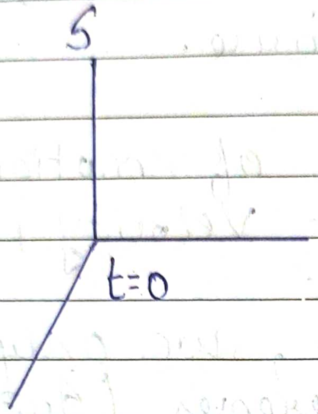
24/11/22

Tutorial-1 (Discussed).

1). $\tan^{-1}(3/4) = \theta$ $\tan\theta = 3/4$. $u = 5\,m/s$.



$u_x = u\cos\theta$

$u_y = u\sin\theta$.

$u_x = 5 \times 4/5 = 4\,m/s$

$u_y = 5 \times 3/5 = 3\,m/s$.

at $t = 2$, u_x = will travel = 8m

u_y will travel = 6m.

$u_x' = u_x - v$ $u_z' = u_z$.

$u_y' = u_y$

$u_x' = 4 - 1 = 3\,m/s$. $u_y' = u_y = 3\,m/s$.

u_x' will travel = 6m

u_y' will travel = 6m.

$\theta' = \tan^{-1}(3/3) = 45°$

2). $\tan\theta = 3/4$ $\cos\theta = 4/5$.

$\sin\theta = 3/5$ $u = c = 3 \times 10^8\,m/s$.

$u_x = 3 \times 10^8 \times 4/5$

$= \frac{4}{5}c$.

$$U_y = 3\times10^8\times 3/5 = \frac{3}{5}c.$$

$$U_x \text{ will move} = \frac{4}{5}c\times t = \frac{4}{5}c\times 2\times10^{-6}$$

$$U_y \text{ will move} = \frac{3}{5}c\times 2\times10^{-6}$$

$$U_x \text{ will move} = \frac{4}{5}\times 3\times10^8\times 2\times10^{-6} = \frac{24}{5}\times 100^{20}$$

$$= 480\,m$$

$$U_y \text{ will move} = \frac{3}{5}\times 3\times10^8\times 2\times10^{-6} = 360\,m.$$

$$U_x' = U_x - V$$

$$= \frac{4c}{5} - 0.6c = \boxed{0.2c}$$

$$U_y' = U_y = 3/5\,c = \boxed{0.6c}$$

$$S' = (120, 360).\ (U_x'\times t,\ U_y'\times t).$$

$$\theta' = \tan^{-1}\left(\frac{0.6}{0.2}\right) = \tan^{-1}(3).$$

$$u' = \sqrt{(0.2)^2+(0.6)^2}\,c \neq c$$

3). $t = 50$ sec.

$x_1 = 1.3$ km

$x_2 = 0.5$ Km

$x_1' = 0.5$ Km

$x_2' = 1.3$ Km

In S, x_1 t_1 In S',

E_1 (1300, to) E_1' (500, to)

E_2 (500, to+50). E_2' (1300, to+50).

x_2 t_2.

$$(S_2' - S_1') = (S_2 - S_1) - V(t_2 - t_1).$$

$$x_1' = x_1 - Vt_1$$
$$500 = 1300 - V \cdot to.$$

$$1300 = 500 - V(to + 50).$$

$$500 = 1300 - Vto$$
$$1300 = 500 - Vto - 50V.$$
$$Vto = 500 - 1300 - 50V.$$

$$500 = 1300 - (500 - 1300 - 50V)$$
$$500 = 1300 - 500 + 1300 + 50V.$$
$$1000 = 2600 + 50V.$$
$$-1600 = 50V$$
$$V = -\frac{\cancel{1600}^{32}}{\cancel{50}}$$

$$\boxed{V = -32\ m/s}$$

$$\boxed{to = -25\ s.}$$

Tutorial-1 (Continuation).

4) In S frame, $u = 50\,m/s.$

$E_1\,(x_1 = 10m,\ t_1 = 1s)$
$E_2\,(x_2 = 50m,\ t_2 = 3s)$

In S' frame,

$x_1' = x - vt = 10 - 50(1) = (-40)$
$x_2' = x - vt = 50 - 50(3) = (-100).$

Length of Carriage $= |x_1' - x_2'| = (-40) - (-100)$
$= \underline{60m}$

5). $u_x = 4\,m/s.$ U_{net} seen by P on ground $= 5\,m/s.$

$$U^2 = U_x^2 + U_y^2$$
$$U_y = \sqrt{U^2 - U_x^2}$$
$$= \sqrt{25 - 16} = 3\,m/s.$$

For P', $U_y = 3\,m/s.$ $V = u - gt$
For P, $U = 5\,m/s.$ $0 = 3 - gt$
$t = 0.3s.$

In frames, (P)

$X = vt = 4 \times (0.3 + 0.3)$ ← whole t when particle moves down.

$= \boxed{2.4m}$

$h = u^2/2g = \dfrac{9}{2 \times 10} = \boxed{9/20\ m}$

6) rod $\vdash 0.6m \dashv$ $\longrightarrow 30 m/s$ $\longleftarrow 30 m/s$ Particle

$x = u \times t$

$\therefore \; t = x/u = \dfrac{0.6}{(30+30)} = \boxed{0.01 \, sec}$

Distance (the 0.6)

relative speed (the $30+30$)

7). In S frame,

$E_1 \, (0m, 0m, t = 2s)$

$E_2 \, (x_2 m, y_2 m, t = 5s)$

$a = (2\hat{i} + 3\hat{j}) \, m/s^2$

$u = 0$ (initial velocity $= 0$)

In S' frame

$u = 5\hat{i} \, m/s$

(S' is moving wrt S)

$E_1'(x_1', y_1', t_1' = 2s)$

$E_2'(x_2', y_2', t_2' = 5s)$.

$\longrightarrow$ In S frame,

$a = (2\hat{i} + 3\hat{j}) \, m/s^2$

$\vec{V} = u + \vec{a}t$

$\vec{V} = 0 + (2\hat{i} + 3\hat{j}) \times \Delta t \, (5-2)$

$\vec{V} = 6\hat{i} + 9\hat{j} \, m/s.$ $\quad V_x = 6\hat{i}$

$V_y = 9\hat{j}$

In E_1, $X_1 = 0$, $Y_1 = 0$.

In E_2, $X_2 = \frac{1}{2} \times (2)(3)^2 = 9m.$

$Y_2 = \frac{1}{2}(3) \times (3)^2 = 13.5m$

In S' frame,

$X_2' = 9 - 5 \times 5$
$= -16 m.$

$Y_2' = Y_2$
$= 13.5 m.$

$X_1' = X_1 - Vt$
$= 0 - 5 \times 2$
$= -10 m.$

$Y_1' = Y_1$
$= 0 m.$

$U_{x_1}' = U_{x_1} - V$
$= 0 - 5$
$= -5 m/s.$

$U_{x_2}' = U_{x_2} - V$
$= 6 - 5$
$= 1 m/s.$

$U_{y_1}' = U_{y_1}$
$= 0 m/s.$

$U_{y_2}' = U_{y_2}$
$= 9 m/s.$

$(X_2' - X_1') = U_{x_1}'(t_1' - t_2') + \frac{1}{2} a_x' (t_1' - t_2')^2$

$-16 + 10 = (-5)(+3) + \frac{1}{2} a_x' (-3)^2$

$-6 = -15 + \frac{1}{2} a_x' (9)$

$9 = \frac{1}{2} a_x' (9)$

$a_x' = 2 m/s^2$

$(Y_2' - Y_1') = U_{y_1}'(t_1' - t_2') + \frac{1}{2} a_y' (t_1' - t_2')^2$

$13.5 = \frac{1}{2} a_y' (3)^2$

$a_y' = \frac{27}{9} = 3 m/s^2$

$\vec{a} = \vec{a}'$

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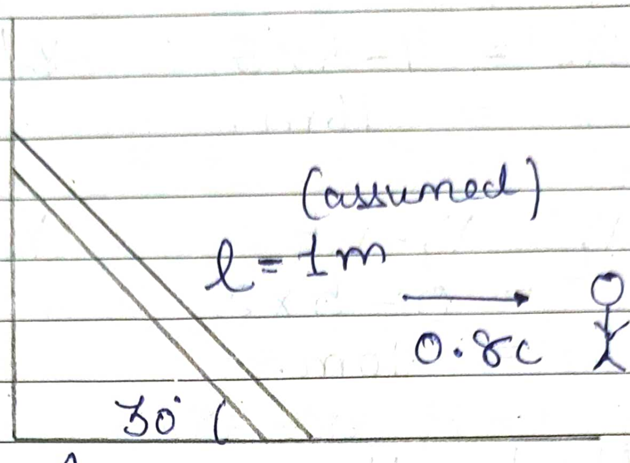
Tutorial-2 (Lorentz Transformation).

5)



$l\sin 30$

$l_y' = l_y = l\sin 30$

$= l/2$

$= 0.5\,l$

$l\cos 30$

$L = \frac{L'}{\gamma} \quad \therefore \; l = \frac{l\cos 30}{\gamma}$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$\therefore l_x' = l\cos 30 \times \sqrt{1 - v^2/c^2}$

$l_x' = l\,\frac{\sqrt{3}}{2}\sqrt{1 - v^2/c^2}$

$= l\,\sqrt{3}/2\,\sqrt{1-(0.8)^2}$

$= 0.865 \times l \times 0.6$

$= (l \times 0.519)$

$$l' = \sqrt{(l'x)^2 + (l'y)^2}$$

$$= \sqrt{(0.5)^2 + (0.519)^2} = \boxed{0.72m}$$

Formulas of Lorentz Transformation & Concept of time Dilation.

1) $x' = B_{xx}(\cancel{x-vt})$

$y' = y$

$z' = z$

$t' = B_{tx}X + B_{tt}(t)$

$B_{tt} = B_{xx} = \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$

$L' = L/\gamma$ or $L = L'/\gamma$ Proper

elongated

$$B_{tx} = \frac{-v \times 1}{c^2 \times \sqrt{1-v^2/c^2}} = \frac{-v(\gamma)}{c^2}$$

$x' = \gamma(x - vt)$

$t' = \gamma\left(t - \dfrac{vx}{c^2}\right)$

IMP.

2) Time Dilation.

Stationary frame: S frame

$E_1\ (X, t_1)$

$E_2\ (X, t_2)$

$\Delta t = t_2 - t_1$ (proper time)

$X_1 = X_2 = X$ (Same location)

Moving frame: S' frame

$\Delta t' = t_2' - t_1' = \gamma(t_2 - t_1) = \gamma \Delta t$

$\Delta t'$: dilated time

Δt: Proper time

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Tutorial-2 (Lorentz Transformation) (Continue).

1)

$S' \longrightarrow v = 0.6c.$ S

(observer B)   ( ) – drops ball

$t = 1.5s$ at rest

Dilated   time

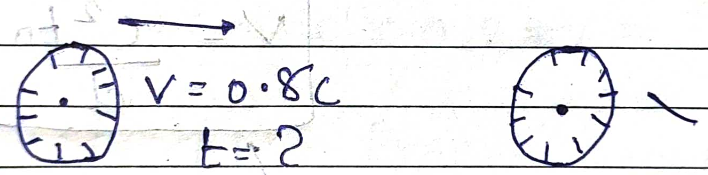
$$\Delta t' = \gamma(\Delta t)$$

$$1.5 = \frac{1}{\sqrt{1-(0.6)^2}}\,\Delta t$$

$$\Delta t = 1.5 \times 0.8 = \boxed{1.2s}$$

Hence, proper time to drop a ball from S frame is 1.2s.

2).



$v = 0.8c$ $t = ?$ $t = 2s$

Clock A   moving   Dilated time

Clock B   rest   Proper time

(observer)

$$\Delta t' = \gamma(\Delta t)$$

$$= \frac{1}{\sqrt{1-(0.8)^2}} \times 2 = \frac{2}{0.6} = \boxed{3.33\text{ sec}}$$

Hence, time interval of ticks of moving clock as seen by observer is 3.33 sec.

3). S frame

$E_1\ (x_1 = 0,\ t_1 = 0)$

$E_2\ (x_2 = d,\ t_2 = t_0)$

S' frame
(lightning is simultaneous).
(Proper time
also $t_1' = t_2'$.
Proper length $= L'$
$= X_2' - X_1'$ $t_1' = t_2'$

In S' frame

$t_1' = t_2'$

$$t_1' = \gamma\left(t_1 - \frac{V x_1}{c^2}\right) = \gamma(0 - 0) = 0.$$

$$t_2' = \gamma\left(t_2 - \frac{V x_2}{c^2}\right) = \gamma\left(t_0 - \frac{Vd}{c^2}\right) = 0$$

$$\therefore\ t_0 = \frac{Vd}{c^2}$$

$$\boxed{V = \frac{c^2 t_0}{d}}$$

Speed of B wrt A.

$$X_2' - X_1' = [\gamma(X_2 - Vt_2)] - [\gamma(X_1 - Vt_1)]$$

distance between positions of two lightnings in S' frame

$$= \gamma(X_2 - X_1 + Vt_1 - Vt_2)$$

$$= \gamma\left(d - 0 + \frac{c^2 t_0}{d} \times 0 - \frac{c^2 t_0 \times t_0}{d}\right)$$

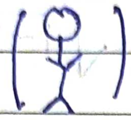
$$= \gamma\left[d - \frac{(ct_0)^2}{d}\right]$$

$$= \boxed{\sqrt{d^2 - (ct_0)^2}}$$

6/.

(100m) A ' (LA) (30m) B (LB')

$\longrightarrow$ $\longleftarrow$ $0.96c = v_B$.



$L = L'/\gamma$ L' = Proper length.

$$L_A' = \frac{L_A}{1} \times \sqrt{1 - v^2/c^2}$$

$$= \frac{100}{1}\left(\sqrt{1 - (0.96)^2}\right)$$

Actual Length

$$= 100 \times 0.28 = \boxed{28\,m}$$

$L = L'/\gamma$ (L' = Proper Length).

(Here, L' = LB).

Contracted Length

$$30 = \frac{L_B}{\gamma}$$

$$L_B = 30 \times \gamma$$

$$= \frac{30}{\sqrt{1 - v^2/c^2}} = \frac{30}{\sqrt{1 - (0.96)^2}}$$

$$= \frac{30}{0.28} = \boxed{107.14\,m}$$

4). $\Delta t' = \gamma \Delta t$ $\qquad \gamma = \frac{25}{20} = 5/4$

$25 = \gamma \times 20$

$$\frac{\Delta t'}{\Delta t} = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\frac{\Delta t}{\Delta t'} = \sqrt{1 - v^2/c^2}$$

$$\left(\frac{\Delta t}{\Delta t'}\right)^2 = 1 - v^2/c^2$$

$$1 - \left(\frac{\Delta t}{\Delta t'}\right)^2 = v^2/c^2$$

$$v = c\sqrt{1 - \left(\frac{\Delta t}{\Delta t'}\right)^2}$$

$$v = c\sqrt{1 - (4/5)^2}$$

$$= c\sqrt{\frac{9}{25}} = \boxed{\frac{3c}{5}}$$

Expanded length $= \frac{3c}{5} \times (25)\,ns.$

Length in S' frame $= \frac{3 \times 3 \times 10^8 \times 25 \times 10^{-9}}{5}$

$= 45 \times 10^{-1}$

$= 4.5\,m = \boxed{45cm}$

$L = L'/\gamma$ $L' =$ Proper length.

$L' = L \times \gamma$

$= \dfrac{45}{\sqrt{1-(0.6)^2}} = \boxed{56.25 \text{ cm}}$

7).

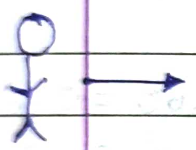
S frame.

Plane (observer) $\longleftrightarrow$ V, $V \cos 60$

$t = 0$

$x = 0$

S' frame $(t_1' = t_2')$



(observer)

$x_A = 0$ $x_B = 1.5$ km

$t = 0$ $t = 1 \mu s$.

In S frame.

$E_1 (x_A = 0, t_A = 0)$

$E_2 (x_B = 1.5 \text{ km}, t_B = 1 \mu s)$

In S' frame, $t_A' = t_B'$.

$$t_A' = \gamma\left(t_A - \frac{V x_A}{c^2}\right) = \gamma(0-0) = 0.$$

$$t_B' = \gamma\left(t_B - \frac{V x_B}{c^2}\right) = \gamma\left(1\times10^{-6} - \frac{V(1.5\times1000)}{c^2}\right) = 0$$

$$\therefore 10^{-6} = \frac{V(1500)}{c^2}$$

$$\boxed{V = \frac{c^2 \times 10^{-6}}{1500}}$$ ← Speed of S' in S frame.

$= 0.2\,c$

7). S S' frame

$(t=0, x=0$ $\quad t_1' = t_2'$

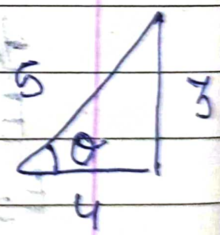
$V\cos 60 = 0.2c$

$V = 0.2 \times 2$

$= 0.4c$

8). $\tan\theta = 3/4$ $\quad \sin\theta = 3/5$

$\theta = 37°$ $\quad \cos\theta = 4/5$



$u\sin\theta = 3/5 \times 0.6c$ (u_y)

$u\cos\theta = 4/5 \times 0.6c$ (u_x)

$X_2 = u_x t_x = 4/5 \times 0.6c \times 2 \times 10^{-6}$

$= \dfrac{8 \times 0.6 \times 3 \times 10^8 \times 10^{-6}}{5}$

$= 288m.$

$Y_2 = U_y t_y = 3/5 \times 0.6c \times 2\times10^{-6}$

$= 216 m.$

Distance travelled $= \sqrt{x^2+y^2}$

$= \sqrt{(288)^2+(216)^2} = \underline{360 m.}$

S'frame

$\gamma = 1.25.$

$X_2' = \gamma(X_2 - vt_2)$

$= 1.25(288 - 0.6c\times2\times10^{-6})$

$= 1.25(288-360)$

$= (-90) m.$

$Y_2' = 216 m.$

$t' = 1.25\left(2\times10^{-6} - \frac{(0.6)c\times288}{9\times10^{16}}\right)$

$= \boxed{1.78 \mu s}$

$V = \sqrt{(V_x)^2+(V_y)^2} = 131.45\times10^6 m/s.$

50.56×10^6 121.34×10^6

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 67.38'$

9). Light Pulse

$x=0, t=0$

(observer)

frame S

$\tan\theta = 3/4$

$t = 2\times10^{-6}s$.

$\sin\theta = 3/5$

$\cos\theta = 4/5$.

$u = c$ (pulse of light)

$u_x = u\cos\theta = c\times 4/5 = 4c/5$.

$u_y = u\sin\theta = c\times 3/5 = 3c/5$.

$$X = u_x \times t = \frac{4c}{5}\times 2\times10^{-6}$$

$$= \frac{4\times3\times10^{8}}{5}\times2\times10^{-6}$$

$$= \underline{\underline{480m}}.$$

$$Y = u_y\times t = \frac{3c}{5}\times2\times10^{-6}$$

$$= \frac{3\times3\times10^{8}}{5}\times2\times10^{-6}$$

$$= \underline{\underline{360m}}.$$

S' S

$\xrightarrow[0.6c]{}$ (observer in S')

$$x' = \gamma(x - vt)$$

$$= 1.25\,(480 - 0.6 \times 3 \times 10^8 \times 2 \times 10^{-6})$$

$$= 1.25\,(480 - 360) = \underline{\underline{150\,m}}.$$

$$y' = y = \underline{\underline{360\,m}}.$$

$$u_x' = \frac{x'}{t'}$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$= 1.25\left(2 \times 10^{-6} - \frac{0.6 \times 3 \times 10^8 \times 480}{(3 \times 10^8)^2}\right)$$

$$= 1.25\,(2 \times 10^{-6} - 96 \times 10^{-8})$$

$$= 1.25 \times 1.04 \times 10^{-6} = 1.3 \times 10^{-6}$$

$$= \boxed{1.3\,\mu s}$$

$$u_x' = \frac{x'}{t'} = \frac{150}{1.3 \times 10^{-6}} = 115.38 \times 10^6$$

$$= \underline{1.15 \times 10^8}.$$

$$u_y' = \frac{y'}{t'} = \frac{360}{1.3 \times 10^{-6}} = 276.92 \times 10^6$$

$$= \underline{\underline{2.76 \times 10^8}}.$$

$$u_{ins}' = \sqrt{(u_x')^2 + (u_y')^2}$$

$$= \sqrt{1.322 \times 10^{16} + 7.617 \times 10^{16}}$$

$$= \boxed{2.98 \times 10^8\ m/s}$$

$$\boxed{c \text{ in } S' = c \text{ in } S}$$

So according to Special theory of relativity, Speed of light (c) should be same in all frames irrespective of diffrent frames.

Hence, proved.

Velocity Transformation

S	S'
(Velocity Components Knowing)	(To find Velocity Components)

$\vec{V} \longrightarrow$ Relative velocity between frames. (Constant as func^n of time)

$\vec{u} \longrightarrow$ Instantaneous velocity of particle in S. (Need not to be Constant)

$\vec{u'} \longrightarrow$ Instantaneous velocity of particle in S'. (Need not to be Constant)

Particle

$\longrightarrow +x$ in S

E_1: Particle at x_1 & t_1

E_2: Particle at x_2 & t_2.

$$\Delta V = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

instantaneous Velocity in S

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Tutorial 3

1)

$0.99c \rightarrow$ u_x $\quad$ $0.99c \leftarrow$ v

$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} = \frac{-0.99c - 0.99c}{1 + (0.99)^2}$$

$$= \boxed{-0.99c}$$

2).

$l \quad \longrightarrow u$

(proper length) x axis $\quad$ S frame

S' frame $\longrightarrow v$

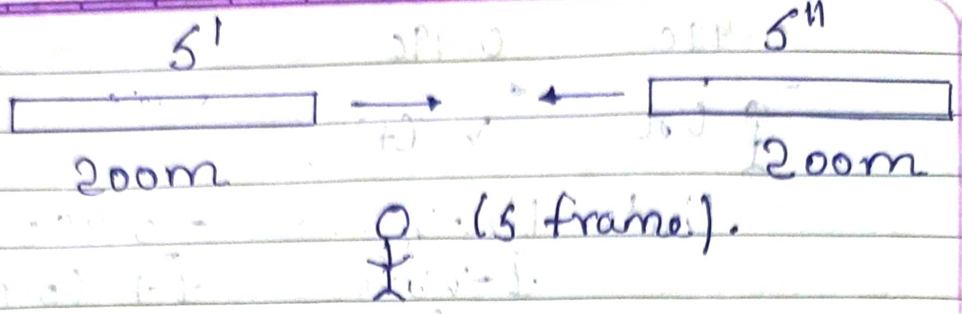
$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \left(\frac{u-v}{\frac{1 - uv/c^2}{c^2}}\right)^2}}$$

l' (Contracted length) $= l/\gamma$

$$= l \times \sqrt{1 - \frac{(u^2 + v^2 - 2uv)}{c^2 \times \left(1 + \frac{u^2v^2}{c^4} - \frac{2uv}{c^2}\right)}}$$

3)



1st Part

$$\gamma = \frac{1}{\sqrt{1-(0.58)^2}} = \frac{1}{0.814} = 1.228.$$

$$L = \frac{L'}{\gamma} = \frac{200}{1.228} = 162.86.$$

Contracted Length.

Total distance as seen by observer in S' (Seeing S'') $= 200 + 162.86$
$= 362.86.$

$$\Delta t = \frac{362.86}{0.58 \times 3 \times 10^8} = \boxed{2.08\,\mu s}$$

2nd Part Lengths are same as seen by observer in S

$\Longrightarrow$ Velocity will also be same.

$$U_x' = \frac{U_x - V}{1 - \frac{VU_x}{c^2}}$$

$$0.58c = \frac{2V}{1 + V^2/c^2}$$

$$0.58\cancel{c} = \frac{2v \cdot c^{\cancel{2}}}{c^2+v^2}$$

$$\therefore \quad 0.58 = \frac{2vc}{c^2+v^2} \qquad \therefore \quad 0.29c^2 + 0.29v^2 = vc.$$

$$\therefore \quad 0.29v^2 - vc + 0.29c^2 = 0.$$

$$v = \left(\frac{c \pm \sqrt{c^2 - 4(0.29)(0.29c^2)}}{2 \times 0.29} \right)$$

$$v = \frac{c \pm 0.814c}{0.58}$$

$$v = 3.12c \quad \text{or} \quad \boxed{v = 0.32c.}$$

As $\boxed{v/c < 1}$ so $v = 3.12c$ is neglected.

$$\gamma_{S(S'S'')} = \frac{1}{\sqrt{1 - \frac{(0.32c)^2}{c^2}}} = \boxed{1.056}$$

↑
1 frame.

$$= \frac{200}{1.056} = \boxed{189.39\,m}$$

Contracted
Length as
Seen by observer
in S frame.

• In every process both energy and momentum have to be conserved.

Tutorial 3 (continue).

4) S' frame

A 1m B

(A before B)

$\Delta t' = 2.5 \times 10^{-9} s$.

(Observer in S frame)

S'' frame

(B before A)

$\Delta t'' = 2.5 \times 10^{-9} s$

→ In S frame,

Length of moving rod $= X_B - X_A$

$= 1m$ at $t_A = t_B$.

In S' frame,

$$t_B' - t_A' = \gamma\left((t_B - t_A) - \frac{v(x_B - x_A)}{c^2} \right)$$

$$2.5 \times 10^{-9} = \gamma\left(0 - \frac{v(1)}{c^2} \right)$$

$$-\frac{\gamma v}{c^2} = 2.5 \times 10^{-9}$$

$$\frac{-v}{c^2\sqrt{1 - v^2/c^2}} = 2.5 \times 10^{-9}$$

Both side square,

$$\frac{v^2}{c^4\left(1-\frac{v^2}{c^2}\right)} = 6.25 \times 10^{-18}$$

$$\frac{v^2 c^2}{c^4(c^2-v^2)} = 6.25 \times 10^{-18}$$

$$\frac{v^2}{c^2(c^2-v^2)} = 6.25 \times 10^{-18}$$

$$v^2 = 6.25 \times 10^{-18} c^4 - 6.25 \times 10^{-18} c^2 v^2$$

$$v^2 + 6.25 \times 10^{-18} c^2 v^2 = 6.25 \times 10^{-18} c^4$$

$$v^2 (1 + 6.25 \times 10^{-18} c^2) = 6.25 \times 10^{-18} c^4$$

$$v^2 = \frac{6.25 \times 10^{-18} c^4}{1 + 6.25 \times 10^{-18} c^2}$$

$$= \frac{6.25 \times 81 \times 10^{-18} \times 10^{32}}{1 + 6.25 \times 10^{-18} \times 9 \times 10^{16}}$$

$$= \frac{506.25 \times 10^{14}}{1 + 56.25 \times 10^{-2}}$$

$$= \frac{506.25 \times 10^{14}}{1.5625}$$

$$v^2 = 324 \times 10^{14}$$

$$v = \pm (18 \times 10^7)$$

$$v = \pm (1.8 \times 10^8)$$

31/12/22

$\boxed{V = \pm (0.6c)}$ $V = -ve$ value taken as +ve value.

Similarly, for S'' frame, (LHS ≠ RHS)

Same as S' frame by seeing in (S–S'' frame)

$\boxed{V = \pm (0.6c)}$ $V = +ve$ value taken.

$$V_{SS''} = \frac{-0.6c - 0.6c}{1 + 0.6 \times 0.6} = \boxed{\frac{-1.2c}{1.36}}$$

Zero Rest Mass Particle. If rest mass is zero,

$E = \gamma_u m_0 c^2$ and $p = \frac{m_0 \vec{u}}{\sqrt{1 - u^2/c^2}}$ are zero unless $u = c$.

$$= \frac{m_0 c^2}{\sqrt{1 - u^2/c^2}}$$

$(0/0) \longrightarrow$ Un-determined

$\boxed{E = K = pc}$

$$E^2 = p^2c^2 + m_0^2c^4$$

Photons.

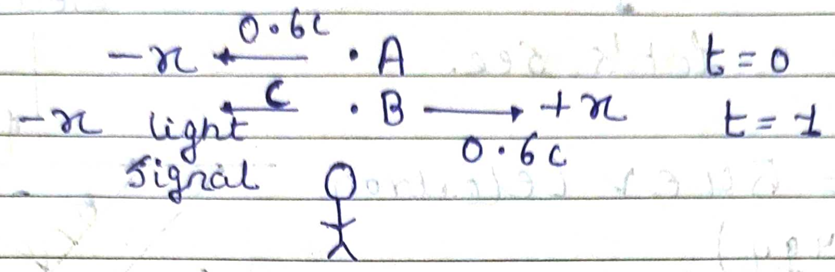
(Zero rest mass, $u = c$)

$E = h\nu$ and $p = \frac{h\nu}{c}$

Physics Tutorial

Tutorial-3 (continue).

5)



(observer in S)

In S frame,

$t_1 = 1s.$

Distance in S $\Rightarrow 0.6c \times 1 + 0.6c \times 1$

$\Rightarrow 1.2c.$

Total dist by light signal = Dist. in S (initially when B starts) after 1s. + $0.6c\Delta t$ (B in $+x$ direction)

$c\Delta t = 1.2c + 0.6c\Delta t$

$\boxed{\Delta t = 3s}$

$\therefore$ Total time in S = $3 + 1 = \boxed{4s}$

A frame.

$t_1' = \gamma\left(t_1 - \frac{vx}{c^2}\right)$ $\quad \gamma = \frac{1}{\sqrt{1-(0.6)^2}}$

$= 5/4\left(1 - \frac{(-0.6c)\,0.6c}{c^2}\right)$ $\quad = 5/4$

$= 1.7s$

$X_1' = \gamma(x_1 - vt)$

$= \frac{5}{4}\left(0.6c - (-0.6c)\times 1\right)$

$= \frac{1.2c \times 5}{4} = 1.5c = 4.5 \times 10^8 \, m.$

$\Delta t' = \frac{4.5 \times 10^8}{3 \times 10^8} = \underline{\underline{1.5s}}.$

$\therefore$ Total time in A frame $= 1.7 + 1.5s$

$= \boxed{3.2s}$

B frame.

$x_1'' = 0$ (In own frame, with that respect, distance $= 0$).

$t_1'' = \gamma\left(t_1 - \frac{v x_1}{c^2}\right)$

$= \frac{5}{4}\left(1 - \frac{(+0.6c)(0.6c \times 1)}{c^2}\right)$

$= \frac{5}{4}(1 - 0.36) = \boxed{0.8s}$

Relative Speed of B & A

$c\Delta t'' = \left(0.8 \times \frac{1.2c}{1.36}\right) + \frac{1.2c \, \Delta t''}{1.36}$

$\boxed{\Delta t'' = 6s}$

$\therefore$ Total time in B frame $= 6 + 0.8s$

$= \boxed{6.8s}$

6).

when $u < c$, $u = c$ — when $u' < c$, $u' = c$.

$$U_x' = \frac{U_x - V}{1 - \frac{VU_x}{c^2}} \qquad U_y', U_z' = \frac{U_y, U_z}{\gamma\left(1 - \frac{VU_x}{c^2}\right)}$$

$$U = \sqrt{U_x^2 + U_y^2 + U_z^2}$$

$$U' = \sqrt{U_x'^2 + U_y'^2 + U_z'^2}$$

$$= \sqrt{\frac{(U_x - V)^2}{\left(1 - \frac{VU_x}{c^2}\right)^2} + \frac{U_y^2}{\gamma^2\left(1 - \frac{VU_x}{c^2}\right)^2} + \frac{U_z^2}{\gamma^2\left(1 - \frac{VU_x}{c^2}\right)^2}}$$

$$= \sqrt{\frac{U_x^2 + V^2 - 2U_xV}{1 + \frac{V^2U_x^2}{c^4} - \frac{2VU_x}{c^2}} + \frac{U_y^2\left(1 - \frac{U_y^2}{c^2}\right)}{1 + \frac{V^2U_x^2}{c^4} - \frac{2VU_x}{c^2}} + \frac{U_z^2\left(1 - U_z^2/c^2\right)}{1 + \frac{V^2U_x^2}{c^4} - \frac{2VU_x}{c^2}}}$$

$$= \sqrt{\frac{U_x^2 + V^2 - 2VU_x + U_y^2 - U_y^4/c^2 + U_z^2 - U_z^4/c^2}{\left(1 - \frac{VU_x}{c^2}\right)^2}}$$

$$= \sqrt{\frac{U^2 - 2VU_x - U_y^4/c^2 - U_z^4/c^2}{\left(1 - \frac{VU_x}{c^2}\right)^2}}$$

$$U'^2 = \frac{(c^2U^2 - 2VUxc^2 - Uy^4 - Uz^4)\, c^{\cancel{4}\,2}}{\cancel{c^2}\,(c^4 + V^2Ux^2 - 2VUxc^2)}$$

$$U'^2 = \frac{(U^2c^2 - 2VUxc^2 - Uy^4 - Uz^4)\, c^2}{c^4 + V^2Ux^2 - 2VUxc^2}$$

$$\frac{c^2 - U'^2}{c^2} = \frac{c^2 - \frac{c^2(U^2c^2 - 2VUxc^2 - Uy^4 - Uz^4)}{c^4 + VUx^2 - 2VUxc^2}}{c^2}$$

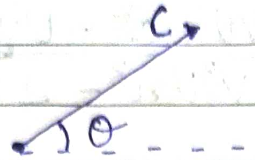
$$= \frac{c^2(c^4 + VUx^2 - 2VUxc^2) - c^2(U^2c^2 - 2VUxc^2 - Uy^4 - Uz^4)}{c^2(c^4 + VUx^2 - 2VUxc^2)}$$

$$= \frac{c^6 + c^2VUx^2 - \cancel{2VUxc^4} - U^2c^4 + \cancel{2VUxc^4} + Uy^4 + Uz^4}{c^2(c^4 + VUx^2 - 2VUxc^2)}$$

$$= \frac{c^6 + c^2VUx^2 + Uy^4 + Uz^4 - U^2c^4}{c^6 + VUx^2c^2 - 2VUxc^4)}.$$

$$\Rightarrow \boxed{\frac{(c^2 - u^2)(c^2 - v^2)}{(c^2 - UxV)^2}}$$

7)



$t = 0$

$x = 0$

$\tan\theta = 3/4$

$\sin\theta = 3/5$

$\cos\theta = 4/5$.

$U_x = 4/5\,c$

$U_y = 3/5\,c$

$$U_x' = \frac{U_x - V}{1 - \frac{V U_x}{c^2}}$$

$$= \frac{0.8c - 0.6c}{1 - 0.8 \times 0.6}$$

$$= \frac{0.2c}{0.52}$$

$$= \boxed{0.384c}$$

$$U_y' = \frac{U_y}{\gamma\left(1 - \frac{V U_x}{c^2}\right)}$$

$$= \frac{3/5\,c\,\sqrt{1-(0.6)^2}}{1 - \frac{0.6c \times 4/5\,c}{c^2}}$$

$$= \frac{0.6c \times 0.8}{1 - 0.6 \times 0.8}$$

$$= \frac{0.48c}{0.52}$$

$$= \boxed{0.923c}$$

Velocity Vector $\vec{V'} = 0.384c\,\hat{i} + 0.923c\,\hat{j}$

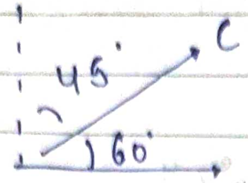
$\Longrightarrow$ c is same in all frames.

Location of pulse in $S' \Rightarrow \vec{x}' = 0.384c\,\hat{i} \times 2\times10^{-6} + 0.923c \times 2\times10^{-6}\,\hat{j}$

$$x' = \sqrt{(0.384c \times 2\times10^{-6})^2 + (0.923c \times 2\times10^{-6})^2}$$

Similar can be found out in S.

8)



V of S' in S frame.

S' frame (rel. to y'-z' frame)

S frame

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 45 + \cos^2 60 + \cos^2\gamma = 1$$

$$\boxed{\gamma = 60^\circ}$$

S frame.

$$U_x = U\cos\alpha = c/2$$

$$U_y = U\cos\beta = c/\sqrt{2}$$

$$U_z = U\cos\gamma = c/2.$$

S' frame. (y'-z' plane)

$x' = 0$ $\quad \alpha' = \pi/2$ $\quad U_x' = 0.$

Also $U_x = V$

$$U_y' = \frac{U_y}{\gamma} \quad \text{and} \quad U_z' = \frac{U_z}{\gamma}$$

19/1/23

Physics Tutorial

Tutorial - 4

1) $E = 4 m_0 c^2$

$E^2 = p^2c^2 + m_0^2 c^4$

$16 m_0^2 c^4 = m_0^2 c^4 + p^2 c^2$

$15 m_0^2 c^4 = p^2 c^2$

$\boxed{p = \sqrt{15}\, m_0 c}$

$p = 2 m_0 c$

$E^2 = 4 m_0^2 c^2 \times c^2 + m_0^2 c^4$

$= 5 m_0^2 c^4$ $\boxed{E = \sqrt{5}\, m_0 c^2}$

2) $KE = 0.1\ MeV$

$E = m_0 c^2 + 0.1 \times 10^6$

$m_0 c^2 = 81.9 \times 10^{-15}\ J = \frac{81.9 \times 10^{-15}}{1.6 \times 10^{-19}}\ eV$

$= 0.511\ MeV.$

$E = 0.511 + 0.1\ MeV$

$= 0.611\ MeV.$

$$0.611 = \gamma_u \times 0.511$$

$$\boxed{\gamma_u = 1.1956}$$

$$(1.195)^2 = \frac{c^2}{c^2 - u^2}$$

$$\boxed{u = 1.64 \times 10^8}$$

relativistic speed.

$$K = \frac{1}{2} m U^2$$

classical.

$$\sqrt{\frac{2K}{m}} = U$$

Joules.

$$\boxed{V = 1.87 \times 10^8}$$

3). $TE = 2 m_o c^2$

$$\gamma_u = 2$$

$$4 = \frac{c^2}{c^2 - u^2}$$

$$\boxed{u = \frac{\sqrt{3}\,c}{2}}$$

4) $1.2 \times 10^8 \longrightarrow 2.4 \times 10^8$

$$\gamma_{u_1} = 1.091 \qquad \gamma_{u_2} = 1.67$$

$$E = (1.67 - 1.091)\, m_o c^2$$

$$= 0.579 \times 81.9 \times 10^{-15}$$

$$= \frac{47.42 \times 10^{-15}}{1.6 \times 10^{-11}} \text{ eV} = \boxed{0.29 \text{ MeV}}$$

5). KE = 100 GeV.
r = 600 km.

$$m_0c^2 \Rightarrow \frac{1.89 \times 10^{-28} \times 3 \times 10^8 \times 3 \times 10^8}{1.6 \times 10^{-19}}$$

$$\Rightarrow \boxed{106.3 \text{ MeV}}$$

$$T.E \Rightarrow 100 \times 10^3 \text{ MeV} + 106.3 \text{ MeV}.$$

Very small value (neglected).

$$100 \times 10^3 = \gamma_u \times 106.3$$

$$\boxed{\gamma_u = 940.7}$$

$u \simeq c$.

$$\text{Time required} = \frac{600 \times 10^3}{3 \times 10^8}$$

$$= \boxed{2 \times 10^{-3} \text{ s}}$$

6). $\text{mass} = 760 \frac{\text{MeV}}{c^2}$

π Meson $\left(140 \frac{\text{MeV}}{c^2}\right)$

$\uparrow$

$\rho = 760 \frac{\text{MeV}}{c^2}$

$\downarrow$

π Meson. $\left(140 \frac{\text{MeV}}{c^2}\right)$

$E = 760$ MeV.

$\pi \; E = 380 \text{ MeV} = \gamma_u m_0 c^2$

$380 = \gamma_u \times 140$ $\boxed{\gamma_u = 2.714}$

$7.365 = \dfrac{c^2}{c^2 - u^2}$ $\boxed{u = 0.93c}$

$V_x' = \dfrac{0.93 \times 2}{1 + 0.93 \times 0.93} \Longrightarrow \boxed{0.997c}$

Tutorial (3)

9).

A · · B

$t_1 = 0.3\,\mu s$ $t_2 = 0.4\,\mu s$

$x_1 = 150m$ $x_2 = 210m$


(S)

$\Delta x = 60$

$c\Delta t = 3 \times 10^8 \times 0.1 \times 10^{-6} = 30$

$\Delta x > c\Delta t$

(Space like separated events)

$\Delta t'$ can be $-ve \Longrightarrow 0$

$\Delta t' = 0$

$\gamma\left(\Delta t - \dfrac{V\Delta x}{c^2}\right) = 0 \Longrightarrow$ Simultaneous.

$\Delta t = \dfrac{V\Delta x}{c^2}$

$0.1 \times 10^{-6} = \dfrac{V \times 60}{9 \times 10^{16}}$ $\boxed{V = c/2}$

Events Cannot Occur at same place.

10). A $\cdot$ $\cdot$ B

$x_A = 150m$ $x_B = 210m$

$t_A = 0.3\mu s$ $t_B = 0.6\mu s.$

$\Delta x = 60$

$c\Delta t = 3\times10^{8}\times 0.3\times10^{-6} = \underline{\underline{90m}}.$

$\Delta x < c\Delta t.$

time like separated events.

$\Delta x' = 0$

$\gamma(\Delta x - v\Delta t) = 0$

$\Delta x = v\Delta t$

$60 = v\times \frac{3}{10}\times10^{-6}$

$\frac{\cancel{600}^{200}}{\cancel{3}}\times10^{6} = v$

$\boxed{v = 2\times10^{8} < c}$

$\Rightarrow$ not simultaneous.

Occur at same place.

$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$

$= 3/\sqrt{5}\left(3\times10^{-7} - \frac{2\times10^{8}\times60}{9\times10^{16}}\right)$

$\Rightarrow \boxed{0.22\mu s}$

2/2/23

Tutorial - 4 (Continue.)

7). $\overset{u \rightarrow}{\underset{m}{\cdot}}$ $\cdot M$

$\bar{P_i} = \gamma_u m_o u$

$= \gamma_u m u + 0 = \bar{P_f} = \gamma_{u_f} M_f U_f$

$\bar{E_i} = \gamma_u m c^2 + M c^2 = \bar{E_f} = \gamma_{u_f} M_f c^2$

$E^2 = p^2 c^2 + m_o^2 c^4.$

$$\frac{\bar{P_f}}{\bar{E_f}} = \frac{U_f}{c^2} = \frac{\gamma_u m u}{\gamma_u m c^2 + M c^2}$$

$$\boxed{U_f = \frac{\gamma_u m u}{\gamma_u m + M}}$$

$\gamma_u m u = \gamma_{u_f} M_f U_f$

$$M_f = \frac{\gamma_u m \cdot u}{\gamma_{u_f} \cdot U_f}$$

$$= \frac{\gamma_u m \cdot u \; (\gamma_u m + M)}{\gamma_{u_f} \cdot \gamma_u m u}$$

$$= \boxed{\frac{\gamma_u m + M}{\gamma_{u_f}}}$$

$u = 0.9c$. $\gamma u = 2.32$.
& $M = m$.

$$U_f = \frac{2.32 \times m \times 0.9c}{2.32 \times m + m}$$

$$= \frac{2.32 \cancel{m} \times 0.9c}{3.32 \cancel{m}} = \boxed{0.628c}$$

Classical :-

$$mu + 0 = (m+M)U_f$$

$$U_f = u/2 = \boxed{0.45c}$$

4/2/23

Tutorial-4 (Continue).

8).

X $(4m_e)$

e $\longrightarrow$ $\longleftarrow$ positron

(m_e) (m_e)

$p\hat{k}$ $-p\hat{k}$

(i) $\bar{P_i} = p\hat{k} - p\hat{k} = 0. = \bar{P_f}$

$\bar{P_f} = \gamma_{U_f} 4m_e . U_f$

$U_f = 0$ & $\gamma_{U_f} = 1.$

$E_f = 4m_e c^2$ (as X is in rest).

$E_i = \gamma_u m_e c^2 + \gamma_u m_e c^2.$

$= 2\gamma_u m_e c^2 = 4m_e c^2.$

$\boxed{\gamma_u = 2}$ $\boxed{u = \sqrt{3}/2\, c}$

$p = \gamma_u m_0 u$

$= 2 \times m_e \times \sqrt{3}/2\, c$

$= \boxed{\sqrt{3}\, m_e \cdot c}$

(ii) e^- is at rest and positron has momentum $q\hat{k}$.

e^- $\longleftarrow$ p

(rest). $q\hat{k}$

Actually e^- is moving but on seeing in his frame, e^- feels that it is at rest.

Speed of positron as seen by e^-'s frame,

$$U_x' = \frac{U_x - V}{1 - \frac{VU_x}{c^2}} = \frac{(-\sqrt{3}/2 - \sqrt{3}/2)c}{1 + \frac{3c^2}{4c^2}}$$

$$= \frac{-\sqrt{3} \times 4\, c}{7}$$

$$= \boxed{\frac{-4\sqrt{3}\,c}{7}}$$

$$\gamma_{u_x'} = \frac{1}{\sqrt{1 - \frac{48c^2}{49c^2}}} \qquad |U_x'| = \frac{4\sqrt{3}c}{7}$$

$$= \boxed{7}$$

$$\vec{q} = \gamma_{u_x'} \times m_e \times U_x'$$

as positron in $-x$ direction So $\vec{q}$ is $(-)$ve

$$= 7 \times m_e \times -\frac{4\sqrt{3}c}{7} \qquad |\vec{q}| = \boxed{4\sqrt{3}m_ec}$$

$$= \boxed{-4\sqrt{3}m_ec}$$

9).

B (100 MeV)

$\uparrow$

A $\cdot$ (140 MeV)

$\downarrow$

C (0 MeV).

$\vec{p_i} = 0$ (As A is in rest)

$\therefore \vec{p_f} = 0 = \vec{p_B} + \vec{p_C}$

$\vec{p_C} = -\vec{p_B} = \boxed{\dfrac{-E_C}{c}}$ — As C is in rest so, $p = E/c$

Energy Conservation $\Longrightarrow$ $140\,MeV = E_B + E_C$.

$E_B = 140 - E_C$.

$\therefore E^2 = p^2c^2 + m_0^2c^4$ (for particle B)

$(140 - E_C)^2 = \left(\dfrac{E_C}{c}\right)^2 c^2 + m_B^2 c^4$

$\therefore (140)^2 - 2(140)E_C + \cancel{E_C^2} = \cancel{E_C^2} + m_B^2 c^4$

$m_B c^2$ = rest mass energy

$\therefore (140)^2 - 280 E_C = (100)^2$

$\dfrac{(140)^2 - (100)^2}{280} = E_C$

$E_C = \boxed{34.2857 \text{ MeV}}$ (240/7 MeV).

After Decaying.

$E_B = 140 - E_C$

$= \boxed{105.7143 \text{ MeV}}$ (740/7)

Momenta of decay products $= E_C/c$.

$E_B = \gamma_u m_0 c^2$

$\frac{740}{7} = \gamma_u \times 100$

$\gamma_u = \frac{740}{700} = 1.0571$

$\Rightarrow \boxed{u = 0.32c}$

2/2/23.

Tutorial Sheet 5.

1).
n=4
n=3
n=2
n=1

$E_n = \frac{-13.6}{n^2}$ eV.

$\underset{(n=1 \text{ to } 4)}{\Delta E} = +13.6\left(1 - 1/4^2\right)$

$= \frac{13.6 \times 15}{16}$ eV

$= \boxed{12.75 \text{ eV}}$

$h\nu + E_R = \Delta E$

emission (Photon) | Kinetic energy.

$\bar{P_i} = 0$ (as hydrogen atom is intially at rest).

$= \bar{P_f} = P_{ph} + \vec{P_R} = 0.$

$\vec{P_R} = -\vec{P_{ph}} = -\frac{h\nu}{c}$ $(P = E/c)$.

$E_R^2 = P_R^2 c^2 + m_n^2 c^4$

(mass of ~~neutron.~~ hydrogen atom)

$m_n = 1.674 \times 10^{-27}$ kg.

$m_n c^2 = \dfrac{1.67 \times 10^{-27} \times 9 \times 10^{16}}{1.6 \times 10^{-19}}$

$1 eV = 1.6 \times 10^{-19} J$ $= \boxed{939.37 \text{ MeV.}}$

$1 J = \dfrac{1 eV}{1.6 \times 10^{-19}}$

$m_e c^2 = \dfrac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-19}}$

$= \boxed{0.5118 \text{ MeV}}$

$KE = \dfrac{P_R^2}{2m} = \dfrac{h^2\nu^2}{2m_n c^2}$ $\qquad h\nu << 2m_n c^2$.

also $\quad m_n c^2 >> m_e c^2$.

$h\nu + \dfrac{(h\nu)^2}{2m_n c^2} = 12.75.$

$h\nu(2m_n c^2) + (h\nu)^2 = 12.75 \times 2m_n c^2$

Very less Value.

$h\nu(\cancel{2m_n c^2}) = 12.75 \times \cancel{2m_n c^2}$

$\boxed{h\nu = 12.75 \text{ eV.}} \simeq \Delta E$

$-\vec{P_{ph}}, \vec{P_R} = \boxed{-\dfrac{12.75}{3 \times 10^8}}$

$\Delta E = \frac{1}{2} m_n v^2$

also

$$V = \frac{h\nu}{c \times m_n} = \frac{12.75 \times 1.6 \times \cancel{10^{-19}}}{3 \times \cancel{10^{8}} \times 1.67 \times \cancel{10^{-27}}}$$

$$\Longrightarrow \frac{20.4}{5.01} = \boxed{4.07 \, m/s}$$

Also, $\frac{3}{2} KT = \frac{1}{2} m v^2$

$3KT = m v^2$

$$T = \frac{mv^2}{3K} = \frac{1.674 \times 10^{-27} \times 16.564}{3 \times 1.38 \times 10^{-23}}$$

$$= \frac{27.7281 \times 10^{-27}}{4.14 \times 10^{-23}}$$

$$= 6.697 \times 10^{-4}$$

$$= \boxed{0.697 \, mK}$$

Kinetic energy <<<< 1eV

as $h\nu \simeq \Delta E$

9/2/23

Tutorial - 5 (Continue).

2) $\lambda_1 = 1850 A^\circ$ $V_1 = 4.62 V$

$\lambda_2 = 5460 A^\circ$ $V_2 = 0.18 V$.

According to Photoelectric effect,

$$KE(max) = h\nu_0 - \phi$$

$$\therefore eV_0 = h\nu_0 - \phi$$

$$h = \frac{eV_0 + \phi}{\nu_0} = \frac{(eV_0 + \phi)\lambda}{c}$$

$$\therefore h = \frac{(1.6 \times 10^{-19} \times 4.62 + \phi) 1850 \times 10^{-10}}{c}$$

$$h = \frac{(1.6 \times 10^{-19} \times 0.18 + \phi) 5460 \times 10^{-10}}{c}$$

On dividing both equations,

$$13675.2 \times 10^{-29} + 1850 \times 10^{-10} \phi$$
$$= 1572.48 \times 10^{-29} + 5460 \times 10^{-10} \phi.$$

$$\therefore \phi = \frac{12102.72 \times 10^{-29}}{3610 \times 10^{-10}}$$

$$= \frac{3.352 \times 10^{-19}}{1.6 \times 10^{-19}} eV = \boxed{2.1 eV}$$

$$\therefore h = \frac{\left(1.6\times10^{-19}\times4.62 + \overset{2.095}{} \times1.6\times10^{-19}\right)1850\times10^{-10}}{3\times10^{8}}$$

$$= \frac{6.72\times10^{-19}\times1850\times10^{-10}\times1.6}{3\times10^{8}}$$

$$= 6630.4\times10^{-37}\ J.S.$$

$$= \boxed{6.63\times10^{-34}\ J\cdot S}$$

$$6.63\times10^{-34} = \frac{\left(1.6\times10^{-19}\times4.62 + 2.1\times1.6\times10^{-19}\right)}{\nu_0}$$

$$\nu_{0_1} = \frac{10.752\times10^{-19}}{6.63\times10^{-34}}$$

$$= \boxed{1.6217\times10^{15}\ Hz}$$

$$\nu_{0_2} = \frac{3.648\times10^{-19}}{6.63\times10^{-34}}$$

$$= \boxed{0.55\times10^{15}\ Hz.}$$

(Photon)

3). X ray $\lambda = 0.1 A^\circ$ → Target e^- → Scattered Photon $(0.11 A^\circ)$, $\phi = ?$, θ, Scattered e^-

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$$

$$\underset{A^\circ}{(0.11 - 0.1)} = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos\phi)$$

$$\therefore\ 0.01\times10^{-10} = \frac{6.63\times10^{-34}}{27.3\times10^{-23}}(1-\cos\phi)$$

$$0.01\times\cancel{10^{-10}} = 0.02428\times\cancel{10^{-10}}(1-\cos\phi)$$

$$\cos\phi = 1 - \frac{0.01}{0.02428}$$

$$\cos\phi = 0.5882.$$

$$\phi = \cos^{-1}(0.5882) = \boxed{53.9^\circ}$$

4). (200 MeV)

$h\nu i \xrightarrow[\text{Photon}]{}$ proton.

Back Scattered $(\phi = 180^\circ)$.

$$KE = h\nu i - h\nu f.$$

$$\lambda f - \lambda i = \frac{h}{m_p \cdot c}(1-\cos180^\circ).$$

$$= \frac{2h}{m_p c}.$$

$$\boxed{\lambda f = \frac{2h}{m_p c} + \lambda i}$$

$$\frac{c}{\nu f} = \frac{2h}{m_p c} + \frac{c}{\nu i}$$

$$c\left(\frac{1}{\nu f} - \frac{1}{\nu i}\right) = \frac{2h}{m_p \cdot c}$$

$$1/\nu f - 1/\nu i = \frac{2h}{m_p \cdot c^2}$$

$$\frac{1}{\nu f} - \frac{1}{48.26 \times 10^{21}} = \frac{2 \times 6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 9 \times 10^{16}}$$

$$\frac{1}{\nu f} = \frac{13.26 \times 10^{-34}}{15.03 \times 10^{-11}} + \frac{1}{48.26 \times 10^{21}}$$

$$= 0.882 \times 10^{-23} + 2.072 \times 10^{-23}$$

$$= 2.954 \times 10^{-23}$$

$$\nu f = \frac{1}{2.954 \times 10^{-23}}$$

$$= \boxed{0.3385 \times 10^{23} \text{ Hz}}$$

$$\underset{(eV)}{h\nu f} = \frac{6.63 \times 10^{-34} \times 0.3385 \times 10^{23}}{1.6 \times 10^{-19}}$$

$= 1.4026 \times 10^{8}\ eV$

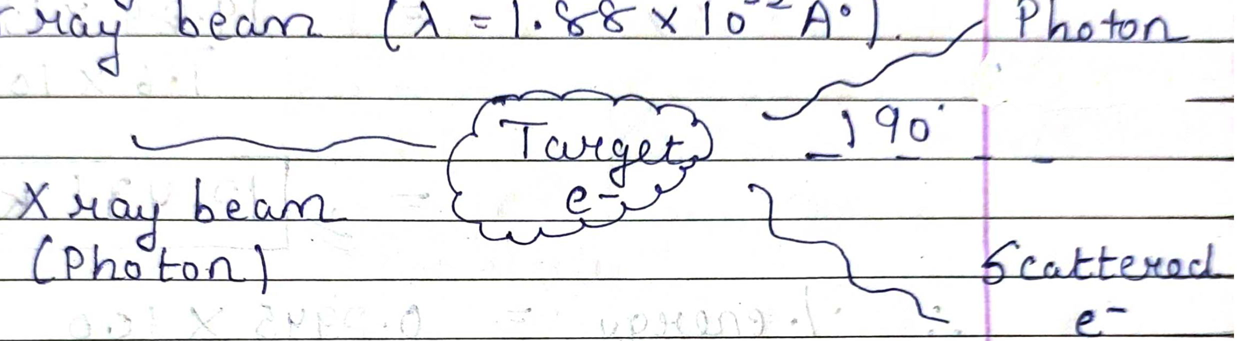
$= \boxed{140.26\ MeV}$

$KE = h\nu_i - h\nu_f$

$= 200 - 140.26 \Longrightarrow \boxed{59.74\ MeV}$

6). X ray beam ($\lambda = 1A°$)

γ ray beam ($\lambda = 1.88 \times 10^{-2} A°$).



X ray Case.

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

$$\lambda_f - \lambda_i = 0.0242\ (1-0)\ A°$$

$$\Delta\lambda = \qquad = \frac{0.0242\ A°}{1}$$

$\lambda_f =$ A°

1.0242 Compton Shift.

$$KE = h\nu_i - h\nu_f$$

$$= h(\nu_i - \nu_f)$$

$$= hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right)$$

$$= 6.63 \times 10^{-34} \times 3 \times 10^{8}\left(\frac{1}{10^{-10}} - \frac{1}{1.0242 \times 10^{-10}}\right)$$

$$= 19.89 \times 10^{-26}\left(1 \times 10^{10} - 0.9763 \times 10^{10}\right)$$

$$= \frac{19.89 \times 10^{-26} \times 0.0237 \times 10^{10}}{1.6 \times 10^{-19}}\ eV.$$

$= \boxed{0.2945 \text{ keV}}$ — Lost Energy in form of KE

Initial Energy given to e^-

$= h\nu_i$

$$= \frac{hc}{\lambda_i} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10}}$$

$$= \frac{19.89 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \boxed{12.431 \text{ keV.}}$$

$\therefore$ % energy lost in Collision $= \frac{0.2945 \times 100}{12.431}$

$= \boxed{2.37\%}$

Similarly, we can found all these values in Case 2.

γ rays

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7). $\text{Max (KE)} = \frac{m_0c^2}{2.5}$ $\qquad \phi = 180^\circ$

$KE = h\nu i - h\nu f$ $\qquad$ also $\Delta\lambda = \frac{h}{mc}(1-\cos\phi)$

$\frac{m_0c^2}{2.5} = \frac{hc}{\lambda i} - \frac{hc}{\lambda f}$ $\qquad \Delta\lambda = \frac{2h}{mc}$

$\frac{m_0c^2}{2.5} = \frac{hc}{\lambda i} - \frac{hc}{\frac{2h}{mc} + \lambda i}$ $\qquad \lambda f - \lambda i = \frac{2h}{mc}$

$\lambda f = \frac{2h}{mc} + \lambda i$

$\frac{m_0c}{2.5} = \frac{h}{\lambda i} - \frac{h(mc)}{2h + \lambda imc}$

$\frac{h}{\lambda i} = \frac{m_0c}{2.5} + \frac{h(mc)}{2h + \lambda imc}$

$\frac{h}{\lambda i} = \frac{2hm_0c + \lambda i m_0^2c^2 + 2.5hmc}{5h + 2.5\lambda imc}$

$h(5h + 2.5\lambda imc) = 2hmc\lambda i + (\lambda mc)^2 + 2.5\lambda ihmc$

$5h^2 + \cancel{2.5\lambda ihmc} = 2\lambda ihmc + \cancel{2.5\lambda ihmc} + (\lambda imc)^2$

$(\lambda imc)^2 + 2\lambda ihmc - 5h^2 = 0$

$\lambda i = \frac{-2hmc \pm \sqrt{(2hmc)^2 + 4(5h^2)(m^2c^2)}}{2m^2c^2}$

$= \frac{-2hmc \pm \sqrt{24h^2m^2c^2}}{2m^2c^2}$

$$= \frac{-2hmc \pm 4.898\,hmc}{2m^2c^2}$$

$$= \frac{4.898\,hmc - 2hmc}{2m^2c^2} \quad (\because \lambda i \text{ cannot be } -ve).$$

$$= \frac{2.898\,hmc}{2m^2C^2}$$

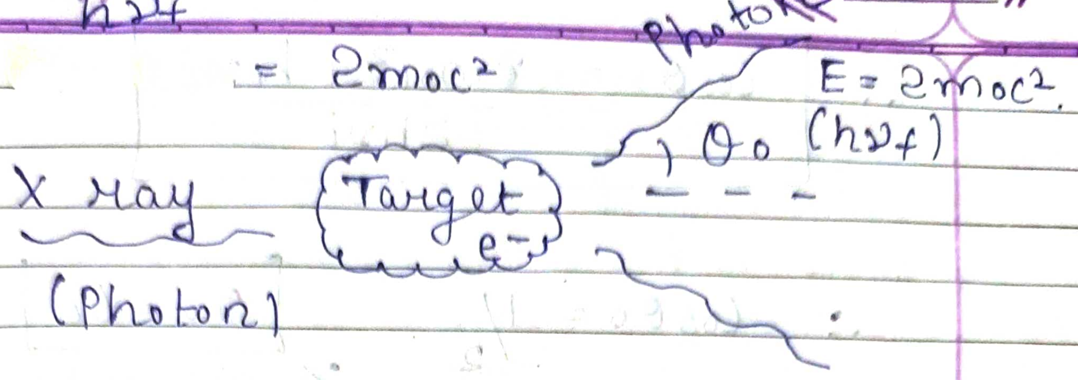
$$= \boxed{\frac{1.449\,h}{mc}}$$

$$h\nu i = \frac{hc}{\lambda i} = \frac{hc \times mc}{1.449\,h}$$

$$= \boxed{0.6901\,m_0c^2}$$

incident X ray energy

8) $h\nu_f = 2m_oc^2$



$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta_o)$$

$$\lambda f - \lambda i = \frac{h}{mc}(1-\cos\theta_o)$$

$$\frac{c}{\nu f} - \frac{c}{\nu i} = \frac{h}{mc}(1-\cos\theta_o).$$

$$\frac{1}{h\nu f} - \frac{1}{h\nu i} = \frac{1}{mc^2}(1-\cos\theta_o)$$

$$\frac{1}{h\nu f} = \frac{1}{h\nu i} + \frac{1-\cos\theta_o}{mc^2}$$

$$\boxed{h\nu f = \frac{mc^2 \cdot h\nu i}{mc^2 + h\nu i(1-\cos\theta_o)}}$$

Also, $\boxed{h\nu f = 2m_oc^2}$

$$\cancel{2m_oc^2} = \frac{h\nu i \times \cancel{mc^2}}{mc^2 + h\nu i - h\nu i\cos\theta_o}$$

$$\therefore 2m_oc^2 = h\nu i\,(2\cos\theta_o - 1).$$

$$\therefore \boxed{\cos\theta_0 = \frac{2m_0c^2 + h\nu_i}{2h\nu_i}}$$

$$\therefore \cos\theta_0 = 1/2 \qquad \boxed{\theta_0 = 60^\circ}$$

9). Bohr's condition of quantization,

$mvr = \frac{nh}{2\pi}$ & $\lambda = \frac{h}{mv}$ (de-Broglie wavelength).

$$\vec{L} = \frac{nh}{2\pi} = \vec{r} \times \vec{p} = r \times mv = mvr.$$

angular momentum (axial Vector)

$$m \cdot v \cdot r = \frac{nh}{2\pi}$$

$$2\pi r = n\left(\frac{h}{mv}\right)$$

$$\boxed{2\pi r = n\lambda}$$

This equation gives info. of Standing wave of e^- along circumference in (Bohr-Model of H-atom).

16/2/23

★ Tutorial - 5 (Continue.) :-

10). (i) $m = 2000 kg.$

$v = 100 km/h = 100 \times 5/18 \, m/s.$

$\lambda = h/p = \frac{6.63 \times 10^{-34} \times 18}{2000 \times 100 \times 5}$

$= \frac{119.34 \times 10^{-34}}{10^6}$

$= 119.34 \times 10^{-40} \, m$

$= \boxed{1.193 \times 10^{-38} m}$

(ii) $m = 0.28 kg.$

$v = 40 m/s.$

$\lambda = h/p = \frac{6.63 \times 10^{-34}}{0.28 \times 40}$

$= 0.591 \times 10^{-34}$

$= \boxed{5.9 \times 10^{-35} m}$

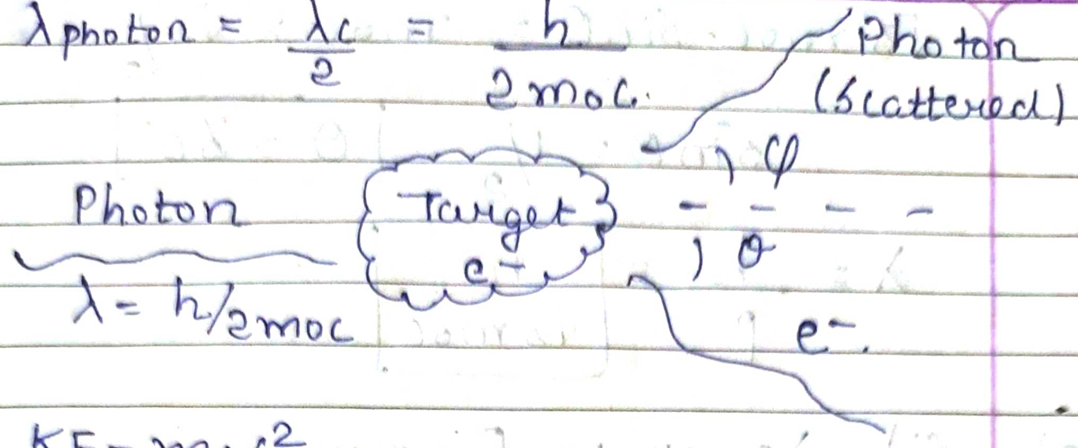
(iii) $m_e = 9.1 \times 10^{-31} kg$

$v = 10^7 m/s.$

$\lambda = h/p = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^7}$

$= \boxed{0.728 \, A^\circ}$

11). $\lambda_{photon} = \frac{\lambda_c}{2} = \frac{h}{2m_0c}$



$KE = m_0c^2$

$TE = K + m_0c^2$

$= 2m_0c^2 \; (h\nu i)$

$KE = h\nu i - h\nu f$ $\qquad \frac{hc}{\lambda} = m_0c^2$

$\underline{\underline{h\nu f = m_0c^2}}$ $\qquad \lambda_f = \frac{h}{m_0c}$

$\lambda i = \frac{h}{2m_0c}$

$\therefore \; \lambda f - \lambda i = \frac{h}{m_0c}(1 - \cos\phi).$

$\therefore \; \frac{h}{2m_0c} = \frac{h}{m_0c}(1 - \cos\phi)$

$\cos\phi = 1/2$ $\qquad \boxed{\phi = 60^\circ}$

$E_i^2 = P_i^2c^2 + m_0^2c^4$

$4m_0^2c^4 - m_0^2c^4 = Pi^2c^2$

$\boxed{Pi = \sqrt{3}\, m_0c}$ $\qquad$ (Momentum Conservation).

$\therefore \; Pi \sin\theta = \frac{m_0c^2}{c}\sin\phi.$

$\sqrt{3}\, m_0 c \cdot \sin\theta = m_0 c \times \sqrt{3}/2$

$\sin\theta = 1/2$ $\boxed{\theta = 30^\circ}$

$\lambda = \frac{h}{p} = \boxed{\frac{h}{\sqrt{3}\, m_0 c}}$

12). (i) $\lambda_{max,1} = 500\,nm$
$\lambda_{max,2} = 700\,nm.$

As, $\lambda_{max} \cdot T = b$, weins constant.

$b = 2.898 \times 10^{-3}$ metre. K.

$\therefore T_1 = \frac{b}{\lambda_{max(1)}} = \frac{2.898 \times 10^{-3}}{500 \times 10^{-9}}$

$= \boxed{5796\,K}$

$T_2 = \frac{b}{\lambda_{max(2)}} = \frac{2.898 \times 10^{-3}}{700 \times 10^{-9}}$

$= \boxed{4140\,K}$

(ii) Stefan's Law,

$e = \sigma T^4$ e = emissivity.
σ = Stefan's constant
$= 5.7 \times 10^{-8}\, W/m^2K^4$

$A e_1 = A \sigma T_1^4$ $\therefore$ $P_1 = A \sigma T_1^4$
$A e_2 = A \sigma T_2^4$ $P_2 = A \sigma T_2^4$

$$\therefore \frac{P_1}{P_2} = \frac{T_1^4}{T_2^4} = \frac{1.128 \times 10^{15}}{2.937 \times 10^{14}}$$

$$= \boxed{3.84}$$

★ Tutorial 6 :-

1). (i) $m_n = 1.67 \times 10^{-27}$ Kg.

$$KE = \frac{3}{2} KT = \frac{1}{2} m v^2$$

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

T = room temperature.
$= 25^\circ C = 298 K.$

$$KE = \frac{3}{2} \times 1.38 \times 10^{-23} \times 298$$

Boltzmann Constant

$$= \boxed{616.86 \times 10^{-23} J}$$

$$\frac{p^2}{2m} = K \qquad p^2 = 2mK$$

$$= 2 \times 1.67 \times 10^{-27} \times 616.86 \times 10^{-23}$$

$$= 2060.3124 \times 10^{-50}$$

$$= 2060.312 \times 10^{-50} \ Kg^2 m^2/s^2.$$

$$P = 45.39 \times 10^{-25} \ Kg \frac{m}{s}.$$

$$\lambda = h/p = \frac{6.63 \times 10^{-34}}{45.39 \times 10^{-25}} \frac{J.s.s}{kg\ m} = \frac{Js^2}{m.kg}$$

$$= \boxed{0.1460 \times 10^{-9}\ m}$$

$$= \boxed{1.460\ A^\circ}$$

$3A^\circ - 15A^\circ$ } Both values in A° (Same order)

∴ Diffraction by solid is possible.

(ii). $\lambda_{photon} = 1\ A^\circ$
(rest mass = 0)

$$\therefore h/p = 10^{-10}$$

$$p = \frac{h}{10^{-10}}, \quad \text{also } E = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 10^{-10}}$$

$$= 19.89 \times 10^{-16}\ J$$

$$\Longrightarrow \frac{19.89 \times 10^{-16}}{1.67 \times 10^{-19}}\ eV$$

$$\Longrightarrow \boxed{11.910\ KeV.}$$

$E^2 = p^2c^2 + m_0^2c^4 \quad (m_0 \neq 0)$.

As λ is very small, $E = p^2/2m$.

$p = \sqrt{2mE}$

$\lambda = \frac{h}{\sqrt{2mE}}$ $\quad E = \frac{hc\sqrt{2mE}}{h}$

$E = hc/\lambda$

$$E = \frac{h^2}{2\lambda^2 m_0}$$

$$E_e = \frac{6.63\times10^{-34} \times 6.63\times 10^{-34}}{2\times 10^{-20} \times 9.1\times10^{-31}} \text{ J}$$

$$= \frac{43.95 \times 10^{-68}}{18.2 \times 10^{-51}}$$

$$= 2.414 \times 10^{-17} \text{ J}$$

$$= \frac{2.414 \times 10^{-17}}{1.67\times10^{-19}} \text{ eV}$$

$$= 1.445 \times 10^2$$

$$= \boxed{144.5 \text{ eV}}$$

$$E_n = \frac{h^2}{2\lambda_n^2 m_n} = \frac{6.63\times10^{-34}\times 6.63\times10^{-34}}{2\times10^{-20}\times 1.67\times10^{-27}} \text{ J}$$

$$= \frac{43.95\times10^{-68}}{3.34\times10^{-47}} \text{ J}$$

$$= \frac{13.15 \times 10^{-21}\ eV.}{1.67 \times 10^{-19}}$$

$$= 7.874 \times 10^{-2} eV$$

$$= \boxed{0.0787\ eV}$$

(iii). $KE = 3/_2 K_B.T = \frac{hc}{\lambda}$ $\lambda = 10^{-6} m.$

$$T = \frac{2K}{3K_B.} \qquad T = \frac{2hc}{3\lambda K_B.}$$

also, $\lambda = \frac{h}{\sqrt{2mKE}}$

$$= \frac{h}{\sqrt{2m \times 3/_2 K_B T}}$$

$$\lambda = \frac{h}{\sqrt{3mK_BT}}$$

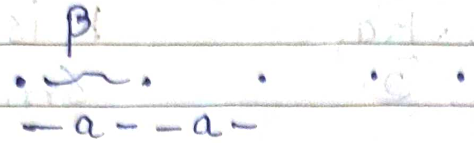
$$10^{-6} = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times T}}$$

$$10^{-12} = \frac{43.95 \times 10^{-68}}{3 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times T}$$

$$T = \frac{43.95 \times 10^{-68}}{3 \times 1.67 \times 1.38 \times 10^{-62}} \quad K.$$

$$\boxed{T = 6.356 \times 10^{-6}\ K}$$

2).



$$\omega = \omega_0 \sin\left(\frac{ka}{2}\right)$$

Also, $\omega_0 = \sqrt{\frac{4\beta}{m}}$

also, dispersion relation of photon.

$$E = h\nu = h\left(\frac{\omega}{2\pi}\right)$$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$$= \hbar\omega$$

$$P = E/c = \frac{\hbar\omega}{c} = \hbar k$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

$\lambda = h/p$ $\quad P = h/\lambda = \boxed{\hbar k}$

Also, $\omega/c = k$ $\quad \omega = kc$

Linear dispersion relation of photon.

(i) Long wavelength.

$\lambda \longrightarrow \infty$

$k = \frac{2\pi}{\lambda} \longrightarrow 0.$

$\sin\theta \simeq \theta$

when θ is very small.

$\therefore \; \omega = \omega_0 \sin\left(\frac{ka}{2}\right)$ when k is very small.

$$\omega = \omega_0 \cdot \frac{ka}{2}$$

Non dispersive as $\lambda \longrightarrow \infty$.
linear medium

(b) $V_{phase} = \omega/k$ at $k = \pi/a$

$$= \frac{\omega_0 \sin}{k}(ka/2)$$

$$= \frac{a\,\omega_0 \sin}{\pi}\left(\pi/\cancel{a} \times \cancel{a}/2\right)$$

$$= \frac{\omega_0 \cdot a}{\pi}$$

$V_{group} = \frac{d\omega}{dk}$ at $k = \pi/a$

$$= a/2 \cdot \omega \cos(ka/2)$$

$$= a/2 \cdot \omega \cos\left(\pi/a \cdot a/2\right)$$

$$= 0$$

3). $E = \gamma_u m_0 c^2 = \hbar\omega$

free particle
|
No force acting on particle.

$$\omega = \frac{\gamma_u m_0 c^2}{\hbar}$$

$$p = \gamma_u m_0 u = \hbar k.$$

$$\omega = \frac{\gamma_u m_0 c^2}{\hbar} \qquad k = \frac{\gamma_u m_0 u}{\hbar}$$

$$V_{phase} = \omega/k = \boxed{c^2/u}$$

$$\omega = \frac{m_0 c^2}{\hbar\sqrt{1-u^2/c^2}} \qquad k = \frac{m_0 u}{\hbar\sqrt{1-u^2/c^2}}$$

As $V_{group} = \frac{d\omega}{dk}$

$$= \frac{d\omega}{du} \cdot \frac{du}{dk}$$

$$= \boxed{\frac{\left(\frac{d\omega}{du}\right)}{\left(\frac{dk}{du}\right)}}$$

$$\frac{d\omega}{du} = \frac{\partial}{\partial u}\left(\frac{m_0 c^2}{\hbar}\left(1-u^2/c^2\right)^{-1/2}\right)$$

$$= \frac{m_0 c^2}{\hbar}\left(-1/\cancel{2}\left(1-u^2/c^2\right)^{-3/2} \times -\frac{\cancel{2}u}{c^2}\right)$$

$$= \frac{m_0 \cancel{c^2}}{\hbar}\left(\frac{u}{\cancel{c^2}\left(1-u^2/c^2\right)^{3/2}}\right)$$

$$= \frac{m_o u}{\hbar \left(1-u^2/c^2\right)^{3/2}}$$

$$\frac{dk}{du} = \frac{\hbar\left(1-u^2/c^2\right)^{1/2} \frac{d}{du}\left(m_o \cdot u\right) - m_o u \cdot \frac{d}{du}\hbar\left(1-u^2/c^2\right)^{1/2}}{\hbar^2 \left(1-u^2/c^2\right)}$$

$$= \frac{\hbar \cdot m_o \left(1-u^2/c^2\right)^{1/2} - m_o u \cdot \left[\hbar \frac{1}{\cancel{2}} \left(1-u^2/c^2\right)^{-1/2} \left(\frac{-\cancel{2}u}{c^2}\right.\right.}{\hbar^2\left(1-u^2/c^2\right)}$$

$$= \frac{\hbar m_o \sqrt{1-u^2/c^2} + \frac{\hbar m_o u^2}{c^2 \times \sqrt{1-\frac{u^2}{c^2}}}}{\hbar^2 \left(1-u^2/c^2\right)}$$

$$= \frac{m_o}{\hbar} \left[\frac{\left(1-u^2/c^2\right)^{1/2} + \frac{u^2}{c^2 \hbar} \left(1-u^2/c^2\right)}{\left(1-u^2/c^2\right)} \right.$$

$$= \frac{m_0}{\hbar}\left[\frac{1}{\sqrt{1-u^2/c^2}} + \frac{u^2}{c^2\hbar \times (1-u^2/c^2)^{3/2}}\right].$$

$$\therefore \frac{dw}{dk} = \frac{\dfrac{m_0 \cdot u}{\hbar\,(1-u^2/c^2)^{3/2}}}{\dfrac{m_0}{\hbar}\left[\dfrac{1}{(1-u^2/c^2)^{1/2}} + \dfrac{u^2}{c^2\hbar\,(1-u^2/c^2)^{3/2}}\right]}$$

$$= u.$$

$$= \frac{\dfrac{u}{(1-u^2/c^2)^{3/2}}}{\left[\dfrac{c^2(1-u^2/c^2)}{\hbar} + \dfrac{u^2}{c^2\hbar\,(1-u^2/c^2)^{3/2}}\right]}$$

$$= \frac{u\left(c^2\hbar\,(1-u^2/c^2)^{3/2}\right)}{(1-u^2/c^2)^{3/2}\cdot c^2\hbar} = u.$$