CAD Laboratory (CE4P001) — Assignment 1

Session: Autumn 2025

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https://github.com/Deekshansh-sapper/CAD-Laboratory-CE4P001

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Contents

1. Introduction

This report explains the assignment steps, the Julia code used for every question, and the meaning of the outputs (plots and numerical results). For each question I give:

- the underlying mathematical problem,
- a short, line-by-line explanation of the code blocks, and
- an interpretation of the generated outputs (what the plots show and how they verify the mathematics).

All plotting uses `Plots.jl`. Automatic symbolic/automatic differentiation operations use `CalculusWithJulia.jl` where noted.

2. Question 1 — Hill height scalar field

Given:

$$[\ h(x, y) = 200 - x^2 - 2y^2.\ ]$$

(a) 3D surface and 2D contour plots

Goal: Visualize the scalar field as a surface and contours.

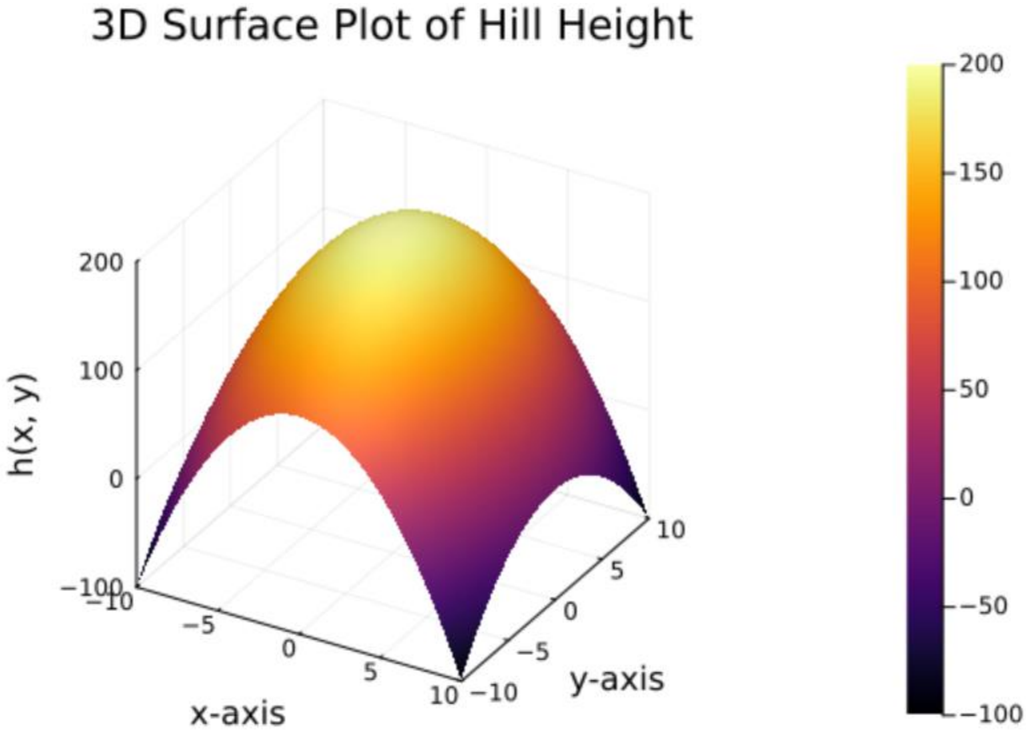
What the code does (stepwise):

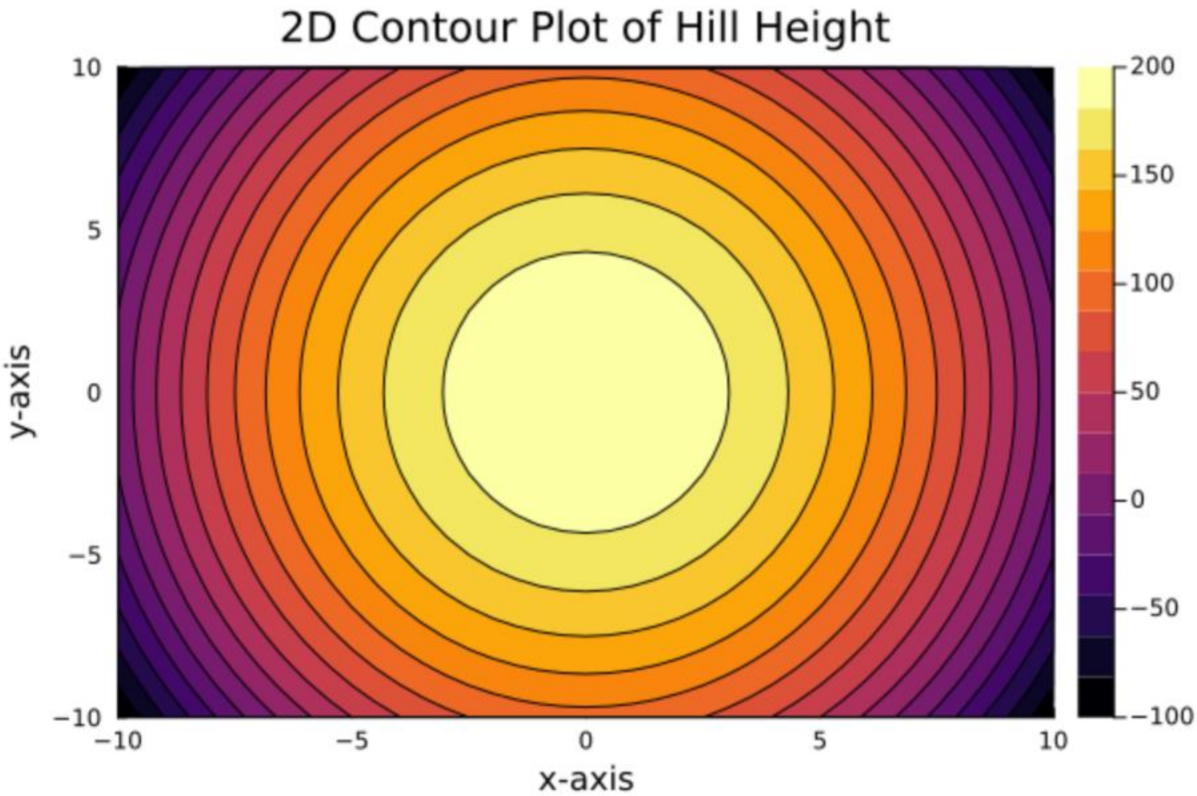
- Define the function `h(x,y)` that returns `200 - x^2 - 2y^2`.
- Create ranges for x and y (`xrange`, `yrange`), here typically from -10 to 10 with a step.
- Build a matrix `z = [h(x1,y1) for x1 in xrange, y1 in yrange]` to store values on the grid.
- Call `surface(xrange, yrange, z, ...)` to produce the 3D surface plot.
- Call `contour(xrange, yrange, z, fill=true, cbar=true, ...)` to produce the 2D contour map.

  Interpretation of outputs:
- The surface plot is an inverted elliptical paraboloid (a hill with a peak). The maximum height occurs at (0,0) with `h(0,0)=200`.
- Contour lines are concentric ellipses centered at the origin because the level sets satisfy `x^2 + 2y^2 = constant`.

- The z-axis range and camera angle affect the visual emphasis; the code sets camera = (30,30) for a clear perspective.

Mathematical check: level curves defined by x^2 + 2y^2 = C → ellipses, consistent with plots.





(b) Automatic gradient (quiver using CalculusWithJulia.jl)

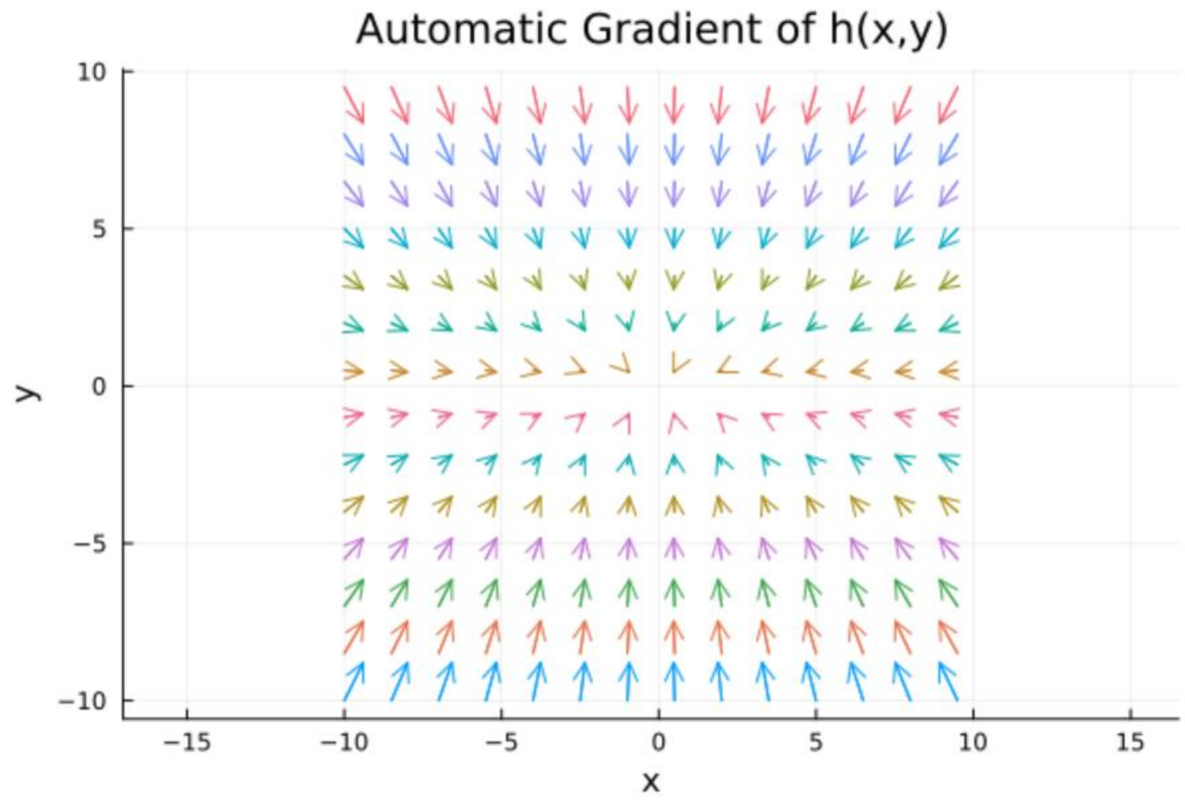
Goal: Plot the gradient vector field using an automatic gradient routine.

What the code does:

- Re-defines h accepting a vector $h(vec) = 200 - vec[1]^2 - 2 * (vec[2]^2)$ so the automatic gradient routine can call it.
- Uses `grad_h(x,y) = gradient(h,[x,y])` from `CalculusWithJulia.jl` to compute the 2-component gradient at each point.
- Builds grid arrays X and Y for plotting quiver arrows.
- Extracts components u and v from the automatic gradient and scales them for better visualization.
- Uses `quiver(X, Y, quiver=(u_scaled, v_scaled), ...)` to show arrows representing ∇h.

Interpretation of outputs:

- The gradient `∇h = (∂h/∂x, ∂h/∂y)` points in direction of maximum increase of h. For this hill:
    - `∂h/∂x = -2x`
    - `∂h/∂y = -4y`
- Arrows point toward the origin when x or y are positive (since derivatives are negative away from origin), meaning the steepest ascent from a point moves towards the peak at the origin.
- Arrow lengths reflect slope magnitude; farther from origin arrows are longer (steeper slope).



(c) Manual gradient and plotting

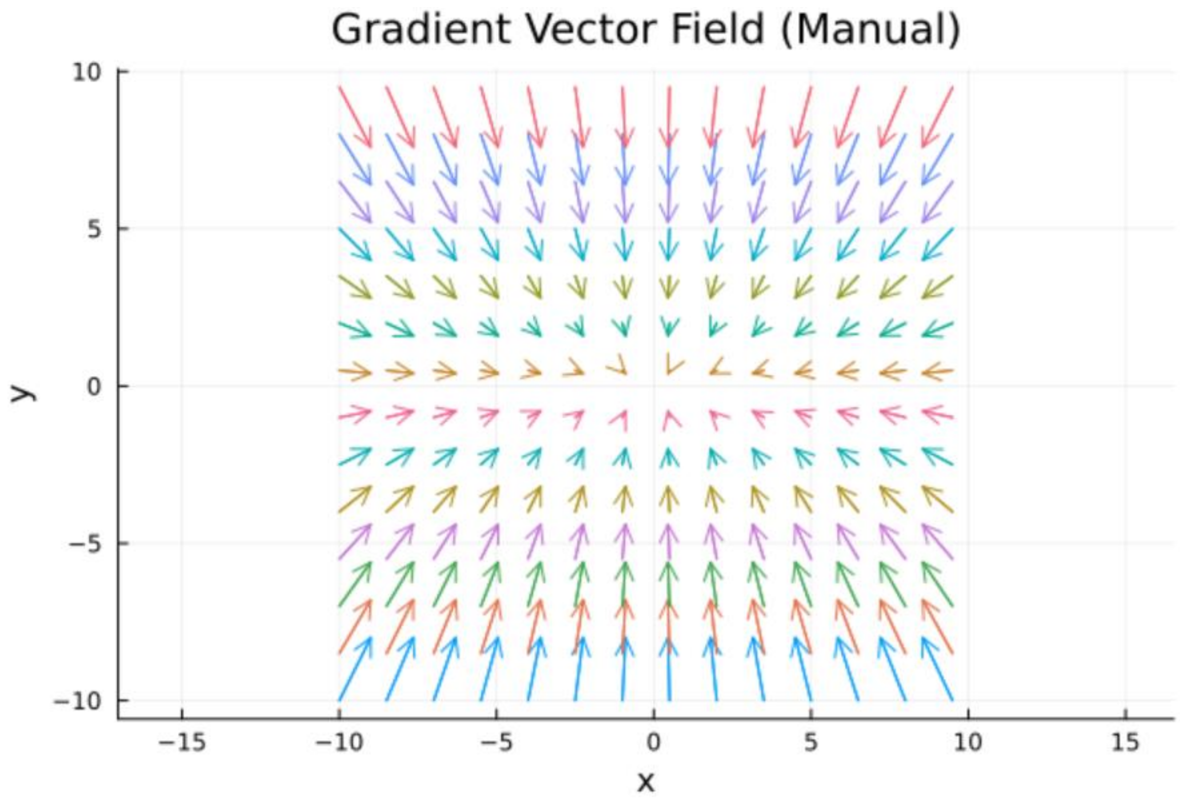
Goal: Compute the gradient analytically and plot it to compare with automatic gradient.

What the code does:

- Declares `A(x,y) = -2*x` and `B(x,y) = -4*y` which are the components of the gradient.
- Builds grid X and Y and component arrays U and V from A and B.
- Scales vectors (`U_scaled`, `V_scaled`) and uses `quiver` to plot.

Interpretation and verification:

- The manual quiver field must match the automatic quiver field from (b). They should be identical up to scaling used for plotting.
- This confirms the correctness of both the symbolic derivative of h and the autograd routine.



3. Question 2 — Cyclone vector field

Given:

$$[\, v(x, y) = x\, e_1 - y^2\, e_2. \,]$$

This represents a 2D vector field: (v(x,y) = (x, -y^2)).

(a) Plot the vector field

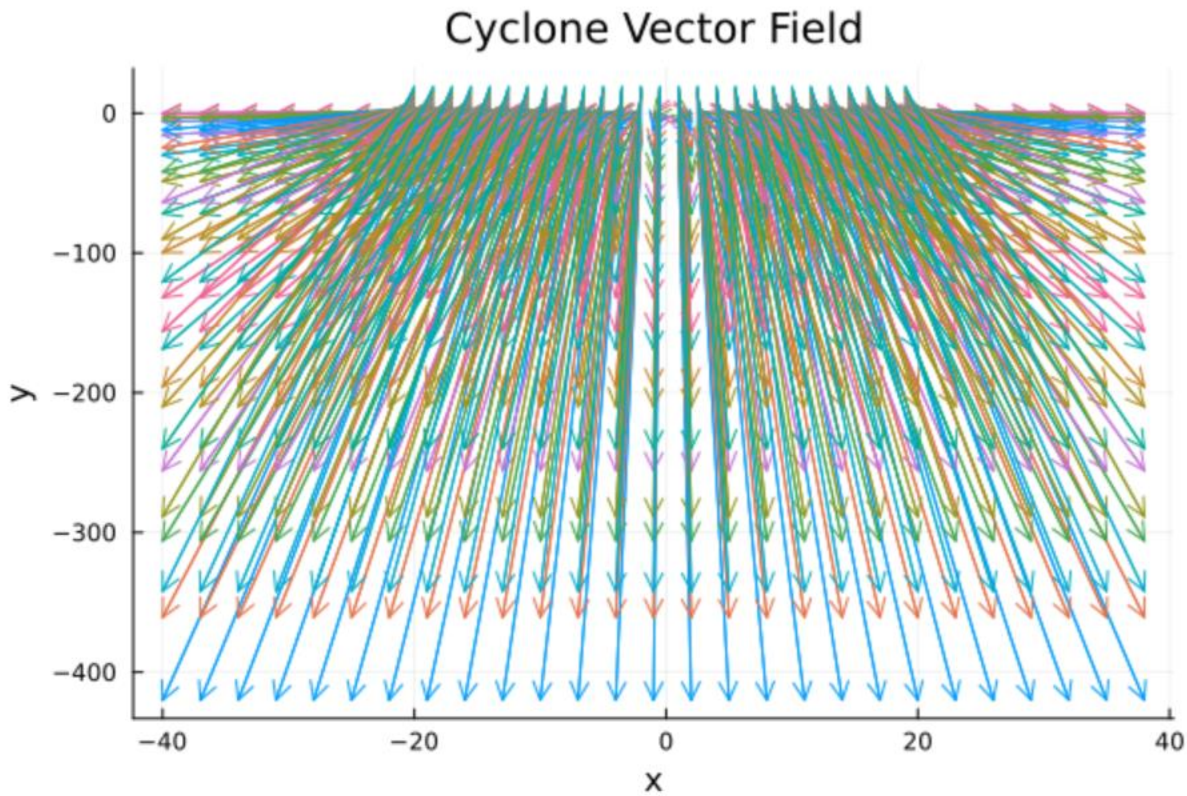
What the code does:

- Defines `vx(x,y) = x` and `vy(x,y) = -y^2`.
- Builds grid ranges (in example -20:1.5:20) and calls `quiver` to plot the vectors at each grid point.

Interpretation:

- The x-component grows linearly with x; arrows point to the right for x>0 and to the left for x<0.
- The y-component is always non-positive (`-y^2 ≤ 0`) — pointing downward or zero on the y-axis.

- Combined, this produces a flow where horizontal motion depends on x, and vertical motion always points downward with magnitude growing with |y|.



(b) Divergence — automatic and manual

Mathematical divergence:

$$\left[\nabla \cdot v = \frac{\partial}{\partial x(x)} + \frac{\partial}{\partial y(-y^2)} = 1 - 2y. \right]$$

Automatic calculation: the code defines `velocity(v) = [v[1], -v[2]^2]` and calls `divergence(velocity,[x,y])` from `CalculusWithJulia.jl`.

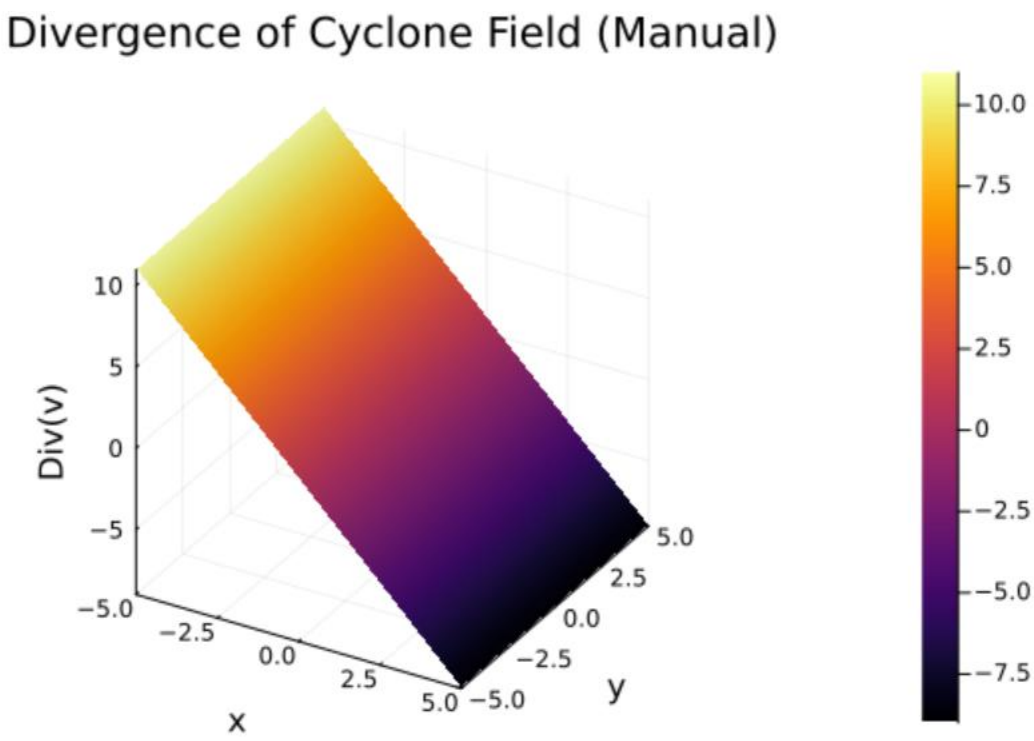
Manual calculation and plot:

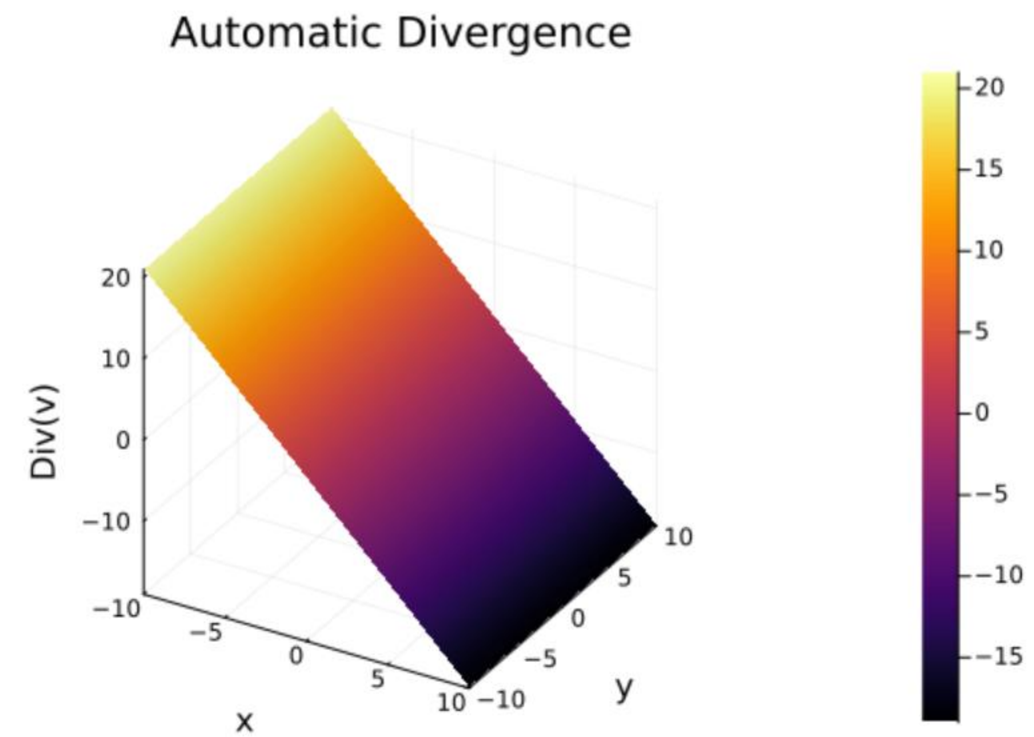
- A function `div_manualq2(x,y) = 1 - 2*y` is evaluated on a grid and surface-plotted.

Interpretation and comparison:

- Both the automatic and manual divergence surfaces should match numerically: a plane that varies linearly with y with a unit offset in x.

- Where `y < 0.5`, divergence is positive (net source); for `y > 0.5` it becomes negative (net sink). This tells how the cyclone field expands or contracts locally depending on the y-coordinate.





(c) Curl — automatic and manual

Mathematical curl (2D scalar curl / out-of-plane):

For 2D vector field (v=(v_x,v_y)) the scalar curl is

$$\left((\nabla \times v)_z = \frac{\partial\, v_y}{\partial x} - \frac{\partial\, v_x}{\partial y}. \right)$$

Here:

[$\frac{\partial v_y}{\partial x}$ = 0, \quad \partial v_x/\partial y = 0, \Rightarrow \text{curl} = 0.]

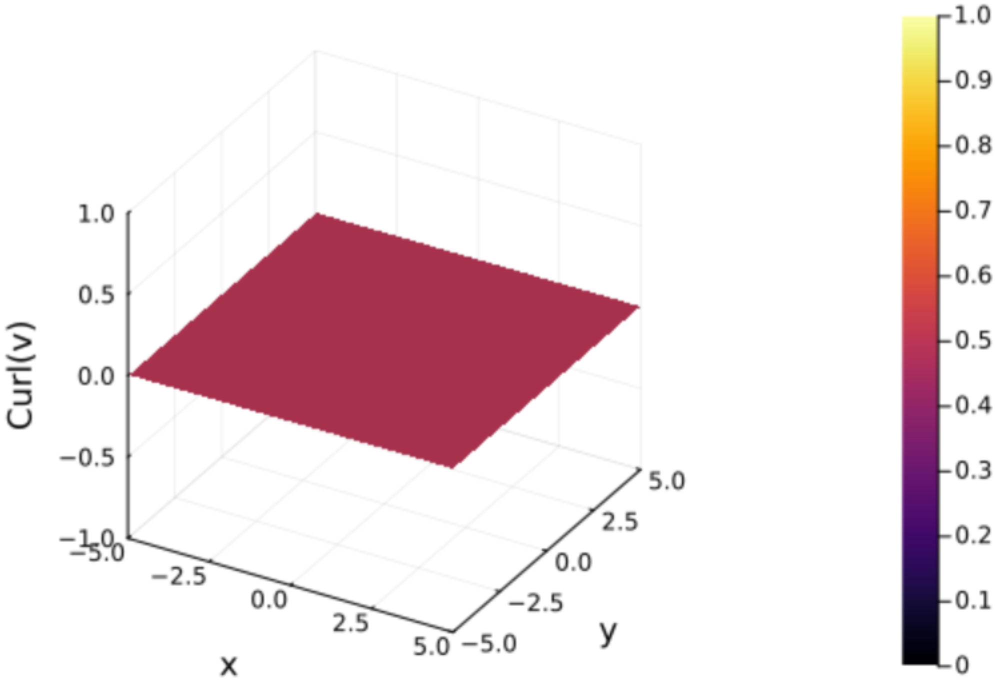
What the code does:

- Uses `curl(vel,[x,y])` to compute curl automatically (returns a scalar field for 2D case) and plots it.
- Implements `curl_manual(x,y) = 0.0` and plots a zero surface.

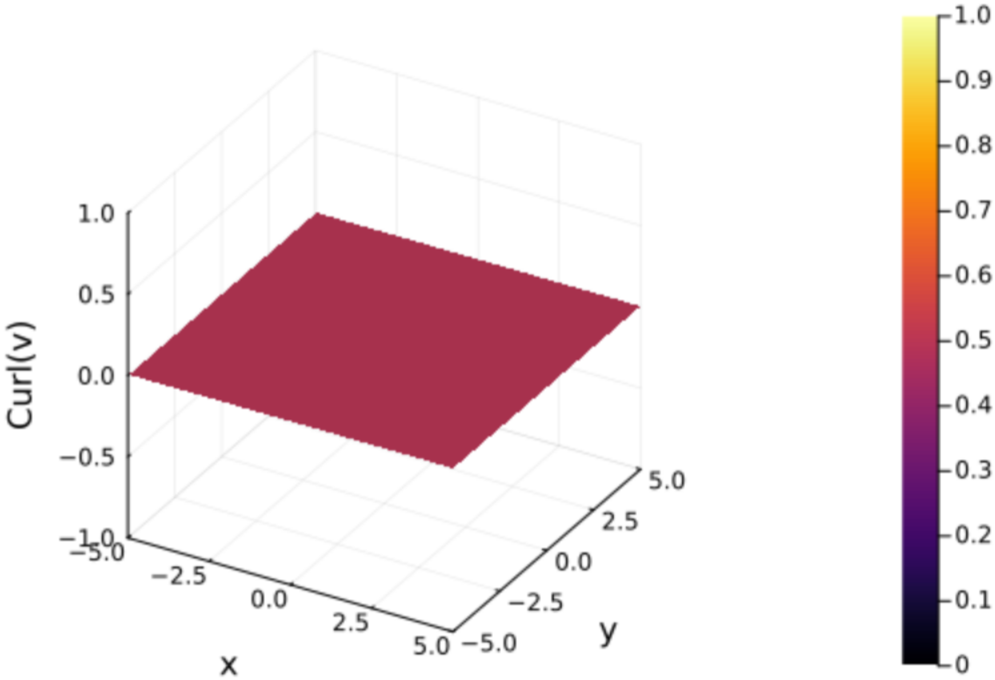
Interpretation:

- Both the automatic and manual calculations show nearly zero curl across the plane, confirming the field is irrotational (no local rotation), even though the name "cyclone" might suggest rotation — this particular model has no curl.

Automatic Curl



Curl (Manual Calculation)



4. Question 3 — River velocity field

Given:

$$\left[\, f(x,y) \;=\; e^{\{x\}}\, y^2\, e_1 + \;(x \;+\; 2y)e_2. \,\right]$$

So $\left(f = \left(e^{\{x\}}y^2,\, x + 2y\right).\, \right)$

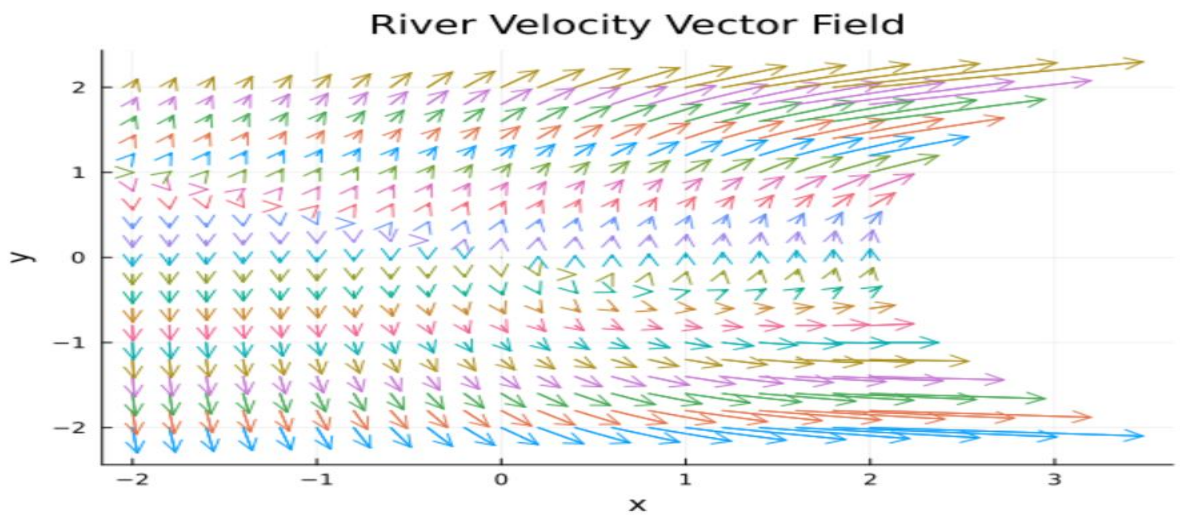
(a) Vector field plot

What the code does:

- Defines `fx(x,y)=exp(x)*y^2` and `fy(x,y)=x+2*y`.
- Builds grid and uses a scaled `quiver` to plot the flow field.

Interpretation:

- The x-component depends exponentially on x and quadratically on y — so for positive x and larger |y| the horizontal velocity grows fast.
- The y-component is linear in x and y and contributes to vertical motion.
- The pattern will show strong rightward flow in regions of positive x and significant vertical variation due to y.



(b) Divergence — automatic and manual

Mathematical divergence:

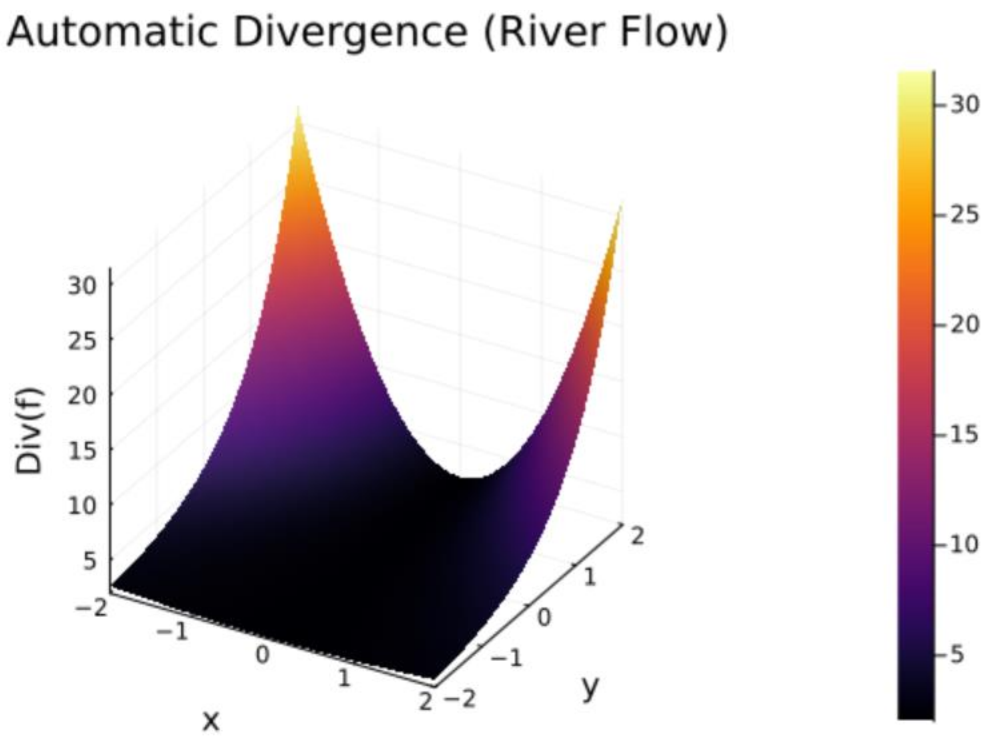
$$\left[\nabla \cdot f \ = \frac{\partial}{\partial x}\left(e^{\{x\}} y^2\right) + \frac{\partial}{\partial y}(x + 2y) = \ e^{\{x\}} y^2 + \ 2. \right]$$

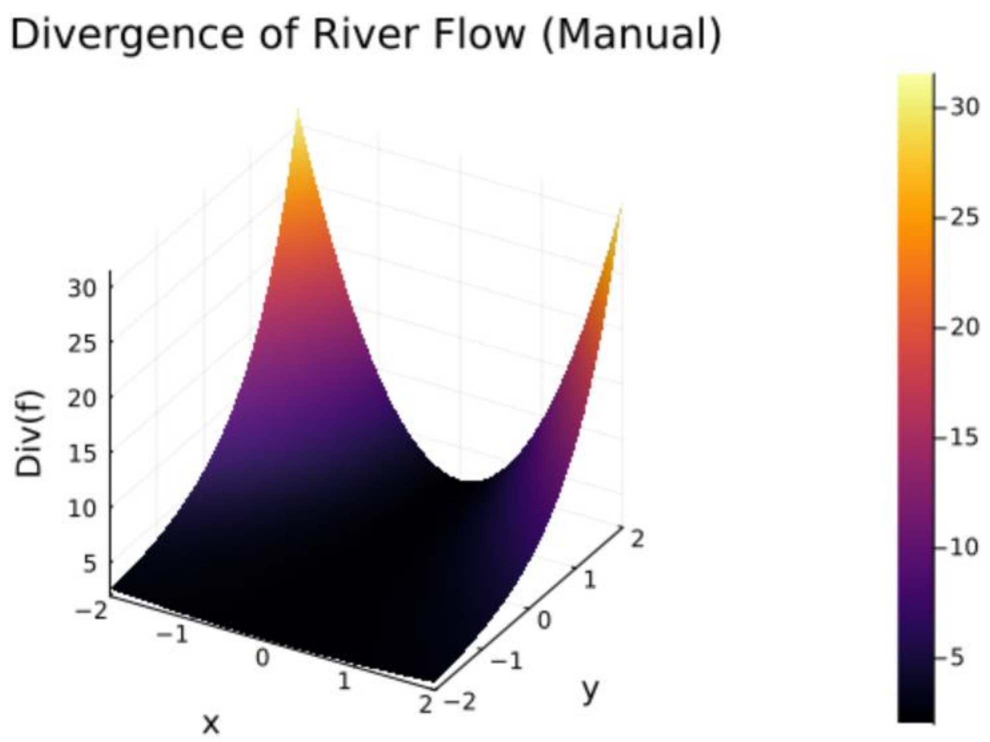
What the code does:

- Uses `divergence(f_vec, [x,y])` for automatic computation and surface plot.
- Implements manual function `div_f_manual(x,y) = exp(x)*y^2 + 2` and surface plots it.

Interpretation:

- Divergence is always shifted by +2 and increases with `exp(x)*y^2`. Regions of large positive x and large |y| show strong positive divergence (sources); negative x reduces the exponential term.
- Agreement between automatic and manual plots confirms the analytic differentiation.





(c) Curl — automatic and manual

Mathematical curl (2D scalar z-component):

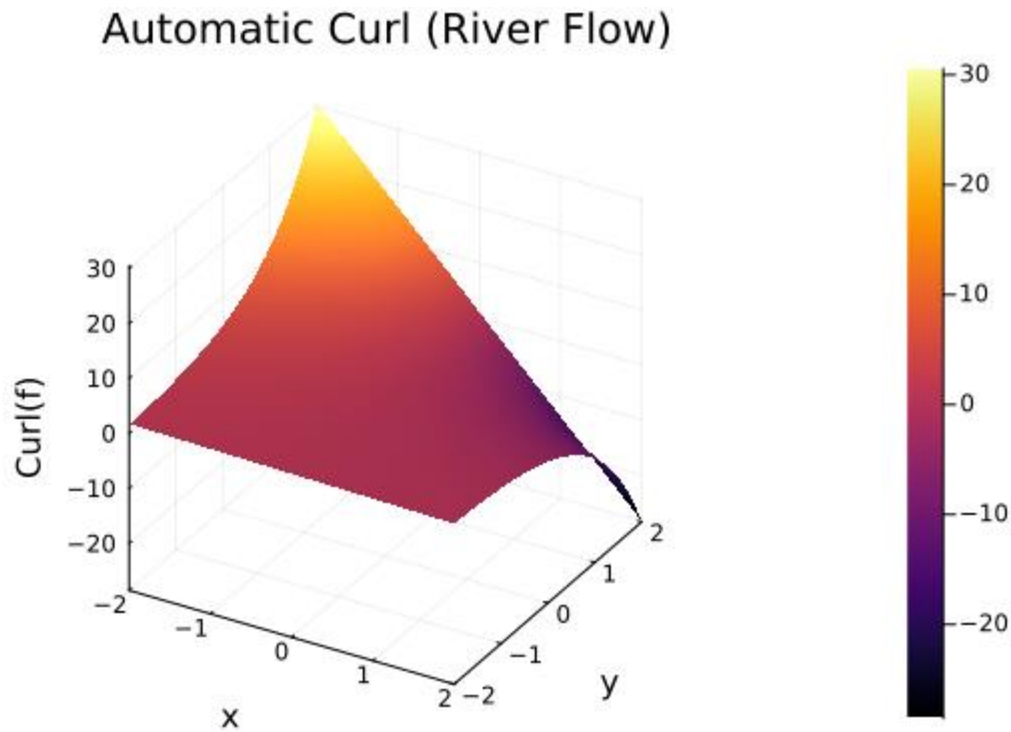
$$\left[(\nabla \times f)_z = \left[\frac{\partial}{\partial x}(x+2y), -\frac{\partial}{\partial y}\left(e^{\{x\}}y^2\right)\right] = \left[1\,, -2y\, e^{\{x\}}.\ \right] \right]$$

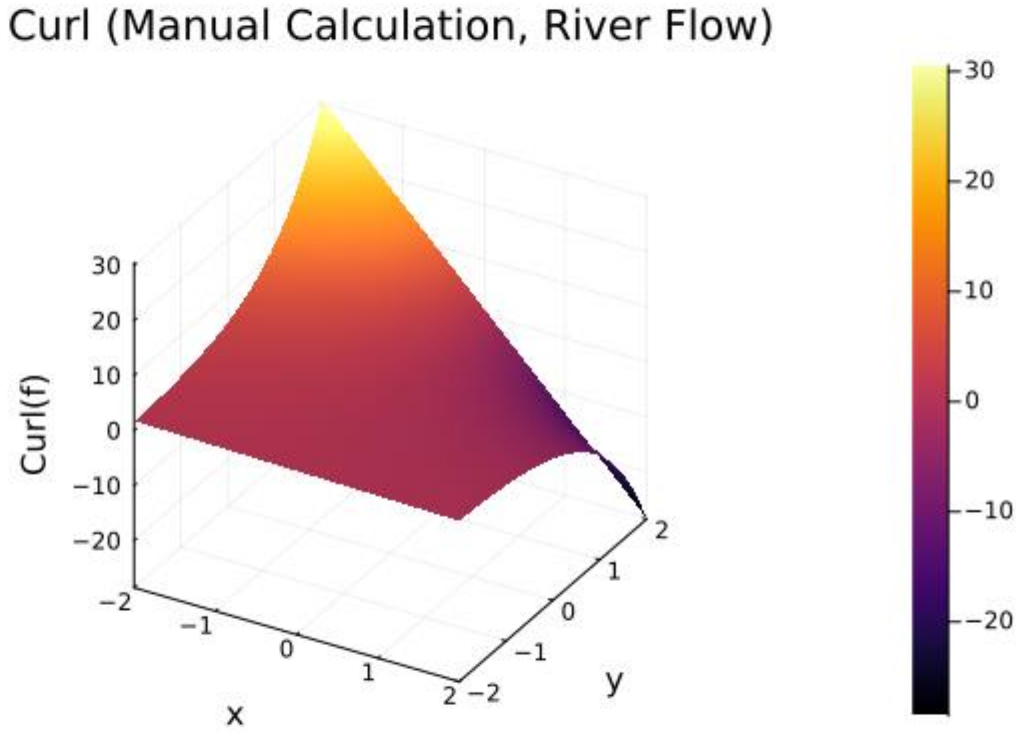
What the code does:

- Uses `curl(f_vec,[x,y])` to compute the curl automatically and creates a surface plot.
- Implements `curl_f_manual(x,y) = 1 - 2*y*exp(x)` and plots.

Interpretation:

- Curl is a function of both x and y. It generally decreases with increasing `y*e^{x}`. For small y and small/negative x, curl near 1 indicates a local clockwise/anticlockwise sense depending on sign convention; where `2y e^{x} > 1`, curl changes sign.
- Matching automatic and manual plots demonstrates correctness.


Automatic Curl (River Flow)
Curl(f)
x
y
30
20
10
0
−10
−20
−2
−1
0
1
2


Curl (Manual Calculation, River Flow)
Curl(f)
x
y
30
20
10
0
−10
−20
−2
−1
0
1
2

5. Question 4 — Beam problem (Fig 1)

Problem description: A beam of length `L_total = 1.25 * l` is loaded with a uniformly distributed load q over part or whole of beam and reactions RA and RB computed accordingly. The code is written to accept parameters `l` and q.

What the code does:

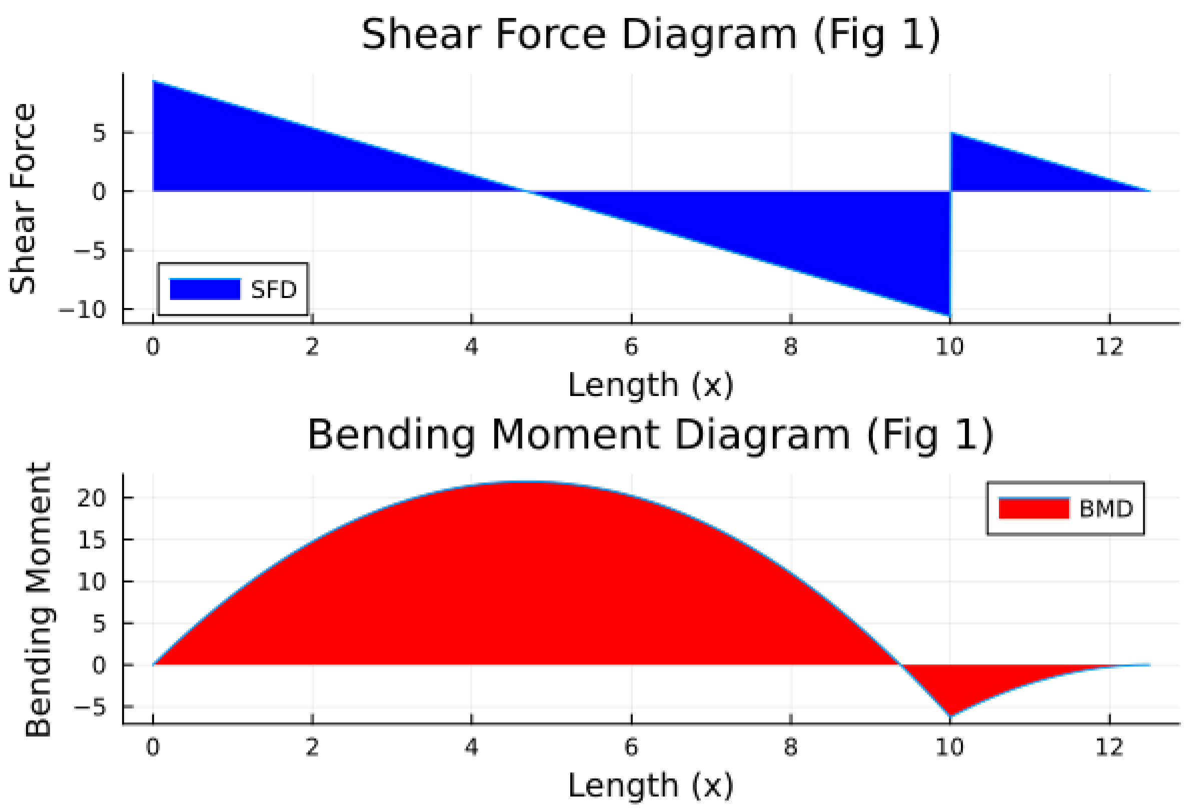
- `solve_beam_1(l,q)` constructs the geometry: `L_total = 1.25*l`.
- Reaction RB is computed by static equilibrium for the portion loaded. RA is found by subtracting RB from total vertical load.
- Defines local functions `shear_force(x)` and `bending_moment(x)` using piecewise expressions depending on x region.
- Evaluates `V_vals = shear_force.(x_vals)` and `M_vals = bending_moment.(x_vals)` on a dense x grid and plots SFD and BMD stacked vertically.

Interpretation of outputs:

- The Shear Force Diagram (SFD) shows linear segments between supports and load discontinuities caused by supports or concentrated loads.
- The Bending Moment Diagram (BMD) is quadratic in regions under uniform load (since integration of a linear shear yields a quadratic moment) and shows maxima/minima where shear crosses zero.
- The code uses fill colors to highlight area under the curves, and uses `display(plot(...,layout=(2,1)))` to show both diagrams together for easy reading.

Verification:

- Check equilibrium: `RA + RB = q * L_total` and moment equilibrium about a point should hold to numerical precision.
- Check where shear zero-crossing occurs; that x is where bending moment reaches an extremum.



6. Question 5 — Beam problem (Fig 2)

Problem description: A more complex beam with a concentrated load `P = 0.8*q*l` at `x_E = 0.4*l`, an UDL of length `l_BC = x_C - x_B` between x_B and x_C, and supports at positions x_B and x_C (and other geometry as shown in the figure).

What the code does:

- Computes P, locations `x_E, x_D, x_B, x_C` as fractions of `l`.
- Computes reactions RA, RC, and RB using static equilibrium (sum of moments and forces); details shown in code formulae.
- Implements piecewise `shear_force(x)` and `bending_moment(x)` according to the intervals: before x_E, between x_E and x_B, and between x_B and x_C.
- Evaluates arrays over `x_vals` and plots SFD and BMD similarly to Q4.

Interpretation:

- The SFD shows jumps at concentrated loads (P) and ramps over UDL regions.
- The BMD shows piecewise-polynomial shapes: linear in regions with no distributed load, quadratic under UDL, and local extrema at locations where shear equals zero.

- Reaction calculations should be validated by substituting numeric l and q and verifying equilibrium.