predicate logic

propositional logic The logic based on the analysis of propositional (statements) is called propositional logic

predicate logic: The logic based on the analysis of predicates is called predicate logic

Ex: x is an even number — it has two parts ① subject ② predicate

x → subject or object or variable

is an even number → predicate

E(x) is called as the propositional function at x

pred → E, variable → x

⇒ once a value has been assigned to the variable x, the statement E(x) becomes a proposition and has a truth value

E(10) = true

E(5) = False

II example

x is taller than y

Here x, y are subjects

is taller than is a predicate

it is denoted by T(x, y)

1-place predicate. The predicates that belongs to one subject is called 1-place predicate $E(x)$

2 place - predicate The predicate that belong to two subjects is called 2-place predicate $T(x, y)$

Ex: Amulya is a Student and This painting is blue

where is a Student is The predicate

is blue " "

This painting is The subject

$S(A) \wedge \cancel{Q}\, B(p)$

Quantifiers

certain Statements containing The words such as 'all', 'every', 'some', 'There exists", 'None", 'There is at least' associated with the idea of quantity

Such words are called quantifiers

Examples:

1. Some men are tall
2. All birds have ~~two~~ wings
3. No air ballon is perfectly round
4. There is a real number less than 11

Universal quantifier — The quantifier "all", every, each, every thing is called universal quantifier.

it is denoted by the symbol $\forall$

Universal quantifier ($\forall$) represents each of the following phrases, all ~~pro~~ phrases have the same meaning

for all x

for every x

for each x

every thing x such that

~~every~~ each thing x such that

Existential Quantifier

The quantifier 'some', "there exist" and there is atleast is called existential quantifier

it denoted by the symbol $\exists$

Existential quantifier ($\exists$) represents each of the following phrases all phrases have the same meaning for Some x

Some x Such That

There exist an x Such That

There is atleast one x Such That

Write down the following quantified Statements in Symbolic form

Ex: For The universe of all integers let

$p(x)$; $x > 0$

$q(x)$; x is even

$r(x)$: x is perfect Square

$s(x)$: x is Divisible by 3

$t(x)$: x is " by 7

(1) At least one integer is even

$(\exists x)\, q(x)$

(2) There exist a +ve integer That is even

$(\exists(x))\,(p(x) \wedge q(x))$

(3) Some even integers are divisible by 3

$(\exists(x)) \cdot (q(x) \wedge s(x)]$

(4) Every integer is either even or odd

$[\forall(x)]\,[\ q(x) \oplus \sim q(x)]$

(5) If x is even and Perfect Square Then x is not divisible by 3

$(\forall(x))\,[(q(x) \wedge r(x)) \rightarrow \sim s(x)]$

Universal quantifier

The quantifier 'all', every, each, every thing is called universal quantifier.

it is denoted by the Symbol $\forall$

Universal quantifier ($\forall$) represents each of the following phrases, all ~~pro~~ phrases have the Same Meaning

for all x

for every x

for each x

every thing x such that

~~every~~ each thing x such that

Existential Quantifier

The quantifier 'Some', "there exist" and there is atleast is called Existential quantifier

it denoted by the Symbol $\exists$

Existential quantifier ($\exists$) represents each of the following phrases all phrases have the Same Meaning for some x

Some x such that

There exist an x such that

There is atleast one x such that

Write down the following quantified statements in symbolic form

Ex: For the universe of all integers let

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$(\exists(x)) \cdot (q(x) \wedge s(x)]$

(4) Every integer is either even or odd

$[\forall(x)]\, [\, q(x) \oplus \sim q(x)]$

(5) If x is even and perfect square then x is not divisible by 3

$(\forall(x))\, [(q(x) \wedge r(x)) \rightarrow \sim s(x)]$

Now consider the following statements. Represent them in symbolic form

(i) All monkey have tails

(ii) No monkey has a tail

(iii) Some monkey have tails

(iv) Some monkey have no tails

(i) Let us consider $M(x)$: x is a monkey

$T(x)$: x has a tail

for all x if x is a monkey then x has a tail

(i) $(\forall x)[M(x) \rightarrow T(x)]$

for all x if x is a monkey then x has a no tail

(ii) $\forall(x)[M(x) \rightarrow \sim T(x)]$

(iii) $\exists(x)[M(x) \wedge T(x)]$ there at least one x such that x is a monkey and x has a tail

(iv) $\exists(x)[M(x) \wedge \sim T(x)]$

there is at least one x such that x is monkey and x has no tail

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Write the following sentences in symbolic form

(i) Some people who trust others are rewarded

(ii) If any one is good then John is good

(iii) Some one is teaching

(iv) It is not true that all roads leads to Rome

$P(x)$: x is a person

$T(x)$: x trust others

$R(x)$: x is rewarded

$G(x)$: x is good

$Q(x)$: x is teasing

$S(x)$: x is road

$L(x)$: x leads to room

(i) $(\exists(x))[P(x) \wedge T(x) \wedge R(x)]$

(ii) $[\exists(x)]\, [\cancel{G(x)}\; P(x) \wedge G(x)] \rightarrow G(John)]$

(iii) $[\exists(x)]\, [P(x) \wedge Q(x)]$

(iv) $[\exists(x)]\, [S(x) \wedge \sim L(x)]$

write the following Sentences in the Symbolic form

(i) Every living thing is a plant or animal

(ii) John's gold fish is alive and it is not plant

(iii) All animals have hearts

(iv) John's gold fish has a heart

Sol

x is a living Thing

$H(x)$: x has heart

$p(x)$: x is plant

$A(x)$: x is a animal

g : John's gold fish

(i) $(\forall x)\left[(p(x) \vee A(x)\right]$

(ii) $\sim p(g)$

(iii) $\forall(x)\left[A(x) \rightarrow H(x)\right]$

(iv) $H(g)$

Q) Write the following sentence in symbolic form

(i) All integers are rational number

(ii) Some integers are power of 3

(iii) Some rational numbers are power of 3

Sol)

$I(x)$: x is an integer

$R(x)$: Rational number

$P(x)$: x is a power of 3

(i) $(\forall(x))\,[I(x) \rightarrow R(x)]$

(ii) $(\exists(x))\,[I(x) \wedge P(x)]$

(iii) $(\exists(x))\,[R(x) \wedge P(x)]$

Write the following sentences in symbolic form

1. All men are mortal

(ii) Socrates is a man

(iii) Socrates is a mortal

Sol

$H(x)$: x is a man

$M(x)$: x is a mortal

s : Socrates

(i) $(\forall(x))\,[H(x) \rightarrow M(x)]$

(ii) $H(s)$

(iii) $M(s)$

Free Variable, bounded Variable, Scope of the Quantifier

ex All Men are tall

$M(x)$: x is Men

$T(x)$: x is Tall

$\forall(x)\left[M(x) \rightarrow T(x)\right]$ If x is Men then x is a tall

the bounded Variable x

bounded Variable: An occurence of Variable that is bounded by a Quantifier (either universal Quantifier or Existential quantifier) is said to be bounded Variable

$\forall(x)\left[M(x) \rightarrow T(x)\right]$

free Variable: An occurence of Variable that is not bounded by a Quantifier (either universal Quantifier or Existential quantifier is said to be free Variable

$M(x) \rightarrow T(x)$

Scope of the Quantifier: Scope of the quantifier is the formula immediately following the Quantifier

Example

predicate formula	Bound Variable (B)	free Variable (F)	Scope of quantifier (S)

① $\forall x\, p(x,y)$

B: x

F: y

S: $p(x,y)$

② $\forall x\,[p(x) \to q(x)]$

B: x

F: —

S: $p(x) \to q(x)$

③ $\forall x\,[p(x) \to (\exists y)\, R(x,y)]$

B: x, y

F: —

S: $\forall x$: $p(x) \to \exists y\, R(x,y)$

$\exists y$: $R(x,y)$

④ $\forall x\,[p(x) \wedge q(x)] \vee \forall y\, R(y)$

B: x, y

F: —

S: $\forall x$: $p(x) \wedge q(x)$

$\forall y$: $R(y)$

⑤ $\exists(x)\,[p(x) \wedge q(x)]$

B: x

F: —

S: $[p(x) \wedge q(x)]$

⑥ $\exists(x)\, p(x) \wedge q(x)$

B: x first

F — x second

S: $p(x) \wedge q(x)$

Inference Rules in predicate logic

→ Rule-P, Rule-T and Rule-CP are same as earlier

→ Additional Rules are required to deal with the formula involving quantifiers

→ Elimination of quantifiers can be done by using rules of specification called Rule US and Rule ES

→ To prefix the correct quantifier we rules of Generalization called Rule-UG and Rule EG

Rule US (universal specification)

If a statement of the form $\forall x\, p(x)$ is assumed to be true then the universal quantifier can be dropped to obtain $p(t)$ is true for an object t in the universe

Notation : $\dfrac{(\forall x)\ p(x)}{p(t) \text{ for all } t}$

Rule- ES (Existential Specification)

If $\exists(x)\ p(x)$ is assumed to be true

then there is an element 't' in the universe such that $p(t)$ is true

$$\frac{(\exists x)\ p(x)}{p(t) \text{ for some } t}$$

Rule - UG (universal Generalisation)

If a statement $p(t)$ is true then each element 't' of the universe. then the universal quantifier may be prefixed to obtain $(\forall x)\ p(x)$

Notation : $$\frac{p(t) \text{ for all } t}{(\forall x)\ p(x)}$$

Rule - EG (Existential Generalisation)

If $p(t)$ is true for some element 't' in the universe, then there exist $\exists(x)\ p(x)$ is true

Notation $$\frac{p(t) \text{ for some } t}{\exists(x)\ p(x)}$$

(7) Verify the Validity of the following argument

All men are mortal

Socrates is a Man

Therefore Socrates is a Mortal

Sol Letus take

$H(x)$: x is Man

$M(x)$: x is Mortal

S : Socrates

(i) All Men are Mortal

for all x if x is a Man Then x is Mortal

$$\forall_x [H(x) \rightarrow M(x)]$$

(ii) Socrates is Man

$$H(S)$$

(iii) Socrates is a Mortal

$$M(S)$$

$\therefore$ the given premises

$$\begin{array}{c} \forall x [H(x) \rightarrow M(x)] \\ H(S) \\ \hline \therefore M(S) \\ \hline \end{array}$$

[1] (1) $\forall x [H(x) \rightarrow M(x)]$ Rule p

[1] (2) $H(S) \rightarrow M(S)$ Rule US

[3] (3) $H(S)$

[1,3] (4) $M(S)$ Rule T (2)(3) I_{11} $P, P \to V \Rightarrow V$

$\therefore$ $M(S)$ is a Valid conclusion from the given premisis

(ii) Verify Every Living thing is a plant or an animal. John's gold fish alive and it is not a plant. All animal have hearts Therefore John's gold fish has a heart

Sol

x = Every living thing

$P(x)$: x is plant

$A(x)$: x is animal

J : John's gold fish

$H(x)$: x has heart

The Symbolical form is

(1) Every Living thing is a plant or an animal

$(\forall x) [P(x) \vee A(x)]$

(2) John's gold fish is not a plant

$\neg P(x) \Leftrightarrow \sim P(J)$

(3) All animals have hearts

$(\forall x) [A(x) \to H(x)]$

i.e. for all x if x is animal then x has a heart

(4) John's gold fish has heart

$H(x) \Rightarrow H(J)$

The premises are

$$\forall x\ [P(x) \vee A(x)]$$
$$\sim P(J)$$
$$\forall x\ (A(x) \rightarrow H(x))$$
$$\therefore\ H(J)$$

[1]	(1)	$\forall x\ [P(x) \vee A(x)]$	Rule P
[1]	(2)	$P(J) \vee A(J)$	Rule US
[3]	(3)	$\sim P(J)$	Rule P
[1,3]	(4)	$A(J)$	$P \vee Q, \sim P \Rightarrow Q$
[5]	(5)	$\forall x\ (A(x) \rightarrow H(x))$	Rule P
[5]	(6)	$A(J) \rightarrow H(J)$	Rule US, (1)
[1,3,5]	(7)	$H(J)$	Rule T $\quad P, P \rightarrow Q \Rightarrow Q$

$\therefore$ $H(J)$ is valid conclusion.

(iii) prove the implication

$\forall x\,[P(x) \rightarrow Q(x)], \quad \forall x\,[R(x) \rightarrow \sim Q(x)]$

$\Rightarrow \forall x\,[R(x) \rightarrow \sim P(x)]$

{1} (1) $\forall x\ P(x) \rightarrow Q(x)$ Rule P

{1} (2) $P(a) \rightarrow Q(a)$ Rule US

{3} (3) $\forall x\,[R(x) \rightarrow \sim Q(x)]$ Rule P

[3] (4) $R(a) \rightarrow \sim Q(a)$ Rule US

[3] (5) $\sim\sim Q(a) \rightarrow \sim R(a)$ Rule T $P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$

[1, 3] (6) $P(a) \rightarrow \sim R(a)$ Rule T I_{12}

[1, 3] (7) $\sim(\sim R(a)) \rightarrow \sim P(a)$ $P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$

$R(a) \rightarrow \sim P(a)$

[1, 3] (8) $\forall x\,[R(x) \rightarrow \sim P(x)]$ Rule UG (8)

It is a valid conclusion

(17) Show that the premises "one student in this class knows how to write programs in Java and Every one who knows how to write programs in Java can get a high-paying Job" Imply the conclusion "Some one in this class can get a high paying Job"

Sol: Let us consider

x: one Student

$C(x)$: x is in this class

$J(x)$: x knows how to write a Java program

$P(x)$: x can get high paying Job

Symbolic form

$\exists x \, [C(x) \wedge J(x)]$

There is at least one x, such that x is in the class and he knows how to write java program

$\forall x \, [J(x) \rightarrow P(x)]$ for all x if x knows how to write a Java program then x can get high paying

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$\exists x \, [C(x) \wedge P(x)]$

There is at least one x such that x is in the class, x can get high paying Job

The premiss are

$$\exists x\ [C(x) \wedge J(x)]$$

$$\forall x\ [J(x) \rightarrow P(x)]$$

$$\therefore\ \exists x\ [C(x) \wedge P(x)]$$

[1]	(1)	$\exists x\ [C(x) \wedge J(x)]$	Rule P
[1]	(2)	$C(a) \wedge J(a)$	Rule ES (1) I_1 $P \wedge Q \Rightarrow P$
[1]	(3)	$C(a)$	
~~[1]~~	~~(4)~~	~~$\forall x\ [J(x) \rightarrow P(x)]$~~	~~Rule P~~
~~[3]~~	~~(4)~~	~~$J(a) \rightarrow P(a)$~~	Rule US (2)
[1]	(4)	$J(a)$	Rule T $P \wedge Q \Rightarrow Q$ Rule
[5]	(5)	$\forall x\ [J(x) \rightarrow P(x)]$	Rule P
[5]	(6)	$J(a) \rightarrow P(a)$	Rule US (5)
[1, 5]	(7)	$P(a)$	Rule T (4) [6] I_{11}
[1, 5]	(8)	$C(a) \wedge P(a)$	Rule $P, Q \Rightarrow P \wedge Q$ I_9
[1, 5]	(9)	$\exists(x)\ [C(x) \wedge P(x)]$	Rule EG

$\therefore$ This is Valid Conclusion

<u>Direct proof</u>: We assume that p is true and use axiom, Definitions and previously proven theorem, together with rules of inference to show that q must also be true

Definition

An integer n is even if $n = 2k$ for some integer k

An integer n is odd if $n = 2k+1$ for some integer k

If a and b are integers a/b if there exists an integer z such that $az = b$

for $3/6$ $3 \cdot 2 = 6$

direct Method

proposition if p then q

proof Suppose p (True)

⋮

therefore q

EX1

if n is even then n^2 is even
p q

proof

Suppose n is even (hypothesis)

this Means $n = 2k$ for some integer k

consider n^2

$$n^2 = n \cdot n$$
$$= 2k\,2k$$
$$n^2 = 4k^2$$
$$= 2 \cdot (2k^2)$$
$$= 2w \quad \text{for some integer } w = 2k^2$$

$\therefore$ n^2 is even

Let a, b and c be integers. If a/b and b/c then a/c

$5/20$ $20/100$ $5/100$

Suppose a/b and b/c

This Mean $b = aw$ for some integer w

and $c = bz$ for " z

Substitute for b

$$c = bz$$
$$c = awz$$
$$c = ay \quad \text{for some integer } y = wz$$

Therefore a/c

proof by contraposition

Ex1 prove that if n is an integer and $3n+2$ is even then n is even

$p \to q$

$\Leftrightarrow \sim q \to \sim p$

If n is odd then $3n+2$ is odd

proof

let n be an integer. Assume for contraposition n is odd then by Def of odd integer $n = 2k+1$ for some integer k

$n = 2k+1$

$3n+2 = 3(2k+1)+2$

$= 6k+3+2$

$= 6k+5+1+1$

$= 6k+4+1$

$= 2(3k+2)+1$

since $k, 3, 2$ are integers, $3n+2$ is 2 times some integer $+1$. By Def of odd integer

$3n+2$ is odd

Ex2 prove

If n^2 is even then n is even

contrapositive is

If n is odd then n^2 is odd

proof

Suppose n is odd

ie $n = 2k+1$ for some integer k

consider

$n^2 = n \cdot n$

$= (2k+1)(2k+1)$

$= 4k^2+4k+1$

$= 2(2k^2+2k)+1$ For some integer $W = 2k^2+2k$

$= 2W+1$

$\therefore$ n^2 is odd

EX3 Let a and b be integer If $5/ab$, then $5/a$ and $5 \nmid b$

contra positive is

Let a and b integer if $5/a$ or ~~and~~ $5/b$ then $5/ab$

proof Suppose $5/a$ or $5/b$

Case① Suppose $5/a$

this means $a = 5w$ for some integer w

Substituting this we get $ab = 5bw$ for some integer

$\Rightarrow ab = 5x$ for some integer $x = bw$

$\therefore 5/ab$

Case2 Suppose $5/b$

this means $b = 5z$ for some integer z

Substituting this we get $ab = 5az$ for some integer z

$ab = 5y$ for some integer $y = az$

$\therefore 5/ab$

In either case $5/ab$

proof by contradiction — if p is true and ¬q is also true

prove that if n is an integer and $3n+2$ is even then n is even

proof: let n be an integer. Assume for proof by contradiction that $3n+2$ is even and n is odd

if n is odd, By Def of integer

$n = 2k+1$ for some integer k

if n is an integer and $3n+2$ is even then n is odd

if n is odd then $3n+2$ is even

$n = 2k+1$

$3(2k+1)+2$

$6k+5$

$6k+4+1$

$2(3k+2)+1$

$$3n+2 = 3(2k+1)+2$$
$$= 6k+3+2$$
$$= 6k+5-1+1$$
$$= 6k+4+1$$
$$= 2(3k+2)+1$$

Then $3n+2$ is ~~t~~ time some integer plus 1

So by Def. odd integer $3n+2$ is odd

This contradicts our assumption that $3n+2$ is even. Hence it is not the case that $3n+2$ is even and n is odd

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We have to prove if $3n+2$ is even then n is even

if ~~$3n+2$ is even then n is odd~~

if

prove that $\sqrt{2}$ is ~~a~~ irrational (not Rational)

proof by Contradiction

$\sqrt{2}$ is Rational

Assume $\sqrt{2} = a/b$ with $gcd(a,b) = 1$

(a and b have no common factor

$2 = a^2/b^2$

b must be odd $2b^2 = a^2$ a must be even

~~Choose~~ ~~an~~ odd numb

$a = 2k$

$1/3$

$2b^2 = a^2$

$2b^2 = (2k)^2$

a^2 is an even

$2b^2 = 4k^2$

a is also even ~~b is~~

$b^2 = 2k^2$

b is even

Since a is even

Since a, b are both even

$gcd(a,b) \neq 1$

$a = 2k$

$a^2 = 4k^2$ which is contradiction

$a^2 = 2(2k^2)$

$2b^2 = 4k^2$

$\cancel{2}b^2 = \cancel{2}(2k^2)$

$b^2 = 2k^2$

b^2 is even

b is even

a, b is factor of 2

$2 = \frac{a^2}{b^2}$

$b^2 = a^2/2$

Now take $a = 2c$

$b^2 = \frac{4c^2}{2}$

$\cancel{2}b^2 = \cancel{4}^{2}c^2$

~~b2~~ $c^2 = b^2$

$c^2 = b^2/2$

if $x \in R$, $x^3 + 4x = 0$ then $x = 0$

let $x \neq 0$

$x^3 + 4x = 0$

$x(x^2 + 4) = 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x =$

Which is Contradiction

that which also if $x \neq 0$

$\therefore x \neq 0$

Direct proofs

The product of two odd numbers is odd

(i)

Proof Let $x = 2m+1$ $y = 2n+1$

$$xy = (2m+1)(2n+1)$$

$$= 4mn + 2m + 2n + 1$$

$$= 2(2mn + m + n) + 1$$

$$= 2k + 1$$

for an integer where $2mn + m + n$ is $= k$

$$= \text{odd}$$

(ii) If m and n are perfect square then

$m + n + 2\sqrt{mn}$ is a perfect square

Proof Let $m = a^2$ $n = b^2$ for some integer a and b

Then $m + n + 2\sqrt{mn}$

$$= a^2 + b^2 + 2\sqrt{a^2b^2}$$

$$= a^2 + b^2 + 2ab$$

$$= (a+b)^2$$

$\therefore$ $m + n + 2\sqrt{mn}$ is a perfect square

Show that the premises " Every one in the D.M class has taken a course in computer science and pavan is a students in this class imply the conclusion " pavan has taken a course in computer science "

Let D(x) : x is in this DM class

c(x) : x has taken in computer science

p : pavan

$\forall x [D(x) \rightarrow c(x)]$

$D[p]$

$c[p]$

(11) Show that primises " A student in the class has not read the book and every one in this class passed the first exam imply the conclusion

" Some one who passed the first exam has not read the book

C(x) : x is in the class

B(x); x has read the book

p(x): x passed the first exam

$\boxed{\exists(x)} \, [c(x) \wedge \sim B(x)]$

$\boxed{\forall(x)} \, [c(x) \rightarrow p(x))$

Conclusion $\exists(x) \, [p(x) \wedge \sim B(x)]$

Implication Rule

I_9, $P, Q \Rightarrow P \wedge Q$

I_{10}, $\sim P, P \vee Q \Rightarrow Q$

I_{11}, $P, P \Rightarrow Q \Rightarrow Q$

I_{12}, $\sim Q, P \Rightarrow Q \Rightarrow \sim P$

I_{13} $P \Rightarrow Q, Q \Rightarrow R \Rightarrow P \Rightarrow R$

I_{14} $P \vee Q, P \Rightarrow R, Q \Rightarrow R \Rightarrow R$

Equivalence

E_1: $\sim(\sim P) \Leftrightarrow P$

E_{10} $P \vee P \Leftrightarrow P$

E_{11} $P \wedge P \Leftrightarrow P$

E_{16}: $P \Rightarrow Q \Leftrightarrow \sim P \vee Q$

E_{17} $\sim(P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

E_{18} $P \Rightarrow Q \Leftrightarrow \sim Q \Rightarrow \sim P$ (contrapositive)

E_{19}: $P \Rightarrow (Q \Rightarrow R) \Leftrightarrow (P \wedge Q) \Rightarrow R$

E_{21} $P \rightleftarrows Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

E_{21} $P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$

(1) Demonstrate the R is valid inference from the premises

$P \Rightarrow Q$, $Q \Rightarrow R$ and P

[1]	(1)	P	Rule P
[2]	(2)	$P \Rightarrow Q$	Rule P
[1,2]	(3)	Q	Rule T (1),(2) I_{11} $P, P \Rightarrow Q \Leftrightarrow Q$
[4]	(4)	$Q \Rightarrow R$	Rule P
[1,2,4]	(5)	R	Rule T (3) (4) I_{11} $P, P \Rightarrow Q \Leftrightarrow Q$

(ii) Show that $r \vee s$ follows logically from the premises

$c \vee d$, $c \vee d \to \sim h$

$\sim h \to (a \wedge \sim b)$

$(a \wedge \sim b) \to r \vee s$

[1]	(1)	$c \vee d$	Rule p
[2]	(2)	$c \vee d \to \sim h$	Rule p
[1, 2]	(3)	$\sim h$	Rule T (1)(2) I_{11} $P, P \to Q \Rightarrow Q$
[4]	(4)	$\sim h \to (a \wedge \sim b)$	Rule p
[1, 2, 4]	(5)	$(a \wedge \sim b)$	Rule T (3)(4) I_{11}
[6]	(6)	$(a \wedge \sim b) \to r \vee s$	Rule p
[1, 2, 4, 6]	(7)	$r \vee s$	Rule T (5), (6) I_{11}

(iii) Show that $r \wedge (p \vee q)$ is valid conclusion from the premises

$p \vee q$, $q \Rightarrow r$, $p \Rightarrow m$, and $\sim m$

[1]	(1)	$\sim m$	Rule p
[2]	(2)	$p \Rightarrow m$	Rule p
[1, 2]	(3)	$\sim p$	Rule T I_{12} (1)(2) $\sim q;\ p \to q \Rightarrow \sim p$
[4]	(4)	$p \vee q$	Rule p
[1, 2, 4]	(5)	q	Rule T (3)(4) $\sim p, p \vee q \Rightarrow q$
[6]	(6)	$q \Rightarrow R$	Rule p
[1, 2, 4, 6]	(7)	R	Rule T (5)(6) I_{11} $P, P \Rightarrow Q \Rightarrow Q$
[1, 2, 4, 6]	(8)	$R \wedge (p \vee q)$	Rule T I_9, $P, Q \Rightarrow P \wedge Q$

function $F(n) = F_n$ is Defined recursively

(IV) Show $s \vee r$ is Tautologically Implied by

$(p \vee q) \wedge (p \to r) \wedge (q \to s)$

[1] (1) $p \vee q$ Rule p

$p \to q \Leftrightarrow \sim p \vee q$

[1] (2) $\sim p \to q$ Rule T (1)

$\sim p \to q \Leftrightarrow \sim(\sim p) \vee q \Leftrightarrow p \vee q$

[3] (3) $\cancel{p \to r}$ $q \to s$ Rule p

[1, 3] (4) $\sim p \to s$ Rule T I_{12} $p \to q, q \to R \Rightarrow P \to R$

[1, 3] (5) $\sim s \to p$ Rule T E_{18} $p \to q \Leftrightarrow \sim q \to \sim p$ (Contrapositive)

[6] (6) $p \to r$ Rule P

[1, 3, 6] (7) $\sim s \to r$ Rule T I_{12}

[1, 3, 6] (8) $\sim(\sim s) \vee r$ Rule T $p \to q \Leftrightarrow \sim p \vee q$

$\underline{s \vee r}$

(5) Show That ~~R ∧ (p ∨ q)~~ S is Valid Inference for The following premises

$p \to \sim q,\ q \vee R,\ \sim s \to P,\ \sim R$

[1] (1) $q \vee R$ Rule p

[1] (2) $R \vee q$ Rule T (1) $P \vee q \Leftrightarrow q \vee P$

[3] (3) $\sim R$ Rule p

[1, 3] (4) q Rule T (2), (3) $P \vee q, \sim P \Rightarrow q$

[5] (5) $p \to \sim q$ Rule p

[5] (6) $q \to \sim p$ Rule T (4) (5) E_{18} $P \to q \Leftrightarrow \sim q \to \sim p$

[7] (7) $\sim s \to P$ Rule p

[7] (8) $\sim p \to s$ Rule T (7) E_{18}

[5, 7] (9) $q \to s$ Rule T (6), (8) I_{13}

[1, 3, 5, 7] (10) s Rule T (3) (9) $P, P \to q \Rightarrow q$

(i) If there was a ball game then the travelling was difficult

(ii) if they arrived on time then travelling was difficult

(iii) They arrived on time

(iv) There fore there was no ball game

Sol p: There was a ball game

q: travelling was difficult

r: They arrived on time

$p \to q$

$r \to \sim q$

r

$\therefore \sim p$

$p \to q,\ r \to \sim q,\ r \Rightarrow \sim p$

[1] (1) $p \to q$ Rule P

[1] (2) $\sim p \vee q$ Rule T, $p \to q \Rightarrow \sim p \vee q$

[3] (3) $r \to \sim q$ Rule P

[3] (4) $\sim r \vee \sim q$ Rule T $p \to q \Rightarrow \sim p \vee q$

[3] (5) $\sim q \vee \sim r$ Rule T $p \vee q = q \vee p$

[1, 3] (6) $\sim p \vee \sim r$ Rule T (2) (5) I_{10}

[7] (7) r Rule P

[1, 3, 7] (8) $\sim p$

[1] (1) r Rule P

[2] (2) $r \Rightarrow \sim q$ Rule P

[1, 2] (3) $\sim q$ Rule T I_{11}

[4] (4) $p \to q$ Rule P

[1, 2, 4] $\sim p$ Rule T I_{12} (3) (4)