Bayesian

Max Di Luca

Mind, **B**rain, and **M**odels 2022/23

15/2/23

Tentative schedule

Week		Lec	Lecturer	Lecture topic	Workshop topic
	1			Preparation	Matlab
08-Feb	2	1	Max	Models	2IFC experiment
15-Feb	3	2	Max	Bayesian	Distributions
22-Feb	4	3	Max+Min Li	Multisensory	Causal inference
01-	5	4	Max	Control Theory	Inverted
08-	6	5	Dietmar	Agent based	Social
15-	7	6	Alan Wing	Synchronizatio	Clapping
22-	8	7	Joe Galea	Cerebellum	Learning
19-Apr	9	8	Max	Touch	Fibers
26-Apr	10	9	Howard	Neural fields	Dynamics
03-	11	10	Max	Visual	Convolution
9-May	12	11	Max	Project	Your choice

Module aims

- Translate computational models into computer programs
- Test hypothesis using simulations
- Produce reports about quantitative model testing

Today's topics

- Probabilistic inference
- Bayesian Brain Hypothesis
- Optimality

Ma, 2012

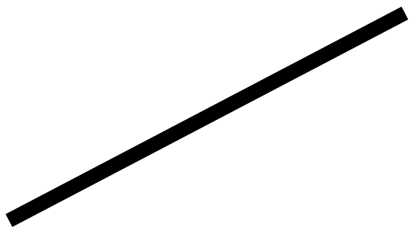
Ma and Jazayeri, 2014

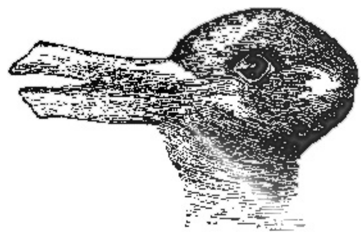
Maloney and Mamassian, 2009

Sensed information

Imprecise: Noise limits the precision

Ambiguous: Not uniquely specified state





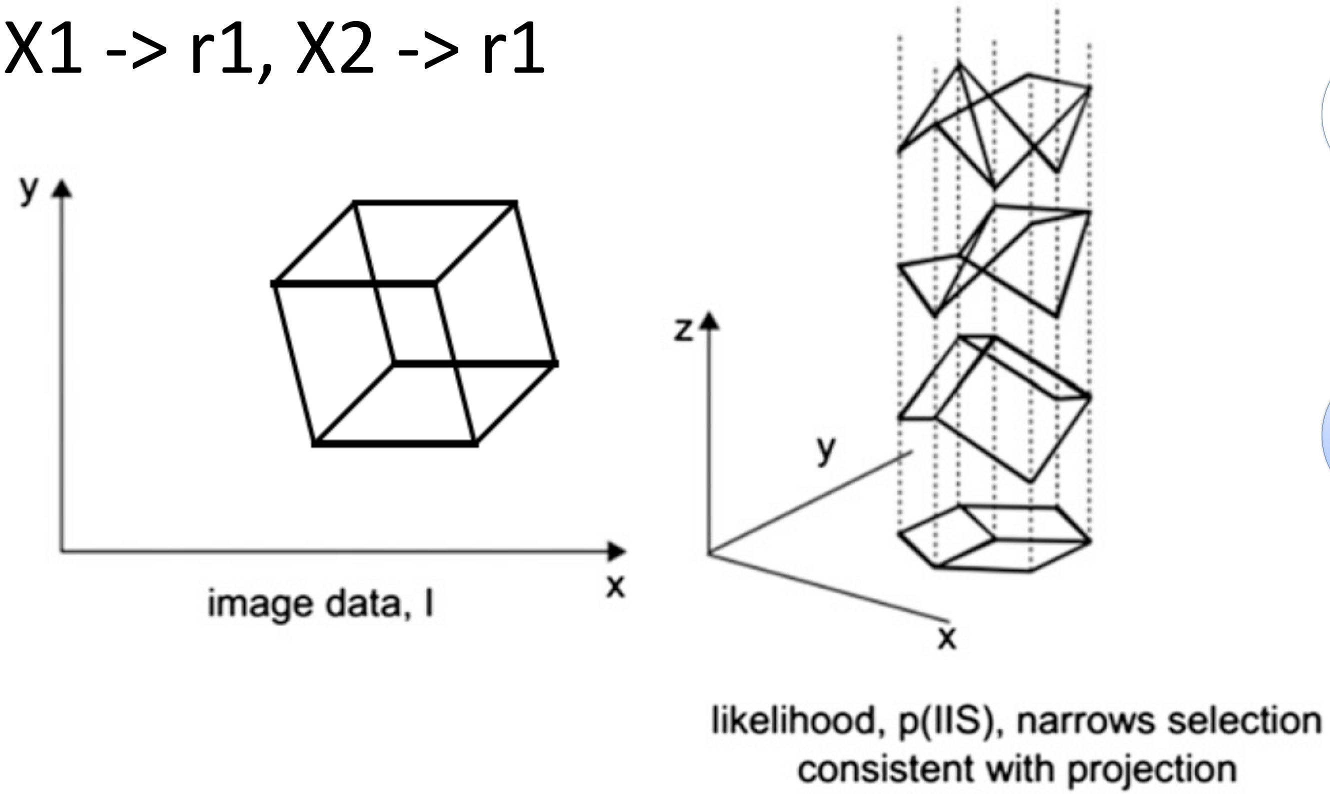
Jastrow (1899, p. 312)

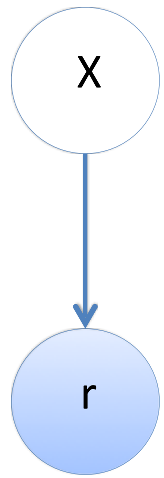
Statistical models

- Noise
  - Even if world is deterministic, from the viewpoint of observer it seems stochastic
- Inference
  - Stochastic environment -> probabilistic framework
  - $P(r1|X1)$

Statistical models

- ill-posed problem: Ambiguity
  - No perfect one-one mapping X -> r,
  - I.e. X1 -> r1, X2 -> r1



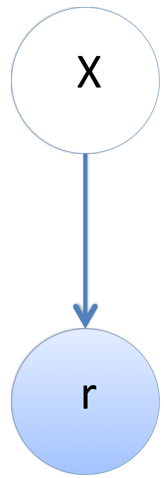


Optimal organism

- How to design an optimal organism
    - Optimal typically means $\min(X - \hat{X})^2$
- Assumptions:
    – Stochastic environments
        - Sensory inputs
        - Motor outputs
    – Goal of organism to manipulate environment to improve utility function

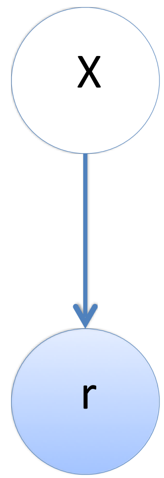
Generative model

- Ex. “Given an urn with 5 red and 3 white balls, what is the probability of randomly drawing a red ball”
- $P(r|X)$
- Has causal structure, X causes r



Bayesian Inference

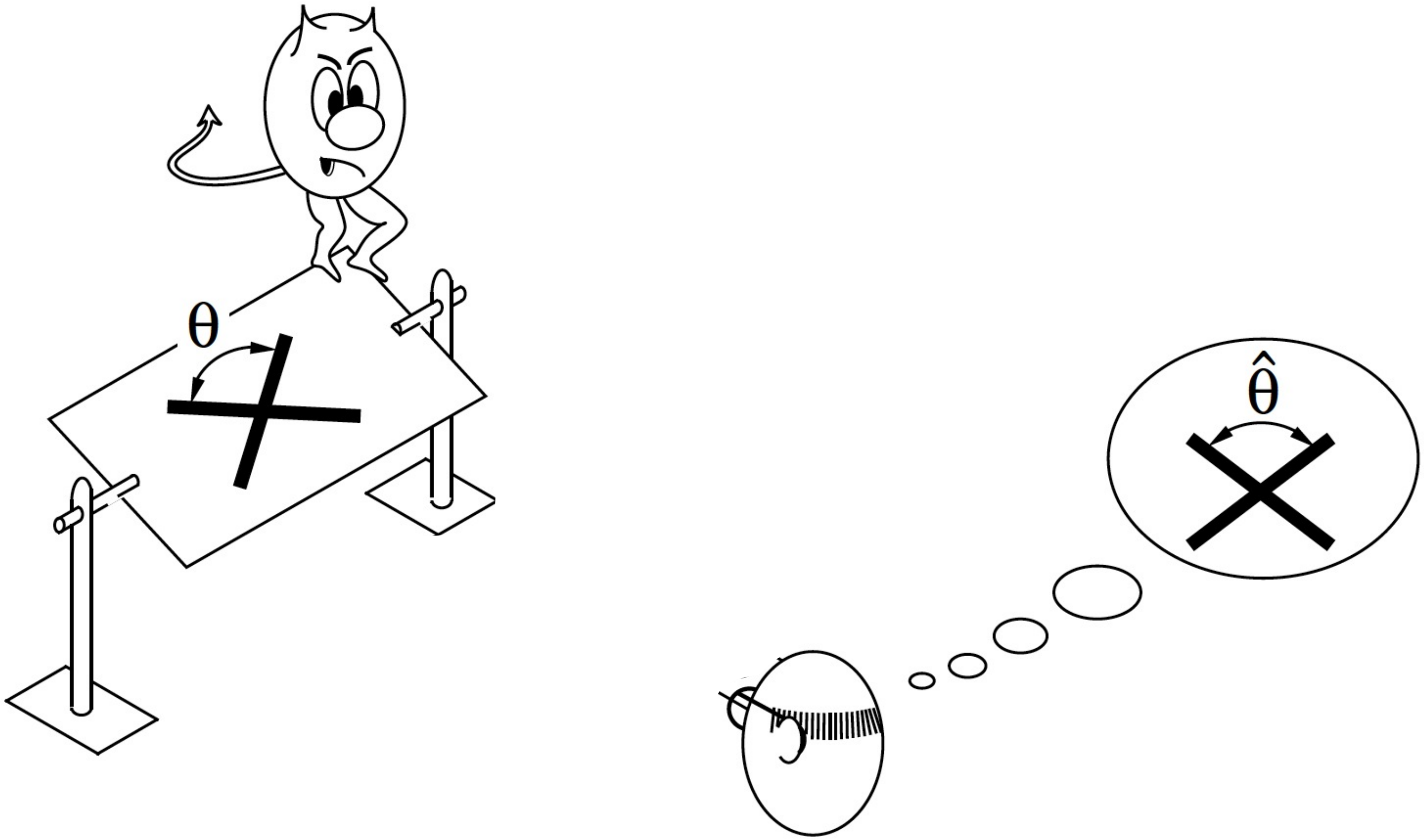
- Inverse problem "Given that you have drawn a red ball 5 times and a white ball 3 times, what can we say about the distribution of balls"
- Infer properties of X, given stimulus r



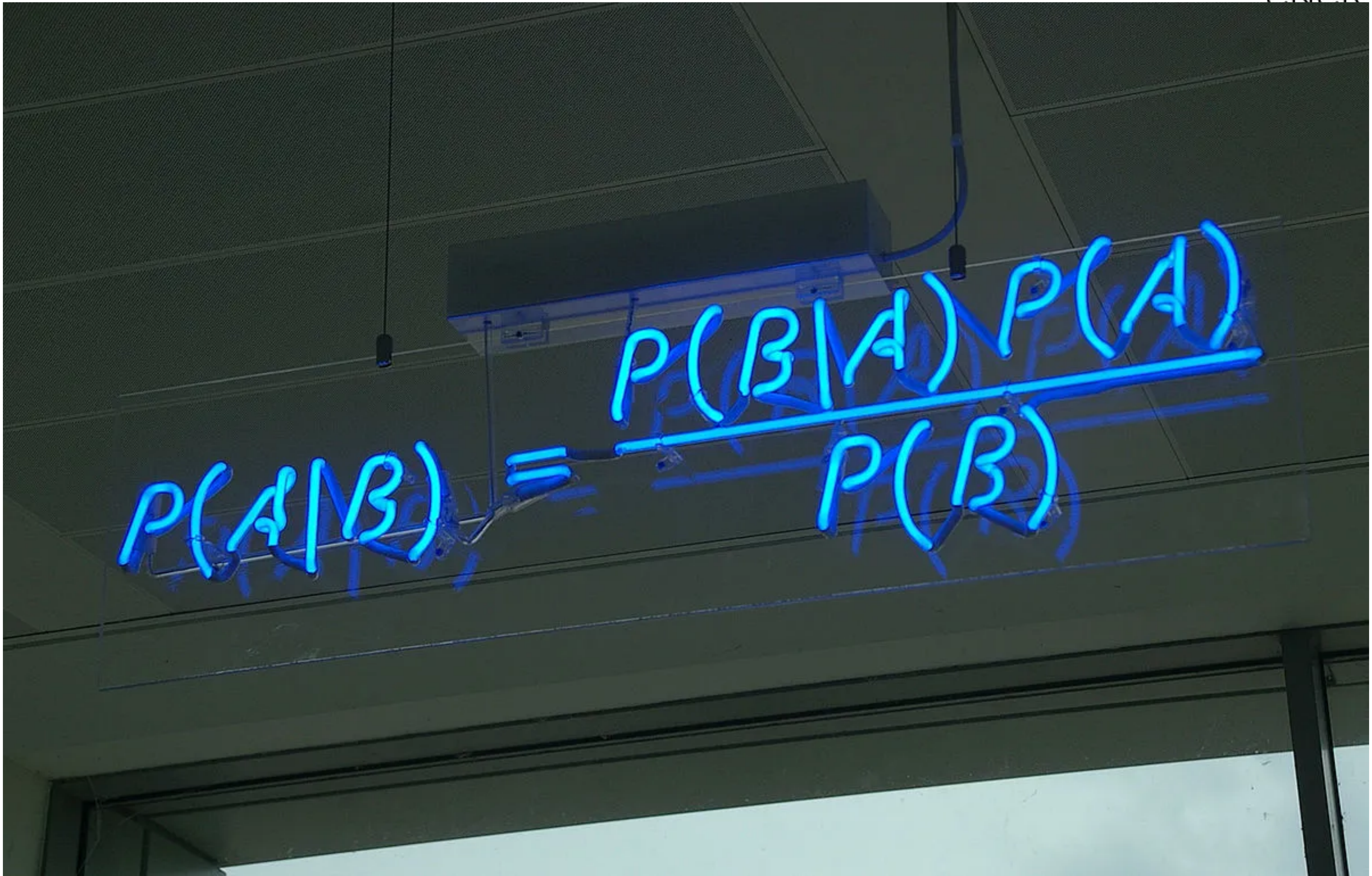
Helmholtz: ‘unconscious inference’

- Find an explanation for the sensory evidence
- Infer (estimate) the state of the world in the best possible way

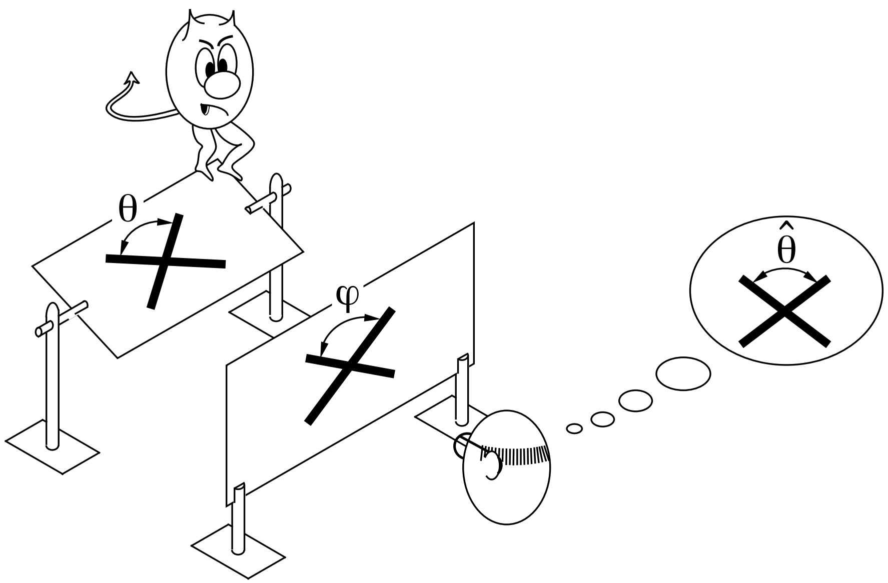
Bayesian Inference



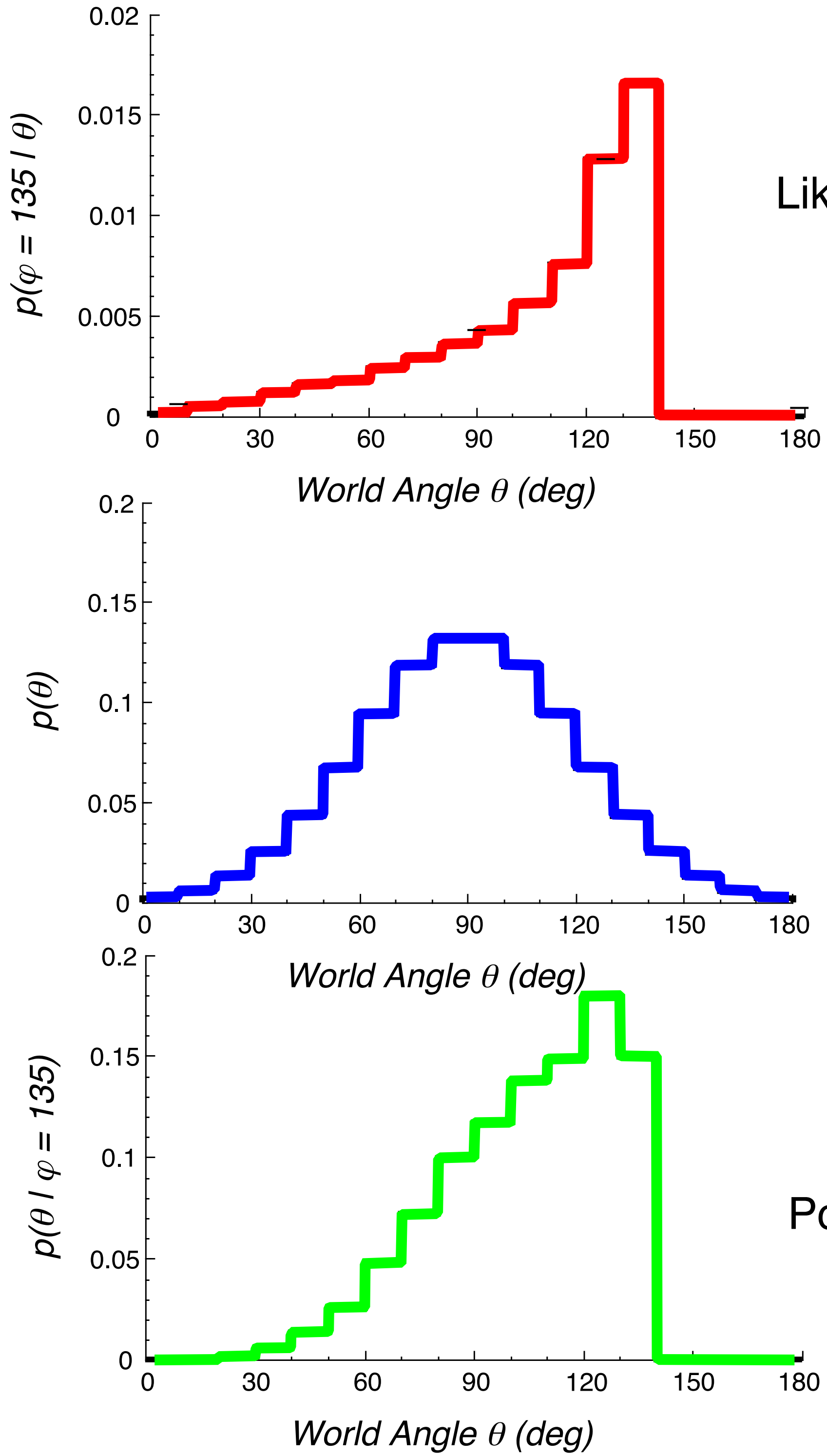
Mamassian Landy Maloney (2002) in Rao Olshausen Lewicki



Bayes' theorem. Credit to mattbuck [CC BY-SA 2.0]



$$p(\theta \mid \varphi) = p(\varphi \mid \theta) \times p(\theta) / p(\varphi)$$



Mamassian et al. (2002)

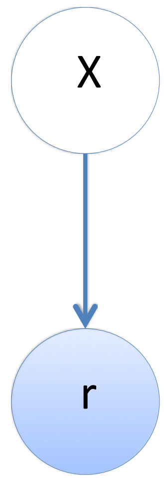
Bayesian Inference

Likelihood

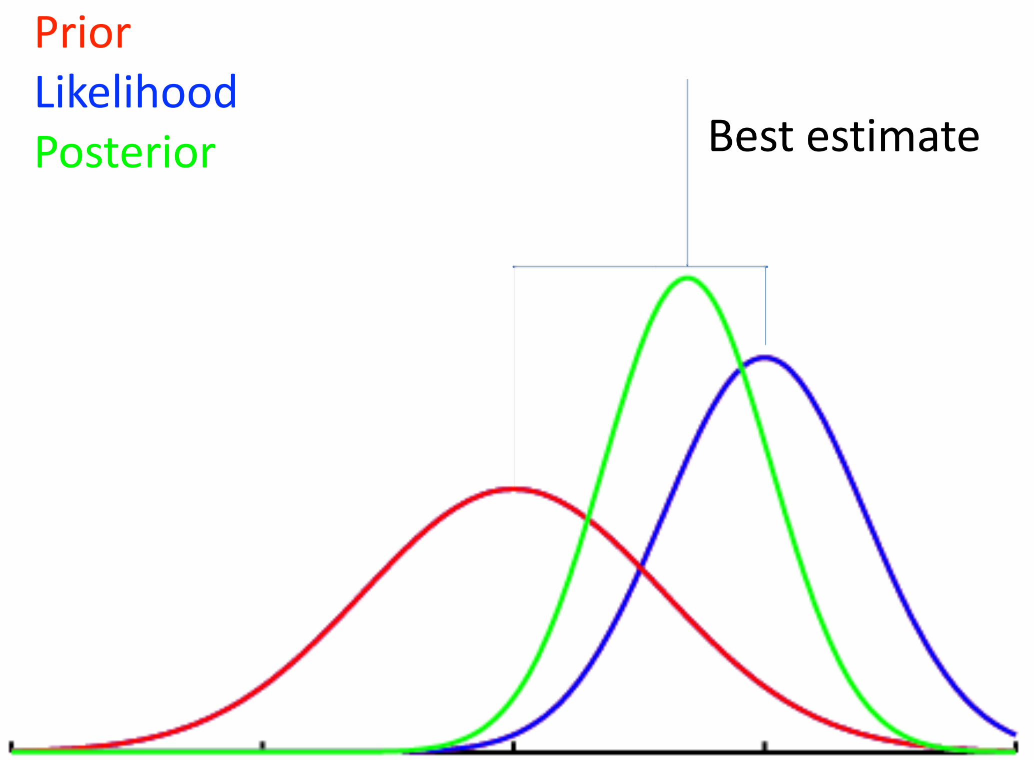
Prior

Posterior

$$P(X \mid r) = \frac{P(r \mid X)\,P(X)}{P(r)}$$



Optimal estimate



$$P(X\mid r)=\frac{P(r\mid X)P(X)}{P(r)}$$

$$\hat{X}=w_1\hat{X}_{Like}+w_2\hat{X}_{Prior}$$

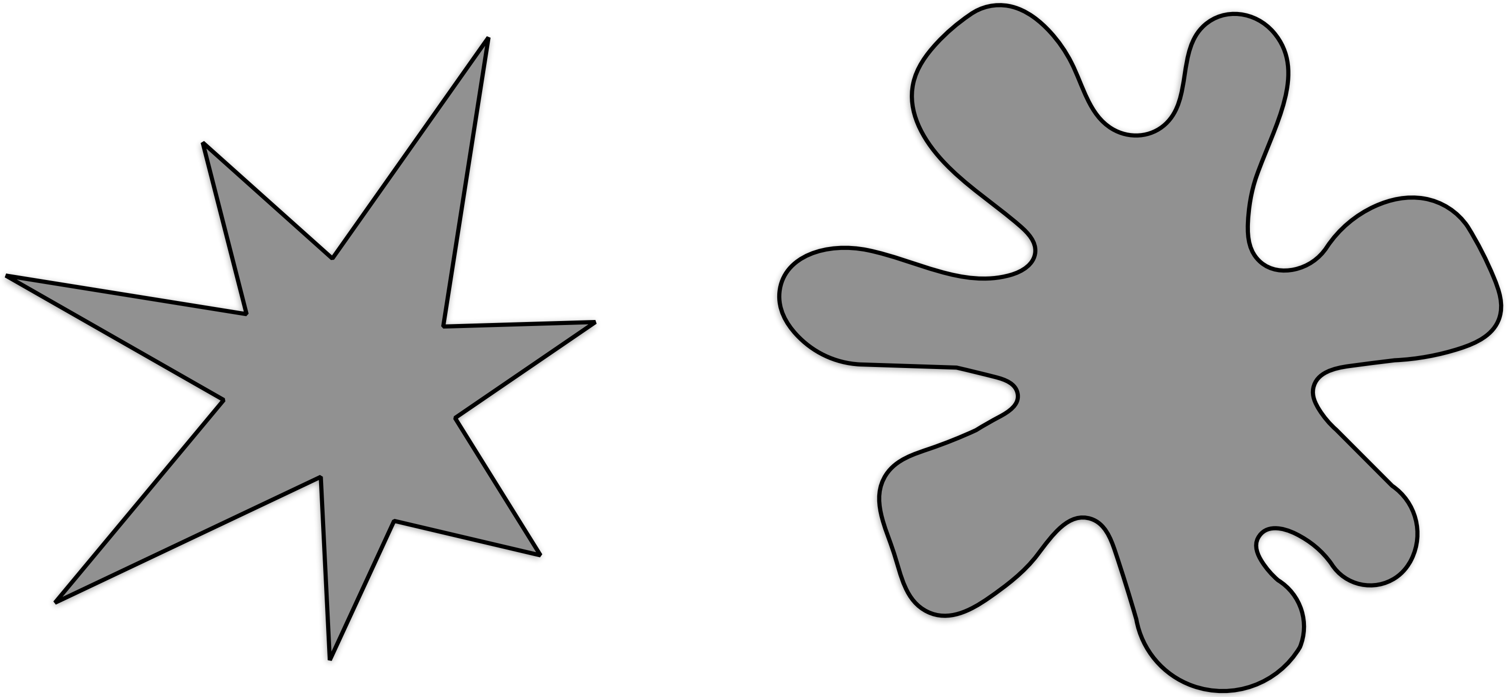
$$w_1=\sigma_{Prior}^2/(\sigma_{Like}^2+\sigma_{Prior}^2)$$

$$w_2=\sigma_{Like}^2/(\sigma_{Like}^2+\sigma_{Prior}^2)$$

Illumination

Ernst & Di Luca (2010)

Intrinsic mapping



Takete / Maluma
Kiki / Bouba

Köhler (1929)

(Preview to lecture 3)

Hypo-priors

Autistic atypicalities

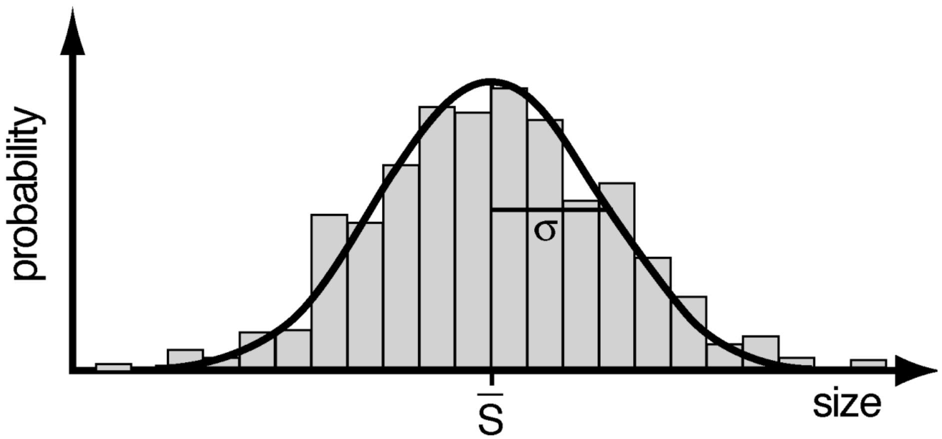
Could occur in the creation of the prior (broader) or ineffective combination of the prior with the likelihood

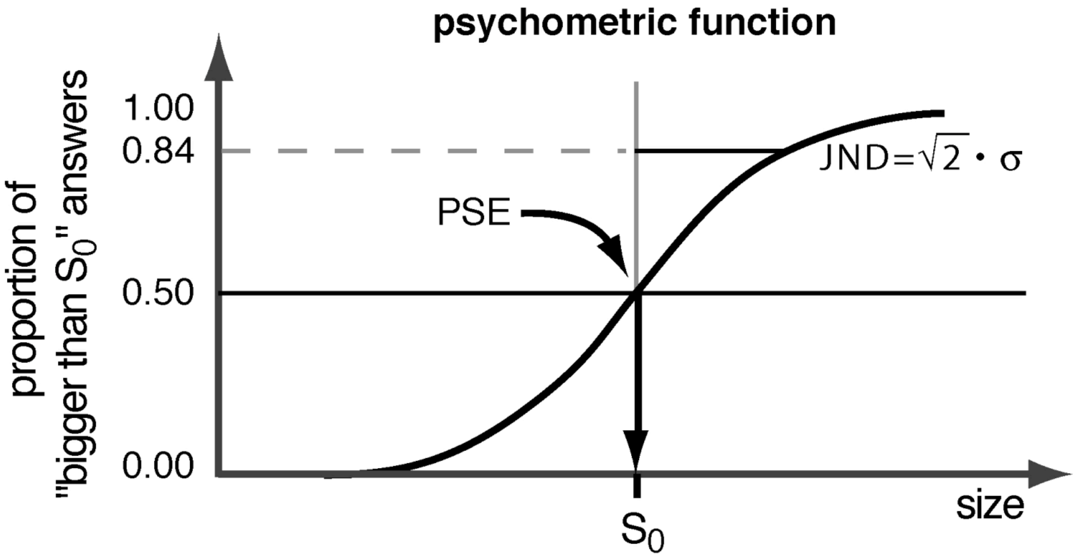
- Unprepared for changes from the norm
- Reduced generalisation in learning
- False or limited expectations
- Sensory seeking behaviour and hypersensitivity
- Poor generation of predictions (desire for sameness)
- Repetitive behaviours as a method of reducing uncertainty by exercising control over the environment

Stream-bounce

Sekuler et al. (1997)

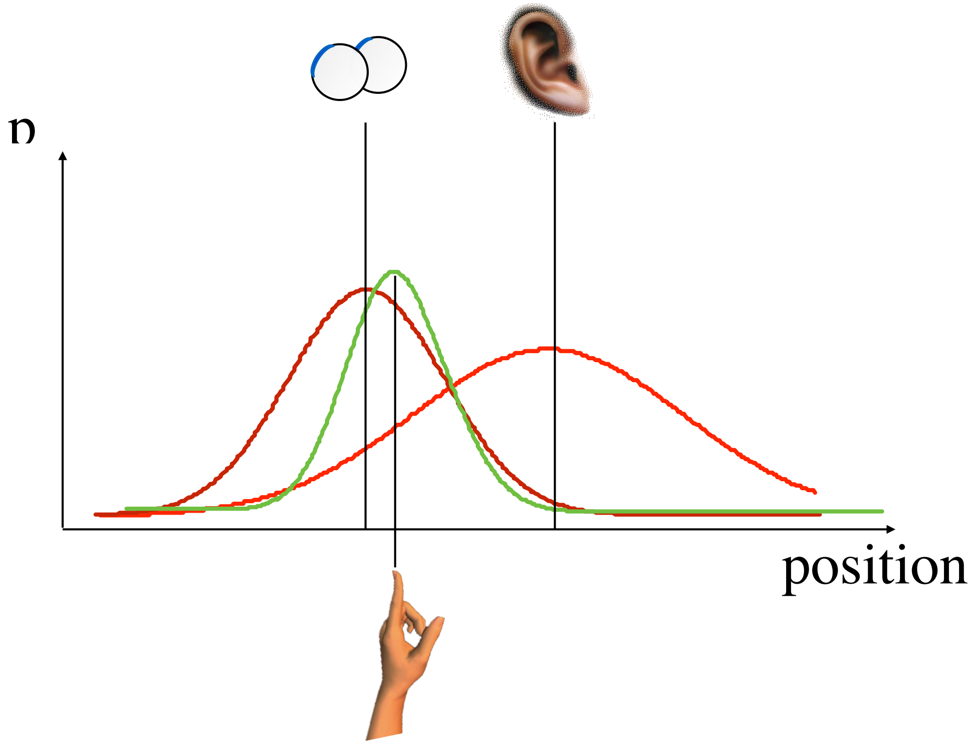
Psychophysics



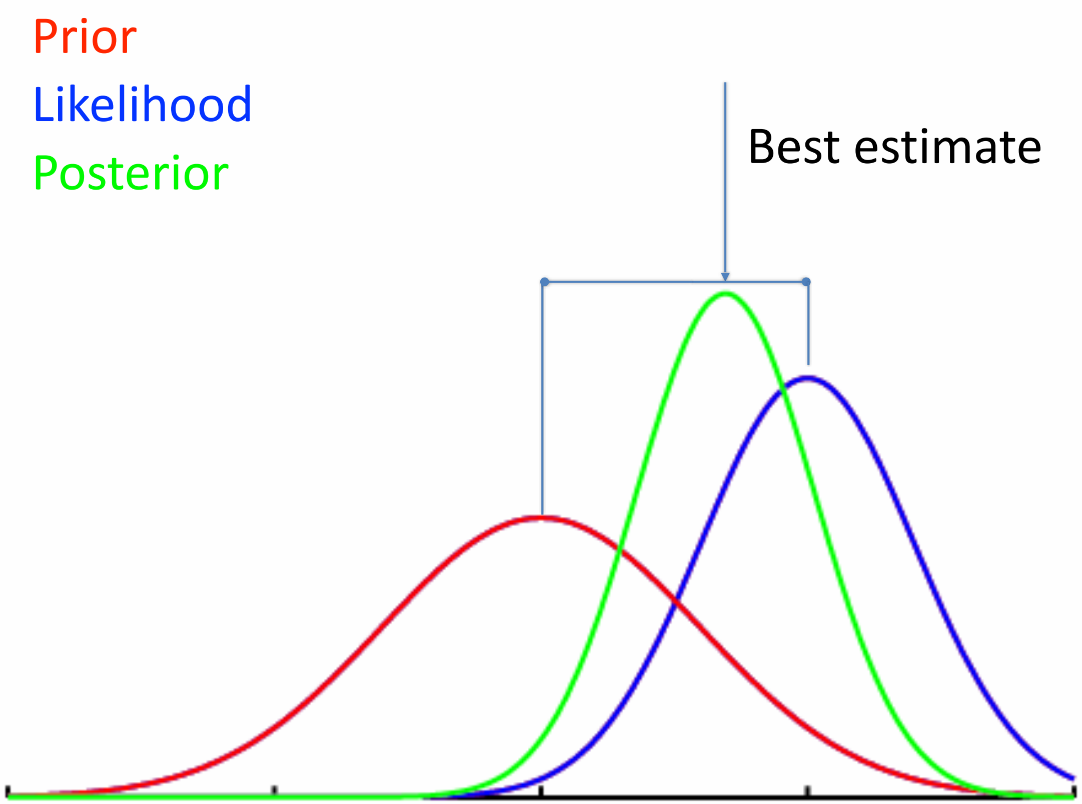


(Again)


Edgar Bergen with Charlie McCarthy


p
position

Optimal estimate



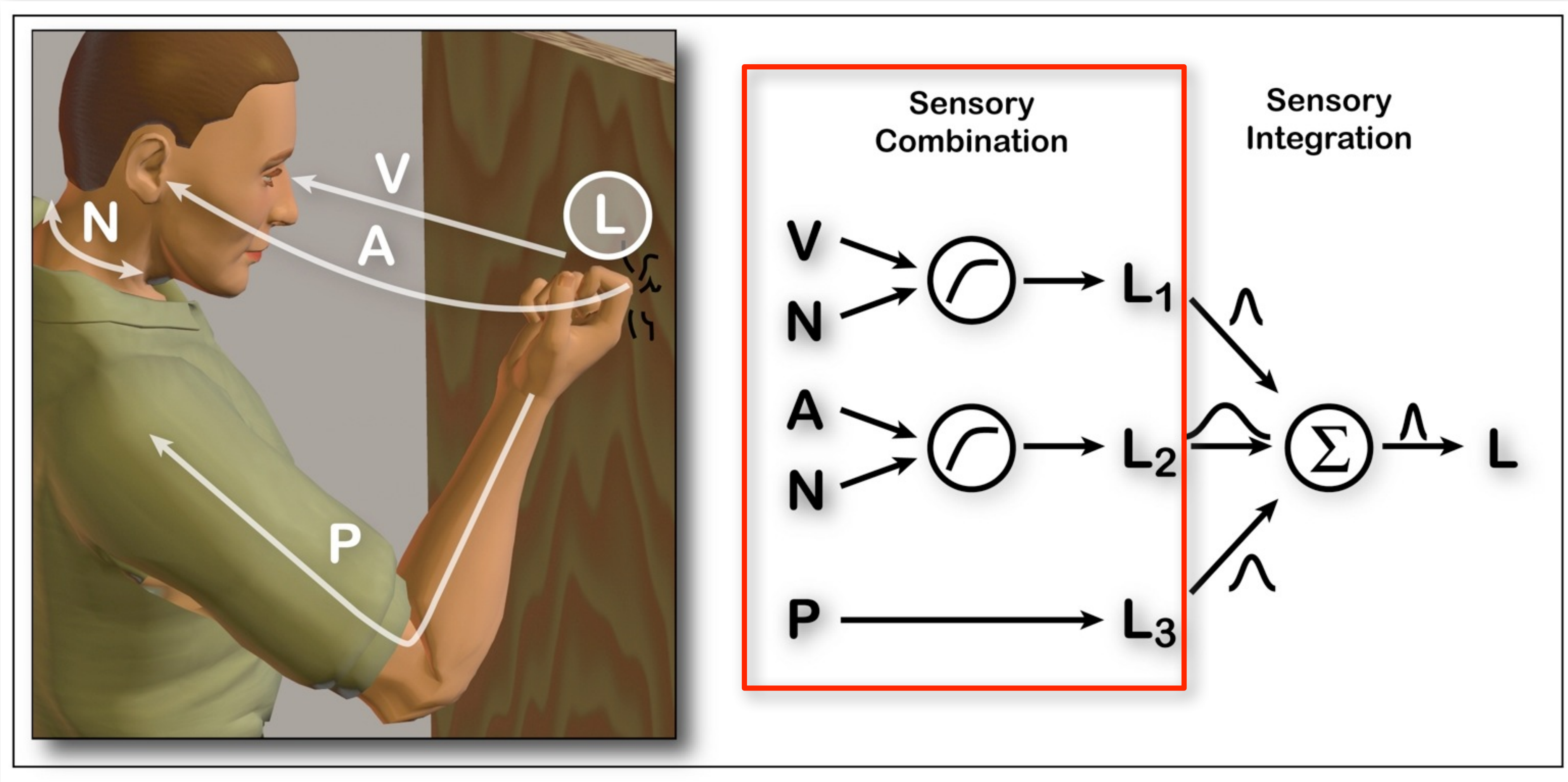
$$\hat{X} = w_1 \hat{X}_{Like} + w_2 \hat{X}_{Prior}$$

$$w_1 = \sigma_{Prior}^2 / (\sigma_{Like}^2 + \sigma_{Prior}^2)$$

$$w_2 = \sigma_{Like}^2 / (\sigma_{Like}^2 + \sigma_{Prior}^2)$$

$$P(X \mid r) = \frac{P(r \mid X) P(X)}{P(r)}$$

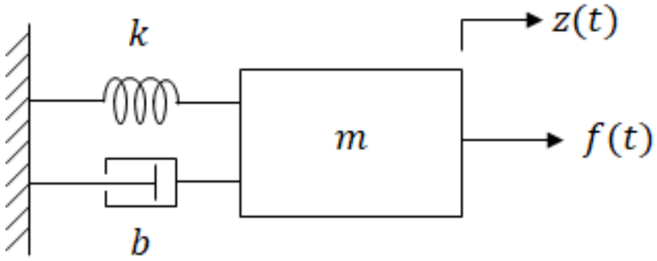
Combination and Integration



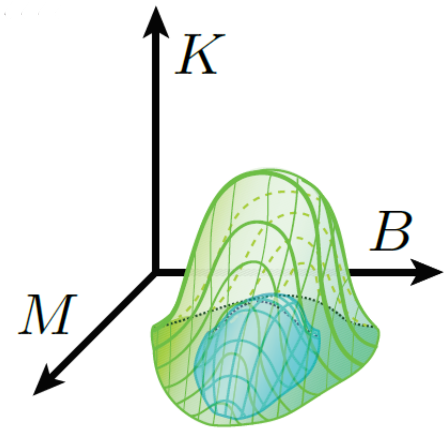
Ernst & Bülthoff (2004)

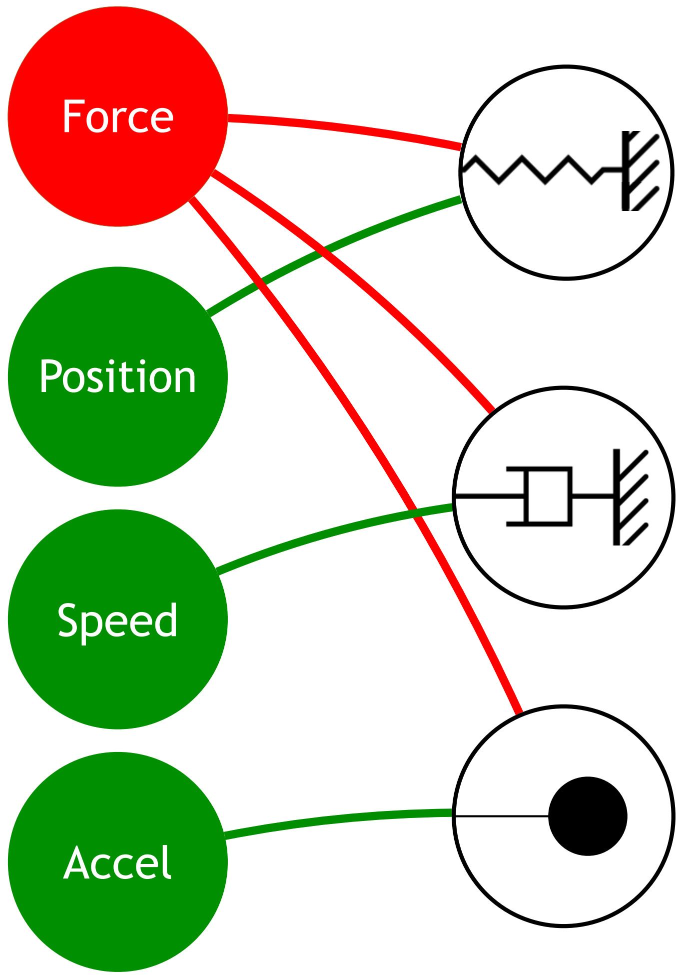
Difference between senses, signals, cues, estimates, perceptions?

Difference between senses, signals, cues, estimates, perceptions?



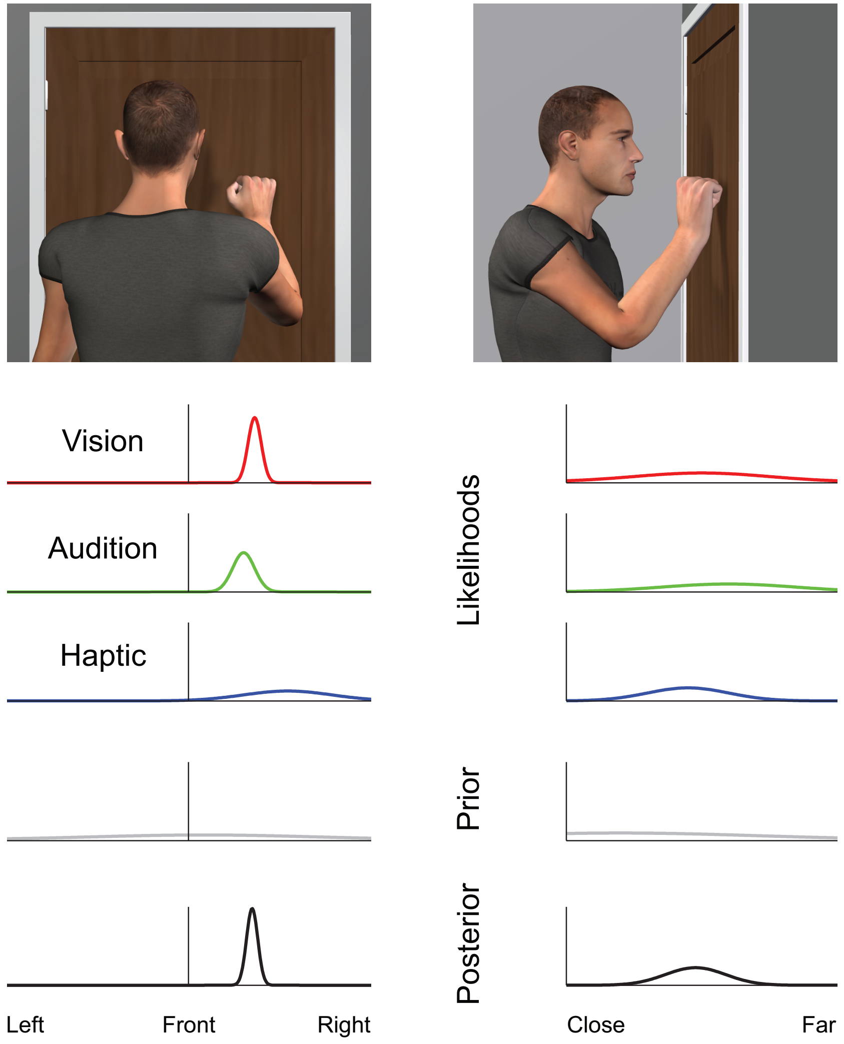
spring	damper	mass
$f(t) = k\,x(\mathrm{t})$	$f(t) = b\,v(\mathrm{t})$	$f(t) = m\,a(\mathrm{t})$





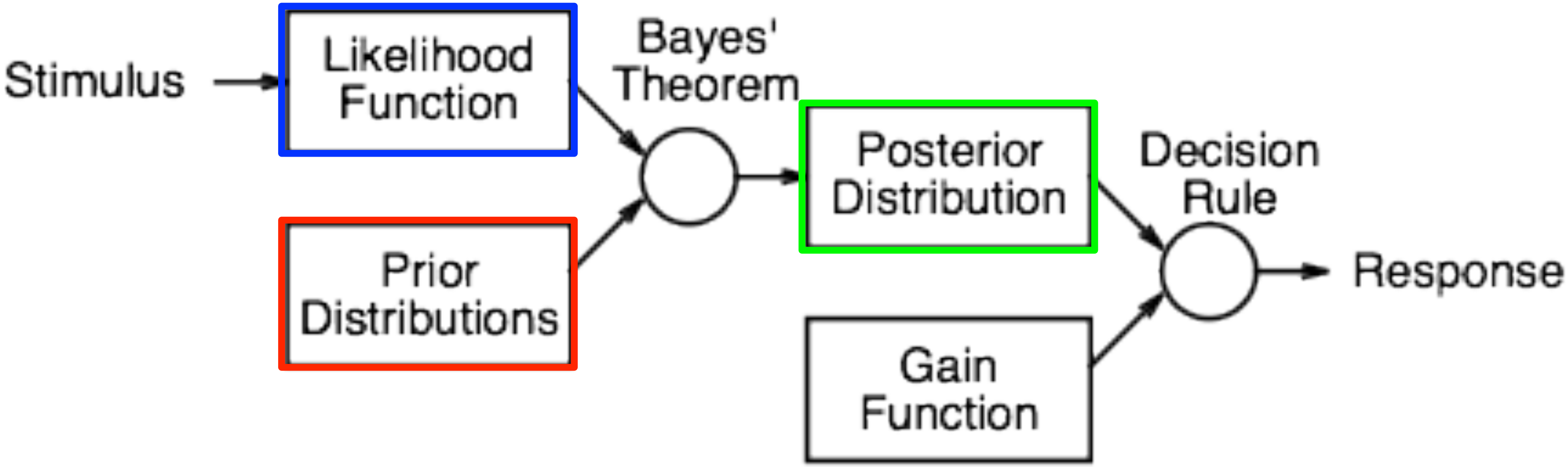
(Preview to lecture 4)

Priors



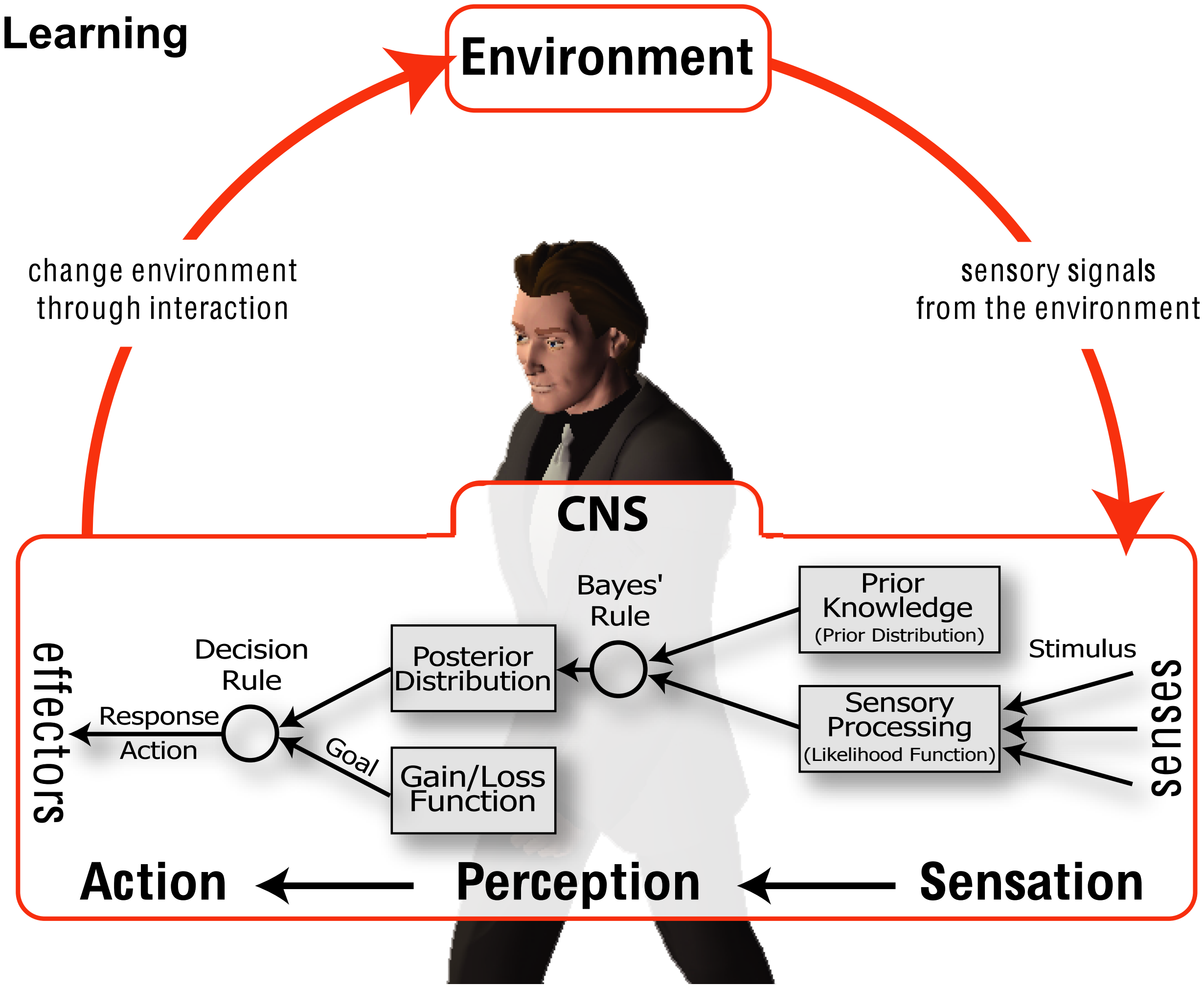
Mohler et al. (2012)

Action planning (Decision)



Perceptual Learning

1. **Multisensory Integration**
2. **Perception for Action**
3. **Perceptual Learning**



The Bayesian Brain

- The brain is endowed with an internal **representation** of the external world.
- The brain is trying to **infer the causes** of sensations. The brain is a statistical organ of hierarchical inference that predicts current and future events based on past experience and generative models of the world.
- The brain is capable of **simulating** the outside world, **assigning probabilities** to hypotheses that best explain sensory data – and continually updating these hypotheses according to **rules of inference**.
- The fine-tuning of perception and action operates under the single imperative of **minimising surprise (free energy) and uncertainty**; thereby maximising statistical and thermodynamic efficiency.
- Continual **optimisation** of the models enables efficient exchange with the environment in a self-organised, self-evidencing and unsupervised fashion.

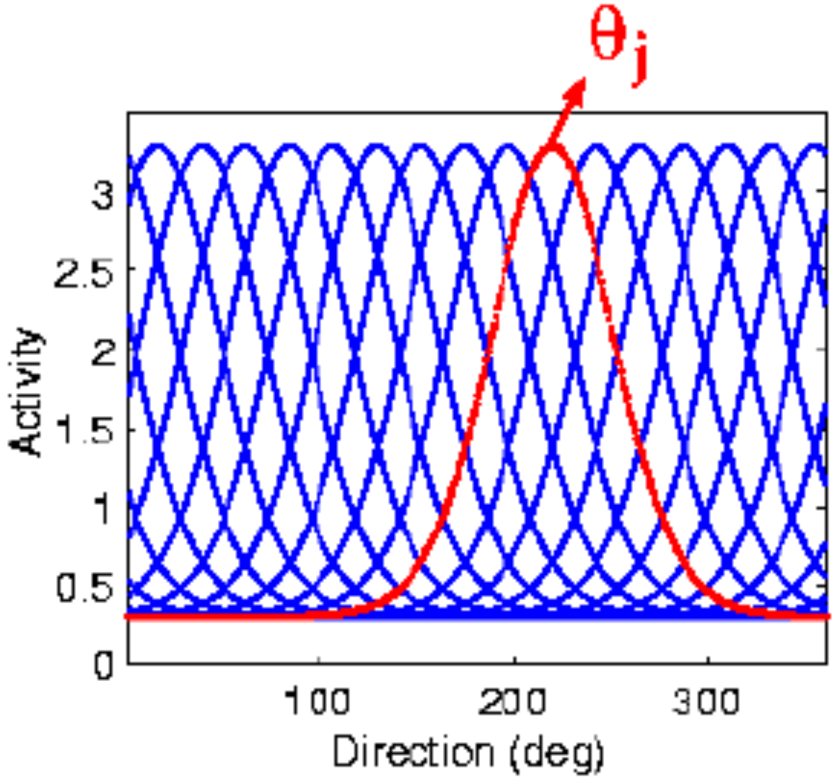
Key Ideas

- The nervous systems represents probability distributions. i.e., it represents the uncertainty inherent to all stimuli.
- The nervous system stores generative models, or forward models, of the world (e.g. $P(I|V)$).
- Biological neural networks can perform complex statistical inferences.

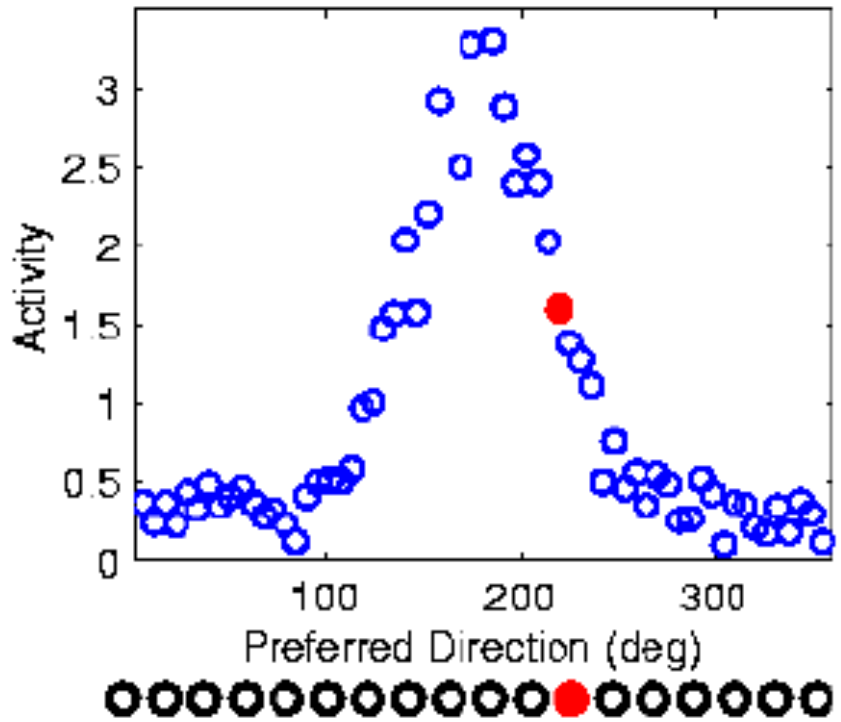
A simple problem

- Estimating direction of motion from a noisy population code

Population Code



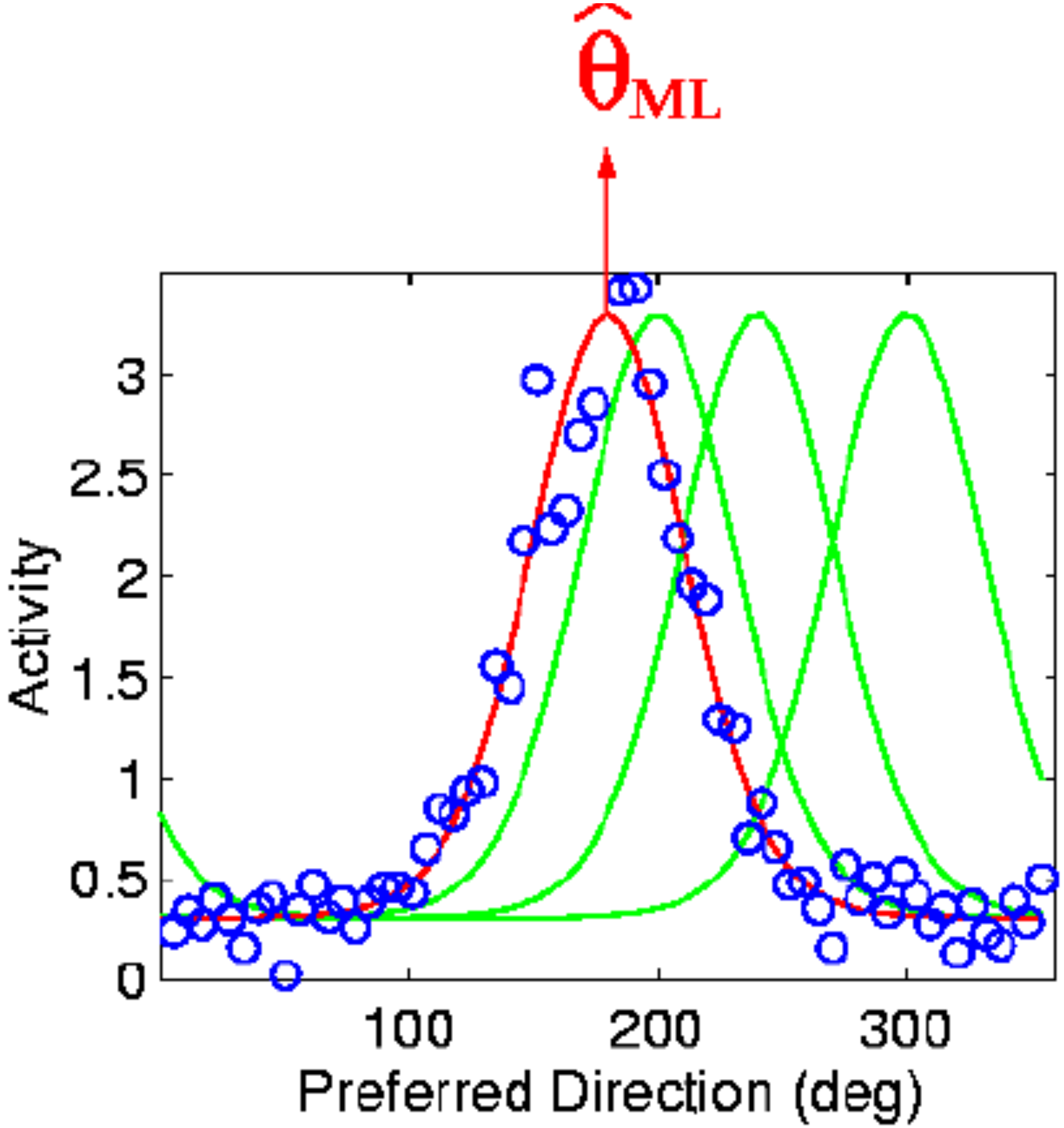
Tuning Curves



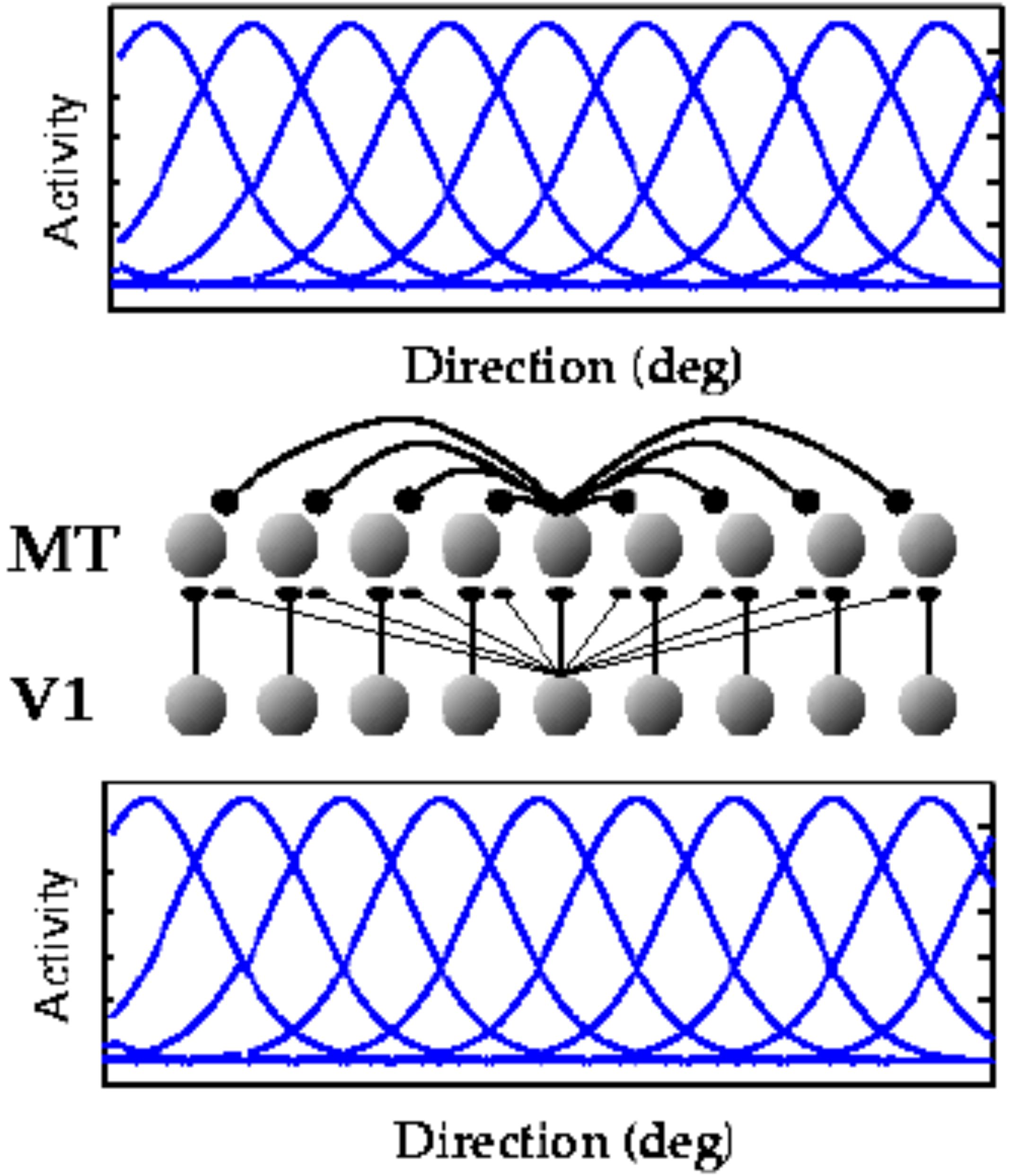
Pattern of activity (A)

(Preview to lecture 9)

Maximum Likelihood

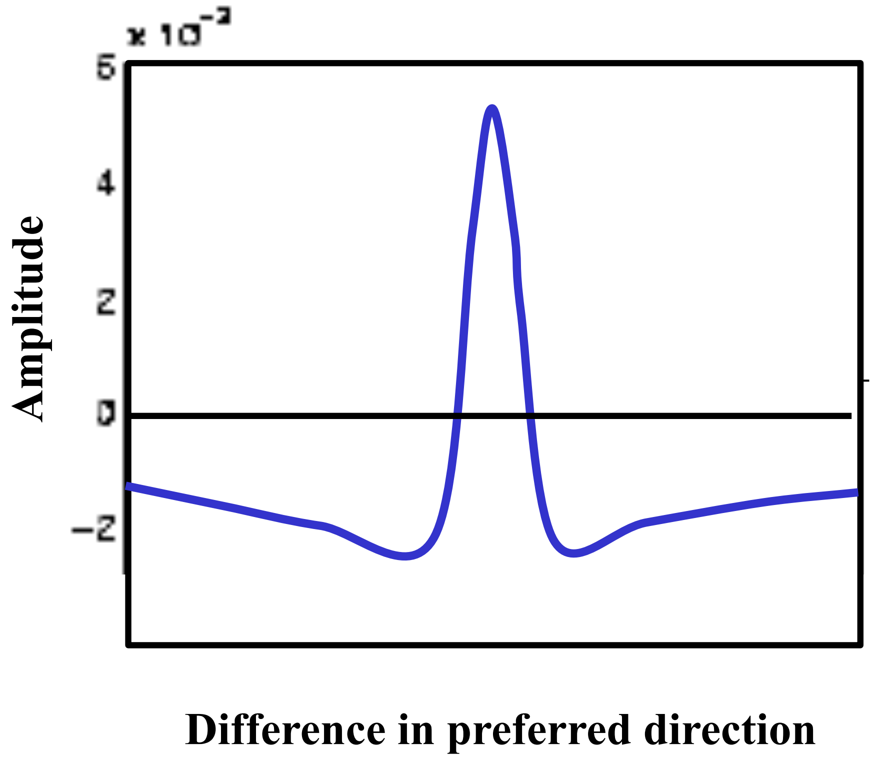


(Preview to lecture 9)



(Preview to lecture 9)

Weight Pattern



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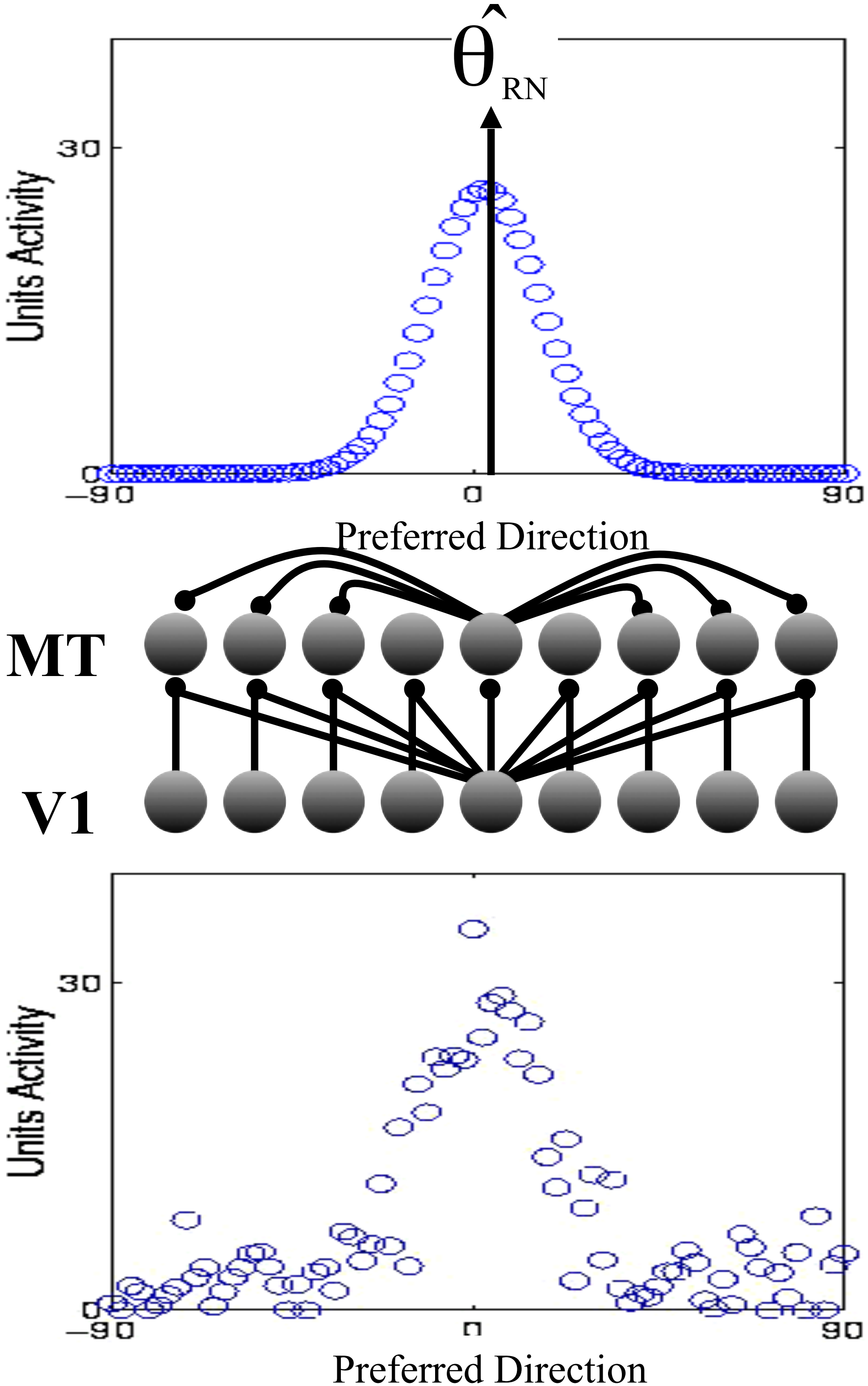
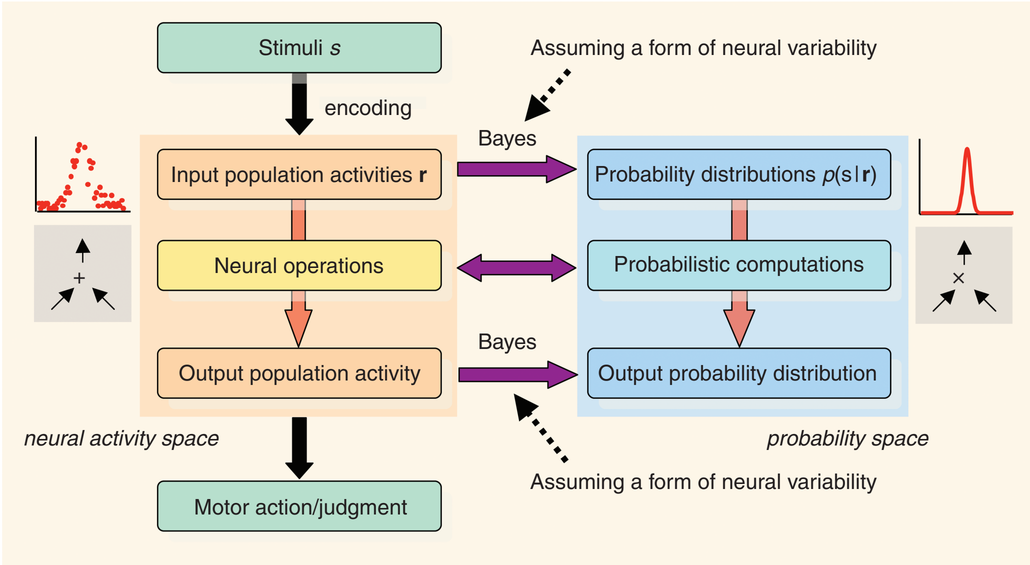
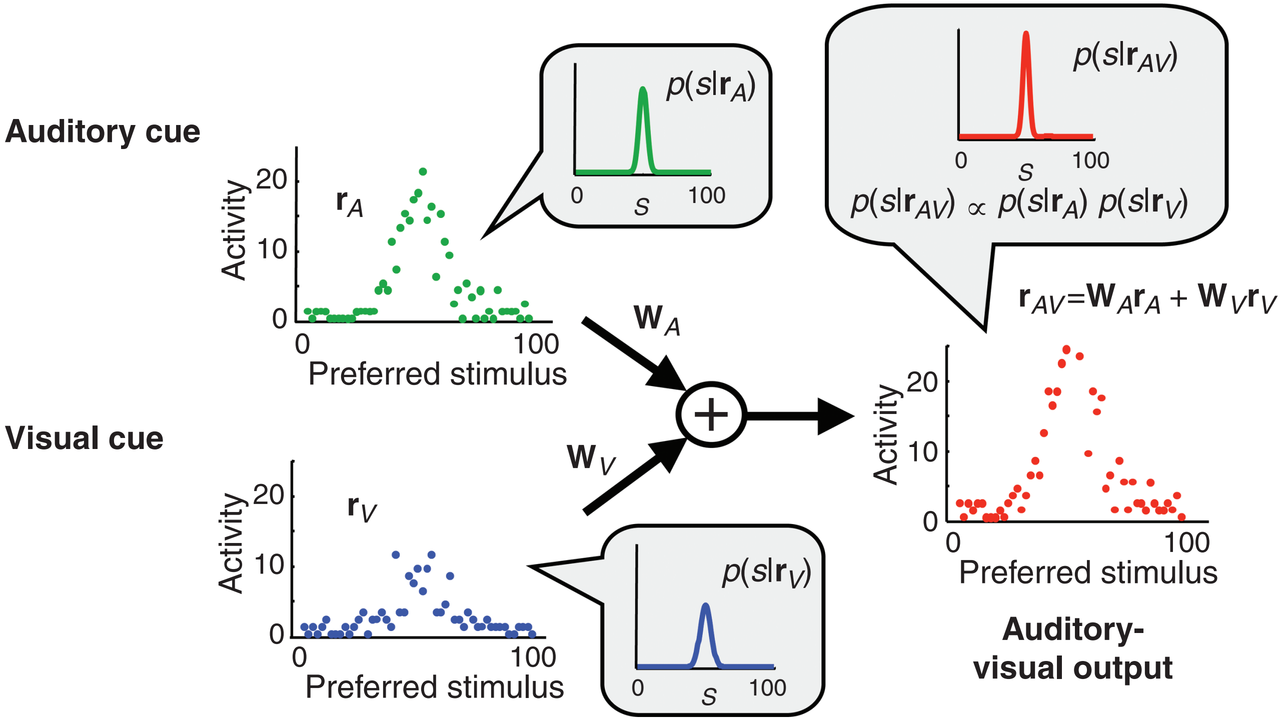


(Preview to lecture 9)


Stimuli s
encoding
Assuming a form of neural variability
Bayes
Input population activities r
Probability distributions p(s|r)
Neural operations
Probabilistic computations
Bayes
Output population activity
Output probability distribution
neural activity space
probability space
Assuming a form of neural variability
Motor action/judgment


Auditory cue
Visual cue
Activity
Preferred stimulus
$\mathbf{r}_A$
$\mathbf{r}_V$
$p(s|\mathbf{r}_A)$
$p(s|\mathbf{r}_V)$
$p(s|\mathbf{r}_{AV})$
$p(s|\mathbf{r}_{AV}) \propto p(s|\mathbf{r}_A)\, p(s|\mathbf{r}_V)$
$\mathbf{W}_A$
$\mathbf{W}_V$
$\mathbf{r}_{AV}=\mathbf{W}_A\mathbf{r}_A + \mathbf{W}_V\mathbf{r}_V$
Auditory-visual output

Free energy

sensations – predictions

Prediction error

Change sensations

Action

Change predictions

Perception

Action and perception minimise surprise

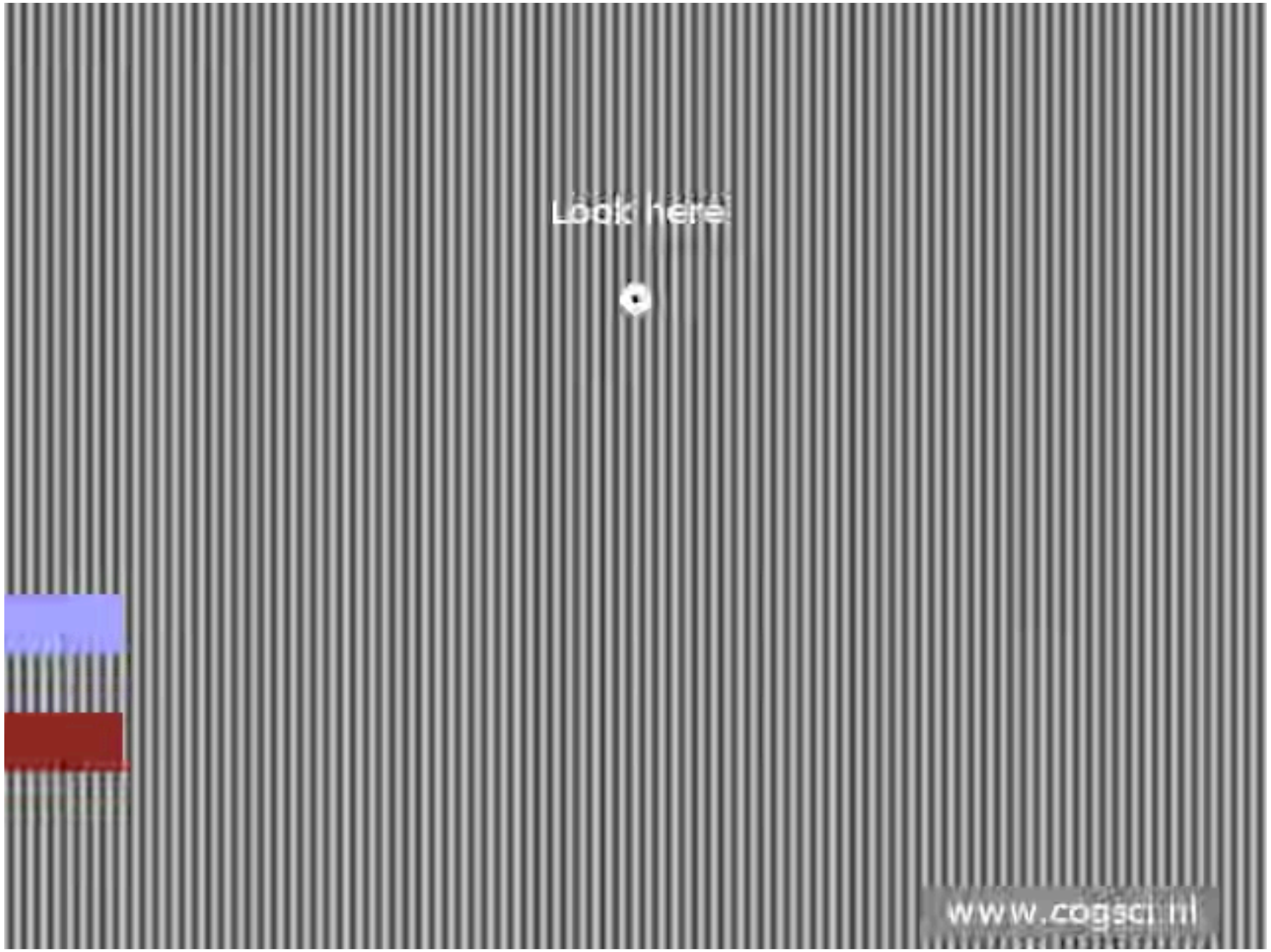
Food for thought

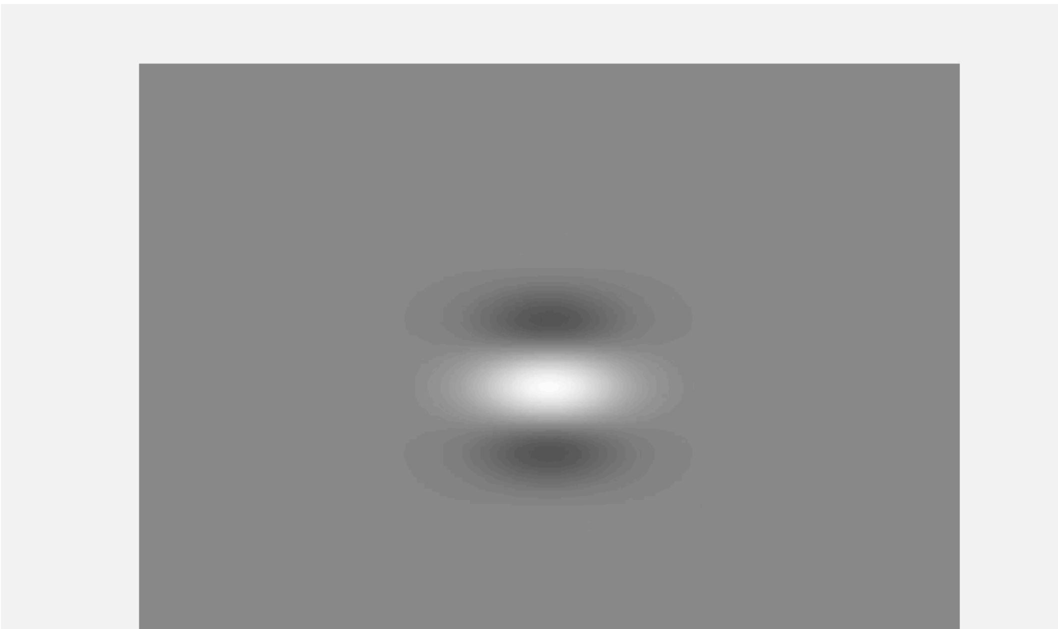
- Does the brain actually use Bayesian rules or are Bayesian models mere approximate descriptions of behaviour?
- Irrationality in cognition
- Implausibly uniform view of the mind
- Near-trivial due to their many degrees of freedom
- Relationship between perception, cognition, rationality, and consciousness
- Priors can be used to accommodate discrepancies
- Often constructed in a post-hoc manner
- Theoretical motivation of Bayesian theories is often unclear

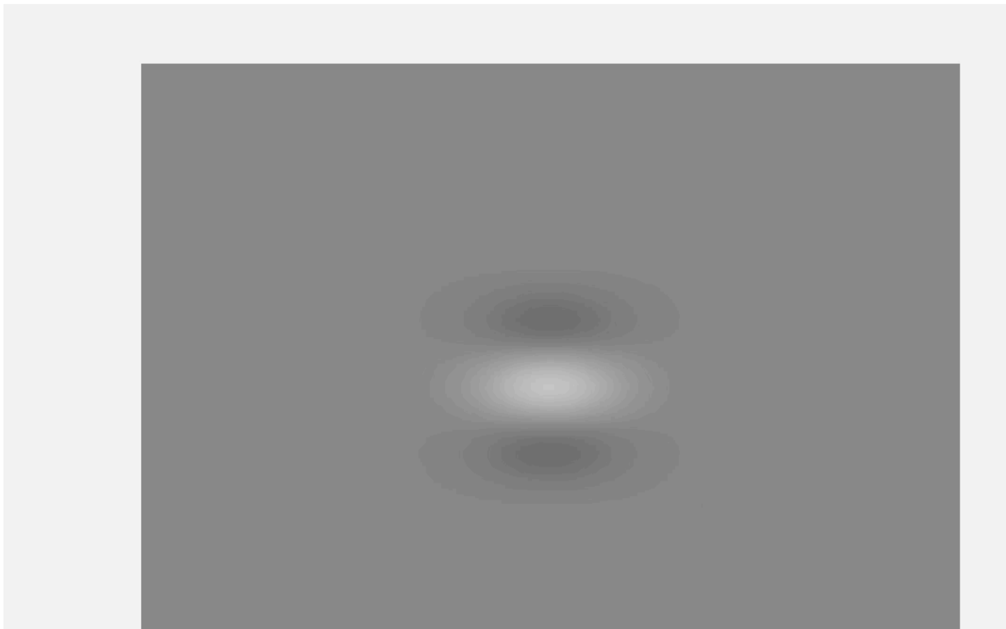
Lab 2

Max Di Luca

Mind, Brain, and Models 2022/23 15/02/2023


Look here!
www.cogsci.nl

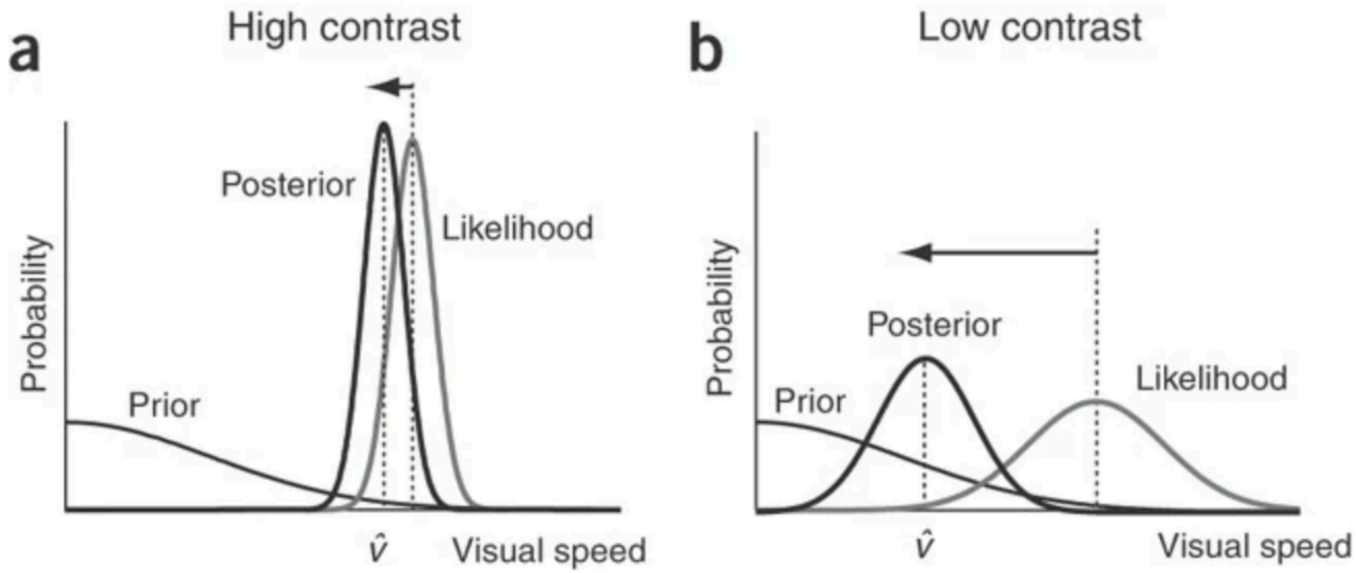




Noise characteristics and prior expectations in human visual speed perception

Alan A Stocker & Eero P Simoncelli

Nature Neuroscience **9**, 578–585 (2006) | Cite this article


a
High contrast
Posterior
Likelihood
Probability
Prior
$\hat{v}$
Visual speed
b
Low contrast
Posterior
Likelihood
Probability
Prior
$\hat{v}$
Visual speed