Discrete Mathematics

1. The foundations logics and proofs
2. Sets and Relations With Matrices
3. Algorithms Induction & Recurresiation & Mathematical induction
4. Discrete probability & Advanced Counting techniques
5. Graph theory (Graphs and trees)

Unit I propositional logic and proofs

1. propositional logic
2. propositional Equalivance
3. predicates & quantifiers
4. Nested quantifiers
5. Rules of inference
6. Induction to proofs
7. proofs Methods and strategy

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propositional logic: A proposition is a deklarative statement that is either true or false but not both

Delhi is a the capital of India (T)

Sachin is not a cricketer (F)

Delhi is the capital of America (F)

$2+3=5$ (T)

$2+3=7$ (F)

Not Delaritive statement like Explaning
order
request.

compound propositions: When two or more propositions are connected through various connectives is called compound propositions

(1) Negation
(2) conjunction
(3) Disjunction
(4) conditional
(5) Biconditional
(6) Tautology
(7) contridaction

To day is raining , To day is sunday

Negation: Let p is a proposition then negation of p is denoted by $\sim p$, $\neg p$

Ex: p: To is a sunday

$\sim p$: To is not a sunday

Truth table

p	$\sim p$
T	F
F	T

conjunction: let p and q be two propositions The conjunction of p and q denoted by $p \wedge q$

⇒ The conjunction $p \wedge q$ is true (T) when p and q are true.

otherwise it is false (F)

P	Q	$P \wedge Q$
B T	BG T	T
T	F	F
F	T	F
F	F	F
Bride	Bride Groom	

Ex p: To is a Sunday

q: it is raining day

To day is a sunday and it is raining day disjunction

let p, q be two propositions

The disjunction of p and q be denoted by

$p \vee q$

or

→ The disjunction of $p \vee q$ is false when p & q are false otherwise it is true

p	q	$p \vee q$
T	F	~~F~~ T
F	T	T
T	T	T
F	F	F

Ex: p: To day is Sunday

q it is raining day

$p \vee q$: To day is Sunday or it is raining day

conditional

If p and q are any two statements then the statement "$p \to q$" which is read as "if p then q" is called conditional statement

p hypothesis q conclusion

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

→ only the conclusion is false then the Result is false otherwise true

Biconditional

Let p and q be propositions the Biconditional statement $p \leftrightarrow q$ is the proposition which is read as,

p if and only if q

it is also known bi implification

$p \leftrightarrow q$ is true when p and q have the same truth value otherwise it false

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

conjunction $\wedge$ disjunction $\vee$ conditional $\rightarrow$ Biconditional

P Bride q-Bride groom

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p=left eye q=Right eye

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p= question paper q>Answer

p= +
q= −

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	F	T
F	T	T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Ex find the truth table

$p \wedge (p \vee q)$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

(ii) $p \wedge (\sim p \vee q)$

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \wedge q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

(iii) $\sim p \wedge (\sim p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim p \wedge (\sim p \wedge \sim q)$
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

(vi) $\sim p \wedge (\sim p \vee \sim q)$

find the truth

$p \wedge (p \Rightarrow q)$

p	q	$p \Rightarrow q$	$p \wedge (p \Rightarrow q)$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

$(p \wedge q) \Leftrightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \Leftrightarrow p \vee q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T

1. $(p \Rightarrow q) \vee (q \Rightarrow p)$
2. $(p \wedge q) \Leftrightarrow (p \vee q)$
3. $(\sim p \Rightarrow \sim q) \wedge (p \Rightarrow q)$
4. $[p \Rightarrow (p \vee q)] \Rightarrow [p \wedge (p \Rightarrow q)]$

(3) propositional Equivalence (Imp)

1. tautalogy
2. contridiction
3. contingency
4. logical Equivalence

tautology A compound proposition That is always true

$(p \vee q) \vee (p \Leftrightarrow q)$; TO

p	q	$p \vee q$	$p \Leftrightarrow q$	$(p \vee q) \vee (p \Leftrightarrow q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

contridiction A compound proposition That is always false (F) is Called con

Ex: $p \wedge (\sim p \wedge q)$

p	q	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

contingency:

A compound proposition that is neither tautology nor a contradiction is called contingency

Ex: $(\sim p \wedge q) \vee (p \rightarrow \sim q)$

p	q	$\sim p$	$\sim p \wedge q$	$\sim q$	$p \rightarrow \sim q$
T	T	F	F	F	F
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	F	T	T

problems to verify

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$\Rightarrow$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T

logical Equivalence

The compound propositions That have Same truth Values in all possible Cases are called logical Equivalence

The compound proposition p and q are Called logical Equivalent if $p \Leftrightarrow q$ is a tautology

The logical Equivalent of a compound proposition p and q is denoted by $p \equiv q$ or $p \Leftrightarrow q$

Ex $(\sim p \wedge q) \equiv \sim(p \vee q)$

(4) logical Equivalence

(1) $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

$\therefore \sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

$\therefore \sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically E

For any three propertions p, q, r

$$[(p \vee q) \Rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \Rightarrow r)]$$

Show that $p \wedge (\sim q \vee r)$ and $p \vee (q \wedge \sim r)$ are not logicall Equallnt

prove the followiy logical Equivalence

$$[(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \wedge (r \Leftrightarrow p)] \Leftrightarrow [(p \wedge q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$$

prove that for any proposition p and q the compound proposition $p \vee q$ and $(p \vee q) \wedge (\sim p \vee \sim q)$ are logically Equivalent

Show that $p \wedge (q \vee r)$, $(p \wedge q) \vee (p \wedge r)$ are logically Equivalence

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	T	T	T
F	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F

$p \wedge (q \vee R) \Leftrightarrow (p \wedge q) \wedge (p \wedge r)$

T
T
T
T
T
T
T
T

it is tautology

(5)

F_0 is Contradiction

① law of double negation

For any proposition p

$\sim(\sim p) \Leftrightarrow p$

② Idempotent law

For any proposition p

$(p \vee p) \Leftrightarrow p$ $(p \wedge p) \Leftrightarrow p$

③ Inverse law

$(p \vee \sim p) \Leftrightarrow T_0$ $(p \wedge \sim p) \Leftrightarrow F_0$

④ Domination law

$(p \vee T_0) \Leftrightarrow T_0$ $(p \wedge F_0) \Leftrightarrow F_0$

⑤ Commutative law

$(p \vee q) \Leftrightarrow (q \vee p)$ $(p \wedge q) \Leftrightarrow (q \wedge p)$

⑥ Absorption law

$[p \vee (p \wedge q)] \Leftrightarrow p$

$[p \wedge (p \vee q)] \Leftrightarrow p$

(7) Demorgan laws

$$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$$

$$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$$

(8) Associative law:

$$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$$

$$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$$

(9) Distributive law:

$$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \quad \times$$

$$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \quad \times$$

(10)

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

$$p \vee (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

① Simplify compound propositions by law of logic

$(p \vee q) \wedge [\sim(\sim p) \wedge q]$

$\therefore$ demorgans law

$(p \vee q) \wedge [\sim(\sim p) \vee \sim q]$

$= (p \vee q) \wedge [p \vee \sim q]$ $\therefore$ Double negation law

$= (a \vee b) \wedge (a \vee c) = a \vee (b \wedge c)$

$p \vee [q \wedge \sim q]$ [$\therefore$ distributive law]

$= p \vee F_0$ (inverse law)

$= p$

② $\sim[\sim(p \wedge q) \wedge r] \vee \sim q$

$\sim p \vee \sim q$

$(p \wedge \cancel{q}) \vee q$

$\sim p \vee \sim q = \sim (p$

$\sim[\sim[(p \wedge q)) \wedge r] \wedge q$

$=$

Logical Equivalence involving Conditional Statements

$$p \to q \equiv \sim p \vee q$$

$$p \to q \equiv \sim q \to \sim p$$

$$p \vee q \equiv \sim p \to q$$

$$p \wedge q \equiv \sim(p \to \sim q)$$

$$\sim(p \to q) \equiv p \wedge \sim q$$

$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$

$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$

$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$

$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

Logical Equivalence involving Bi Conditional Statements

$$p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

$$p \Leftrightarrow q \equiv \sim p \Leftrightarrow \sim q$$

$$p \Leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

$$\sim(p \Leftrightarrow q) \equiv p \Leftrightarrow \sim q$$

(i) Show that $\sim(p \to q) \equiv p \wedge \sim q$ without constructing truth table

$\sim(p \to q)$ since $p \to q \equiv \sim p \vee q$

$\sim(\sim p \vee q)$ By De Morgan law

$p \wedge \sim q$

(ii) Show that $(\sim p \wedge (p \vee q)) \to q$ is tau[tology] without constructing truth [table]

$\underbrace{\sim p \wedge (p \vee q)}_{p} \to q$

$\sim[\sim p \wedge (p \vee q)] \vee q$

$\sim[(\sim p \wedge p) \vee (\sim p \wedge$

$\sim(F \vee (\sim p \wedge q)] \vee$

$\sim(\sim p \wedge q) \vee$

$(p \vee \sim q)$

$p \vee ($

p

$\therefore \sim p \wedge (p \vee q)$

(iii) $\sim[p \vee ($

$\sim[p \vee (\sim p \wedge q)] = \sim[(p \vee \sim p) \wedge (p \vee q)]$

$= \sim[T \wedge p \vee q]$

$= \sim(p \wedge q)$ By Demorgan law

$= \sim p \vee \sim q$

(iv) Show that $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to r) \wedge (q \to r)$

$(a \vee b) \wedge (a \wedge c)$

$a \vee (b \wedge c)$

$(\sim p \vee r) \wedge (\sim q \vee r)$

$[(\sim p \vee r) \wedge \sim q] \vee [(\sim p \vee r) \wedge r]$ distributive law

$\boxed{p \Rightarrow q \Leftrightarrow \sim p \vee q}$

(i) Show that $(p \wedge q) \rightarrow (p \vee q)$ is totology

proof

$(p \wedge q) \Rightarrow (p \vee q)$

$\because p \Rightarrow q \Leftrightarrow \sim p \vee q$

$\Leftrightarrow \sim(p \wedge q) \vee p \vee q$

$(\sim p \vee \sim q) \vee (p \vee q)$ By Demorgan's law

$(\sim p \vee p) \vee (\sim q \vee q)$ (inverse law)

$T \vee T$

T

$(\sim p \vee \sim q) \vee ($

F T

$(\sim p \vee \sim q) \vee$

$(A \cup B)$

$A \cup ($

$\sim p \vee ($

$\sim p \vee$

$\sim p$

(ii) $(p \vee q) \vee (\sim p \wedge \sim q \wedge R) \Leftrightarrow p \vee q \vee R$

$(p \vee q) \vee ((\sim p \wedge \sim q) \wedge R)$ — associative law

$\underset{a}{(p \vee q)} \underset{\vee}{\vee} (\sim \underset{b}{(p \vee q)} \underset{\wedge c}{\wedge R})$ By Demorgan's law

~~$(p \vee q)$~~ $\vee (p$

$(p \vee q \vee \sim (p \vee q)) \wedge (p \vee q \vee R)$

$T \wedge (p \vee q \vee R)$

$p \vee q \vee R$

$A \vee (B \wedge C)$

$= (A \vee B) \wedge ($

(iii) $(P \to q) \cap [\sim q \cap (R \vee \sim q)] \Leftrightarrow \sim (q \vee P)$

$(\sim p \vee q) \wedge [\sim q \wedge (R \vee \sim q)]$

$P \wedge (P \wedge q)$ absorption law

$(\sim p \vee q) \wedge [\sim q)$

$(\sim p \wedge \sim q) \vee (q \wedge \sim q)$

$(\sim p \wedge \sim q) \vee T$

$\sim (p \vee q) \vee T$

$\sim (p \vee q)$

(iv) $(P \wedge q) \vee (\sim p \wedge q) \vee (P \wedge \sim q) \equiv P \vee Q$

$(P \wedge \vee \sim p) \wedge q \vee (P \wedge \sim q)$

$(T \wedge q) \vee (P \wedge \sim q)$ (By distribution)

$q \vee (P \wedge \sim q)$ (By inverse law)

(By identity law)

$(q \vee P) \wedge (q \vee \sim q)$

$(P \vee q)$ ... (By ...

(v) $(\sim p \wedge (\sim q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

$\sim p \wedge (\sim q \wedge R) \vee (q \wedge R) \vee (P \wedge R)$

$\sim p \wedge (\sim q \wedge R) \vee (Q \vee P) \wedge R$ (distribution

$(\sim p \wedge \sim q) \wedge R \vee (Q \vee P) \wedge R$ By associative

$(\sim p \wedge \sim q) \wedge R \vee (P \vee Q) \wedge R$ By commutative

$\sim (P \vee q) \wedge R \vee (P \vee q) \wedge R$ By Demorgan

$(\sim (P \vee q) \vee P \vee q) \wedge R$

$T \wedge R$

R

(6) Show that $(P \vee q) \wedge \sim (\sim p \wedge (\sim q \vee \sim R)) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim R)$ is tautology

$\sim (\sim p \wedge (\sim q \vee \sim R))$ (Double negation & Demorgan law)

$P \vee (q \wedge R)$

Now $(P \vee q) \wedge P \vee (q \wedge R)$

$(P \vee q) \wedge (P \vee q) \wedge (P \vee R)$ associative law

$(P \vee q) \wedge P \vee q) \wedge (P \vee R)$ By idempotent)

$(P \vee q) \wedge (P \vee R)$

$(\sim p \wedge \sim q) \vee (\sim p \wedge \sim R)$

$\sim(p \vee q) \vee \sim(p \vee R)$ $\qquad \sim[(p \vee q) \wedge p \vee R]$ $\qquad (\wedge$

$(p \vee q) \wedge (p \vee R) \vee \sim(p \vee q) \wedge p \vee R)$ $\qquad (\cancel{(p \vee q) \vee \sim(p \vee q)}] \wedge p \vee R$

$[(p \vee q) \vee \sim(p \vee q)] \wedge (p \vee R \vee \sim p \vee R)$ $\qquad (\wedge$

$T \wedge T = T$

(7) $p \to (q \to R) \Leftrightarrow (p \wedge q) \to R$

$p \to q \Leftrightarrow \sim p \vee q$

$p \to (q \to R) \Leftrightarrow \sim p \vee (q \to R)$

$\Leftrightarrow \sim p \vee (\sim q \vee R)$ By associative law

$\Leftrightarrow (\sim p \vee \sim q) \vee R$

$\Leftrightarrow \sim(p \wedge q) \vee R$ By Demorgan law

$\Leftrightarrow (p \wedge q) \to R$

(8) $(p \to r) \wedge (q \to r) \Leftrightarrow (p \vee q) \to r$

$\Leftrightarrow (p \to r) \wedge (q \to r)$ $\qquad p \to q \Leftrightarrow \sim p \vee q$

$\Leftrightarrow (\sim p \vee r) \wedge (\sim q \vee r)$ By ~~associative~~ Distributive law

$\Leftrightarrow (\sim p \wedge \sim q) \vee r$

By Demorgan law

$\Leftrightarrow \sim(p \vee q) \vee r$

$\Leftrightarrow (p \vee q) \to r$

(9) $p \to (q \to p) \Leftrightarrow \sim p \to (p \to q)$

$p \to (q \to p) \Leftrightarrow \sim p \vee (q \to p)$

$\sim p \vee (\sim q \vee p)$ By associative

$\cancel{(\sim p \vee \sim q)} \vee p$

$\sim p \vee (p \vee \sim q)$ $A \vee (B \cup C)$

$(\sim p \vee p) \vee \sim q$ $(A \cup B) \cup C$

$T \vee \sim q$

$\Rightarrow T$

$\sim (\sim p \to (p \to q)$

$\sim (\sim p \vee p \to q$

$\sim (\sim p \vee (\sim p \vee q)$

$(\sim p \vee \sim p) \vee q$ $p \vee p = p$

$\sim p \vee q$

$= T$

Show that $(\sim p \wedge (p \vee q)) \to q$ is tautology without constructing truth table

$\sim (p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$

(I)

$\sim(p \to q) \equiv p \wedge \sim q$

$\sim(p \to q) \Leftrightarrow \sim(\sim p \vee q)$

$= \underline{p \wedge \sim q}$

(II) $[\sim p \wedge (p \vee q)] \to q$

$p \to q \Leftrightarrow \sim p \vee q$

~~$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$~~

$(\sim p \wedge p) \vee (\sim p \wedge q)$

$F \vee (\sim p \wedge q)$

$\sim p \vee q$

$\sim p \wedge q$

$\sim[\sim p \wedge (p \vee q)] \vee q$

$[p \vee \sim(p \vee q)] \vee q$

$(\cancel{p \vee \sim p}) \vee q$

$(p \vee (\sim p \wedge \sim q)) \vee q$

$A \cup (B \wedge C)$

$(A \wedge B) \vee C$

$(A \vee C) \wedge (B \vee C)$

$(p \vee \sim p) \wedge (p \wedge \sim q) \vee q$

$T \wedge (p \wedge \sim q) \vee q$

$(p \wedge \sim q) \vee q$

$(A \wedge B) \vee C$

$(A \vee C) \wedge (B \vee C)$

$(p \vee q) \wedge (\sim q \vee q)$

$(p \vee q) \wedge T$

$= T$

$(\sim p \wedge (p \vee q)) \to q$

$\sim((\sim p \wedge (p \vee q)) \vee q$

$\sim((\sim p \wedge p) \vee (\sim p \wedge q)) \vee q$

$\sim(F \vee (\sim p \wedge q)) \vee q$

$\sim((\sim p \wedge q)) \vee q$

$(p \vee \sim q) \vee q$

$p \vee (\sim q \vee q)$

$p \vee (T)$

$= \underline{T}$

$\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim q$

$\sim((p \vee \sim p) \wedge (p \vee q))]$

$\sim((T) \wedge (p \vee q))]$

$F \vee \sim(p \vee q)$

$F \vee \sim p \wedge \sim q$

$\underline{\sim p \wedge \sim q}$

Rules of inference

→ Argument is a sequence of statements that end with conclusion

→ A sequence of statements is an argume... are called premises

→ Argument is said to be valid when all the premises are true followed by conc... statement is true

→ The Reasoning used in deriving ... conclusion from premises follows certain accepted rul... of logical inferen...

→ Any conclusion which is arrived followin...

$\Rightarrow$ $P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_n \Rightarrow C$ is an argument

Here $P_1, P_2 \cdots P_n$ are premises of argument ; C is called conclusion of argument

$\rightarrow$ it is practice to write above argument as follows

$$\begin{array}{c} P_1 \\ P_2 \\ \vdots \\ P_n \\ \hline C \\ \hline \end{array}$$

Introduction to Inference theory

consider the following argument and conclusion

(I)

(a) If vijai solved seven problem correc[tly] then vijai obtains Grade A

(b) vijai solved seven problems correctly

(c) There fore vijai obtained the Grade A

Discussion

(a), (b) are premises and (c) is conclu[sion]

Let P : vijai solved seven problems cor[rectly]

Q : vijai obtained Grade A

(a) is denoted $P \Rightarrow Q$

(b) is " P

...validity of argument

There are two methods

(i) Truth table

(ii) Rule of inference

(Tautological inference)

Validity of an argument through truth table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

when $p \to q$ and p are true then q is a...

then the argument

$$\begin{array}{c} p \to q \\ p \\ \hline \therefore q \end{array}$$

is valid

Determine whether conclusion follows logically from premises H_1 and H_2

(a) $H_1: p \rightarrow q$

$H_2: p$

$\therefore c: q$

Argument is Valid
check Truth table

(b) $H_1: p \rightarrow q$

$H_2: \sim p$

$\therefore c: q$

Argument is in Valid

(c) $H_1: p \rightarrow q$

$H_2: \sim (p \wedge q)$

$\therefore c: \sim p$

Argument is Valid

(d) $H_1: \sim p$

$H_2: p \rightarrow q$

$c: \sim (p \wedge q)$

Argument is V

(e) $H_1: p \rightarrow q$

$H_2: q$

$\therefore c: p$

Argument is in Valid

p	q	$p \rightarrow q$	$\sim p$	$p \wedge q$	$\sim (p \wedge q)$
T	T	T	F	T	F
T	F	F	F	F	T
F	T	T	T	F	T
F	F	T	T	F	T

Note: it is possible to draw truth table when no of atomic variable in premises and conclusion is small

⇒ When atomic variable in premises and conclusions is large it would be difficult to draw truth table

Hence we use Rules of inference (Tautological Implication) to check validity of argument

I_1: $p \wedge q \Rightarrow p$ (conjunctive simplification)

i.e if $p \wedge q$ is true then p is true

I_2 $p \Rightarrow p \vee q$ (Disjunctive Amplification)

I_3 $p \to q, q \to r \Rightarrow p \to r$ (Hypothetical syllogism)

i.e
$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

I_4 Modus pones

$p \to q$ and $p \Rightarrow q$

i.e
$$\begin{array}{l} p \\ p \to q \\ \hline \therefore\ q \end{array}$$

I5 Modus Tollens

$$p \Rightarrow q$$
$$\sim q$$
$$\therefore \sim p$$

$(p \Rightarrow q) \wedge (\sim q) \Rightarrow \sim p$

I6 Rule of Disjunctive syllogism

$$p \vee q$$
$$\sim p$$
$$\therefore q$$

i.e. $(p \vee q) \wedge (\sim p) \Rightarrow q$

) Rule of Contradiction

$$\sim p \rightarrow F_0 \Rightarrow p$$

$$\sim p \rightarrow F_0$$
$$\therefore p$$

Test whether the following argument is valid

If I drive to work then I will arrive tired

I am not tired (when I arrive at work)

$\therefore$ I dont drive to work

p: I drive to work

Q: I ~~will~~ arrive tired

$$p \rightarrow q$$
$$\sim q$$
$$\sim p$$

- Modus tollens rule Thus it is valid

(1) Test whether the following is valid argument

If Sachin hits a century then he gets a free car

Sachin hits a century

$\therefore$ Sachin gets a free car

Sol Let p: Sachin hits century

q: Sachin get free car

then the argument reads

$$p \to q$$

$$p$$

$$\therefore q$$

By Modus ponens Rule this is valid argument

(2) If a Sachin hits a century then he gets a free car

Sachin doesn't get a free car

$\therefore$ Sachin has not hit a century

$$\sim p \vee q$$

$$\sim q$$

$$p \to q$$

$$\sim q$$

$$\sim p$$

By Modus Tollens the argument is valid.

(3) Test whether the following is a Valid argument

If Sachin hits a century then he gets a free car

Sachin gets a free car

$\therefore$ Sachin has hit a century

$$\begin{array}{l} p \to q \\ \quad q \\ \hline \therefore p \end{array}$$

Argument is Invalid

If $p \to q$ and q is true

T	T	T
T	F	F
F	T	T
F	F	T

$\sim p \vee q$

Determine whether the conclusion C follows logically from the given premises

$H_1 : p \to q$

$H_2 : p$ $\to$ Valid conclusion

$C : q$

$H_1 : p \to q$

$H_2 : q$

$C : p$

it is not a valid conclusion

$H_1 : p \to q$ $H_2 : \sim p$

$C : q$

it is not valid conclusion

$H_1 : \sim q$

$H_2 : p \to q$

$C : \sim p$

it is Valid conclusion

$H_1 : \sim p$ $H_2 : p \Leftrightarrow q$

$C : \sim (p \wedge q)$

it is a Valid conclusion

→ Test whether the following is a Valid argument

If I study, then I donot fail in the Exam

If I dont fail in the Exam, my father gifts a two-wheeler to me

---

∴ If I Study then father gifts a two wheeler to me

p: I Study q: I dont fail in the Exam

r: My father gifts a two wheeler to me

The argument read as

$p \to q$

$q \to r$

$\therefore p \to r$ is the Rule of Syllogism

This is Valid argument

→ test the Validity of the following argument

If I study, I will not fail in the Exam

If I donot Watch TV in the evening, I will study

I failed in the Examination

Inference Rules:

I_1 $P \wedge Q \Rightarrow P$ Conjunctive Simplification

I_2 $P \wedge Q \Rightarrow Q$ disjunction amplification

I_3 $P \Rightarrow P \vee Q$

I_4 $Q \Rightarrow P \vee Q$

I_5 $\sim P \Rightarrow P \rightarrow Q$

I_6 $Q \Rightarrow P \rightarrow Q$

I_7 $\sim(P \rightarrow Q) \Rightarrow P$

I_8 $\sim(P \rightarrow Q) \Rightarrow \sim Q$

I_9 $P, Q \Rightarrow P \wedge Q$ (Conjun…

I_{10} $\sim P, P \vee Q \Rightarrow Q$ (dis…

I_{11} $P, P \rightarrow Q \Rightarrow Q$ (Mod…

I_{12} $\sim Q, P \rightarrow Q \Rightarrow \sim P$

I_{13} $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$

I_{14} $P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$

Rules of inference

Rule P: A premise may be introduced at any point in the derivation

Rule T: A formula 'S' may be introduced in a derivation if S is tautologically implied by any one or more of preceding formulas derivation

Note: The truth table technique becomes difficult when the no of statement variables present in all the formulae representing the premises and conclusion is large.

$p \to q \Rightarrow \sim q \to \sim p$

Syllogism

ponens

tollens

(Hypothetical syllogism)

(Dilemma)

Equivalence Implication

E_1 : $\sim(\sim p) \Leftrightarrow p$

E_{10} : $p \vee p \Leftrightarrow p$

E_{11} : $p \wedge p \Leftrightarrow p$

E_{16} : $p \to q \Leftrightarrow \sim p \vee q$

E_{17} : $\sim(p \to q) \Leftrightarrow p \wedge \sim q$

E_{18} : $p \to q \Leftrightarrow \sim q \to \sim p$ (contra positive)

E_{19} : $p \to (q \to R) \Leftrightarrow (p \wedge q) \to R$

E_{21} $p \rightleftarrows q \Leftrightarrow (p \to q) \wedge (q \to p)$

E_{21} $p \rightleftarrows q \Leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$

① Demonstrate that r is valid inference from the premises $p \to q$, $q \to r$ and p

{1}	(1)	p	Rule P
{2}	(2)	$p \to q$	Rule P
{1,2}	(3)	q	Rule T (1),(2) $p, p \to q \Rightarrow q$
{4}	(4)	$q \to r$	Rule P
{1,2,4}	(5)	r	Rule T (3),(4) $p, p \to q \Rightarrow q$

(11) Show that $r \vee s$ follows from the premises

$c \vee d$, $c \vee d \rightarrow \sim h$,

$\sim h \rightarrow (a \wedge \sim b)$

$(a \wedge \sim b) \rightarrow r \vee s$

P

$P \rightarrow Q$

{1}	(1)	$c \vee d$	Rule P	
{2}	(2)	$c \vee d \rightarrow \sim h$	Rule P	
{1,2}	(3)	$\sim h$	Rule T (1),(2)	$P, P \rightarrow Q \Rightarrow Q$
{4}	(4)	$\sim h \rightarrow (a \wedge \sim b)$	Rule P	
{1,2,4}	(5)	$(a \wedge \sim b)$	Rule T	$P, P \rightarrow Q \Rightarrow Q$
{6}	(6)	$(a \wedge \sim b) \rightarrow r \vee s$	Rule P	
{1,2,4,6}	(7)	$r \vee s$	Rule T (5),(6)	$P, P \rightarrow Q \Rightarrow Q$

$r \vee s$ is logically valid conclusion

(iii) Show that $s \vee r$ is Tautologically implied by

$$(p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow s)$$

{1}	(1)	$p \vee q$	Rule p		
{1}	(2)	$\sim p \Rightarrow q$	Rule T		$p \Rightarrow q \Leftrightarrow \sim p \vee q$ $= $ $\sim p \Rightarrow q$ $\Rightarrow \sim(\sim p) \vee q$ $= p \vee q$
{3}	(3)	$q \Rightarrow s$	Rule p		
{1, 3}	(4)	$\sim p \Rightarrow s$	Rule T	(2) (3)	$p \Rightarrow q,\ q \Rightarrow r$ $\Rightarrow p \Rightarrow r$
{1, 3}	(5)	$\sim s \Rightarrow p$	Rule T		$p \Rightarrow q \Leftrightarrow \sim q \Rightarrow \sim p$
{6}	(6)	$p \Rightarrow r$	Rule p		
{1, 3, 6}	(7)	$\sim s \Rightarrow r$	Rule T	(5), (6)	$p \Rightarrow q,\ q \Rightarrow r \Rightarrow p \Rightarrow r$
{1, 3, 6}	(8)	$s \vee r$	Rule T	(7),	$p \Rightarrow q \Rightarrow \sim p \vee q$ $\sim s \Rightarrow r$ $= \sim(\sim s) \vee r$

(14) Show that $r \wedge (p \vee q)$ is valid conclusion from the premises $p \vee q$, $q \rightarrow r$, $p \rightarrow m$, and $\sim m$

$\sim q$

$\sim p \vee q$

{1}	(1)	$\sim m$	Rule P	
{2}	(2)	$p \rightarrow m$	Rule P	
{1,2}	(3)	$\sim p$	Rule T	$\sim q$, $p \rightarrow q \Rightarrow \sim p$
{4}	(4)	$p \vee q$	Rule P	
{1,2,4}	(5)	q	Rule T	$\sim p$, $p \vee q \Rightarrow q$
{6}	(6)	$q \rightarrow r$	Rule P	
{1,2,4,6}	(7)	r	Rule T	p, $p \rightarrow q \Rightarrow q$
{1,2,4,6}	(8)	$r \wedge (p \vee q)$		$p, q \Rightarrow p \wedge q$

Example problem on Rule - C.P
(conditional proof)

(1) Show that $R \Rightarrow S$ can be derived from the premises
$P \Rightarrow (Q \Rightarrow S)$, $\sim R \vee P$, Q

LHS
$R \Rightarrow S$ RHS

R can be considered as additional premises

This additional premises and given premises should produce S

{1} (1) R Rule-P (Additional premise)

[2] (2) $\sim R \vee P$ Rule - P

[2] (3) $R \Rightarrow P$ Rule T, (2) and E16. $\sim R \vee P \Leftrightarrow R \Rightarrow P$ (E16)

[1, 2] (4) P Rule T (1)(3) and I_{11} $P, P \Rightarrow Q \Rightarrow Q$

[5] (5) $P \Rightarrow (Q \Rightarrow S)$ Rule P

[1, 2, 5] (6) $Q \Rightarrow S$ Rule T (4)(5) and I_{11} $P, P \rightarrow Q \Rightarrow Q$

[7] (7) Q Rule P

[1, 2, 5, 7] (8)

③ Show that $p \to s$ can be derived from premises

$\sim p \vee q$, $\sim q \vee r$, $R \Rightarrow S$

Now we have to show $p \Rightarrow s$. Introduce p as additional premise. That additional and given premises should produce S as conclusion.

[1] (1) P Rule p (additional premise)

[2] (2) $\sim p \vee q$ Rule p

[2] (3) $p \Rightarrow q$ Rule -T, (2) and E_{16} $p \Rightarrow q \Leftrightarrow \sim p \vee q$

~~[4] (4) $\sim q \vee r$~~

[1, 2] (4) q Rule -T (1)(3) I_{11} $p, p \Rightarrow q \Rightarrow q$

[5] (5) $\sim q \vee R$ Rule p

[5] (6) $q \Rightarrow R$ Rule T (4)(5) E_{16}, $\sim q \vee R \Leftrightarrow q \Rightarrow R$ ~~I_{11} $p, p \Rightarrow q \Rightarrow q$~~

[1, 2, 5] (7) R Rule T (4), (6) I_{11}: $p, p \Rightarrow q \Rightarrow q$

[8] (8) $R \Rightarrow S$ Rule p

[1, 2, 5, 8] (9) S Rule (7), (8) I_{11}, $R, R \Rightarrow S \Rightarrow S$

[1, 2, 5, 8] (10) $p \Rightarrow S$ Rule CP

Derive $P \to (Q \to S)$ using Rule CP if necessary from the premises

$P \to (Q \to R)$, $Q \to (R \to S)$

Given premises

$P \to (Q \to R)$, $Q \to (R \to S)$

[1] (1) P Rule P (additional premise)

[2] (2) $P \to (Q \to R)$ Rule P

[1,2] (3) $Q \to R$ Rule T (1),(2) I_{11}

[1,2] (4) $\sim Q \vee R$ Rule T E_{16}: $Q \to R \Leftrightarrow \sim Q \vee R$

[5] (5) $Q \to (R \to S)$ ~~$P \to (Q \to$~~ Rule P

[5] (6) $\sim Q \vee (\sim R \vee S)$ Rule T (5) and E_{16}

[1,2,5] (7) ~~$\sim Q \vee (\sim R \vee S$~~

$(\sim Q \vee R) \wedge (\sim Q \vee (\sim R \vee S))$

$\sim Q \vee (R \wedge (\sim R \vee S))$ Rule T (4) (6) and E_9

$\sim Q \vee ((R \wedge \sim R) \vee S)$

$\sim$

P can be taken as additional premise and given premises together should produce $Q \Rightarrow S$

$P, P \to Q \Rightarrow Q$