SET Theory

Set :- Set is a collection of objects (or) Elements.

* set is denoted by 'capital letters'.
* Elements are denoted by 'small letters'.
* If A is a set and x is a element of A then we can write $x \in A$.
* the set can be represented in two ways
  1. Roaster form
  2. set builder form

1. Roastar form :-

In this form a set is described by list out the elements with in brackets.

Ex :- If A is a set of odd number which is lessthan 10

$A = [1, 3, 5, 7, 9]$

2. set builder form :-

In this form a set is described by characteristic property.

Ex :- If A is a set of odd number which is less than 10

$A = \{x / x \text{ is a odd number} < 10\}$

Types of sets

1. Null set :-

A set is said to be null set if it has no elements. The null set is written as

$\phi = \{ \}$

2. Singleton set :-

A set contains only one element is called singleton set.

Ex:- set $A = \{1\}$

$B = \{3\}$

3. Finite set :-

A set contains finite values then it is called finite set.

Ex :- $A = \{1, 2, 3\}$

4. Infinite set :-

A set contains infinite values then it is called infinite set

Ex :- A = set of natural numbers

set of integers.

5. Equal set :-

Let, A and B are two sets the elements of set A is same as the elements of set B is called equal set

Ex :- $A = \{3, 4, 5\}$ $B = \{3, 4, 5\}$

6. Sub set :-

Let A and B are said to be subset two sets every element of set A is also an element of set B. Then A is called a sub set of B.

It is denoted by $A \subseteq B$

Ex:- $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5\}$

7. proper subset :-

A set 'A' is called proper subset of B if $A \subseteq B$ But $B \nsubseteq A$

Ex :- $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5\}$

$A \subseteq B$ but $B \nsubseteq A$

Universal set :-

the set of all elements of all the sets under consideration is called universal set.

* if set $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$ and $C = \{0, 1\}$

the universal set u is given by

$u = \{0, 1, 2, 3, 4, 5, 6\}$

Power set :- set of all subsets of a set is called power set

for example set $A = \{1, 2\}$

subsets are $\{1\}, \{2\}, \{1, 2\}, \{\}$ (or) ϕ

formulae :- 2^n

the no of subsets in a power set is given by 2^n.

Operation on set (or) set operations

Union :-

Let A and B be the two sets. The union of the A & B is the Combine the all the Elements of the A and B.

It is denoted by $A \cup B$.

$\Rightarrow \{x / x \in A$ or $x \in B\}$ or

$\{x / x \in A \cup B\}$

Intersection of the set :-

Let A and B be the two sets. the intersection of the two sets A & B. is the Common elements in the set A & set B

It is denoted by $A \cap B$

$\Rightarrow \{x / x \in A$ and $x \in B\}$

Complement of the set :-

Let u be the universal set and A be the one set. the Complement of the set A denoted by A is the elements of the set A not Contained in the universal set

i.e $A' = \{x / x \in u$ and $x \notin A\}$

$\boxed{\text{i.e } A' = u - A}$

Relative

Let A

Complem

A - B

B - A

symme

Let

(i) If

find

(iii) fin

(iv) B'

(v) $\overline{(A \cup B)}$

(vi) $A' \cap$

(vii) $A' \cup$

(viii) $\overline{A \cap}$

(ix) $A-B$

(x) $A \Delta B$

Relative Complement :-

Let A and B are two sets the relative complement of A and B are given by

$A - B = \{x / x \in A \text{ and } x \notin B\}$

$B - A = \{x / x \in B \text{ and } x \notin A\}$

Symmetric difference :-

Let A and B are two sets the symmetric difference b/w two sets is given by $A \Delta B$ and is given by

$$\boxed{A \Delta B = (A - B) \cup (B - A)}$$

(1) If set $A = \{1, 2, 3, 5, 6, 8\}$ and set $B = \{0, 1, 2, 4, 8\}$, $\mu = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

(i) find $A \cup B$

(ii) find $A \cap B$

(iii) find A^c (or) A'

(iv) B'

(v) $\overline{(A \cup B)}$

(vi) $A' \cap B'$

(vii) $A' \cup B'$

(viii) $\overline{A \cap B}$

(ix) $A - B$, $B - A$

(x) $A \Delta B$

(i) $\{0, 1, 2, 3, 4, 5, 6, 8\} = A \cup B$

(ii) $A \cap B = \{1, 2, 8\}$

(iii) $A^c = U - A$

where $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{1, 2, 3, 5, 6, 8\}$

$\therefore U - A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 3, 5, 6, 8\}$

$A' = \{0, 4, 7\}$

(iv) $B^c = U - B$

$\therefore U - B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} - \{0, 1, 2, 4, 8\}$

$= \{3, 5, 6, 7\}$

(v) $(\overline{A \cup B}) = U - (A \cup B)$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8\} - \{0, 1, 2, 3, 4, 5, 6, 8\}$

$= \{7\}$

(vi) $A' \cap B'$

$A' \cap B' = \{0, 4, 7\} \cap \{3, 5, 6, 7\}$

$= \{7\}$

(vii) $A' \cup B'$

$\{0,4,7\} \cup \{3,5,6,7\}$

$= \{0,3,4,5,6,7\}$

(viii) $\overline{A \cap B}$

$= U - (A \cap B)$

$= \{0,1,2,3,4,5,6,7,8\} - \{1,2,8\}$

$= \{0,3,4,5,6,7\}$

(ix) $A - B = \{3,5,6\}$

$B - A = \{0,4\}$

(x) $A \Delta B = (A-B) \cup (B-A)$

$= \{3,5,6\} \cup \{0,4\}$

$= \{0,3,4,5,6,\}$

Theorem :-

For any three sets A, B and C prove that $A \cup (B \cup C) = (A \cup B) \cup C$

Proof :- Let $x \in A \cup (B \cup C)$

$x \in A$ (or) $x \in (B \cup C)$

$x \in A$ (or) ($x \in B$ (or) $x \in C$)

($x \in A$ (or) $x \in B$) or $x \in C$

$x \in A \cup B$ or $x \in C$

$x \in (A \cup B) \cup C$

$A \cup (B \cup C) \subseteq (A \cup B) \cup C \longrightarrow (1)$

Let $x \in (A \cup B) \cup C$

$x \in (A \cup B)$ or $x \in C$

$x \in A$ or $x \in B$ or $x \in C$

$x \in A$ or ($x \in B$ or $x \in C$)

$x \in A \cup (x \in B \cup C)$

$x \in A \cup (B \cup C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C) \longrightarrow (2)$

$A \cup (B \cup C) = (A \cup B) \cup C$

Theorem:-

For any three sets A, B and C prove that $A \cap (B \cap C) = (A \cap B) \cap C$

Let $x \in A \cap (B \cap C)$

$x \in A$ and $x \in (B \cap C)$

$x \in A$ and $(x \in B$ and $x \in C)$

$(x \in A$ and $x \in B)$ and $x \in C$

$x \in A \cap B$ and $x \in C$

$x \in (A \cap B) \cap C$

$A \cap (B \cap C) \leq (A \cap B) \cap C \rightarrow$ (1)

Let $x \in (A \cap B) \cap C$

$x \in (A \cap B)$ and $x \in C$

$(x \in A$ and $x \in B)$ and $x \in C$

$x \in A$ and $(x \in B$ and $x \in C)$

$x \in A \cap (x \in B \cap C)$

$x \in A \cap (B \cap C)$

$(A \cap B) \cap C \leq A \cap (B \cap C) \rightarrow$ (2)

$\therefore A \cap (B \cap C) = (A \cap B) \cap C$

Theorem :-

For any two sets A and B prove that $(A \cup B)' = A' \cap B'$

Proof :- Let $x \in (A \cup B)'$

$x \in \mu$ and $x \notin (A \cup B)$

$x \in \mu$ and $(x \notin A$ and $x \notin B)$

$(x \in \mu$ and $x \notin A)$ and $(x \in \mu$ and $x \notin B)$

$x \in A'$ and $x \in B'$

$x \in A' \cap B'$

$\therefore (A \cup B)' \leq A' \cap B' \longrightarrow (1)$

$x \in A' \cap B'$

$x \in A'$ and $x \in B'$

$(x \in \mu$ and $x \notin A)$ and $(x \in \mu$ and $x \notin B)$

$x \in \mu$ and $(x \notin A$ and $x \notin B)$

$x \in \mu$ and $(x \notin A \cup B)$

$\cdot\, x \in (A \cup B)'$

$\therefore A' \cap B' \leq (A \cup B)' \longrightarrow (2)$

from (1) & (2)

we can say that $(A \cup B)' = A' \cap B'$

theorem :-

For any two sets A and B prove that $(A \cap B)' = A' \cup B'$

$x \in (A \cap B)'$

$x \in \mu$ and $x \notin (A \cap B)$

$x \in \mu$ and $(x \notin A$ or $x \notin B)$

$(x \in \mu$ and $x \notin A)$ or $(x \in \mu$ and $x \notin B)$

$x \in A'$ or $x \in B'$

$x \in A' \cup B'$

$(A \cap B)' \subseteq A' \cup B' \longrightarrow$ (1)

$x \in A' \cup B'$

$x \in A'$ or $x \in B'$

$(x \in \mu$ and $x \notin A)$ or $(x \in \mu$ and $x \notin B)$

$(x \in \mu)$ and $(x \notin A$ or $x \notin B)$

$x \in \mu$ and $(x \notin A \cap B)$

$x \in \mu$ and $(x \notin A \cap B)$

$x \in (A \cap B)'$

$A' \cup B' \subseteq (A \cap B)' \longrightarrow$ (2)

$(A \cap B) = A' \cup B'$

Theorem:-

For any two sets A and B. prove that $A-(A\cap B) = A-B$

Proof :-

Let $x \in A-(A\cap B)$

$x \in A$ and $x \notin A\cap B$

$x \in A$ and $(x \notin A$ or $x \notin B)$

$(x \in A$ and $x \notin A)$ or $(x \in A$ and $x \notin B$

ϕ or $x \in A-B$

$x \in A-B$

$\therefore$ $A-(A\cap B) \subseteq A-B \longrightarrow (1)$

$x \in A-B$

$x \in A$ and $x \notin B$

$x \in A$ and $x \notin A\cap B$

since $\boxed{x \notin B \Rightarrow x \notin A\cap B}$

$x \in A-(A\cap B)$

$A-B \subseteq A-(A\cap B) \longrightarrow (2)$

from (1) & (2)

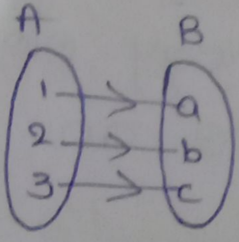
$A-(A\cap B) = A-B$

Functions

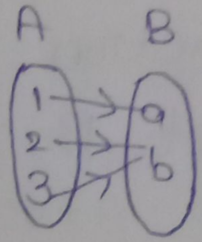
Function :-

~~A function~~
Let A and B are two sets a function f from A to B is denoted by $f : A \rightarrow B$ is a relation from A to B such that every element in set A there is exactly one element in the set B. where
A is a domain
B is a Codomain

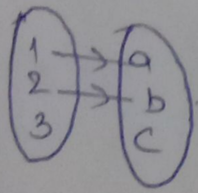
Ex :- *



*



*

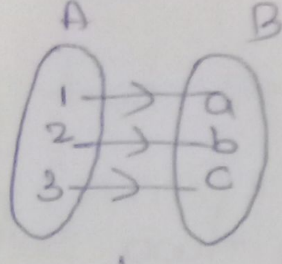


→ it is not a function

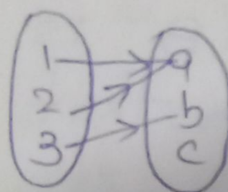
Types of functions :-

1. Injective (or) one to one function :-
A function $f : A \rightarrow B$ is said to be one to one function every distinct elements of set A is mapping to the every distinct elements of set B
* Two elements of set A cannot be mapped with same elements of set B.

Ex:-



one to one function

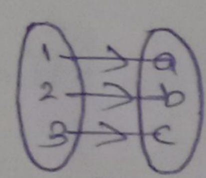


not one to one function.

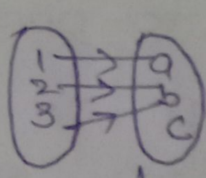
2. onto function:-

A function $f: A \to B$ is called onto function if every element of set B will be an image of set A.

Ex:-



onto function

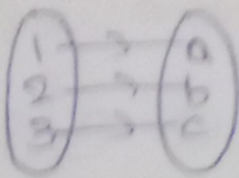


not onto function.

3. Bijective function:-

A function $f: A \to B$ is called bijective function if it is both one to one and onto.

Ex:-

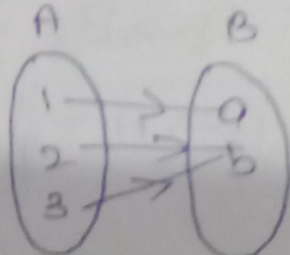


bijective function

Many to one function:-

A function $f: A \to B$ is called many to one function, if many element of set A is mapped to the single elements set B

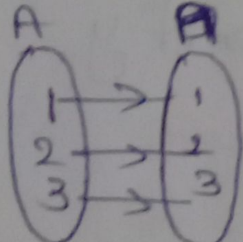
Ex:-



Identity function:-

A function $f: A \to A$ is called the identity function is the image of every elements of set A is itself

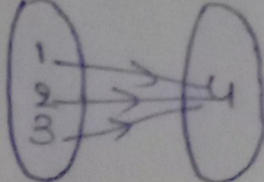
Ex:-



Constant function:-

A function $f: A \to B$ is called constant function if all the elements of set A is mapped with the single element of set B.

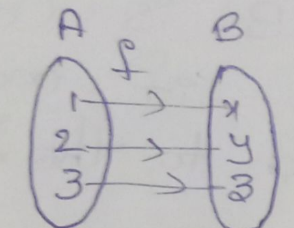
Ex:-



Inverse function:-

A function $f : A \rightarrow B$ is a bijective function then $f^{-1} : B \rightarrow A$ is called inverse of the function.

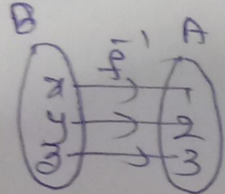
Ex:-



$A = \{1, 2, 3\}$

$B = \{x, y, z\}$

$f : A \rightarrow B = \{(1, x)(2, y)(3, z)\}$



$f^{-1} : B \rightarrow A = \{(x, 1)(y, 2)(z, 3)\}$

To find the inverse function $f(x)$

Step 1:-
Replace $f(x)$ by 'y'

step 2:-
Interchange the x and y

step 3:-
Solve the 'y' value

Step 4:-
Replace y by $f^{-1}(x)$

① Find the inverse of the function

$f(x) = x^3 - 2$

step1 ⇒ $y = x^3 - 2$

step2 ⇒ $x = y^3 - 2$

step3 ⇒ $y^3 = x + 2$

$y = (x+2)^{1/3}$

$y = \sqrt[3]{x+2}$

step4 ⇒ $\boxed{f^{-1}(x) = \sqrt[3]{x+2}}$

② Find the inverse of the function

$f(x) = \frac{5x-3}{2x+1}$

step 1 ⇒ $y = \frac{5x-3}{2x+1}$

step 2 ⇒ $x = \frac{5y-3}{2y+1}$

step 3 ⇒ $2xy + x = 5y - 3$

$2xy + x - 5y = -3$

$2xy - 5y = -3 - x$

$y(2x-5) = -3 - x$

$y = \frac{-3-x}{2x-5}$

step 4 ⇒ $f^{-1}(x) = \frac{-3-x}{2x-5}$

~~Comps~~

Composite of the function

Let x and y, z be are three sets then the function $f: x \to y$ and g $g: y \to z$ then the Composition of the function

$g \circ f : x \to z$

If the function $f(x) = x+2$ $g(x) = x-2$ and $h(x) = 3x$ for $\forall x \in R$ find

① $g \circ f$

② $f \circ g$

③ $f \circ f$

④ $g \circ g$

⑤ $f \circ h$

⑥ $f \circ (h \circ g)$

① $g \circ f = g(f(x))$

$= g(x+2)$

$= \cancel{g(x-2+2)}$

$= x - \cancel{2} + \cancel{2}$

$= x$

② $f \circ g = f(g(x))$

$= f(x-2)$

$= x - \cancel{2} + \cancel{2}$

$= x$

③ $f \circ f = f(f(x))$
$= f(x+2)$
$= x+2+2$
$= x+4$

④ $g \circ g = g(g(x))$
$= g(x-2)$
$= x-2-2$
$= x-4$

⑤ $f \circ h = f(h(x))$
$= f(3x)$
$= \cancel{f(3(x+2))}$ $\cancel{3(x+2)}$ $3x+2$
$= \cancel{3x+6}$

⑥ $f \circ (h \circ g) = f((h(g(x)))$
$= \cancel{f((h(x-2)))}$
$= \cancel{f(3x-2)}$
$= \cancel{3(x+2)} - 2$ $3x+2-2$
$= 3x+\cancel{2}-\cancel{2}$
$= 3x$

(6) $fo(hog)$

$f((h(g(x)))$

$f(h(x-2))$

$f(3x-6)$

$= 3x-6+2$

$= 3x-4$

To find the one to one function of function is given:

Let x_1, x_2 are $\in R$ $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

then it is called one-one function.

(1) Test whether the function $f(x) = x^2$ $x \in R$ is a one-one function are not.

Sol Let $x_1, x_2 \in R$

$$f(x) = x^2$$

By definition $f(x_1) = f(x_2)$

$$x_1^2 = x_2^2$$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) = 0$$

$$x_1 + x_2 = 0$$

$x_1 = -x_2$ but $x_1 = x_2$

So, it is not a one-one function

(2) Test whether $f(x) = 2x+3$ is one one or not.

Sol $f(x) = 2x+3$

By definition $f(x_1) = f(x_2)$

$$2x_1+3 = 2x_2+3$$
$$2x_1 - 2x_2 = 0$$
$$x_1 - x_2 = 0$$
$$x_1 = x_2$$

$\therefore$ the given function is a one-one fn.

(3) Test whether the ~~fn~~ $f(x) = 2x+3$ $f: R \to R$ onto function or not?

Sol $f(x) = 2x+3$

$$f: \overset{x}{R} \to \overset{y}{R}$$
$$2x+3 = y$$
$$2x = y-3$$
$$x = y-3/2$$

In the codomain if $y=3$ then $x=0$
if $y=1$ then $x=-1$

i.e., y is a real numbers then we get x is also real number

$\therefore f(x) = 2x+3$ is a onto function.

(4) Test Whether $f(x) = 2x+3$ the function starts from $f: N \to R$ is onto function or not.

$$f(x) = 2x+3$$
$$f: N \to R$$
$$y = 2x+3$$
$$2x = y-3$$
$$x = \frac{y-3}{2}$$

If $y = 4 \in R$ then $x = \frac{1}{2} \notin N$

$\therefore$ this is not a onto function.

Recursive function :-

the process of repeating a function each time apply it to the result of the previous stage (or) recursive fn is the technique of getting the soln from using the previous value of the same function.

Ex:- $f(x) = 4^x$

$$f(0) = 4^0 = 1$$
$$f(1) = 4^1 = 1 \times 4 = 4 \times f(0)$$
$$f(2) = 4^2 = 4 \times 4 = 4 \times f(1)$$
$$\boxed{\therefore f(n) = 4f(n-1)}$$

$f(n) = n!$

$\cancel{f(0) = 0! = \bullet = f(0)}$

$\cancel{f(2) = 2! = 2\times1 = f(0)\times 2}$

$f(0) = 0! = 1 =$

$f(1) = 1! = 1\times1 = 1\times f(0)$

$f(2) = 2! = 2\times1 = 2\times f(\bullet)$

$f(3) = 3! = 3\times2\times1 = 3\times f(2)$

$\vdots$

$f(n) = n! = n\times f(n-1)$

$$\boxed{f(n) = n\times f(n-1)}$$

(Find the recursive $\underline{\underline{fn}}$ for the $f(n) = 5n$

$f(n) = 6^n$

$\cancel{f(n) = 5n} \Rightarrow \cancel{f(n) = an}$

$\cancel{f(0) = 5(0)} = 0$

$\cancel{f(1) = 5 =}$

$\cancel{f(2) = 5\times2 =}$

Given $f(n) = 5n$

Consider $a_n = 5n$

$a_0 = 5\times0 = 0$

$a_1 = 5$

$a_2 = 10 = 5+5 = 5+a_1$

$a_3 = 15 = 10+5 = a_2+5$

$a_4 = 20 = 15+5 = a_3+5$

$\vdots$

$$\boxed{a_n = a_{n-1} + 5}$$

Given $f(n) = 6^n$

$n=0$ $\quad f(0) = 6^0 = 1$

$n=1$ $\quad f(1) = 6^1 = 6\times 1 = 6\times f(0)$

$n=2$ $\quad f(2) = 6^2 = 36 = 6\times 6 = 6\times f(1)$

$n=3$ $\quad f(3) = 6^3 = 216 = 36\times 6 = 6\times f(2)$

$$\boxed{f(n) = 6\times (n-1)}$$

(or)

$a_n = 6^n$

$a_0 = 6^0 = 1$

$a_1 = 6^1 = 6 = 6\times 1 = 6\times a_0$

$a_2 = 6^2 = 36 = 6\times 6 = 6\times a_1$

$a_3 = 6^3 = 216 = 36\times 6 = 6\times a_2$

$$\boxed{a_n = 6\times a_{n-1}} \quad n\geq 1$$

(2) A function $f(n) = a_n$ is defined recursively $a_0 = 4$, $a_n = a_{n-1} + n$ for $n \geq 1$. Find $f(n)$ in explicit form.

Sol:- Given $a_n = a_{n-1} + n$

$$a_{n-1} = a_{n-2} + (n-1)$$

$$a_{n-2} = a_{n-3} + n-2$$

$$a_{n-3} = a_{n-4} + n-3$$

$$a_n = a_{n-2} + n-1 + n$$

$$a_n = a_{n-3} + n-2 + n-1 + n$$

$$a_n = a_{n-4} + n-3 + n-2 + n-1 + n$$

$$a_n = a_1 + 2 + 3 + \dots + n$$

$4 = n-1$
$3 = n-2$
$2 = n-3$

$$a_n = (a_0 + 1) + 2 + 3 + \dots + n$$

$$a_n = a_0 + 1 + 2 + 3 + \dots + n$$

$f(n) = a_n$
$f(n) = a_{n-1} + 1$
$f(0) = a_0 + 1$

$$a_n = 4 + \frac{n(n+1)}{2}$$

Sum of natural numbers $= \frac{n(n+1)}{2}$

(3) A function $f(n) = a_n$ is defined recursively $a_0 = 3$, $a_n = 2a_{n-1} + 1$ for $n \geq 1$ find $f(n)$ in explicit form.

Sol:- Given $a_n = 2a_{n-1} + 1$

$$a_{n-1} = 2a_{n-2} + 1$$

$$a_{n-2} = 2a_{n-3} + 1$$

$$a_{n-3} = 2a_{n-4} + 1$$

$$a_n = 2a_{n-1} + 1$$

$$a_n = 2[2a_{n-2} + 1] + 1$$

$$= 2^2 a_{n-2} + 2 + 1$$

$$= 2^2[2a_{n-3} + 1] + 2 + 1$$

$$= 2^3 a_{n-3} + 2^2 + 2 + 1$$

$$= 2^3[2a_{n-4} + 1] + 2^2 + 2 + 1$$

$$= 2^4 a_{n-4} + 2^3 + 2^2 + 2^1 + 1$$

$4 = n-1 \quad 2 = n-3$

$3 = n-2 \quad 1 = n-4$

$$= 2^{n-1} a_1 + 2^{n-2} + 2^{n-3} + \dots 2 + 1$$

$f(n) = a_n$

$f(n) = 2a_{n-1} + 1$

$f(1) = 2a_0 + 1$

$= 2(3) + 1$

$= 7$

$$= 2^{n-1} 7 + 1 + 2 + \dots\dots 2^{n-2}$$

Geometric series $= \dfrac{1 - a^{n+1}}{1-a}$ $\quad n = n-2$, $a = 2$

$$= (7)2^{n-1} + \frac{1 - 2^{n-2+1}}{1-2}$$

$= 7(2^{n-1}) -1 + (2^{n-1})$

$= 8\,2^{n-1} - 1$

(4) A function $f(n) = a_n$ is defined recursively by $a_0 = 1$, $a_1 = 1$, $a_2 = 1$,

$a_n = a_{n-1} + a_{n-3}$ for $n \geq 3$

Prove that $f(n+2) \geq (\sqrt{2})^n$ for $n \geq 0$

Sol Given $a_0 = 1$, $a_1 = 1$, $a_2 = 1$

$a_n = a_{n-1} + a_{n-3}$

$a_3 = a_2 + a_0$

$= 1 + 1$

$= 2$

$a_4 = a_3 + a_1$

$= 2 + 1$

$= 3$

$f(n) = a_n$

$f(n+2) = a_{n+2}$

$f(n+2) \geq (\sqrt{2})^n$

$a_{n+2} \geq (\sqrt{2})^n$

Take $n = 1$

$a_3 \geq (\sqrt{2})^1$

$a_3 \geq \sqrt{2}$

$2 \geq \sqrt{2}$

This is true for $n=1$

we assume that this is true for $n=k$

$$a_{k+2} \ge (\sqrt{2})^k$$

Now, we have to show that this is true for $n = k+1$

$$\cancel{a_{k+3} \ge (\sqrt{2})}$$

$$a_{k+3} = a_{k+3-1} + a_{k+3-3}$$

$$a_{k+3} = a_{k+2} + a_k$$

$$a_{k+3} \ge (\sqrt{2})^k + (\sqrt{2})^{k-2}$$

$a_{k+2} \ge (\sqrt{2})^k$

$k = k-2$

$a_{k-2+2} \ge (\sqrt{2})^{k-2}$

$a_k \ge (\sqrt{2})^{k-2}$

$$a_{k+3} \ge (\sqrt{2})^{k-2}\left[1 + (\sqrt{2})^2\right]$$

$$a_{k+3} \ge (\sqrt{2})^{k-2}[3]$$

$$a_{k+3} \ge 3\,\frac{(\sqrt{2})^k}{(\sqrt{2})^2}$$

$$a_{k+3} \ge \frac{3}{2}(\sqrt{2})^k$$

$$a_{k+3} \ge \frac{3}{2}\,\frac{(\sqrt{2})^k}{\sqrt{2}}(\sqrt{2})$$

$$a_{k+3} \ge \frac{3(\sqrt{2})^{k+1}}{2\sqrt{2}} \ge \sqrt{2}^{k+}$$

this is true for $n = k+1$
Hence, by the mathematical induction it is true for all positive integer values.

Fibonacci Function:-
Fibonacci function is denoted by F_n.
It is defined recursively by $F_0 = 0$, $F_1 = 1$

$$\boxed{F_n = F_{n-1} + F_{n-2}}$$

$$F_2 = F_1 + F_0 = 1+0 = 1$$
$$F_3 = F_2 + F_1 = 1+1 = 2$$
$$F_4 = F_3 + F_2 = 2+1 = 3$$
$$F_5 = F_4 + F_3 = 3+2 = 5$$
$$F_6 = F_5 + F_4 = 5+3 = 8$$
$$F_7 = F_6 + F_5 = 8+5 = 13$$
$$F_8 = F_7 + F_6 = 13+8 = 21$$
$$F_9 = F_8 + F_7 = 21+13 = 34$$
$$F_{10} = F_9 + F_8 = 34+21 = 55$$

(1) If F_0, F_1, F_2 are fibonacci number Prove that $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$, for all +ve integers.

Proof:- Let $n = 1$

$$\sum_{i=0}^{1} F_i^2 = F_0^2 + F_1^2 \qquad = F_n \times F_{n+1}$$
$$= 0 + 1^2 = 1 \qquad = F_1 \times F_2$$
$$1 \qquad = 1 \times 1$$

this theorem is true for $n=1$

we assume that these theorem is true for $n=k$

$$\sum_{i=0}^{k} F_i^2 = F_k \times F_{k+1}$$

Now, we have to prove that this is true for $n=k+1$

$$\sum_{i=0}^{k+1} F_i^2 = \sum_{i=0}^{k} F_i^2 + F_{k+1}^2$$

$$= (F_k \times F_{k+1}) + F_{k+1}^2$$

$$= F_{k+1}\left[F_k + F_{k+1}\right]$$

$$= F_{k+1} \times F_{k+2}$$

$$\boxed{\begin{array}{l} F_n = F_{n-1} + F_{n-2} \\ F_{k+2} = F_{k+1} + F_k \end{array}}$$

This is true for $n=k+1$.

Hence, by the mathematical induction it is true for all integer values of n.

(2) If F_0, F_1, F_2 are fibonacci numbers prove that $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$ for positive values of n.

Given, $\displaystyle\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$

Take $n = 1$

$$\Rightarrow \frac{F_0}{2^1} = 1 - \frac{F_3}{2^1}$$

$$\Rightarrow \frac{0}{2} = 1 - \frac{2}{2}$$

$$0 = 0$$

this is true for $n=1$

we assume that this theorem is true for $n = k$

$$\sum_{i=1}^{k} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{k+2}}{2^k}$$

Now, we have to show prove that this is true for $n = k+1$

$$\sum_{i=1}^{k+1} \frac{F_{i-1}}{2^i}$$

$$= \sum_{i=1}^{k} \frac{F_{i-1}}{2^i} + \frac{F_{k\cancel{+1}\cancel{-1}}}{2^{k+1}}$$

$$= \left(1 - \frac{F_{k+2}}{2^k}\right) + \frac{F_k}{2^{k+1}}$$

$$= \frac{2^{k+1} - 2F_{k+2} + F_k}{2^{k+1}}$$

$$= 1 - \left(\frac{F_{k+2}}{2^k} - \frac{F_k}{2^{k+1}} \right)$$

$$= 1 - \left(\frac{2F_{k+2} - F_k}{2^{k+1}} \right)$$

$$= 1 - \left[\frac{(F_{k+2} - F_k) + F_{k+2}}{2^{k+1}} \right]$$

$F_n = F_{n-1} + F_{n-2}$

$F_{k+2} = F_{k+1} + F_k$

$F_{k+2} - F_k = F_{k+1}$

$$\text{same} \left\{ \begin{aligned} &= 1 - \left[\frac{F_{k+1} + F_{k+2}}{2^{k+1}} \right] \\ &\qquad \text{(or)} \\ &= 1 - \left[\frac{[F_{k+1} + \cancel{F_k} - \cancel{F_k}] + F_{k+2}}{2^{k+1}} \right] \end{aligned} \right.$$

$$= 1 - \left[\frac{F_{k+1} + F_{k+2}}{2^{k+1}} \right]$$

$F_n = F_{n-1} + F_{n-2}$

$F_{k+3} = F_{k+2} + F_{k+1}$

$$= 1 - \frac{F_{k+3}}{2^{k+1}}$$

This is true for $n=k+1$
Hence, by mathematical induction it is true for all positive values of n.

Imp
(3) For the fibonacci numbers f_0, f_1, f_2 prove that $f_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$

Given,

$$f_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Take $n=1$

$$F_1 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{\cancel{1+}\sqrt{5} - \cancel{1+}\sqrt{5}}{2}\right]$$

$$= \frac{1}{\cancel{\sqrt{5}}}\left[\frac{\cancel{2}\cancel{\sqrt{5}}}{\cancel{2}}\right]$$

$$F_1 = 1$$

$\because F_1 = 1$

$$1 = 1$$

$\therefore$ this is true for $n=1$
we assume that this is true for $n=k$

$$F_k = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right]$$

Now, we have to show that $n=k+1$

$$\cancel{F_{k+1} = \frac{1}{\sqrt{5}}}$$

$$F_n = F_{n-1} + F_{n-2}$$

$$F_{k+1} = F_k + F_{k-1}$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right] + \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\right]$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right] + \left(\frac{1+\sqrt{5}}{2}\right)^{k-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\right]$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[\left[\left(\frac{1+\sqrt{5}}{2}\right)^k + \left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\right] - \left[\left(\frac{1-\sqrt{5}}{2}\right)^k + \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\right]\right]$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left[1 + \left(\frac{1+\sqrt{5}}{2}\right)\right] - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left[1 + \left(\frac{1-\sqrt{5}}{2}\right)\right]\right]$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left[\frac{3+\sqrt{5}}{2}\right]-\left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left[\frac{3-\sqrt{5}}{2}\right]\right]$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left(\frac{6+2\sqrt{5}}{4}\right)-\left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left[\frac{6-2\sqrt{5}}{4}\right]\right]$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left[\frac{(1+\sqrt{5})}{2}\right]^{2}-\left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left(\frac{1-\sqrt{5}}{2}\right)^{2}\right]$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right]$$

This is true for $n=k+1$

Hence, by mathematical induction it is true for all values of n.

Lucas function :-

The Lucas function "l_n" is defined recursively by $l_0 = 2$ $l_1 = 1$ and

$$l_n = l_{n-1} + l_{n-2} \text{ for } n \geq 2$$

$$l_2 = l_1 + l_0$$

$$= 1 + 2$$

$$\boxed{l_2 = 3}$$

$$l_3 = l_2 + l_1$$

$$= 3 + 1$$

$$\boxed{l_3 = 4}$$

$$l_4 = l_3 + l_2$$

$$= 4 + 3$$

$$\boxed{l_4 = 7}$$

$$l_5 = l_4 + l_3$$

$$= 7 + 4$$

$$\boxed{l_5 = 11}$$

$$l_6 = l_5 + l_4$$

$$= 11 + 7$$

$$\boxed{l_6 = 18}$$

$l_7 = l_6 + l_5$

$= 18 + 11 =$

$=$

$\boxed{l_7 = 29}$

$l_8 = l_7 + l_6$

$= 29 + 18$

$\boxed{l_8 = 47}$

$l_9 = l_8 + l_7$

$= 47 + 29$

$\boxed{l_9 = 76}$

$l_{10} = l_9 + l_8$

$= 76 + 47$

$= 123$

If F_i's are Fibonacci numbers and L_i's are locus numbers

then PT

$L_n = F_{n-1} + F_{n+1}$ for all +ve integers

Proof:-

$n = 1$

$L_1 = F_0 + F_2$

$= 0 + 1$

$L_1 = 1$

$1 = 1$

LHS = RHS

This is true for $n = 1$

we assume that this is true for $n = k$

$L_k = F_{k-1} + F_{k+1}$

We have to P.T $n = k+1$

$L_{k+1} = l_k + l_{k-1}$ $[\because l_n = l_{n-1} + l_{n-2}]$

$L_{k+1} = F_{k-1} + F_{k+1} + F_{k-2} + F_k$

$(F_n = F_{n-1} + F_{n-2}) = \underbrace{F_{k-1} + F_{k-2}}_{F_k = F_{k-1} + F_{k-2}} + \underbrace{F_{k+1} + F_k}_{F_{k+2} = F_{k+1} + F_k}$

$= F_k + F_{k+2}$

$\therefore$ This is true for $n = k+1$

Hence, by mathematical induction it is true for all +ve integer values.

If F_i's are fibonacci numbers and L_i's are locus numbers then, P.T

$l_{n+4} - l_n = 5F_{n+2}$

Proof :-

Take $n = 1$

LHS $l_5 - l_1 = 11 - 1$

$= 10$

RHS: $5F_{1+2} = 5F_3$

$= 5(2)$

$= 10$

$\therefore$ LHS = RHS.

This is true for $n = 1$

we assume that this is true for $n = k$

$l_{k+4} - l_k = 5F_{k+2}$

we have to P.T $n=k+1$

$l_{k+5} - l_{k+1}$

$\boxed{\text{w.k.T} \quad l_n = l_{n-1} + l_{n-2}}$

$\therefore (l_{k+4} + l_{k+3}) - (l_k + l_{k-1})$

$= (l_{k+4} - l_k) + (l_{k+3} - l_{k-1})$

$= \underbrace{(l_{k+4} - l_k)}_{l_{k+4} - l_k = 5F_{k+2}} + \underbrace{l_{(k-1)+4} - l_{k-1}}_{l_{(k-1)+4} - l_{k-1} = 5F_{k+1}}$

$= 5F_{k+2} + 5F_{k+1}$

$= 5[F_{k+2} + F_{k+1}]$

$= 5F_{k+3}$ $\quad \begin{cases} F_n = F_{n-1} + F_{n-2} \\ F_{k+3} = F_{k+2} + F_{k+1} \end{cases}$

$\therefore$ This is true for $n=k+1$

Hence by mathematical induction it is true for all +ve integer values.

Relation

Cartesian Product / Cross product

Let A and B are two sets then the set of all ordered pairs (a,b) where $a \in A$ $b \in B$ is called Cartesian product / cross product.

It is denoted by $A \times B$.

For ex :-

If $A = \{1,2,3\}$ $B = \{3,4,5\}$

$A \times B = \{(1,3)\ (1,4)\ (1,5)\ (2,3)\ (2,4)\ (2,5)$
$(3,3)\ (3,4)\ (3,5)\}$

Relation :-

Relation is defined as collection of all ordered pairs.

An order pair $(x,y) \in R$ where R is a relation.

(or)

Let A and B two sets the a subset of $A \times B$ is called relation from A to B

If 'R' is a relation from A to B and the ordered pair $(a,b) \in R$

then it is denoted by aRb

for ex:-

* Let $A = \{2, 3, 4\}$ $B = \{3, 4, 5, 6, 7, 8\}$

if $(x, y) \in R$ x divides y

$R = \{(2,4)\ (2,6)\ (2,8),\ (3,3)\ (3,6)\ (4,4)\ (4,8)\}$

* If $A = \{1, 2, 3, 4\}$ if $(x, y) \in R$

then $x \leq y$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,2)\ (2,3)\ (2,4)\ (3,3),\ (3,4)\ (4,4)\}$

Types of Relations:-

① Reflexive Relation:

A relation ~~one~~ R on a set 'x' is reflective if $(x, x) \in R$ for any $x \in X$

for ex:-

The set $x = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\}$

Then the relation 'R' is called reflextive relation.

② IRReflexive Relation:-

A relation 'R' on a set 'x' is a irreflexive relation if $(x, x) \notin R$ for any $x \in X$

for ex :-

(1) $x = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\}$

then the relation 'R' is called irreflexive relation

(2) $x = \{a, b, c\}$

$R = \{(a,a)\ (b,b)\}$

then the relation 'R' is called Irreflexive relation.

Symmetric Relation :-

A Relation 'R' on a set x is symmetric relation if $(x, y) \in R$ then $(y, x) \in R$ for any $x, y \in X$

for ex:-

$R = \{(1,2)\ (2,1)\ (1,1)\}$

Anti-symmetric Relation:-

A Relation 'R' on a set x is Anti-symmetric if $(x, y) \in R$ $x \neq y$ then $(y, x) \notin R$

for ex:-

$R = \{(1,2)\ (3,2)\ (1,1)\}$

if $(x, y) \in R$ and $x = y$ then $(y, x) \in R$
this is also called Anti-symmetric Relation

Ex:- $R = \{(1,1)\ (2,2)\ (3,3)\}$

Transitive Relation :-

A Relation 'R' on a set x is a transitive Relation.
if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$
for any $x, y, z \in X$

Ex:- $R = \{(1,2)\ (2,3)\ (1,3)\ (2,5)\ (5,2)\ (2,2)\ (3,2)\}$

Representation of the Relations:

there are two types of Representation of the Relations:

1. Relation of Matrix
2. Diagraph

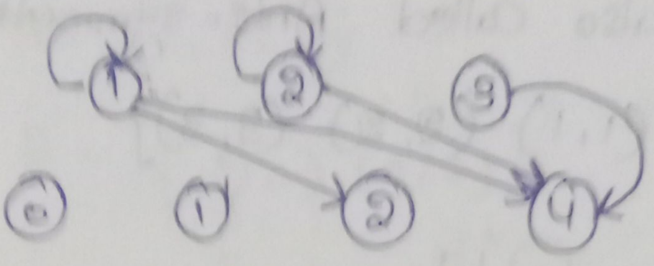
Ex:- if $X = \{1, 2, 3\}$ $Y = \{0, 1, 2, 4\}$
$x, y \in R$ if $x \leq y$

$R = \{(1,1)\ (1,2)\ (1,4)\ (2,2)\ (2,4)\ (3,4)\}$

$$\begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow$ Representation of relation of matrix

(3) Representation of Diagraph



Ex: 2

The set $X = \{1, 2, 3, 4\}$ ~~with~~

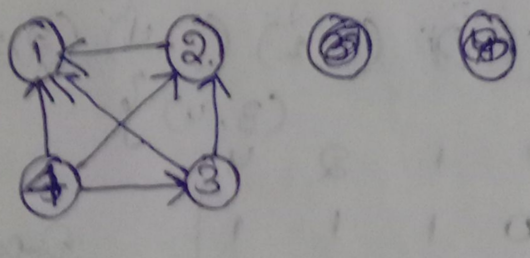
$R = \{(x, y) / x > y\}$

$R = \{(2,1)\ (3,2)\ (3,1)\ (4,3)\ (4,2)\ (4,1)\}$

(1) Matrix Representation

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(2) Diagraph Representation



③ Let $A = \{1, 2, 3, 4\}$ and R be the relation on set A defined by $x R y$ only x/y then write the relation R and matrix and diagraph.

Sol Given $A = \{1, 2, 3, 4\}$

$$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,2)\ (2,4)\ (3,3)\ (4,4)\}$$

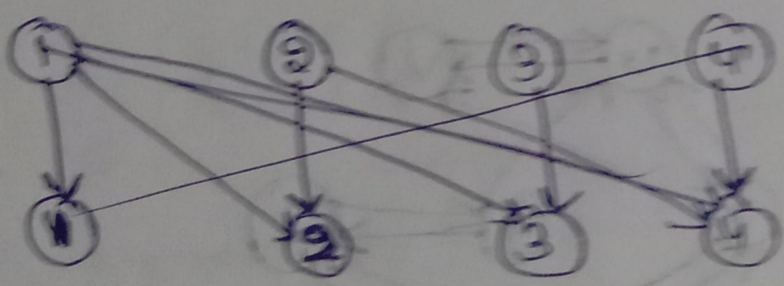
$x/y = 4/28 = 4)28(7$
$\quad 28$
$\quad \underline{\ \ 0\ \ }$

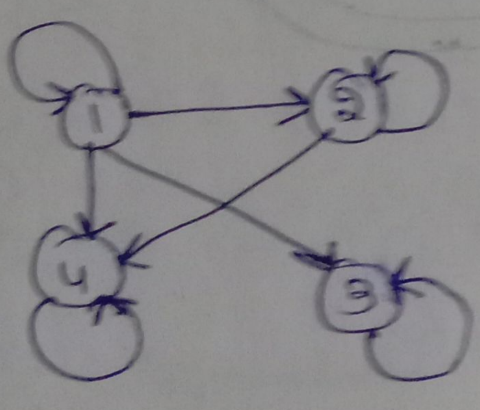
Remainder becomes '0'

Matrix Representation

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Diagrap Representation





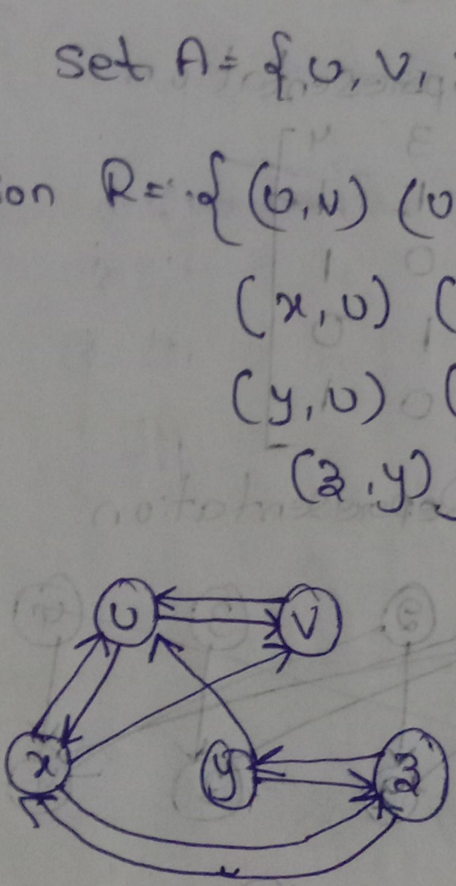
(4) Let $A = \{u, v, x, y, z\}$ and R is the relation on A whose matrix is

$$\begin{array}{c|ccccc} & u & v & x & y & z \\ \hline u & 0 & 1 & 1 & 0 & 0 \\ v & 1 & 0 & 1 & 0 & 0 \\ x & 1 & 1 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 & 1 \\ z & 0 & 0 & 1 & 1 & 0 \end{array}$$

Determine the relation and draw its diagraph.

Sol Let set $A = \{u, v, x, y, z\}$

Relation $R = \{(u,v)\ (u,x),\ (v,u),\ (v,x)$
$(x,u)\ (x,v)\ (x,z)$
$(y,u)\ (y,z)\ (z,x)$
$(z,y)\}$



Equivalence Relation:-

A Relation 'R' and the set A is said to be Equivalence relation if

1. R is Reflexive
2. R is a Symmetric
3. R is a Transitive

Example

1) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,4)\ (4,3)\ (3,3)\ (4,4)\}$ verify that R is a equivalence relation or not.

Sol :- Given $A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,4)\ (4,3)\ (3,3)\ (4,4)\}$

R is a Reflexive

$(a,a) \in R$

$(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$(4,4) \in R$

$\therefore$ R is a reflexive

R is symmetric

$(a,b) \in R \longrightarrow (b,a) \in R$

$(1,2) \in R \longrightarrow (2,1) \in R$

$(3,4) \in R \longrightarrow (4,3) \in R$

$(1,1) \in R \longrightarrow (1,1) \in R$

$\therefore$ R is symmetric

R is transitive

$(a,b) \in R, \longrightarrow (a,c) \in R$
$(b,c) \in R$

$(1,2) \in R$
$(2,1) \in R \longrightarrow (1,1) \in R$

$(1,2) \in R$
$(2,2) \in R \longrightarrow (1,2) \in R$

$(3,4) \in R$
$(4,3) \in R \longrightarrow (3,3) \in R$

$(4,3) \in R$
$(3,4) \in R \longrightarrow (4,4) \in R$

$\therefore$ R is transitive Relation

$\therefore$ R is a equivalence relation

(2) Let $A = \{1,2,3,4\}$ and $R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,1)\ (3,3)\ (1,3)\ (4,1)\ (4,4)\}$

check the relation is equivalence or not

Given $A = \{1,2,3,4\}$

$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,1)(3,3)\ (1,3)\ (4,1)\ (4,4)\}$

R is a reflexive

$(a,a) \in R$
$(1,1) \in R$
$(2,2) \in R$

$(3,3) \in R$

$(4,4) \in R$

$\therefore$ R is a reflexive

R is a symmetric

$(a,b) \in R \longrightarrow (b,a) \in R$

$(1,2) \in R \longrightarrow (2,1) \in R$

$(1,3) \in R \longrightarrow (3,1) \in R$

$(4,1) \in R \longrightarrow (1,4) \notin R$

$\therefore$ R is not a symmetry

$\therefore$ the Given relation 'R' is not a equivalence relation.

(3) show that $R = \{(x,y), x+y$ must be even $\forall x,y \in N\}$ is equivalence or not

Given $R = \{(x,y), x+y = \text{even}$
$\forall x,y \in N\}$

i.e $R = \{(1,1)\ (1,3)\ (1,5)\ (2,2)\ (2,4)\ (2,6)\}$

R is a Reflexive

$(a,a) \in R\ \forall a \in N$

$(1,1) \in R$ ......

$\therefore$ R is a reflexive

R is a symmetric

$(a,b) \in R \longrightarrow (b,a) \in R$

$(1,3) \in R \longrightarrow (3,1) \in R$

$(1,5) \in R \longrightarrow (5,1) \in R$

$(2,4) \in R \longrightarrow (4,2) \in R$ ......

$\therefore$ R is Symmetric

R is a transitive

$(a,b) \in R$
$(b,c) \in R$ $\longrightarrow$ $(a,c) \in R$

$(1,3) \in R$
$(3,3) \in R$ $\longrightarrow$ $(1,3) \in R$

$(1,5) \in R$
$(5,5) \in R$ $\longrightarrow$ $(1,5) \in R$

$\therefore$ R is a transitive

$\therefore$ the Given relation 'R' is a equivalence relation.

(4) Determine the nature of the following relations $A = \{1, 2, 3\}$ and $R = \{(1,2), (2,1) (1,3) (3,1)\}$

(ii) $R = \{(1,1) (2,2) (3,3) (2,3) (3,2)\}$

(i) Give $A = \{1,2,3\}$

$R = \{(1,2) (2,1) (1,3) (3,1)\}$

R is a reflexive

$(a,a) \in R$

$(1,2) \notin R$

$(2,1) \notin R$

$(1,3) \notin R$

$(3,1) \notin R$

$\therefore$ R is not a reflexive

$\therefore$ the given relation 'R' is not equivalence relation.

(iii) $A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (2,3)\ (3,2)\}$

R is a reflexive

$(a,a) \in R$

$(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$\therefore$ R is reflexive

R is a symmetric

$(a,b) \in R \rightarrow (b,a) \in R$

$(1,1) \in R \rightarrow (1,1) \in R$

$(2,2) \in R \rightarrow (2,2) \in R$

$(3,3) \in R \rightarrow (3,3) \in R$

$(2,3) \in R \rightarrow (3,2) \in R$

$\therefore$ R is a symmetric

R is a trasitive

$\left.\begin{array}{l}(a,b) \in R \\ (b,c) \in R\end{array}\right\} \longrightarrow (a,c) \in R$

$\left.\begin{array}{l}(3,2) \in R \\ (2,2) \in R\end{array}\right\} \longrightarrow (3,2) \in R$

$\therefore$ R is a trasitive

$\therefore$ The given relation 'R' is equivalence relation.

(5) Find whether the given relation $R = \{(a,b) / a \le b \ \forall \ a,b \in Z^+\}$ is an equivalence or not.

Given $R = \{(a,b) / a \le b \ \forall \ a,b \in Z^+\}$

i.e., $R = \{(1,2)\ (1,1)\ (1,3)\ (1,4)\ (2,2)$
$(2,3)\ (2,4)\ (3,3)\ (3,4)\ (3,5)\}$

R is a Reflexive

$(a,a) \in R$

$(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$\therefore$ R is reflexive

R is a symmetric

$(a,b) \in R \rightarrow (b,a) \in R$

$(1,2) \in R \rightarrow (2,1) \notin R$

$\therefore$ R is not a symmetric

R is a trasitive

$(a,b) \in R$
$(b,c) \in R$ $\longrightarrow$ $(a,c) \in R$

$(1,2) \in R$
$(2,3) \in R$ $\longrightarrow$ $(1,3) \in R$

$\therefore$ R is trasitive

$\therefore$ the give relation 'R' is not a equivalence relation.

Let set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ on this set define the relation 'R' by $(x, y) \in R \Rightarrow x - y$ is a multiple of 5. Verify that R is a equivalence Relation.

Sol Given

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

R is a Reflexive

$(x, x) \in R \Rightarrow x - x$ is a multiple of 5

0 is a multiple of 5

$\Rightarrow$ for $(x, x) \in R$

$\therefore$ R is a reflective

R is a symmetric

If $(x, y) \in R$

$= x - y$ is a multiple of 5

$-(y - x)$ is a multiple of 5

$\therefore y - x$ is also multiple of 5

$\Rightarrow (y, x) \in R$

$\therefore$ R is a symmetric

R is a Transitive

if $(x, y) \in R$, $(y, z) \in R$

$x - y$ is a multiple of 5

$x - y = 5k_1 \longrightarrow$ $20 - 10 = 10$ $= 5 \times 2 = 5 \times k_1$

$y-z$ is multiple of 5

$y-z = 5k_2$

$x-\cancel{y}+\cancel{y}-z = 5k_1 + 5k_2$

$x-z = 5k$ where $k = k_1 + k_2$

$\therefore$ $x-z$ is multiple of 5

$\Rightarrow (x,z) \in R$

$\therefore$ R is a Transitive

$\therefore$ R Satisfies Reflexive, symmetric, transitive

$\therefore$ The given relation is equivalence relation

(7) for a fixed integer $n>1$ prove that the relation Congruence to modulo m is equivalence relation on the set of all integers Z.

Remainder

Sol for $(a,b) \in Z$

a is Congruence to b modulo n

$a \equiv b \pmod n$

$(a-b)$ is multiple of n

$(a-b) = kn \ \forall \ k \in Z$

Let us define the relation R so that

$aRb \Leftrightarrow a \equiv b \pmod n$

R is a reflexive

aRa

$(a,a) \in R$

$a \equiv a \pmod n$

$(a-a)$ is a multiple of n

0 is a multiple of n

$\therefore R$ is a reflexive

R is a symmetric

if aRb

$\Rightarrow a \equiv b \pmod n$

$(a-b)$ is multiple of n

$-(b-a)$ is also multiple of n

$b-a$ is also a multiple of n

bRa

$aRb \Rightarrow bRa$

$\therefore R$ is a symmetric

R is transitive

if aRb, bRc

$\Rightarrow a \equiv b \pmod n$

$(a-b)$ is a multiple of n

$b \equiv c \pmod n$

$(b-c)$ is a multiple of n

$a-\cancel{b}+\cancel{b}-c$ is also a multiple of n

$(a-c)$ is a multiple of n

$a \equiv c \pmod n$

aRc

aRb, $bRc \Leftrightarrow aRc$

$\therefore R$ is a transitive.

$\therefore$ The given relation 'R' is equivalence relation for all integers Z.

Operations on Relations

1. Union Relation:-

Given the relations R_1 & R_2 from a set A to set B then the union of R_1 & R_2 is denoted by $R_1 \cup R_2$ is defined as a relation from A to B with the following conditions

$(a,b) \in R_1 \cup R_2 \Leftrightarrow (a,b) \in R_1$ or $(a,b) \in R_2$

2. Intersection of the Relation:-

Given the relations R_1 & R_2 from a set A to set B then the intersection of R_1 & R_2 is denoted by $R_1 \cap R_2$ is defined as a relation from A to B with the following conditions

$(a,b) \in R_1 \cap R_2 \Leftrightarrow (a,b) \in R_1$ and $(a,b) \in R_2$

3. Compliment of the Relation:

$\overline{R}$ is the Compliment of the set of the relation R and universal cross product $A \times B$ i.e $\boxed{\overline{R} = A \times B - R}$

4. Converse of the Relation:-

Let R be a relation from A to B the Converse of the relation R is denoted by R^c it is defined as a relation

from B to A with the Condition
$(a,b) \in R^c \Rightarrow (b,a) \in R$

Problem :-

(1) set $A = \{a, b, c\}$ set $B = \{1, 2, 3\}$ and
$R = \{(a,1)\ (b,1)\ (c,2)\ (c,3)\}$ and
$S = \{(a,1)\ (a,2)\ (b,1)\ (b,2)\}$ Find $\overline{R}$, $\overline{S}$
$R \cup S$, $R \cap S$, R^c, S^c

Sol $A \times B = \{(a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)$
$(b,3)\ (c,1)\ (c,2)\ (c,3)\}$

$\overline{R} = A \times B - R$

$= \{(a,2)\ (a,3)\ (b,2)\ (b,3)\ (c,1)\}$

$\overline{S} = A \times B - S$

$= \{(a,3)\ (b,3)\ (c,1)\ (c,2)\ (c,3)\}$

$R \cup S$

$= \{(a,1)\ (a,2)\ (b,1)\ (b,2)\ (c,2)\ (c,3)\}$

$R \cap S$

$= \{(a,1)\ (b,1)\}$

$R^c = \{(1,a)\ (1,b)\ (2,c)\ (3,c)\}$

$S^c = \{(1,a)\ (2,a)\ (1,b)\ (2,b)\}$

Composite Relation

Let R be a relation from A to B and the relation S from B to C then we define a new relation called the Product (or) the Composition of R & S is denoted by RoS from A to C

$R : A \to B$

$S : B \to C$

$RoS : A \to C$

Let set $A = \{1, 2, 3, 4\}$ and R, S are relations on A defined by $R = \{(1,2)(1,3)(2,4)(4,4)\}$ & $S = \{(1,1)(1,2)(1,3)(1,4)(2,3)(2,4)\}$ then find (i) RoS

(ii) SoR

(iii) RoR (R^2)

(iv) SoS (S^2)

(v) Ro(SoR)

(vi) (RoS)oR

(vii) (RoR)oR

(viii) (SoS)oS

Given $A = \{1, 2, 3, 4\}$

$R = \{(1,2)(1,3)(2,4)(4,4)\}$

$S = \{(1,1)(1,2)(1,3)(1,4)(2,3)$ (2

(i) $RoS = \{(1,2)(1,3)(2,4)(4,4)\}o\{(1$

$(1,2)(1,3)(1,4)(2,3)$ (2

* $(1,2) \in R$
$(2,3) \in S$ $\rightarrow (1,3) \in RoS$
delete

* $(1,2) \in R$
$(2,4) \in S$ $\rightarrow (1,4) \in RoS$
delete

* $(1,4) \in R$
$(4,4) \in R$ $\rightarrow (1,4) \in RoS$

$RoS = \{(1,3)\ (1,4)\}$

(ii) SoR

$= \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,3)\ (2,4)\} \circ \{(1,2)\ (1,3)\ (2,4)\ (4,4)\}$

* $(1,1) \in S$
$(1,2) \in R$ $\quad (1,2) \in SoR$

* $(1,1) \in S$
$(1,3) \in R$ $\quad (1,3) \in SoR$

* $(1,2) \in S$
$(2,4) \in R$ $\quad (1,4) \in SoR$

* $(1,4) \in S$
$(4,4) \in R$ $\quad (1,4) \in SoR$

* $(2,4) \in S$
$(4,4) \in R$ $\quad (2,4) \in SoR$

$\therefore SoR = \{(1,2)\ (1,3)\ (1,4)\ (2,4)\}$

(iii) RoR

$\Rightarrow \{(1,2)\ (1,3)\ (2,4)\ (4,4)\} \circ \{(1,2)\ (1,3)\ (2,4)\ (4,4)\}$

* $(1,2) \in R$
$(2,4) \in R$ $\quad (1,4) \in RoR$

* $(2,4) \in R$
$(4,4) \in R$ $\quad (2,4) \in RoR$

$* \ (4,4) \in R \quad (4,4) \in R \circ R$
$(4,4) \in R$

$\therefore R \circ R = \{(1,4)\ (2,4)\ (4,4)\}$

(iv) $S \circ S$

$\Rightarrow \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,3)\ (2,4)\} \circ \{(1,1)$
$(1,2)\ (1,3)\ (1,4)\ (2,3)\ (2,4)\}$

$* \ (1,1) \in S$
$(1,1) \in S \rightarrow (1,1) \in S \circ S$

$* \ (1,1) \in S$
$(1,2) \in S \rightarrow (1,2) \in S \circ S$

$* \ (1,1) \in S$
$(1,3) \in S \rightarrow (1,3) \in S \circ S$

$* \ (1,1) \in S$
$(1,4) \in S \rightarrow (1,4) \in S \circ S$

$* \ (1,2) \in S$
$(2,3) \in S \rightarrow (1,3) \in S \circ S$

$* \ (1,2) \in S$
$(2,4) \in S \rightarrow (1,4) \in S \circ S$

$*$

$\therefore S \circ S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\}$

(v) $R \circ \{S \circ R\}$

$\Rightarrow \{(1,2)\ (1,3)\ (2,4)\ (4,4)\} \circ \{(1,2)\ (1,3)$
$(1,4)\ (2,4)\}$

* $(1,2) \in R$ $\quad$ $(1,4) \in R\circ(S\circ R)$
$(2,4) \in S\circ R$

$\therefore R\circ(S\circ R) = \{(1,4)\}$

(vi) $(R\circ S)\circ R$

$\Rightarrow \{(1,3)\ (1,4)\} \circ \{(1,2)\ (1,3)\ (2,4)\ (4,4)\}$

* $(1,4) \in R\circ S$ $\quad$ $(1,4) \in (R\circ S)\circ R$
$(4,4) \in R$

$\therefore (R\circ S)\circ R = \{(1,4)\}$

(vii) $(R\circ R)\circ R$

$\Rightarrow \{(1,4)\ (2,4)\ (4,4)\} \circ \{(1,2)(1,3)(2,4)\ (4,4)\}$

* $(1,4) \in R\circ R$ $\quad$ $(1,4) \in (R\circ R)\circ R$
$(4,4) \in R$

* $(2,4) \in R\circ R$ $\quad$ $(2,4) \in (R\circ R)\circ R$
$(4,4) \in R$

* $(4,4) \in R\circ R$ $\quad$ $(4,4) \in (R\circ R)\circ R$
$(4,4) \in R$

$\therefore (R\circ R)\circ R = \{(1,4)\ (2,4)\ (4,4)\}$

(viii) $(S\circ S)\circ S$

$\{(1,1)(1,2)\ (1,3)\ (1,4)\} \circ \{(1,1)\ (1,2)(1,3)$
$(1,4)\ (2,3)\ (2,4)\}$

* $(1,1) \in S\circ S$ $\quad$ $(1,1) \in (S\circ S)\circ S$
$(1,1) \in S$

* $(1,2) \in S o S$
  $(2,3) \in S$ $\quad (1,3) \in (S o S) o S$

* $(1,2) \in S o S$
  $(2,4) \in S$ $\quad (1,4) \in (S o S) o S$

* $(1,1) \in S o S$
  $(1,2) \in S$ $\quad (1,2) \in (S o S) o S$

$\therefore (S o S) o S = \{(1,1)(1,2)(1,3)(1,4)\}$

Partial order relation:-
A Relation 'R' on a set P is said to be partial order relation then R is a Reflexive, Antisymmetric and transitive set P.
It is denoted by the symbol "$\leq$"

Partial order set (Poset)
A set P with a partial order relation defined on it is called partial order set.
It is denoted by the $(P, R) = (P, \leq)$

Ex:- The set of all integers Z with the relations $\leq$ and $\geq$ are po sets

Total ordered set

Let 'R' be a partial order relation and set P then R is called Total ordered set if $a, b \in P$ either $a \leq b$ (or) $b \leq a$

Hasse diagram

A partial order and set P can be represented by means of diagram is known as Hasse diagram.

Rules for Hasse diagram

1. form all the related pairs without identity pairs.
2. Remove the related pairs obtained by trasitive property.
3. Draw the diagram with remaining pairs.

(1) Draw the Hasse diagram for the set $P = \{2, 3, 6, 12, 24, 36\}$ with partial order relation with divisibility

Sol: Let set $P = \{2, 3, 6, 12, 24, 36\}$

$R = \{(2,2)\ (2,6)\ (2,12)\ (2,24)\ (2,36)$

divisibilies

$(3,3)\ (3,6)\ (3,12)\ (3,24)\ (3,36)$

$(6,6)\ (6,12)\ (6,24)\ (6,36)$

$(12,12)\ (12,24)\ (12,36)$

$(24,24)\ (36,36)\}$

$2 \rightarrow 2, 6, 12, 24, 36$

$3 \rightarrow 3, 6, 12, 24, 36$

$6 \rightarrow 6, 12, 24, 36$

$12 \rightarrow 12, 24, 36$

$24 \rightarrow 24$

$36 \rightarrow 36$

R is a reflexive

$\because (a, a) \in R$

$(2,2) \in R$ $(12,12) \in R$

$(3,3) \in R$ $(24,24) \in R$

$(6,6) \in R$ $(36,36) \in R$

$\therefore$ R is a reflexive

R is a Anti symmetric relation

$\because (a,b) \in R$ $(b,a) \notin R$ $a \neq b$

eg :- $(2,6) \in R$ $(6,2) \notin R$

R satisfies the antisymmetric relation.

R is a transitive

$\because (a,b) \in R$ $(b,c) \in R$

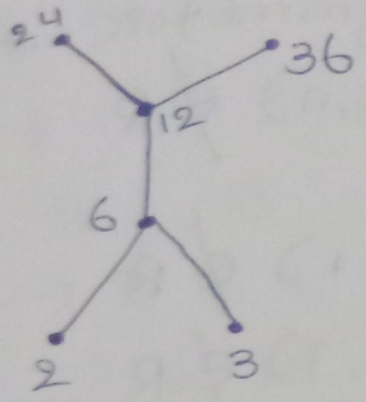
$(a,c) \in R$

eg:- $(3,6) \in R$ $(6,12) \in R$

$(3,12) \in R$

$\therefore$ R satisfies the transitive

Hasse diagram



(2) Draw the Hasse diagram for $[\{1, 2, 3, 4, 6, 9\}, |]$

sol

$1 \rightarrow 1, 2, 3, 4, 6, 9$

$2 \rightarrow 2, 4, 6$

$3 \rightarrow 3, 6, 9$

$4 \rightarrow 4$

$6 \rightarrow 6$

$9 \rightarrow 9$

divisibility

$R = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,9)$
$(2,2)\ (2,4)\ (2,6)$
$(3,3)\ (3,6)\ (3,9)$
$(4,4)\ (6,6)\ (9,9) \}$

R is a reflexive

$\because (a,a) \in R$

Eg:- $(1,1) \in R$, $(2,2) \in R$ $(3,3) \in R$
$(4,4) \in R$, $(6,6) \in R$ $(9,9) \in R$

∴ R satisfies the reflexive

R is a anti symmetric relation

∵ $(a,b) \in R$ $(b,a) \notin R$ $a \neq b$

eg: $(1,2) \in R$ $(2,1) \notin R$

$(1,3) \in R$ $(3,1) \notin R$

∴ R satisfies the anti symmetric relation.

R is a transitive

∵ $(a,b) \in R$ $(b,c) \in R$

$(a,c) \in R$

eg:- $(1,2) \in R$ $(2,2) \in R$

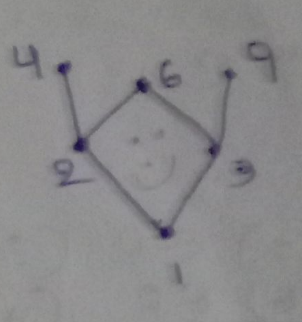
$(1,2) \in R$

$(2,4) \in R$ $(4,4) \in R$

$(2,4) \in R$

∴ R satisfies the transitive

Hasse diagram



1) Draw the Hasse diagram for
$[\{1, 2, 3, 6, 9, 18\}, |]$

$1 \to 1, 2, 3, 6, 9, 18$

$2 \to 2, 6, 18$

$3 \to 3, 6, 9, 18$

$6 \to 6, 18$

$9 \to 9, 18$

$18 \to 18$

$R = \{ (1,1) (1,2) (1,3) (1,6) (1,9) (1,18)$
$(2,2) (2,6) (2,18).$
$(3,3) (3,6) (3,9). (3,18)$
$(6,6) (6,18) (9,9) (9,18)$
$(18,18) \}$

(i) R is a reflexive

$\because (a,a) \in R$

eg: $(1,1) \in R$ $(2,2) \in R$, $(3,3) \in R$
$(6,6) \in R$ $(9,9) \in R$ $(18,18) \in R$

$\therefore$ R Satisfies the reflexive.

(ii) R is a anti symmetric

$\because (a,b) \in R$ $(b,a) \notin R$ $a \neq b$

eg: $(1,2) \in R$ $(2,1) \notin R$

R Satisfies the anti symmetric relation

(iii) R is a transitive

$\because$ $(a,b) \in R$ $(b,c) \in R$

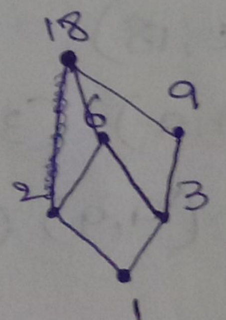
$(a,c) \in R$

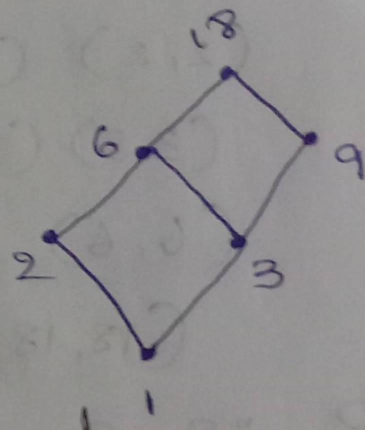
Eg:- $(1,2) \in R$ $(2,6) \in R$

$(1,6) \in R$

$\therefore$ R satisfies the transitive

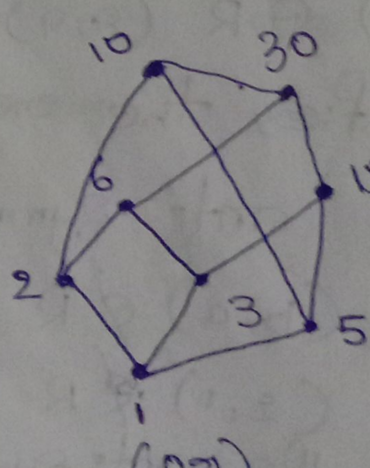
Hasse diagram



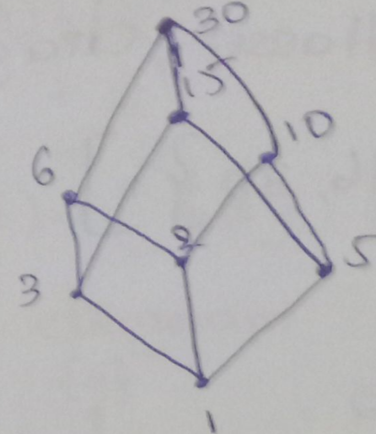


(4) Draw the Hasse diagram

$[\{1, 2, 3, 5, 6, 10, 15, 30\}, /]$



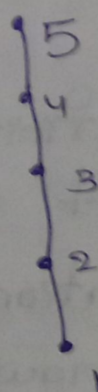
(or)



up ~~lower~~ higher numbers will be place above the lower numbers.

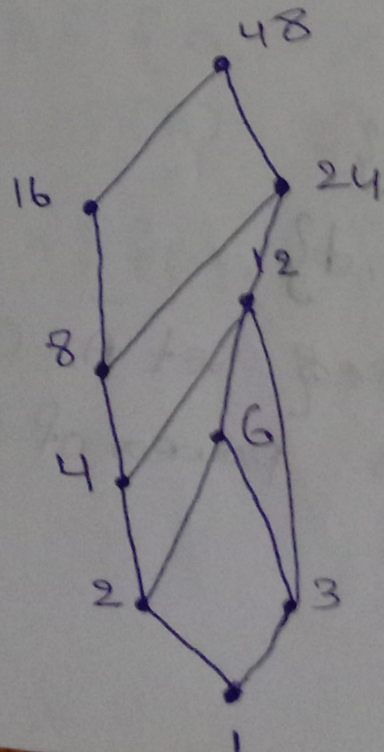
(5) Draw the Hasse diagram

$[\{1, 2, 3, 4, 5\} \leq]$



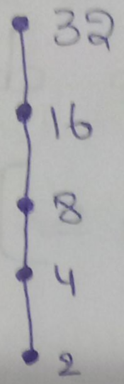
(6) Draw the Hasse diagram

$[\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\} \mid]$



⑦ Draw the Hasse diagram

$[\{2, 4, 8, 16, 32\}, |]$



⑧ Let 'A' be the given finite set and P(A) is the power set Let subset ($\subseteq$) be the inclusion relation on the elements of P(A) Draw the Hasse diagram $[P(A), \subseteq]$ for

① $A = \{a\}$

② $A = \{a, b\}$

③ $A = \{a, b, c\}$

④ $A = \{a, b, c, d\}$

sol :- Suppose the set A contains n elements it's power of set A is 2^n elements

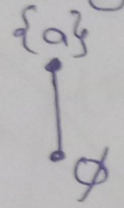
(1) $P(A) = 2^n$

$= 2^1$

$= 2$

$P(A) = [\{\phi\}, \{a\}]$

Hasse diagram



(2) $A = \{a, b\}$

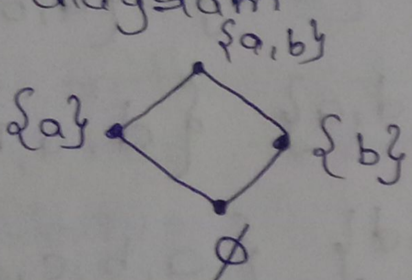
$P(A) = 2^n$

$= 2^2$

$= 4$

$P(A) = [\{\phi\}, \{a\}, \{b\}, \{a, b\}]$

Hasse diagram



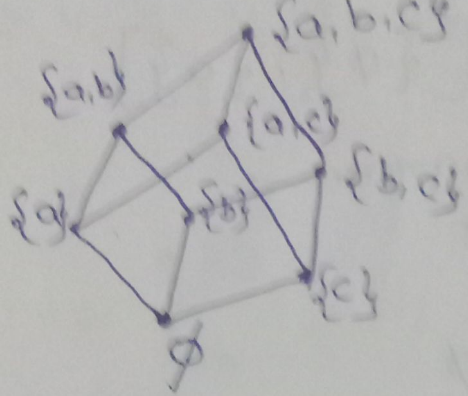
(3) $A = \{a, b, c\}$

$P(A) = 2^n = 2^3 = 8$

$P(A) = [\{\phi\}, \{a\}, \{b\}, \{c\} \; \{a, b\} \; \{b, c\}$

$\{c, a\} \; \{a, b, c\}]$

Hasse diagram

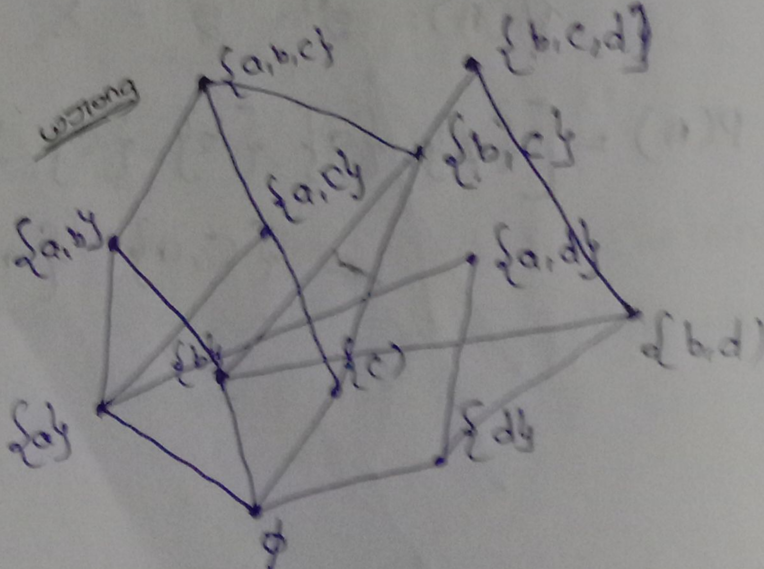


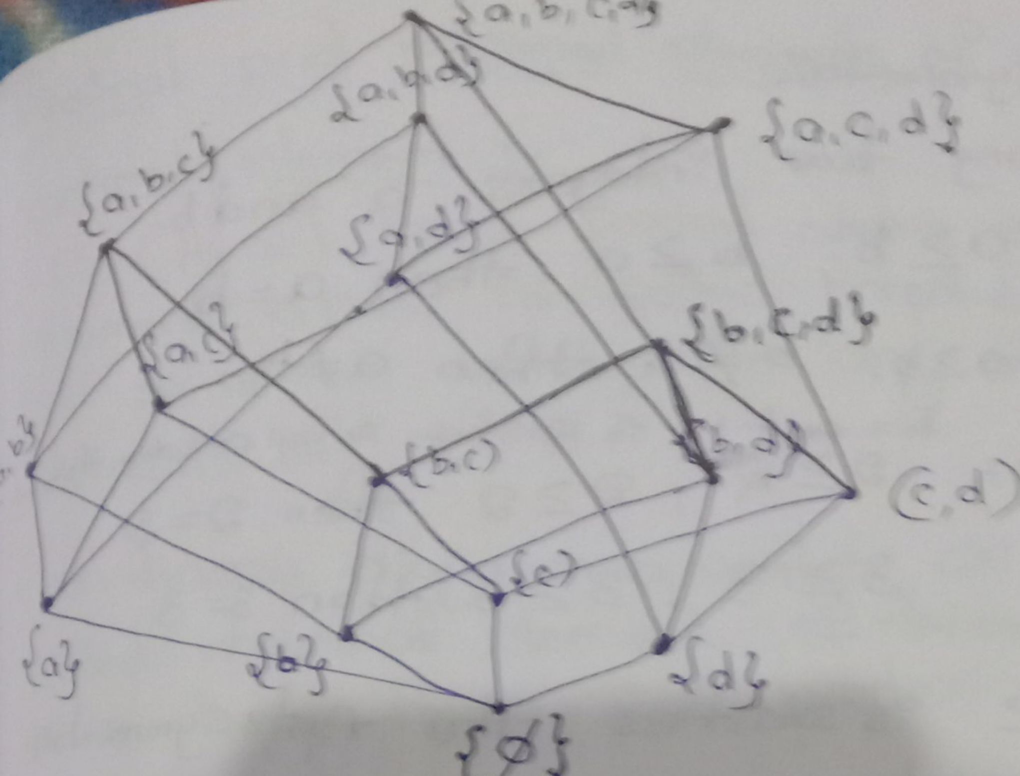
④ $A = \{a, b, c, d\}$

$P(A) = 2^n = 2^4 = 16$

$P(A) = [\{\phi\}\ \{a\}\ \{b\}\ \{c\}\ \{d\}\ \{a,b\}\ \{b,c\}$
$\{c,d\}\ \{d,a\}\ \{a,b,c\}\ \{b,c,d\}$
$\{c,d,a\}\ \{d,a,b\}\ \{a,c\}$
$\{b,d\}\ \{a,b,c,d\}]$

Hasse diagram





(q) show that " $\geq$ " is a partially ordering on the set of integers.

Sol To prove that " $\leq$ " is a partial ordering relation on the set of integers it satisfies the following relation

1. Reflexive
2. Anti symmetric
3. Transitive.

1. Reflexive

for every integer 'a' $a \geq a \ \forall a \in Z$

Hence, " $\geq$ " satisfies the reflexive relation

eg:- $2 \geq 2$

$3 \geq 3$

$4 \geq 4$

$\therefore$ It is a reflexive relation

2. Antisymmetric

for any two integers a and b

cond-1 if $a \geq b$, $b \geq a$ then $a = b$

cond-2 if $a \geq b$, $b \not\geq a$ then $a \neq b$

for cond-1 it satisfies so we consider that only

eg:- $2 \geq 2$, $2 \geq 2$ then $2 = 2$

$3 \geq 3$, $3 \geq 3$ then $3 = 3$

$\therefore \geq$ is satisfies the Antisymmetric relation

3. Transitive

for any three integers $a, b, c \in Z$

if $a \geq b$, $b \geq c$ then $a \geq c$

eg:- $3 \geq 2$, $2 \geq 1$ then $3 \geq 1$

$\therefore \geq$ is satisfies the trasitive relation

Hence it follows that "$\geq$" is a partial order on the set of integers

Maximal and minimal elements of poset

Maximal element :-

Let $(A, \leq)$ be a partial order set an element $a \in A$ is called maximal Element If it is not related to any other Elements of set A

or

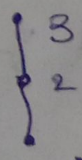
In Hassie diagram a vertex $a \in A$ is called maximal element if there is no edge leving from that vertex

Minimal Element :-

In Hasse diagram a vertex $a \in A$ is called minimal element if there is no edge entering from that vertex.

For example :-

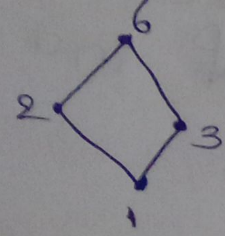
$(A, \leq)$ be a poset and set $A = \{1, 2, 3\}$ with relation $\leq$ find the maximal Element and minimal Element



Maximal = 3

Minimal = 1

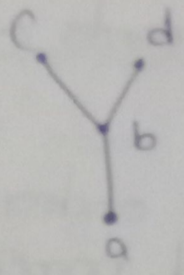
Find the maximal and minimal Elements of the following Hasse diagram.

① 

Maximal = 6

Minimal = 1

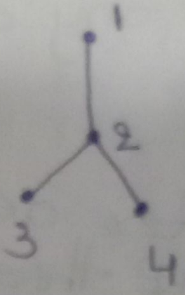
(2)



Maximal = c, d

Minimal = a

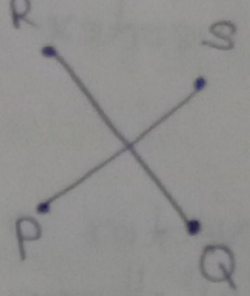
(3)



Maximal = 1

Minimal = 3, 4

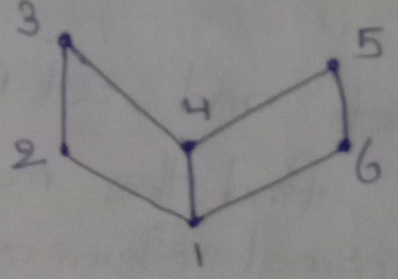
(4)



Maximal = R, S

Minimal = P, Q

(5)



Maximal = 3, 5

Minimal = 1

Components of the poset :- [upper boun
lower boun
least upper boun
Greates lower bo

upper Bound :-

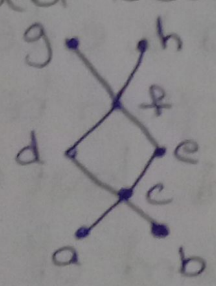
upper bound contains the Elements that are $\leq$ all the Elements ~~of~~ in a subse
of b in a poset

Lower bound :- Lower bound contains the Elements that are $\leq$ all the Elements in a subset [$\subseteq$] of b in a poset

Least upper bound : It is the one of the upper bounds which is < [less than] all other upper bounds.

Greatest lower bound :- It is the one of the lower bound Elements which is greater than [>] all other lower bounds.

① Consider Hasse diagram of a poset [A, $\leq$] given below.
If B = {c, d, e} find out all upper bounds, all lower bounds, least upper bound, Greatest lower bound.



A = {a, b, c, d, e, f, g, h}
B = {c, d, e}

upper of f

f : B → A [Compare B to A]

c → c, d, e, f, g, h.

d → d, f, g, h

e → e, f, g, h

upper bound of c, d, e are is f, g, h

Least upperbound → f

lower bounds

$f : A \to B$

$A = \{a, b, c, d, e, f, g, h\}$

$B = \{c, d, e\}$

$a \to c, d, e$

$b \to c, d, e$

$c \to c, d, e$

$d \to d, e$

$e \to e$

$f \to$ Nil

$g \to$ Nil

$h \to$ Nil

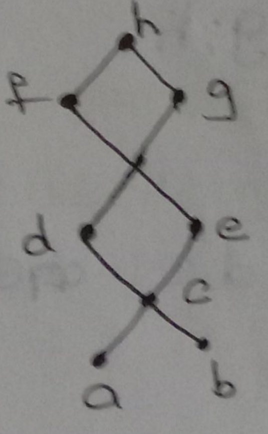
$\therefore$ the lower bound of c, d, e are a, b, c

Greatest lower bound $\to c$

(2) Consider the poset $A = \{a, b, c, d, e, f, g, h\}$ whose hasse diagram is given below. Find upper bound, lower bound, least upper bound, Greatest lower bound of $\subseteq$ subset

(1) $B = \{a, b\}$

(2) $B = \{c, d, e\}$



(1) $A = \{a, b, c, d, e, f, g, h\}$
$B = \{a, b\}$

upper bound of
$f : B \to A$
$a \to a, c, d, e, \cancel{f, g, h}$
$b \to b, c, d, e, \cancel{f, g, h}$

upper bound of a, b are c, d, e, ~~f, g, h~~

Least upper bound - c

lower bounds
$f : A \to B$ $A = \{a, b, c, d, e, f, g, h\}$
$B = \{a, b\}$

$a \to a,$
$b \to b$
$c \to$ Nil
$d \to$ Nil
$e \to$ Nil
$f \to$ Nil
$g \to$ Nil
$h \to$ Nil

No lower bounds $= \phi$

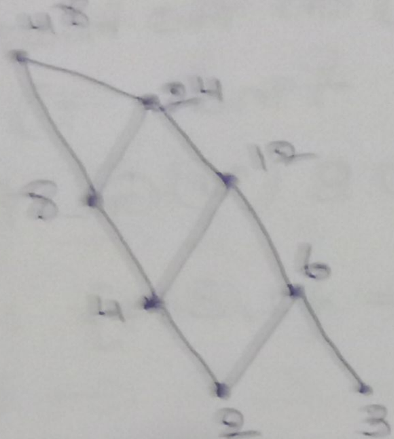
(2) $A = \{a, b, c, d, e, f, g, h\}$
$B = \{c, d, e\}$
upper bound of
$f : B \to A$
$c \to c, d, e$
$d \to d,$
$e \to e$

(3) set $A = \{2, 3, 4, 6, 8, 12, 24, 48\}$ and $\leq$ denotes partial order of divisibility with $\leq$ subset $B = \{4, 6, 12\}$.

Draw the Hassie diagram and also find upper bounds, lower bounds, least upper bound, Greatest lower bound.

Sol $A = \{2, 3, 4, 6, 8, 12, 24, 48\}$

Hassie diagram



A = {2, 3, 4, 6, 8, 12, 24, 48}

B = {4, 6, 12}

upper bounds of

f : B → A

4 → 4, 8, 12, 24, 48

6 → 6, 12, 24, 48

12 → 12, 24, 48

upper bounds of 4, 6, 12 are 12, 24, 48.

Least upper bound → 12

Lower bounds of

f : A → B

2 → 4, 6, 12   12 → 12

3 → 6, 12   24 → Nil

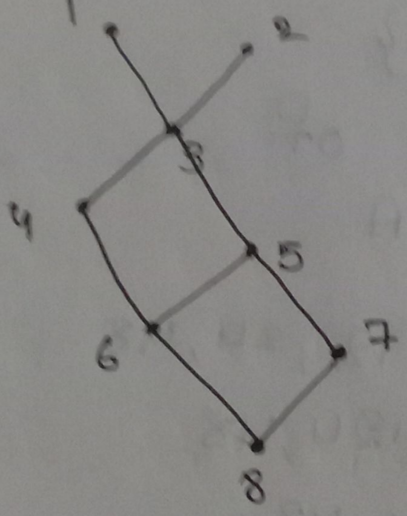
4 → 4, 12   48 → Nil

6 → 6, 12

8 → Nil

Lower bound of 4, 6, 12 are 2
Greatest lower
~~least~~ upper bound is 2

(4) The poset [A, ≤] on its hasse diagram is given below find i) UB ii) L.B (iii) L.U.B (iv) G.L.B for the subset B = {2, 3, 4}



$f : B \to A$

$B = \{2, 3, 4\}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

upper bound of

$2 \to 2$

$3 \to 1, 2, 3$

$4 \to 1, 2, 3, 4$

upper bound = 2

least upper bound = 2

$f: A \to B$

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 3, 4\}$

Lower bound of

$1 \to \phi$

$2 \to 2$

$3 \to 2, 3$

$4 \to 2, 3, 4$

$5 \to 2, 3,$

$6 \to 2, 3, 4$

$7 \to 2, 3$

$8 \to 2, 3, 4$

Lower bound of subset 2, 3, 4 are 4, 6, 8

Greatest lower bound is 8

Theorem:-

(i) Suppose R and S are reflexive relations on a set of A then $R \cap S$ is also reflexive

Let $a \in A$ $\because$ R is a reflexive

$(a, a) \in R \quad \forall a \in A$

$\because$ S is a reflexive

$(a, a) \in S \quad \forall a \in A$

$\therefore (a, a) \in R \,\&\, S$

$\therefore$ R & S is a reflexive

Theorem:
Suppose R & S are symmetric, transitive relation on set A then $R \cap S$ is also a symmetric and transitive

sol Let $(a,b) \in A$

$\because$ R is a symmetric ~~(a,b) ∈ R ⇒ (b,a)~~
$(a,b) \in R \Rightarrow (b,a) \in R$
$(a,b) \in S \Rightarrow (b,a) \in S$

$(a,b) \in R \cap S \Rightarrow (b,a) \in R \cap S$

$\therefore$ $R \cap S$ is a symmetric

~~Let A, B, C ∈~~
~~Let a, b, c ∈ A~~
Let $a, b, c \in A$
since R is transitive
$(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

Since S is a transitive
$(a,b) \in S$, $(b,c) \in S \Rightarrow (a,c) \in S$

$\Rightarrow (a,c) \in R \cap S$

If R be a equivalence relation in set A then R^{-1} are also equivalence relation
Let R be a equivalence relation and set A
it satisfies the

(i) R is reflexive
(ii) R is symmetric
(iii) R is a transitive
we have to show that R^{-1} is also a equivalence relation
(i) the relation R^{-1} is a reflexive
since $(a,a) \in R$ for all $a \in A$
$\Rightarrow (a,a) \in R^{-1}$ for all $a \in A$
$\therefore R^{-1}$ is a reflexive
we have to prove that
(ii) R^{-1} is a symmetric
$(a,b) \in R^{-1} \Rightarrow (b,a) \in R^{-1}$
Now,
$(a,b) \in R^{-1}$
$\Rightarrow (b,a) \in R$
$\Rightarrow (a,b) \in R$ since R is a symmetric
$\Rightarrow (b,a) \in R^{-1}$
$\therefore R^{-1}$ is a symmetric
we have to prove that
(iii) R^{-1} is a transitive
$(a,b), (b,c) \in R^{-1} \Rightarrow (a,c) \in R^{-1}$
$\Rightarrow (a,b)\,(b,c) \in R^{-1}$
$\Rightarrow (b,a)\,(c,b) \in R$
$\Rightarrow (c,b)\,(b,a) \in R$
$(c,a) \in R$
since R is a transitive
$(a,c) \in R^{-1}$
$\therefore R^{-1}$ is a transitive

Hence, $\bar{R}$ satisfies the equivalence relation.

Show that the inclusion subset "$\subseteq$" relation is a partial ordering set of a power set of A

To prove that "$\subseteq$" is a partial ordering on the power set of a set
it satisfies the following relations.

(i) Reflexive

(ii) for every set of A, $A \subseteq A$

where A is a subset "$\subseteq$" of S

Hence "$\subseteq$" satisfies the reflexive relation

(ii) Anti symmetric

for any two set A & B

if $A \subseteq B$ and $B \subseteq A$ then $A = B$

Hence "$\subseteq$" satisfies the anti symmetric relation

(iii) Transitive

for any three sets A, B & C

if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

Hence "$\subseteq$" satisfies the transitive relation.

Hence "$\subseteq$" is a partial ordering set of power set of A.