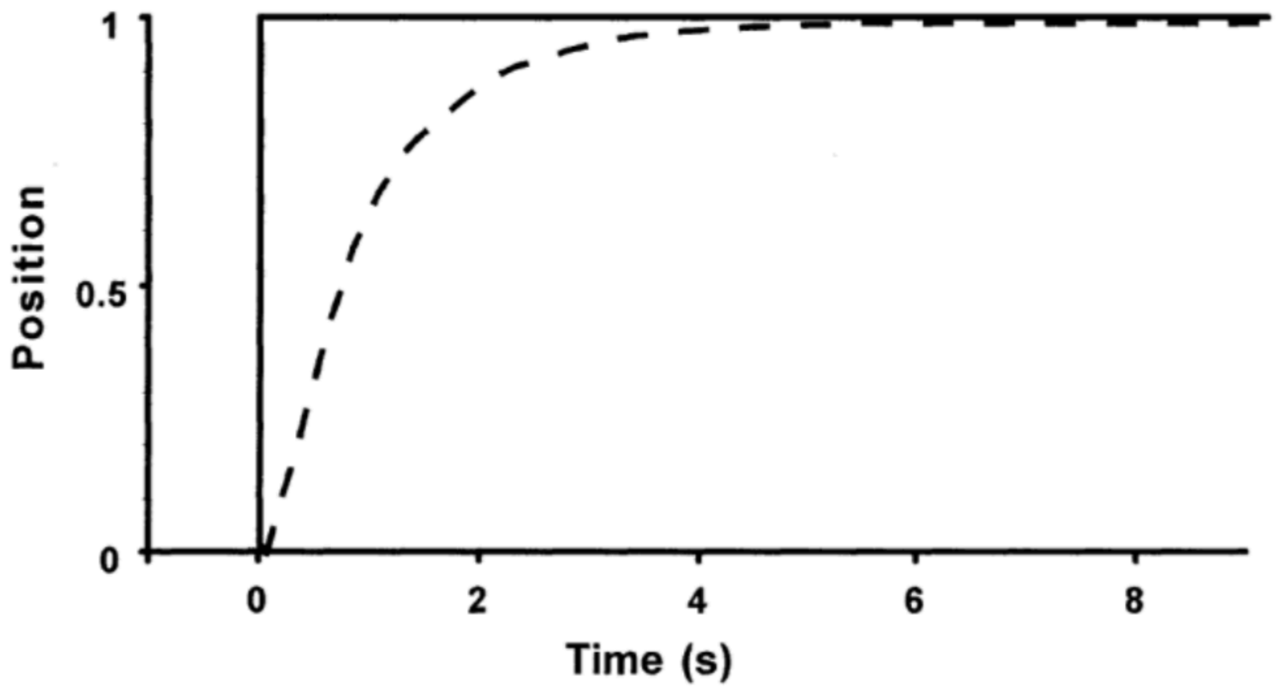


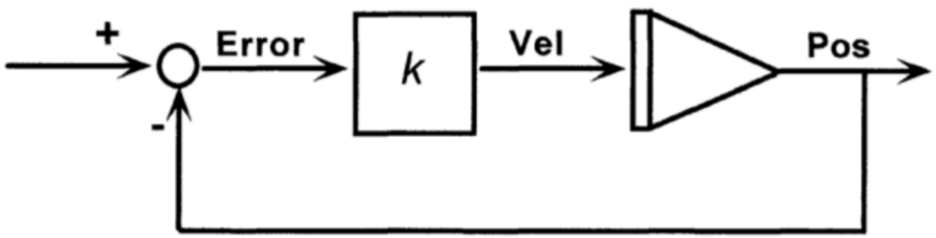
Fitt's law


Target
Amplitude (A)
Target
Width (W)
Width (W)

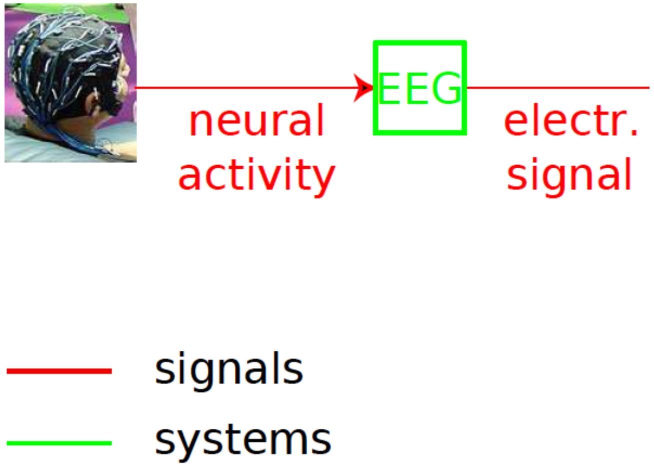

1.2
1
0.8
0.6
0.4
0.2
0
MT (s)
0
5
10
15
Index of Difficulty (bits)
1 Oz Stylus
1 Lb Stylus
Disk
Pin

First-order lag





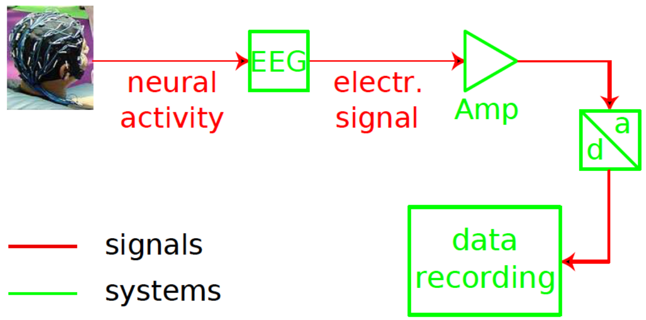
Signals and Systems



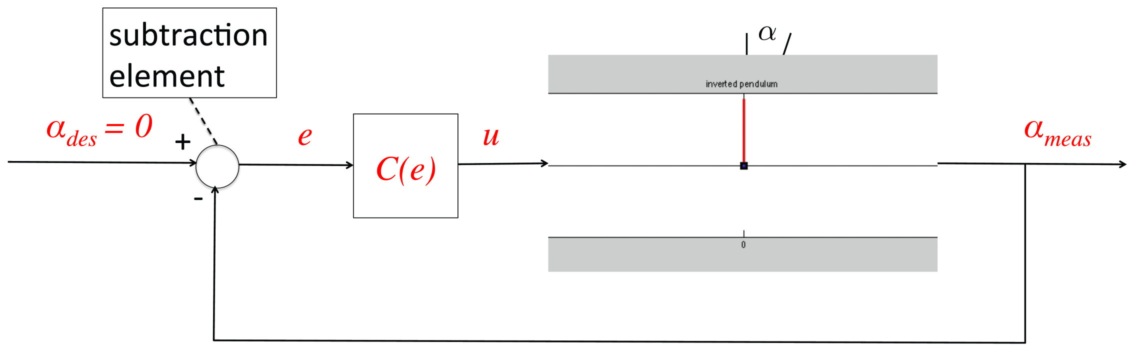
- Signals
  - carry information
  - can be transported in different media, domains, ...
- Systems
  - transform input signals into system outputs
  - can bring information from one domain to another

Overview

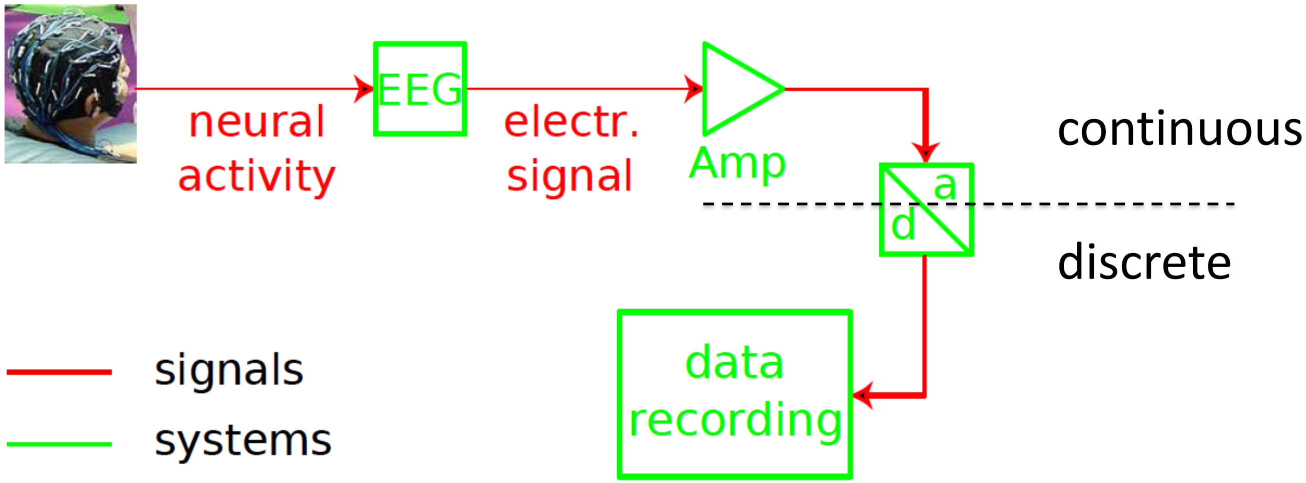
- Signals and Systems
  - Signal flow diagram
  - System description
  - System behaviour
  - Modeling



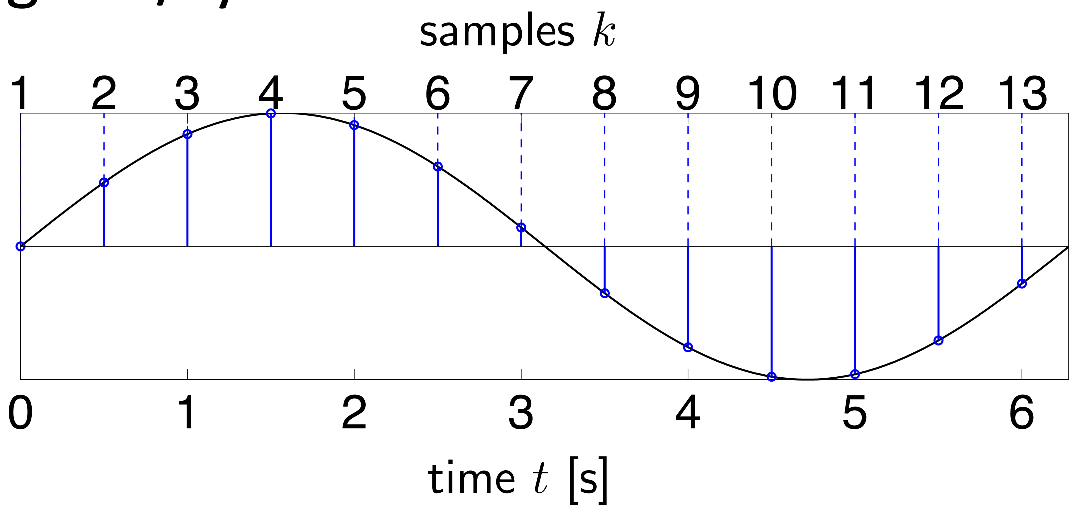
- Control
  - Control loop
  - Controller types
  - Stability



Signal Flow Diagrams

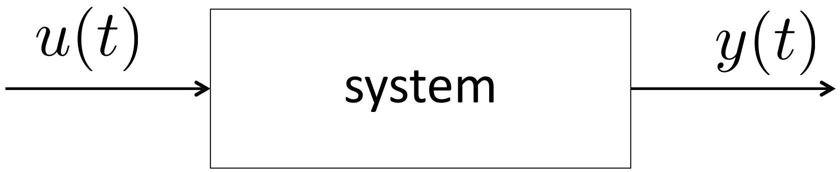


- Graphical representation of multiple systems connected by signals
- Continuous/discrete time signals/systems
  - time-continuous signal $u(t)$
  - time-discrete signal $u[k]$



Static System Description

- Algebraic expression



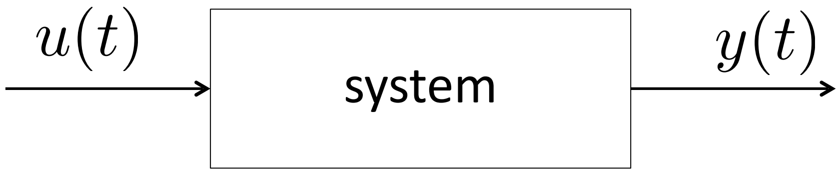
$$y(t) = g(u(t))$$

- Can describe systems “without memory”
  System output $y(t)$ only depends on current input $u(t)$, not history or future of $u(t)$, $y(t)$

- **Examples:** EEG signal amplifier – $V_{out}(t) = K \cdot V_{in}(t)$

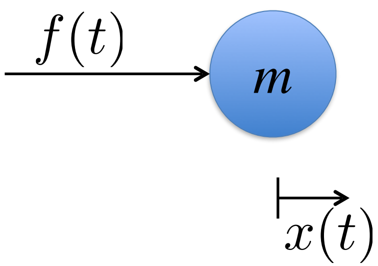
Dynamic System Description

- (Linear) differential equation



$$a_n \frac{d^n}{dt^n} y(t) + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} y(t) + \ldots + a_0 y(t) = b_m \frac{d^m}{dt^m} u(t) + b_{m-1} \frac{d^{m-1}}{dt^{m-1}} u(t) + \ldots + b_0 u(t)$$

- Describes continuous-time systems “with memory”
- Best description form for many physical processes (theoretical)
- **Example:**

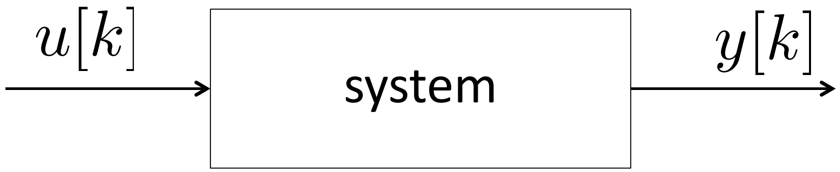


$$f(t) = m \frac{d^2}{dt^2} x(t)$$ (Newton’s second law)

“Memory” effect: Can you tell the position $x(t)$ only from the momentary force input $f(t)$?

System Description

- Difference equation



$$a_n y[k-n] + a_{n-1} y[k-n+1] + \ldots + a_0 y[k] = b_m u[k-m] + b_{m-1} u[k-m+1] + \ldots + b_0 u[k]$$

- Discrete-time system model, can approximate continuous-time characteristics
- Computer simulation, learning algorithms, … wherever the signal value is only known at specific time instances

System Description

- Discrete-time state space model

$$\mathbf{z}[k] = \begin{bmatrix} z_1[k] \\ z_2[k] \\ \vdots \\ z_n[k] \end{bmatrix} = \begin{bmatrix} a_{11}z_1[k-1] + a_{12}z_2[k-1] + \ldots + a_{1n}z_n[k-1] \\ a_{21}z_1[k-1] + a_{22}z_2[k-1] + \ldots + a_{2n}z_n[k-1] \\ \vdots \\ a_{n1}z_1[k-1] + a_{n2}z_2[k-1] + \ldots + a_{nn}z_n[k-1] \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} u[k]$$

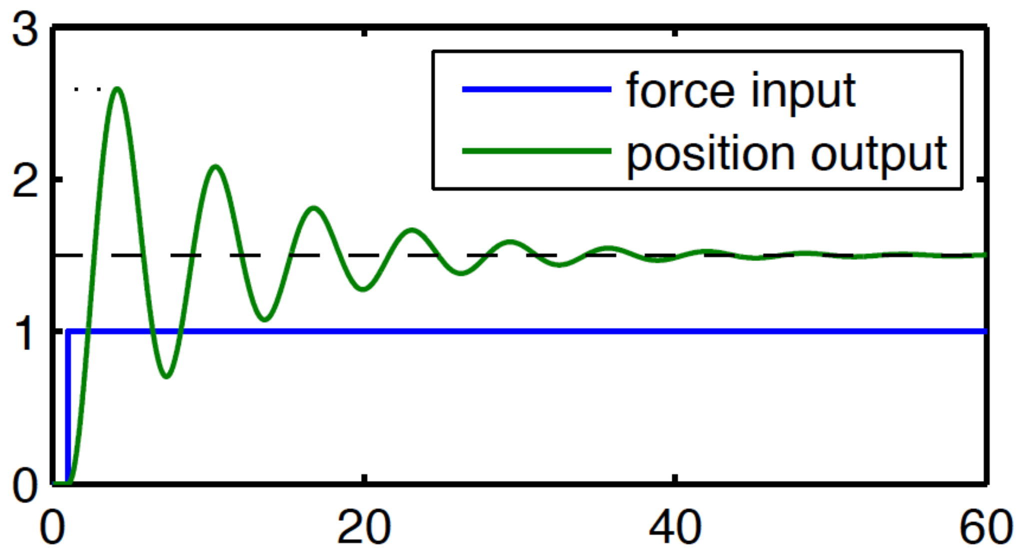
$$y[k] = c_1 z_1[k] + c_2 z_2[k] + \ldots c_n z_n[k] + du[k]$$

- Set of 1st-order difference equations
- Equivalent to one difference equation of order n (see previous)

System Behaviour

- Impulse-/step response:
  Reaction of a dynamic system to a step/impulse input
  Video

- Inferable system properties:
  - Oscillating/non-oscillating
  - Stationary gain
  - Overshoot
  - Onset time

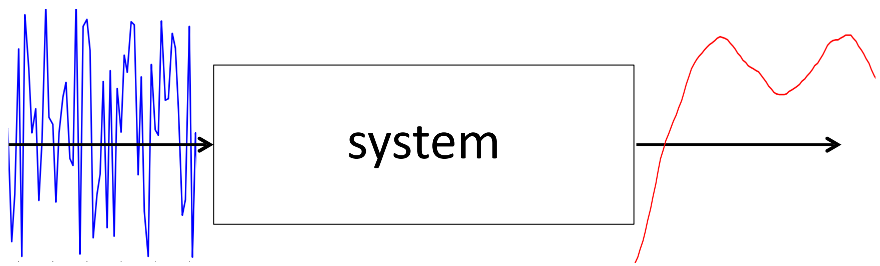


Modeling

- Model structure (order?, oscillating?, linear?)
  - Known
  - Guessed/assumed

$$u(t) \longrightarrow \boxed{\frac{d}{dt}y(t) = a_1\frac{d}{dt}u(t) + a_0u(t)} \longrightarrow y(t)$$

- Parameter identification
  - Select appropriate excitation signal
    - White noise
    - Sinusoids of different frequencies
    - Step signal (not ideal)

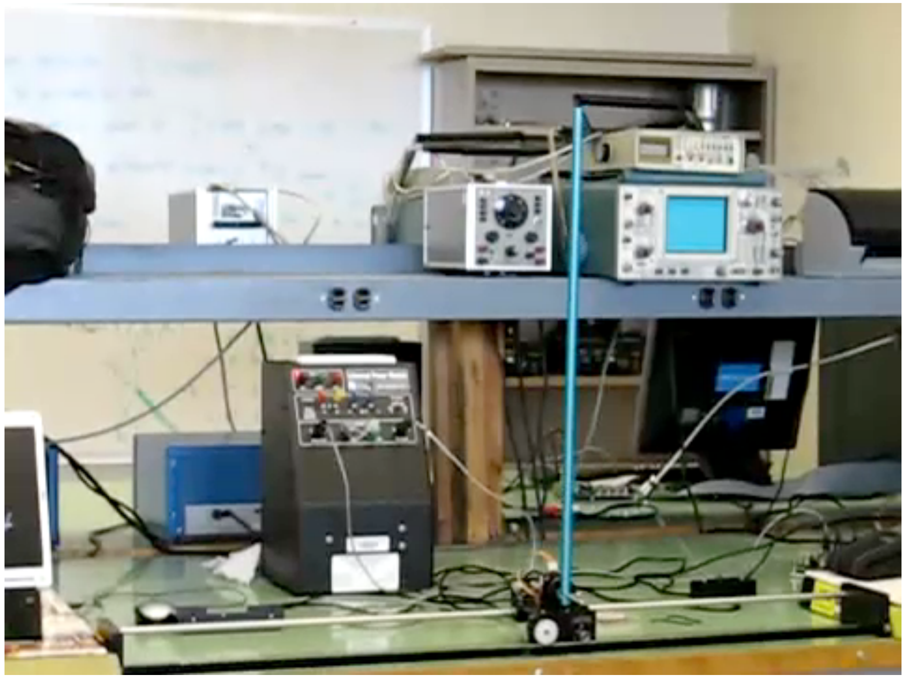


  - Measure input and output data
  - Find parameterization that explains data best

$$\frac{d}{dt}y(t) = 1.5\frac{d}{dt}u(t) + 2u(t)$$

Control

- Change a system's natural behaviour by controlling the input
  Example: Inverted pendulum as abstraction of
  - Upright posture
  - "Stick balancing" task
  - Segway
  - Skyscraper earthquake stabilisation

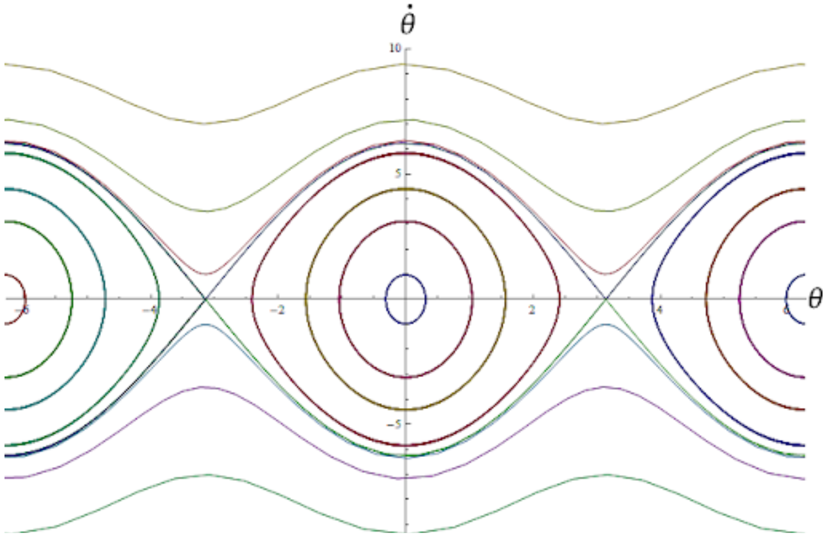


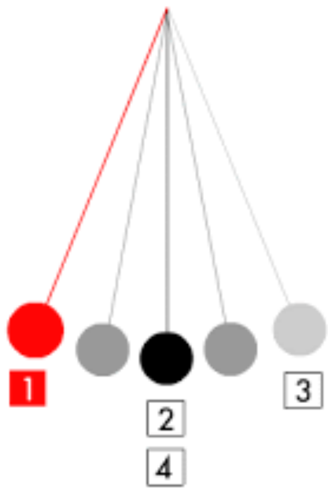
Video

Exp Brain Res. 2012 Sep;221(3):309-28.
A new paradigm for human stick balancing: a suspended not an inverted pendulum.
Lee KY, O'Dwyer N, Halaki M, Smith R.

Simple Pendulum

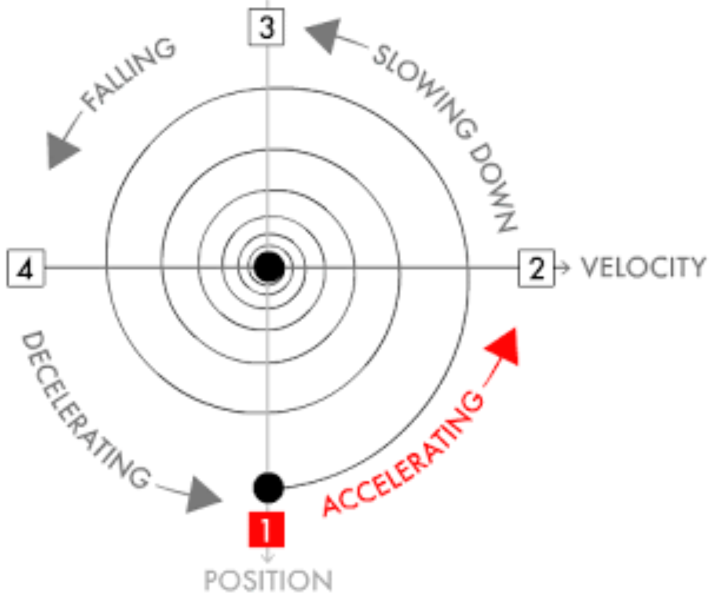
- $\dot{x} = f(x,u)$
  - u is the input command
  - x is the state
  - $\dot{x}$ is the first derivative over time
- Affine control system
  - $\ddot{q} = f_1(q,\dot{q}) + f_2(q,\dot{q})u$
  - q is the position
  - $\dot{q}$ is the first derivative (i.e. velocity)
  - $\ddot{q}$ is the second derivative (i.e. acceleration)

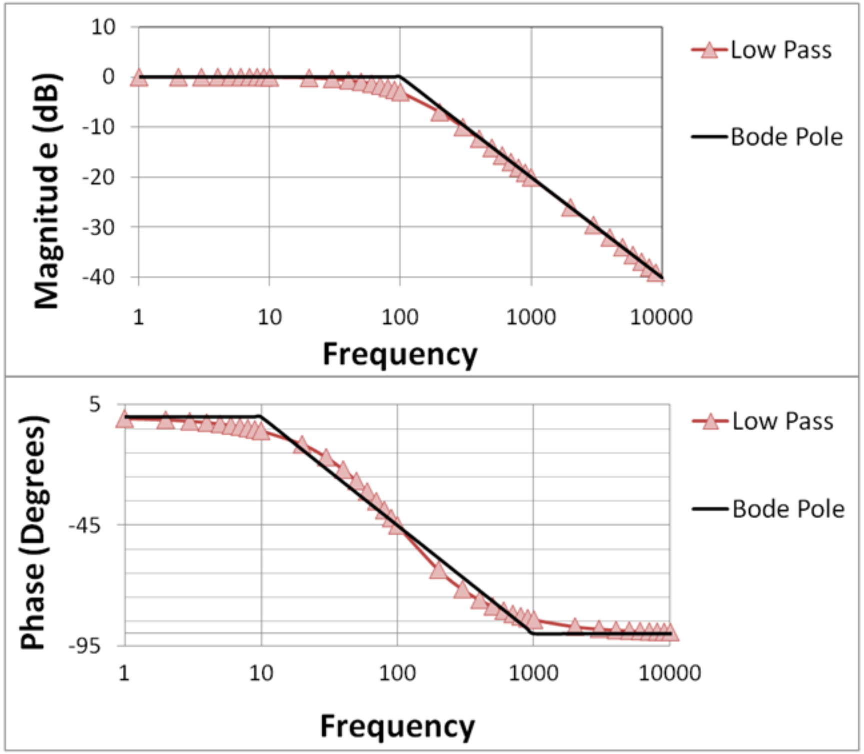




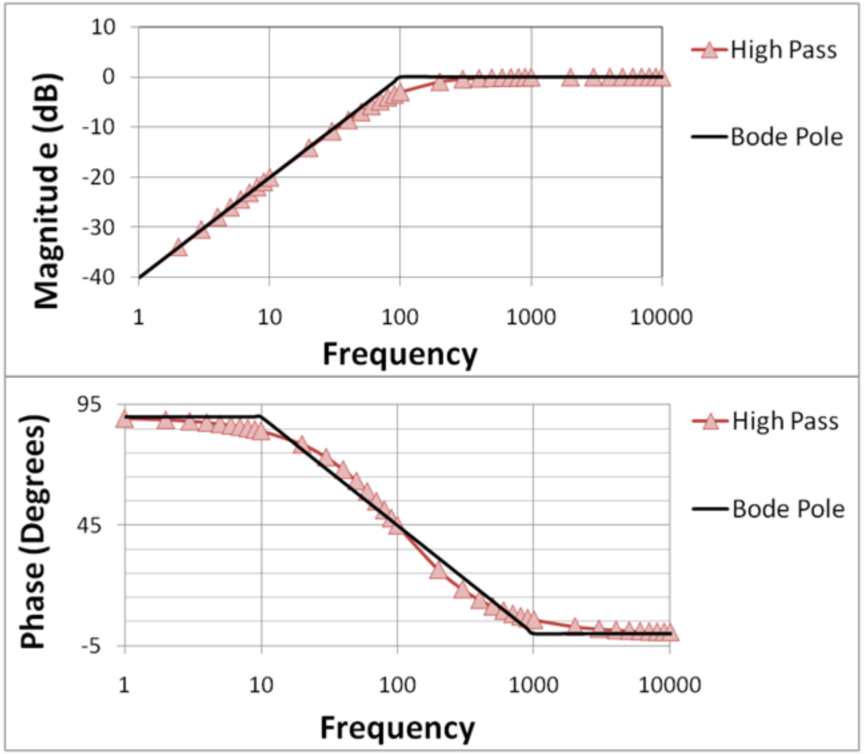
Pendulum

Phase portrait





10
0
-10
-20
-30
-40
Magnitud e (dB)
1
10
100
1000
10000
Frequency
Low Pass
Bode Pole
5
-45
-95
Phase (Degrees)
1
10
100
1000
10000
Frequency
Low Pass
Bode Pole

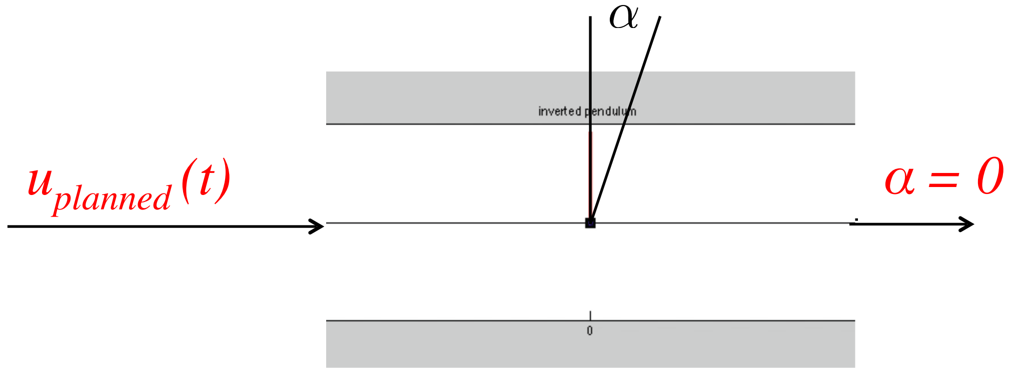

10
0
-10
-20
-30
-40
Magnitud e (dB)
1
10
100
1000
10000
Frequency
High Pass
Bode Pole
95
45
-5
Phase (Degrees)
1
10
100
1000
10000
Frequency
High Pass
Bode Pole
CNCR

Open-loop Control

Control principle

1. Planning – “Which actions must be performed to achieve the goal?”
2. Command planned action to system

+ Very easy to implement
+ Few sensors needed
- No disturbances tolerated
- situation-specific controller

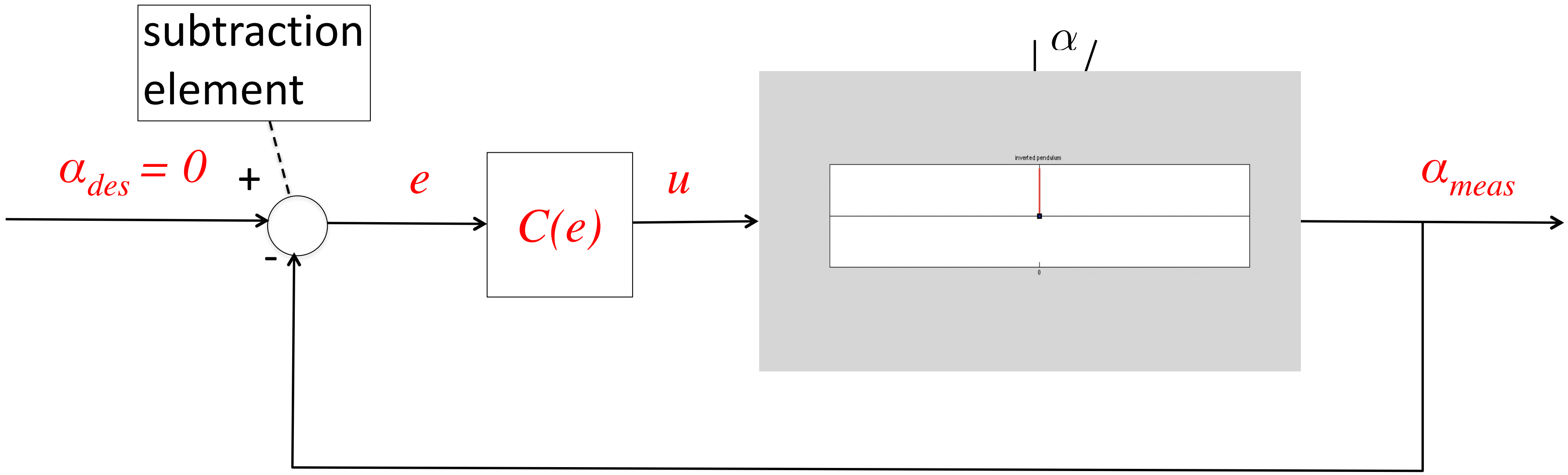


“Trust, but verify”

Closed-loop Control

Control principle:

1. Define your control goal: $\alpha_{des} = 0$
2. Measure the current system output: α_{meas}
3. Feedback loop: Compare goal and measured output: $e = \alpha_{des} - \alpha_{meas}$
4. Influence system based on momentary error: $u = C(e)$



Controller Types

- Proportional (P) Controller

$$u(t) = K_P e(t)$$

System input $u(t)$ directly proportional to control error $e(t)$

- Proportional-Derivative (PD) Controller

$$u(t) = K_P e(t) + K_D \frac{d}{dt} e(t)$$

System input $u(t)$ depends on error $e(t)$ and error derivative $d/dt\ e(t)$

+ Increases reactivity

+ Increases stability margins

- Amplifies measurement noise

Controller Types

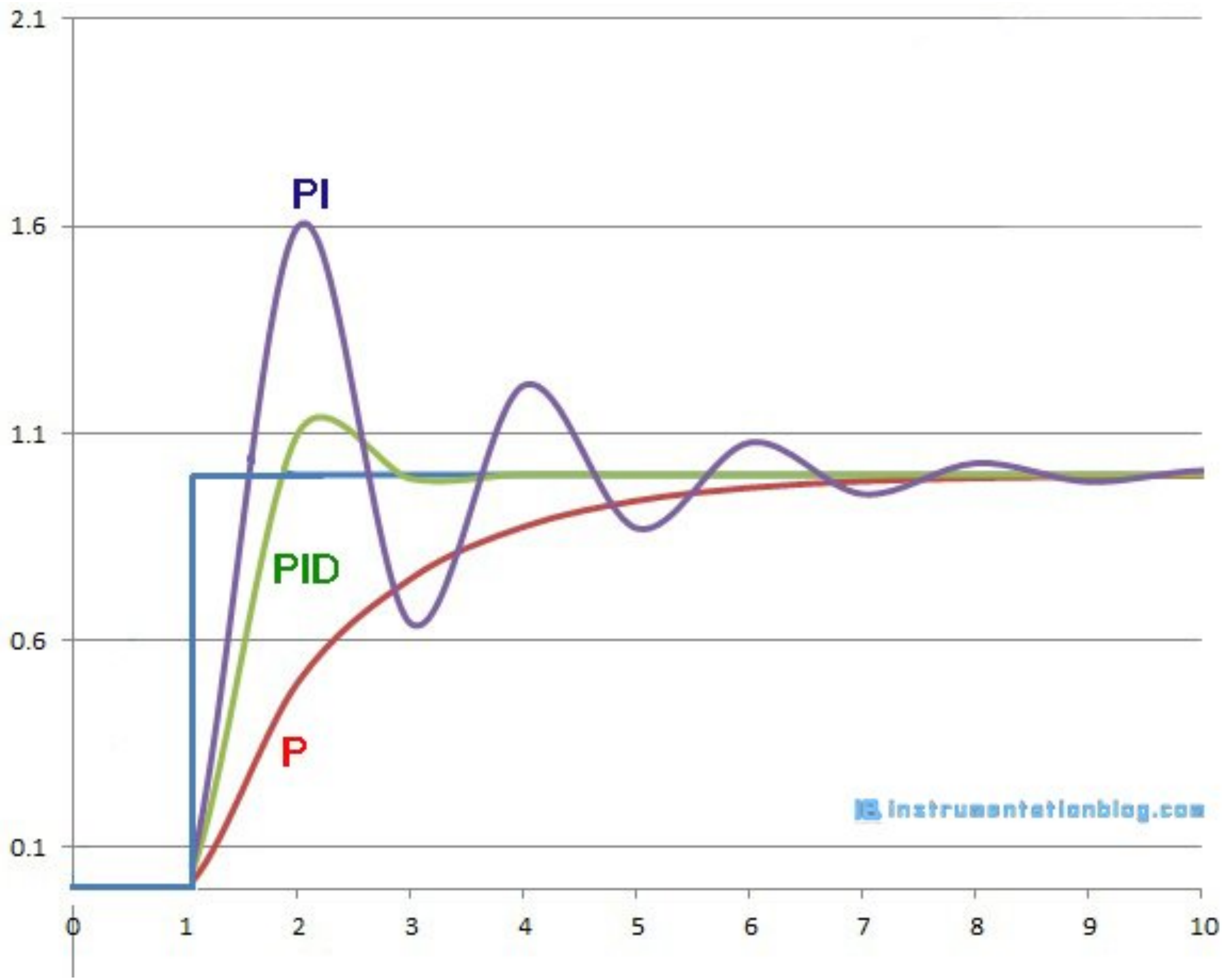
- Proportional-Integral-Derivative (PID) Controller

$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{d}{dt} e(t)$$

System input *u(t)* depends on error *e(t)*, error derivative *d/dt e(t)* and accumulated error *Int(e) dt*

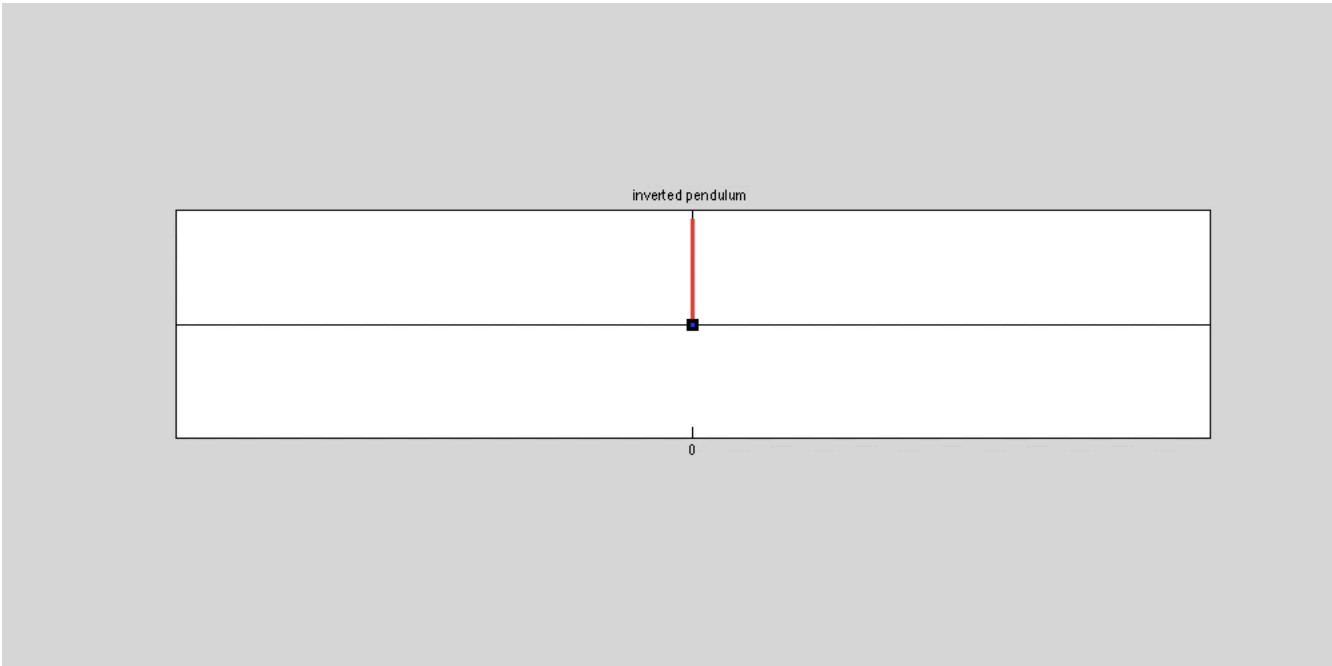
+ **De facto industrial standard**
+ Minimizes control error
- Integral component decreases stability margins

Controller Types



Control Behaviour

- Control gain variations

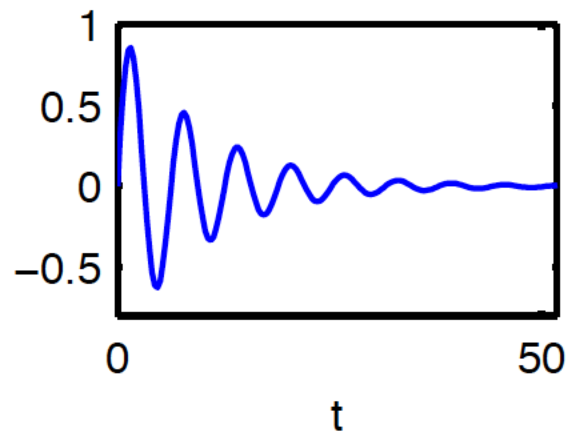


$K_p = 5$
$K_p = 50$
$K_p = 100$
$K_p = 250$
$K_p = 275$

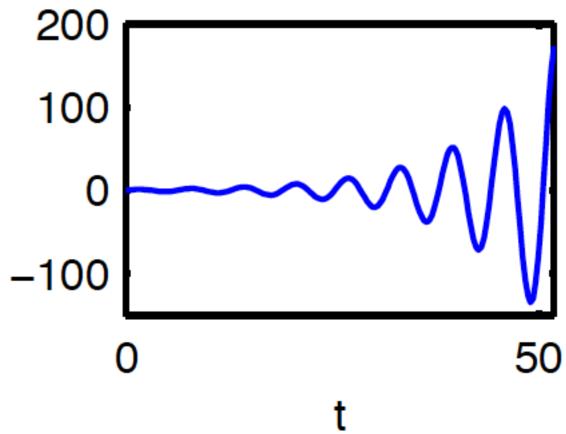
- Gain ↑ → responsiveness ↑ (desired)
- Gain ↑ → system inputs ↑ (Limits in real systems!)

Stability

- Stable system:
  - System energy decreases over time
  - Most natural phenomena/physical systems stable



- Unstable system:
  - Increasing system energy

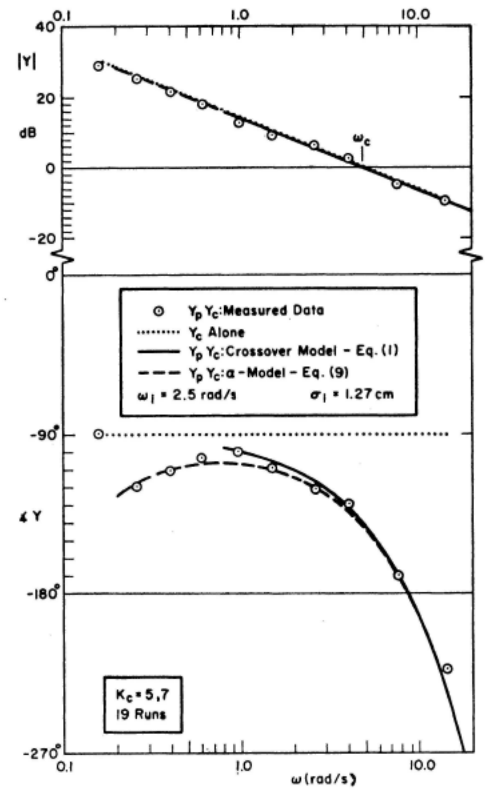


Control can stabilize unstable systems and destabilize stable systems!

Sensorimotor Control Systems

- PD controller approximates human behaviour in visual tracking task

  D. McRuer and H. Jex "A review over quasi-linear pilot models"
  In IEEE Transactions on Human Factors in Electronics, vol. HFE-8,
  pp. 231–249, Sep. 1967.



- Humans can adapt manual control gain to task requirements

  Arne J. Nagengast, Daniel A. Braun and Daniel M. Wolpert "Risk-Sensitive Optimal Feedback Control Accounts for Sensorimotor Behavior under Uncertainty"
  In PLOS Computational Biology, July 2010

- Body sway behaviour similar to PD control of inverted pendulum

  K. Masani, A. H. Vette, and M. R. Popovic, "Controlling balance during quiet standing: proportional and derivative controller generates preceding motor command to body sway position observed in experiments.," *Gait Posture*, vol. 23, no. 2, pp. 164–72, Mar. 2006

Summary

- Introduction to Control Theory
- Modelling of a dynamic system
- Inverted pendulum
- PID controller