University of Waterloo
CS 241 Winter 2018
Review

Formal definition of DFA:

A DFA is a 5-tuple $(\Sigma, Q, q_0, A, \delta)$ where:

- finite alphabet Σ
- finite set of states Q
- start state $q_0 \in Q$
- set of final/accepting states $A \subseteq Q$
- transition function $\delta : Q \times \Sigma \to Q$

Formal definition of NFA:

A NFA is a 5-tuple (Σ, Q, q_0, A, T) where:

- finite alphabet Σ
- finite set of states Q
- start state $q_0 \in Q$
- set of final/accepting states $A \subseteq Q$
- transition relation $T : Q \times \Sigma \to 2^Q$

For example, if $Q = \{A, B\}$, then $2^Q = \{\{\epsilon\}, \{A\}, \{B\}, \{A, B\}\}$. $|2^Q| = 2^Q$.
NFA to DFA: subset construction

ϵ-NFA to NFA:

1. take ϵ shortcuts; replace ϵx with x $\forall x \in \Sigma$
2. pull back final states
3. remove ϵ transitionis
4. remove dead states (states that have no transitions into it)

Regular expressions:

RE is

- $\emptyset$, or
- ϵ, or
- a, where $a \in \Sigma$, or
- E_1E_2 where E_1, E_2 are REs (series), or
- $E_1|E_2$ where E_1, E_2 are REs (parallel), or
- E^* where E is RE (feedback)

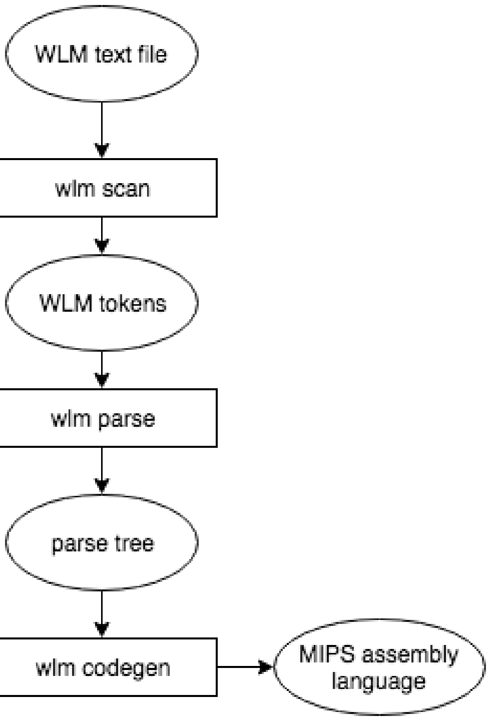
Definition of Regular Language:

A regular language is a language which is

- sepcified by a regular expression
- recognized by an ϵ-NFA
- recognized by an NFA
- recognized by a DFA

Compilation:

- Lexical analysis: scanning/tokenizing
- Syntactic analysis: parsing
- Context-sensitive (semantic) analysis
- Synthesis (code generation)



Context-free grammar(CFG):

G: CFG
$L(G)$: set of words specified by G(i.e. the language specified by G)
N: finite set of non-terminals (non-ending)
T: finite set of terminals (ending)
P: finite set of production rules (rewriting rules)
S: $S \in N$, start symbol

We say that $\alpha A \beta$ directly derives $\alpha\gamma\beta$ if there exists a production rule $A \to \gamma$. Also called a *derivation step.*
We say that $\alpha A \beta$ derives $\alpha\gamma\beta$ if there exist two or more derivation steps such that $\alpha A \beta \to^* \alpha\gamma\beta$.

Leftmost derivation: when there are 2 or more non-terminals in a derivation, we pick the leftmost non-terminal.
Rightmost derivation: when there are 2 or more non-terminals in a derivation, we pick the rightmost non-terminal.

Ambiguity in CFGs:

A string x is ambiguous if $x \in L(G)$ and there are more than one parse trees for x.
A CFG G is ambiguous if some word $w \in L(G)$ is ambiguous.
A grammar is ambiguous if there is a word x such that x has

1. ≥ 2 different parse trees, or
2. ≥ 2 different leftmost derivations, or
3. ≥ 2 different rightmost derivations

left recursion: leftmost symbol of the RHS is the LHS ($E \to E + B$)
right recursion: rightmost symbol of the RHS is the LHS ($E \to B + E$)
Related to associativity.

Top-down parsing with a stack:

Invariant: derivation = input already read + stack (stack is read from the top-down)
Use a rule: pop the stack which has the LHS non-terminal, push RHS in reverse onto the stack.
Accept the input when simultaneously stack and input are empty.
The orcale: Predict$(A, x) = A \to \alpha$
A is on top of the stack, and x is the first symbol of input to be read.

LL(1) Grammar:
$\forall$ non-terminal $A \in N, x \in T,\ |\text{Predict}(A, x)| \leq 1.$
L: Left-to-right input
L: leftmost derivation
1: one token of "lookahead" (in terms of input)

Constructing a Predictor Table: form search trees (search until the first terminal symbol)
Algorithm:
$\alpha, \beta \in (N \cup T)^*,\ x, y \in T, A \in N$

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Empty($\alpha$) = true if $\alpha \to^* \epsilon$
// can $\alpha$ disappear?
First($\alpha$) = $\{x \mid \alpha \to^* x\beta\}$
// starting from $\alpha$, what can I generate as a first terminal symbol?
Follow($A$) = $\{y \mid S' \to^* \alpha A y \beta\}$
// starting from the start symbol, does the terminal $y$ ever appear following the non-terminal $A$?
Predict($A, x$) = $\{A \to \alpha \mid x \in$ First($\alpha$)$\}$ $\cup$ $\{A \to \beta \mid x \in$ Follow($A$) and Empty($\beta$)$\}$
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Algorithm for parsing:

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Input: $w$
Push $S'$
for each $x \in w$
  while(top of stack is some $A \in N$){
    pop $A$
    if Predict($A, x$) = $\{A \to \alpha\}$
      push $\alpha$
    else
       reject
  }
  pop $c$
  if $c \neq x$ reject
end for
accept $w$
```

Bottom-up parsing with a stack:
stack + input to be read = current derivation (stack is read from bottom to top)
shift: push
reduce($A \to ab$): pop RHS, push LHS
oracle: in the form of a DFA

LR(0) ϵ-NFA formal definition:
Given a CFG $G = (N, T, P, S)$, construct an ϵ-NFA$(Q, N \cup T, q_0, F, D)$ as follows:

- $Q = \{A \to \alpha \bullet \beta | A \to \alpha\beta \in P\}$
- $q_0 = \{S' \to \vdash \bullet S \dashv\}$

- $D[A \to \alpha \bullet X\beta, X] = \{A \to \alpha X \bullet \beta\}$
- $D[A \to \alpha \bullet X\beta, \epsilon] = \{B \to \bullet\gamma | B \to \gamma \in P\}$
- $F = \{S' \to \vdash S \bullet \dashv\}$

2 actions:
If you have stack K and input a, and Ka is recognized, you can **shift**.
If you have stack K and input a, and the top of K is a state containing $A \to \alpha\bullet$ and a can follow A, **reduce** $A \to \alpha\bullet$.
If more than one of these are defined, you have a **conflict**.

Building an LR(0) automaton:
An item is a production with a dot ($\bullet$) somewhere on the RHS (which indicates a partially completed rule).
How to construct:

- make the start state the first rule, with the dot ($\bullet$) in front of the leftmost symbol of the RHS
- for each state, label an arc with the symbol that follows $\bullet$ and advance the $\bullet$ one position to the right in the next state
- if the $\bullet$ precedes a non-terminal, add all productions with that non-terminal on the LHS to the current state, with the $\bullet$ in the leftmost position

Using the automaton:

- If there is a transition out of our current state on the current input, then *shift* (push) that input onto the stack
- We know we can *reduce* if the current state has only one item and the $\bullet$ is the rightmost symbol
- To *reduce*, pop the RHS off the stack, reread the stack (from the bottom-up), follow the transition for the LHS and push the LHS onto the stack

Conflict: shift-reduce, reduce-reduce
If such conflicts are present, the grammar is not LR(0)
Adding a lookahead token to the automaton fixes the conflict.
For each $A \to \alpha$, attach Follow(A).
Increasing efficiency: store (state, input) on stack, and start the transducer at the top of the stack.

Outputting a derivation (parse tree):

– Create a "tree stack"

– Each time we reduce, pop the RHS nodes from tree stack

– Push the LHS node and make its children the nodes we just popped

Traversing the tree twice:
- Symbol table: variable values
- Detect semantic errors

Context-sensitive analysis:
Input: variable names, procedure names, parse tree (syntactically valid)
Output: ERROR if there are any context-sensitive errors; (decorated) parse tree otherwise