VALLIAMMAI ENGEINEERING COLLEGE

(S.R.M.NAGAR, KATTANKULATHUR-603 203)

DEPARTMENT OF MATHEMATICS

QUESTION BANK

III SEMESTER

MA8351- DISCRETE MATHEMATICS

Regulation – 2017

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VALLIAMMAI ENGEINEERING COLLEGE

S.R.M.NAGAR, KATTANKULATHUR-603 203

B.TECH. INFORMATION TECHNOLOGY

QUESTION BANK

SUBJECT : MA8351 / DISCRETE MATHEMATICS
SEMESTER / YEAR : III Semester / II year IT

UNIT I - LOGIC AND PROOFS

Propositional Logic – Propositional equivalences - Predicates and Quantifiers – Nested Quantifiers – Rules of inference - Introduction to proofs – Proof methods and strategy.

PART-A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Construct the truth table for the compound proposition $(p \to q) \leftrightarrow (\neg p \to \neg q)$.	BTL -1	Remember
2.	Construct the truth table for the compound proposition $(p \to q) \to (q \to p)$.	BTL -3	Apply
3.	What are the contra positive, the converse and the inverse of the conditional statement "If you work hard then you will be rewarded".	BTL -2	Understand
4.	Find the truth table for the statement $p \to \neg q$.	BTL -1	Remember
5.	Give the truth value of $T \leftrightarrow T \wedge F$.	BTL -1	Remember
6.	Write the symbolic representation and give its contra positive statement of "If it rains today, then I buy an umbrella" .	BTL -2	Understand
7.	When do you say that two compound propositions are equivalent ?	BTL -1	Remember
8.	Show that $(p \to r) \wedge (q \to r) and\ (p \vee q) \to r$ are logically equivalent.	BTL -4	Analyze
9.	Show that the propositions $p \to q\ and\ \neg p \vee q$ are logically equivalent.	BTL -4	Analyze
10.	Without using truth table show that $p \to (q \to p) \Leftrightarrow \neg p \to (p \to q)$.	BTL -3	Apply
11.	Show that $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$ is a tautology.	BTL -3	Apply
12.	Is $\neg p \wedge (p \vee q)) \to q$ a tautology?	BTL -5	Evaluate
13.	Using the truth table, show that the proposition $p \vee \neg(p \wedge q)$ is a tautology.	BTL -3	Apply
14.	Given $P = \{2,3,4,5,6\}$, state the truth value of the statement $(\exists x \in p)(x + 3 = 10)$.	BTL -4	Analyze
15.	Let $E = \{-1,0,1,2\}$ denote the universe of discourse. If $P(x,y) = x + y + 1$, find the truth value of $(\forall x)(\exists y)P(x,y)$.	BTL -4	Analyze
16.	Find a counter example, if possible, to these universally quantified statements, Whose the universe of discourse for all variables consists of all integers. (a) $\forall x \forall y\ (x^2 = y^2 \to x = y)$. (b) $\forall x \forall y\ (xy \geq x)$.	BTL -2	Understand
17.	What are the negations of the statements $\forall x\ (x^2 > x)\ and\ \exists x\ (x^2 = 2)$?	BTL -1	Remember
18.	Write the negation of the statement$(\exists x)(\forall y)P(x,y)$.	BTL -1	Remember
19.	Give an indirect proof of the theorem " If $3n + 2$ is odd, then n is odd.	BTL -2	Understand

20.	Prove that $p, p \to q, q \to r \implies r$	**BTL -6**	Create
	PART – B		
1.(a)	Prove that $((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is a tautology	**BTL -2**	Understand
1. (b)	Show that $(p \to q), (r \to s), (q \to t), (s \to u), \neg(t \wedge u), (p \to r) \implies \neg p$	**BTL -2**	Understand
2. (a)	Check whether $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology?	**BTL -2**	Understand
2.(b)	Show that $(\exists x)(P(x) \wedge Q(x)) \implies (\exists x)P(x) \wedge (\exists x)Q(x)$. Is the converse true?	**BTL -2**	Understand
3. (a)	Show that $\forall x(P(x) \to Q(x)), \forall x(R(x) \to \neg Q(x)) \implies \forall x(R(x) \to \neg P(x))$	**BTL -2**	Understand
3.(b)	Write the symbolic form and negate the following statements. 1. Everyone who is healthy can do all kinds of work. 2. Some people are not admired by everyone. 3. Everyone should help his neighbors or his neighbors will not help him. 4. Everyone agrees with someone and someone agrees with everyone	**BTL -3**	Apply
4. (a)	Without constructing the truth tables, obtain the principle disjunctive normal form of $(\neg p \to r) \wedge (q \leftrightarrow r)$	**BTL -1**	Remember
4.(b)	Show that $r \to s$ can be derived from the premises $p \to (q \to s), \neg r \vee p\ and\ q$	**BTL -3**	Apply
5. (a)	Establish this logical equivalences, where A is a proposition not involving any quantifiers. Show that $(\forall x P(x)) \wedge A \equiv \forall x(P(x) \wedge A) and\ (\exists x P(x)) \wedge A \equiv \exists x(P(x) \wedge A)$	**BTL -3**	Apply
5.(b)	Show that $\exists\, xP(x) \wedge \exists\, x\, Q(x)$ and $\exists\, x(P(x) \wedge\ Q(x))$ are not logically equivalent	**BTL -4**	Analyze
6.(a)	Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ without using truth table.	**BTL -1**	Remember
6.(b)	Show that $\forall\, xP(x) \wedge \exists\, x\, Q(x)$ is equivalent to $\forall\, x\, \exists y\, (P(x)\ \wedge Q(y))$	**BTL -3**	Apply
7. (a)	Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday". "We will go swimming only if it is sunny", " If do not go swimming then we will take a canoe trip" and " If we take a canoe trip, then we will be home by sunset". Lead to the conclusion "we will be home by sunset".	**BTL -4**	Analyze
7. (b)	Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.	**BTL -4**	Analyze
8. (a)	Show that the conclusion $\forall x(P(x) \to \neg Q(x))$ follows from the premises $\exists x(P(x) \wedge Q(x)) \to \forall y(R(y) \to S(y))$ and $\exists y(R(y) \wedge \neg S(y))$	**BTL -2**	Understand
8.(b)	Prove that $(P \longrightarrow Q) \wedge (R \longrightarrow Q) \Leftrightarrow (P \vee R) \longrightarrow Q)$	**BTL -2**	Understand
9. (a)	Show that $\forall x(P(x)\ \vee Q(x)) \implies \forall x P(x) \vee \exists x Q(x)$ by indirect method.	**BTL -1**	Remember
9.(b)	Show that the statement "Every positive integer is the sum of squares of three integers" is false.	**BTL -1**	Remember
10.(a)	Using indirect method of proof, derive $P \longrightarrow \neg S$ from the premises $P \longrightarrow (Q \vee \boldsymbol{R}),\ Q \to \neg P, S \to \neg R\ and\ P.$	**BTL -1**	Remember
10.(b)	Without using truth table find PCNF and PDNF of	**BTL -2**	Understand

	$[P \longrightarrow (Q \wedge R)] \wedge [\neg P \rightarrow (\neg Q \wedge \neg R)]$.		
11. (a)	Prove that the premises $P \longrightarrow Q,\ Q \longrightarrow R,\ \ R \longrightarrow S, S \longrightarrow \neg R$ and $P \wedge S$ are inconsistent.	**BTL -1**	Remember
11. (b)	Show that the premises "One student in this class knows how to write program in JAVA", and "Everyone who knows how to write the programme in JAVA can get a high paying job imply a conclusion "someone in this class can get a high paying job" .	**BTL -1**	Remember
12. (a)	Show that $(\neg P \longrightarrow R) \wedge (Q \leftrightarrow P) \Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$.	**BTL -2**	Understand
12. (b)	Use the quantifiers to express following statements. 1. there is a student in the class who can speak Hindi.2.every student in this class knows how to drive a car.3.some student in this class knows has visited Alaska but not visited Hawaii.4.all students in this class have learned at least one programming language.	**BTL -6**	Create
13. (a)	Obtain the PDNF and PCNF of $(P \wedge Q) \vee (\neg P \wedge R)$	**BTL -1**	Remember
13.(b)	Prove that A$\longrightarrow\ \neg D$ is a conclusion from the premises $A \longrightarrow B \vee C,\ B \longrightarrow \neg A\ \ and\ D \longrightarrow \neg C$by using conditional proof.	**BTL -1**	Remember
14. (a)	Show that $R \wedge (P \vee Q)$is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M, \neg M$	**BTL -4**	Analyze
14. (b)	Show that that $\exists\ x\ P(x) \longrightarrow \forall x\ Q(x) \Rightarrow \forall x\ (P(x) \longrightarrow Q(x))$	**BTL -2**	Understand

UNIT II -COMBINATORICS

Mathematical induction – Strong induction and well ordering – The basics of counting – The pigeonhole principle – Permutations and combinations – Recurrence relations – Solving linear recurrence relations – Generating functions – Inclusion and exclusion principle and its applications .

PART- A

Q.No.	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	State the principle of strong induction.	**BTL -1**	Remember
2	Use mathematical induction to show that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$	**BTL -2**	Understand
3	Use mathematical induction to show that $n! \geq 2^{n+1}, n = 1,2,3, \ldots$	**BTL -2**	Understand
4	What is well ordering principle?	**BTL -1**	Remember
5	How many permutations are there in the word MISSISSIPPI?	**BTL -5**	Evaluate
6	State the Pigeonhole principle.	**BTL -1**	Remember
7	What is the number of arrangements of all the six letters in the word PEPPER?	**BTL -5**	Evaluate
8	How many permutations of {a,b,c,d,e,f,g} starting with a?	**BTL -1**	Remember
9	In how many ways can all the letters in MATHEMATICAL be arranged?	**BTL -3**	Apply
10	Find the minimum of students needs to guarantee that 5 of them belongs to the same subject if there are 5 different major subjects	**BTL -4**	Analyze
11	How many different bit strings are there of length seven?	**BTL -1**	Remember
12	Twelve students want to place order of different ice creams in a ice cream parlor, which has six type of ice creams. Find the number of orders that the twelve students can place.	**BTL -4**	Analyze
13	How many permutations are there in the word MALAYALAM?	**BTL -3**	Apply

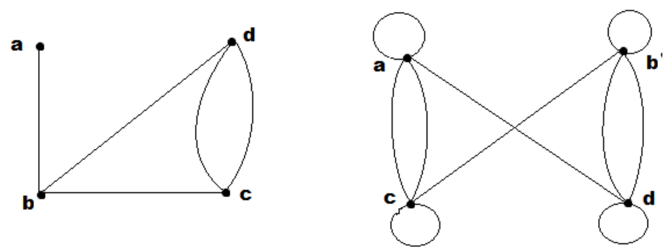
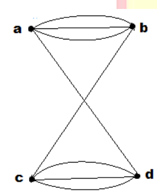
14	Find the recurrence relation for the Fibonacci sequence.	**BTL -2**	Understand
15	Find the recurrence relation satisfying the equation $y_n = A(3)^n + B(-4)^n$	**BTL -2**	Understand
16	Solve the recurrence relation $y(k) - 8y(k-1) + 16y(k-2) = 0,\ k \geq 2,\ where\ \ y(2) = 16\ and\ y(3) = 80.$	**BTL -3**	Apply
17	Find the recurrence relation of the sequence $s(n) = \ a^n, n \geq 1$	**BTL -1**	Remember
18	Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 11$	**BTL -6**	Create
19	Determine whether the sequence $\{a_n\}$ is a solution of the recurrence relation$a_n = 2a_{n-1} - a_{n-2}$, n=2, 3, 4, … where $a_n = 3n$ for every non negative integer n.	**BTL -5**	Evaluate
20	What is the solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}\ with\ a_0 = 1, a_1 = 6$	**BTL -5**	Evaluate
	PART-B		
1.(a)	Using induction principles prove that $n^3 + 2n$ is divisible by 3.	**BTL -3**	Apply
1. (b)	Show that if n and k are positive integers then$\binom{n+1}{k} = (n+1)\binom{n}{k-1}/k$. Use this identity to construct an inductive definition of the binomial co-efficient.	**BTL -1**	Remember
2. (a)	If we select ten points in the interior of an equilateral triangle of side 1 , show that there must be atleast two points whose distance apart less than $1/3$.	**BTL -2**	Understand
2.(b)	From a club consisting of six men and seven women, in how many ways we select a committee of (1) 3 men and 4 women? (2) 4 persons which has at least one woman? (3) 4 persons that has at most one man? (4) 4 persons that has both sexes?	**BTL -2**	Understand
3. (a)	There are three files of identical red, blue and green balls, where each files contains at least 10 balls. In how many ways can 10 balls be selected? (1) If there is no restriction. (2) If at least 1 red ball must be selected. (3) If at least 1 red, at least 2 blue and at least 3green balls must be selected. (4) If at most 1 red ball is selected.	**BTL -2**	Understand
3.(b)	Prove by mathematical induction that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$	**BTL -1**	Remember
4. (a)	State the strong induction (the second principle of mathematical induction). Prove that a positive integer >1 is either a prime number or it can be written as product of prime numbers.	**BTL -4**	Analyze
4.(b)	If n pigeonholes are occupied by $(kn+1)$ pigeons, where k is a positive integer, prove that at least one pigeonhole is occupied by $(k+1)$ or more pigeons. Hence, find the minimum number of m integers to be selected from $S = \{1, 2,3 \ldots 9\}$ so that the sum of two of m integers is even.	**BTL -3**	Apply
5. (a)	Use mathematical induction to show that $\dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + \cdots + \dfrac{1}{\sqrt{n}} > \sqrt{n}, \qquad n \geq 2$	**BTL -2**	Understand
5.(b)	In how many ways can six boys and four girls be arranged in straight line so that no two girls are sit together.	**BTL -4**	Analyze
6. (a)	How many bits of string of length 10 contain 1. Exactly four 1's 2. At most four 1's 3. At least four 1's 4. An equal	**BTL -6**	Create

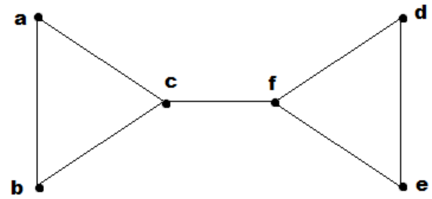
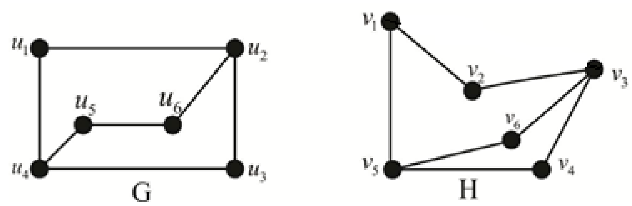
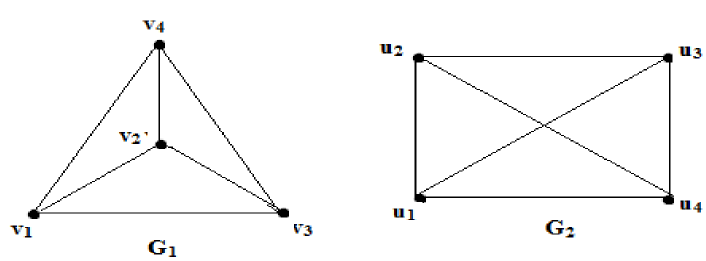
	number of 0's and 1's		
6.(b)	Prove by mathematical induction that $6^{n+2}+7^{2n+1}$ is divisible by 43, for each positive integer n.	**BTL -1**	Remember
7. (a)	Let m be any odd positive integer. Then prove that there exist a positive integer n such that m divides $2^n - 1$	**BTL -1**	Remember
7. (b)	Prove that in a group of six people at least three must be mutual friends or at least three must be mutual strangers.	**BTL -1**	Remember
8. (a)	A total of 1232 students have taken a course in Spanish, 879 have taken a course in French and 114 have taken a course in Russian. Further 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian and 14 have taken courses in both French and Russian. If 2092 students have taken at least one Spanish, French and Russian, how students have taken a course in all three languages?	**BTL -1**	Remember
8.(b)	Prove that the number of subsets of a set having n elements is 2^n	**BTL -1**	Remember
9.(a)	Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5, a_1 = -9$ *and* $a_2 = 15$.	**BTL -3**	Apply
9.(b)	Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with initial conditions $a_0 = 2, a_1 = 5$ *and* $a_2 = 15$	**BTL -1**	Remember
10.(a)	Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n, \quad n \geq 0, \quad a_0 = 3$	**BTL -1**	Remember
10.(b)	Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n;\ n \geq 2$ *given that* $a_0 = 2$ *and* $a_1 = 8$.	**BTL -3**	Apply
11.(a)	Solve $G(k) - 7\,G(k-1) + 10G(k-2) = 8k + 6$, for $k \geq 2$ $S(0) = 1, S(1) = 2$.	**BTL -5**	Evaluate
11.(b)	Solve the recurrence relation $a_n = 3a_{n-1} + 2,\ n \geq 1,$ *with* $a_0 = 1$ by the method of generating function.	**BTL -5**	Evaluate
12.(a)	Find the number of integers between 1 to 250 that are not divisible by any of the integers 2,3,5 and 7	**BTL -5**	Evaluate
12.(b)	Determine the number of positive integer n, $1 \leq n \leq 2000$ that are not divisible by 2, 3 or 5 but are divisible by 7.	**BTL -2**	Understand
13.(a)	Use the method of generating function to solve the recurrence relation $s_n + 3s_{n-1} - 4s_{n-2} = 0$, where $n \geq 2, s_0 = 3, s_1 = -2$	**BTL -3**	Apply
13.(b)	Find the generating function of Fibonacci sequence.	**BTL -5**	Evaluate
14.(a)	A factory makes custom sports cars at an increasing rate. In the 1st Month only one car made, in the 2 nd month two cars are made, and so on with n cars made in the n th month.(1) Set up the recurrence relation for the number of cars produced in the first n month by this factory. (2) How many cars are produced in the 1st year?	**BTL -6**	Create
14.(b)	Find all solutions of the recurrence relation $a_n - 2a_{n-1} = 2n^2, n \geq 1,$ $a_1 = 4$.	**BTL -5**	Evaluate

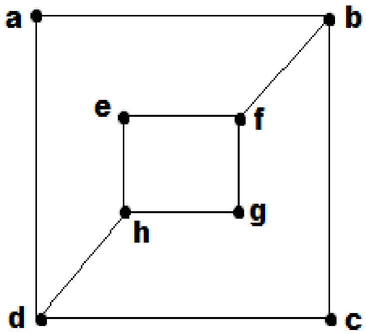
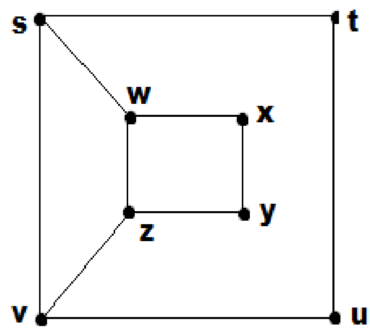
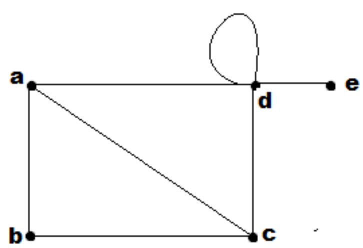
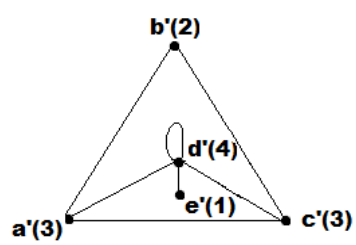
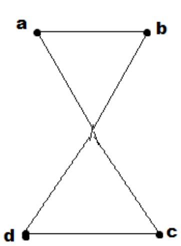
UNIT III -GRAPHS

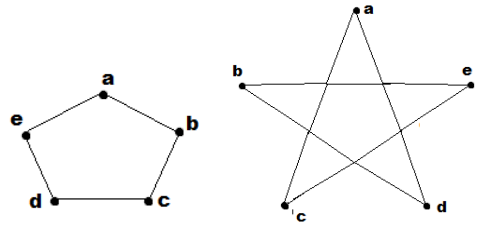
Graphs and graph models – Graph terminology and special types of graphs – Matrix representation of graphs and graph isomorphism – Connectivity – Euler and Hamilton paths.

PART – A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Define complete graph and draw K_5	**BTL -1**	Remember
2.	Define a regular graph. Can a complete graph be a regular graph?	**BTL -1**	Remember
3.	How many edges are there in a graph with 10 vertices each of degree 6?	**BTL -2**	Understand
4.	Define pseudo graphs.	**BTL -1**	Remember
5.	Define strongly connected graph.	**BTL -1**	Remember
6.	State the handshaking theorem.	**BTL -1**	Remember
7.	Prove that the number of odd degree vertices is always even.	**BTL -2**	Understand
8.	Represent the given graph using an adjacency matrix. 	**BTL -5**	Evaluate
9.	Define isomorphism of directed graph	**BTL -1**	Remember
10.	Define connected graph and a disconnected graph with example.	**BTL -1**	Remember
11.	Let G be the graph with 10 vertices. If four vertices has degree four and six vertices has degree five , then find the number of edges of G.	**BTL -1**	Remember
12.	How many edges does a graph have if it has vertices of degree 5, 2, 2, 2, 2, 1? Draw such a graph.	**BTL -3**	
13.	Use an incidence matrix represents the graph. 	**BTL -3**	Apply
14.	Draw the graph with the following adjacency matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	**BTL -2**	Understand
15.	Define a Hamilton path of G.	**BTL -2**	Understand
16.	What should be the degree of each vertex of a graph G if it has Hamilton circuit?	**BTL -4**	Analyze
17.	Define complete bipartite graph .	**BTL -6**	Create
18.	State the necessary and sufficient conditions for the existence of an Eulerian path in a connected graph.	**BTL -4**	Analyze
19.	For which values of n do the graphs K_n and C_n have an Euler path but no Euler circuit?	**BTL -4**	Analyze
20.	Does the graph have a Hamilton path? If so find such a path.	**BTL -6**	Create

			
	PART – B		
1.(a)	Prove that the number of vertices of odd degree in a graph is always even.	**BTL -1**	Remember
1. (b)	Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $(n-k)(n-k-1)/2$	**BTL -2**	Understand
2. (a)	Prove that a connected graph G is Euler graph if and only if every vertex of $G$is of even degree.	**BTL -1**	Remember
2.(b)	Prove that a simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assign the same color.	**BTL -2**	Understand
3. (a)	Examine whether the following pair of graphs are isomorphic or not. Justify your answer. 	**BTL -5**	Evaluate
3.(b)	Let $\delta(G) and\ \Delta(G)$ denotes minimum and maximum degrees of all the vertices of G respectively. Then show that for a non-directed graph G, $\delta(G) \leq \frac{2\|E\|}{\|V\|} \leq \Delta(G)$	**BTL -3**	Apply
4. (a)	Examine whether the following two graphs G and G' associated with the following adjacency matrices are isomorphic. $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$.	**BTL -2**	Understand
4.(b)	Define isomorphism .Establish an isomorphism for the following the  graphs.	**BTL -4**	Analyze
5. (a)	For any simple graph G, the number of edges of G is less than or equal to $\frac{n(n-1)}{2}$, where n is the number of vertices in G.	**BTL -1**	Remember
5.(b)	Derive a connected graph has an Euler trail if and only if it has at most	**BTL -5**	Evaluate

	two vertices of odd degree.		
6. (a)	Prove that the complement of a disconnected graph is connected.	**BTL -2**	Understand
6.(b)	Show that the following graphs G and H are not isomorphic. 	**BTL -3**	Apply
7. (a)	Define Isomorphism between the two graphs. Are the simple graphs with the following adjacency matrices isomorphic? $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$	**BTL -1**	Remember
7. (b)	If a graph G has exactly two vertices of odd degree there is a path joining these two vertices.	**BTL -6**	Create
8. (a)	Define 1. Complete graph. 2. Complete bipartite graph with example.	**BTL -1**	Remember
8.(b)	Show that the following graphs are isomorphic. 	**BTL -4**	Analyze
9. (a)	Give an example of a graph which is 1. Eulerian but not Hamiltonian 2. Hamiltonian but not Eulerian 3. Hamiltonian and Eulerian 4. Neither Hamiltonian nor Eulerian	**BTL -1**	Remember
9.(b)	How many paths of length four are there from a to d in the simple graph G given below. 	**BTL -5**	Evaluate
10.(a)	Show that the complete graph K_n contain $\frac{n(n-1)}{2}$ different Hamilton cycle.	**BTL -4**	Analyze

10.(b)	Briefly explain about sub graph with example.	**BTL -1**	Remember
11.(a)	Using adjacency matrix examine whether the following pairs of graphs G *and* G^1 given below are isomorphism or not. 	**BTL -3**	Apply
11.(b)	Briefly explain about walks with example.	**BTL -1**	Remember
12.(a)	Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete sub graph of K_5 with 4 vertices.	**BTL -6**	Create
12.(b)	The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by finding a permutation matrix.$A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	**BTL -5**	Evaluate
13.(a)	The sum of all vertex degree is equal to twice the number of edges (or) the sum of the degrees of the vertices of G is even.	**BTL -4**	Analyze
13.(b)	Write the adjacency matrix of the digraph $G = \left\{ \begin{matrix} (v_1, v_3), (v_1, v_2), (v_2, v_4), (v_3, v_1), (v_2, v_3), (v_3, v_4), (v_4, v_1), \\ (v_4, v_2), (v_4, v_3) \end{matrix} \right\}$. Also draw the graph	**BTL -1**	Remember
14.(a)	Prove that the following statements are equivalent for a simple connected graph.1.G is Eulerian. 2. Every vertex of G has even degree.	**BTL -2**	Understand
14.(b)	Explain cut edges and cut vertices with suitable example.	**BTL -1**	Remember

UNIT –IV: ALGEBRAIC STRUCTURES

Algebraic systems – Semi groups and monoids - Groups – Subgroups – Homomorphism's – Normal Subgroup and cosets – Lagrange's theorem – Definitions and examples of Rings and Fields.

PART-A

Q.No.	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	Define semigroup and monoid. Give an example of a semigroup which is not a monoid.	**BTL -1**	Remember
2.	Give an example of sub semigroup.	**BTL -2**	Understand
3.	Show that semi-group homomorphism preserves the property of idempotency.	**BTL -3**	Apply
4.	Find the idempotent elements of $G = \{1, -1, i, -i\}$ under the binary operation multiplication	**BTL -2**	Understand
5.	Prove that identity element in a group is unique.	**BTL -1**	Remember
6.	Prove that monoid homomorphism preserves invertibility.	**BTL -2**	Understand
7.	State any two properties of a group.	**BTL -1**	Remember
8.	Let Z be agroup of integers with binary operation * defined by $a * b = a + b - 2$ for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.	**BTL -2**	Understand
9.	Prove that if G is abelian group, then for all $a, b \in G, (a * b)^2 = a^2 * b^2$	**BTL -5**	Evaluate

10.	Show that every cyclic group is abelian.	**BTL -4**	Analyze
11.	Prove or disprove: Every subgroup of an abelian is normal.	**BTL -4**	Analyze
12.	If a is a generator of a cyclic group$G$, then show that a^{-1} is also a generator of G.	**BTL -6**	Create
13.	Show that $(Z_5, +_5)$ is a cyclic group.	**BTL -6**	Create
14.	Find the left cosets of $\{[0],[3]\}$ in the addition modulo group $(Z_6, +_6)$.	**BTL -3**	Apply
15.	Define kernel of a homomorphism.	**BTL -1**	Remember
16.	State Lagrange's theorem.	**BTL -1**	Remember
17.	Define a ring and give an example	**BTL -1**	Remember
18.	Prove that the order of an element a of a group G is the same as that of its inverse (a^{-1})	**BTL -1**	Remember
19.	Give an example of a ring which is not a field.	**BTL -6**	
20.	Define integral domain and give an example.	**BTL -4**	Analyze
	PART – B		
1.(a)	Show that group homomorphism preserves identity, inverse, and sub group	**BTL -1**	Remember
1.(b)	If (G,*) is a finite cyclic group generated by ab element $a\epsilon G$ and is of order n then $a^n = e$ so that $G = \{a, a^2, \dots a^n (= e)\}$. Also n is the least positive integer for which $a^n = e$.	**BTL -2**	Understand
2.(a)	Prove that the intersection of two subgroups of a group G is again a subgroup of G	**BTL -2**	Understand
2.(b)	Let G be a group and $a \in G$. Let $f: G \to G$ be given by $f(x) = axa^{-1}, \forall\, x \in G$. Prove that f is an isomorphism of G onto G	**BTL -2**	Understand
3.(a)	Show that M_2, the set of all $2X2$ non singular matrices over R is a group under usual matrix multiplication. Is it abelian?	**BTL -6**	Create
3.(b)	Show that the union of two subgroups of a group G is again a subgroup of G if and only if one is contained in the other.	**BTL -4**	Analyze
4. (a)	State and prove Lagrange's theorem	**BTL -4**	Analyze
4.(b)	If $S = NXN$, the set of ordered pairs of positive integers with the operation $*$ defined by $(a, b) * (c, d) = (ad + bc,\ bd)$ and if $f: (S,*) \to (Q, +)$ is defined by $f(a, b) = a/b$, show that f is a semi group homomorphism.	**BTL -5**	Evaluate
5. (a)	Prove that the set $Z_4 = \{0,1,2,3\}$ is a commutative ring with respct ot the binary operation $+_4$ and $\times_4$	**BTL -1**	Remember
5.(b)	Determine whether $H_1 = \{0, 5, 10\}$ and $H_2 = \{0, 4, 8, 12\}$are subgroups of Z_{15}.	**BTL -5**	Evaluate
6. (a)	Discuss Ring and Fields with suitable examples.	**BTL -4**	Analyze
6.(b)	State and prove the fundamental theorem of group homomorphism	**BTL -1**	Remember
7. (a)	Find the left cosets of the subgroup $H = \{[0],[3]\}$ of the group $[Z_6, +_6]$.	**BTL -5**	Evaluate
7. (b)	If G is a group of prime order, then G has no proper subgroups.	**BTL -6**	Create
8. (a)	Prove that the necessary and sufficient condition for a non-empty subset H of a group $(G,*)$to be a subgroup is $a, b \in H \Rightarrow a * b^{-1} \in H$	**BTL -1**	Remember
8.(b)	Let $f: G,*) \to (H,\Delta)$ be group homomorphism then show that $Ker(f)$ is a normal subgroup.	**BTL -4**	Analyze
9. (a)	Prove that every subgroup of a cyclic group is cyclic.	**BTL -5**	Evaluate

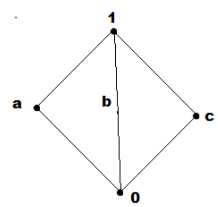
9.(b)	If $(G,*)$ is an abelian group and if $\forall\, a, b\ \in G$. Show that $(a*b)^n = a^n * b^{\,n}, for\ every\ integer\ n$	**BTL -5**	Evaluate
10.(a)	Define a cyclic group. Prove that any group of prime order is cyclic.	**BTL -6**	Create
10.(b)	Let $(H,\ \cdot)$be a subgroup of $(G,\ \cdot)$. Let $N = \{x\ /x \in G, xHx^{-1} = H\}$. Show that $(N,\ \cdot)$is a subgroup of G.	**BTL -3**	Apply
11.(a)	Prove that in a group G the equations $a * x = b\ and\ y * a = b$ have unique solutions for the unknowns x and y as $x = a^{-1} * b, y = b * a^{-1}$ when $a, b \in G$.	**BTL -3**	Apply
11.(b)	Prove that the set of all matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$forms an abelian group with respect to matrix multiplication.	**BTL -1**	Remember
12.(a)	If $f: G \to G'$ be a group homomorphism and H is a subgroup of G', then $f^{-1}(H)$ is a subgroup of G	**BTL -2**	Understand
12.(b)	Show that$(Q^+,*)$ is an abelian group where $*$ is defined as $a * b = ab/2,\ \forall a, b \in Q^+$	**BTL -1**	Remember
13.(a)	Show that the group $(\{1,2,3,4\}, X_5)$ is cyclic.	**BTL -6**	Create
13.(b)	Prove that $G = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\right\}$ forms an abelian group under matrix multiplication.	**BTL -2**	Understand
14.(a)	Prove that the set $\{1, -1, i, -i\}$ is a finite abelian group with respect to the multiplication of complex numbers.	**BTL -1**	Remember
14.(b)	State and prove Cayley's theorem.	**BTL -1**	Remember

UNIT-V LATTICES AND BOOLEAN ALGEBRA

Partial ordering – Posets – Lattices as posets – Properties of lattices - Lattices as algebraic systems – Sub lattices – Direct product and homomorphism – Some special lattices – Boolean algebra.

PART-A

Q.No.	**Question**	**Bloom's Taxonomy Level**	
1.	Draw a Hasse diagram of $\langle X, \leq\rangle$ where $X = \{1,2,3,4,6,8,12,24\}$and R be a division relation. Find the Hasse diagram of the poset <*X*,*R*>	**BTL -1**	Remember
2.	Show that the least upper bound of a subset B in a poset $\langle A, \leq\rangle$is unique if it exists.	**BTL -2**	Understand
3.	Let $A = \{a, b,\ c\}$and $\rho(A)$ be its power set, draw a Hasse diagram of $\langle\rho(A), \subseteq\rangle$.	**BTL -1**	Remember
4.	State distributive lattice.	**BTL -1**	Remember
5.	Check whether the posets $\{(1,3,6,9), D\}$ and $\{(1,5,25,125), D\}$are lattices or not. Justify your claim.	**BTL -4**	Analyze
6.	Show that the absorption laws are valid in a Boolean algebra.	**BTL -1**	Remember
7.	Define sub lattice.	**BTL -1**	Remember
8.	Define lattice homomorphism.	**BTL -1**	Remember
9.	Give an example of a lattice that is not complemented.	**BTL -2**	Understand
10.	In a lattice $(L, \leq)$, prove that $a \wedge (a \vee b) = a, \forall a, b \in L$	**BTL -2**	Understand
11.	When is a lattice called complete?	**BTL -3**	Apply
12.	Give an example of a distributive lattice but not complete.	**BTL -4**	Analyze
13.	Define Boolean Algebra.	**BTL -1**	Remember

14.	Show that in a Boolean Algebra $ab' + a'b = 0 \ iff \ \ a = b$	**BTL -5**	Evaluate
15.	Prove the Boolean identity $a.b + a.b' = a.$	**BTL -5**	Evaluate
16.	Is a Boolean Algebra contains six elements? Justify your answer.	**BTL -4**	Analyze
17.	What values of the Boolean variables $x \ and \ y$ satisfy $xy = x + y$?	**BTL -3**	Apply
18.	State the De Morgan's laws in Boolean algebra.	**BTL -4**	Analyze
19.	Find the values, if any, of the Boolean variable x that satisfy $x.1 = 0$	**BTL -3**	Apply
20.	How many different Boolean functions are there of degree 7?	**BTL -6**	Create
	Part – B		
1.(a)	Show that every non-empty subset of a lattice has a least upper bound and a greatest lower bound.	**BTL -2**	Understand
1. (b)	Prove that in a Boolean Algebra$(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$	**BTL -2**	Understand
2. (a)	Examine whether the lattice given in the following Hasse diagram is distributive or not?  	**BTL -5**	Evaluate
2.(b)	If $P(S)$ is the power set of a non-empty set S, prove that $\{P(S),\cup,\cap,\backslash,\phi,S\}$is a Boolean Algebra.	**BTL -2**	Understand
3. (a)	If S_n is the set of all divisors of the positive integer n and D is the relation of 'division', prove that $\{S_{30}, D\}$ is a lattice. Find also all the sub lattices of $\{S_{30}, D\}$ that contain six or more elements.	**BTL -1**	Remember
3.(b)	Show that every chain is a distributive lattice.	**BTL -4**	Analyze
4. (a)	In a complemented and distributive lattice, then prove that complement of each element is unique.	**BTL -4**	Analyze
4.(b)	If $a, b \in S, \ S = \{1, 2, 3, 6\} \ and \ a + b = LCM\ (a, b),$ $a.b = GCD\ (a, b) \ and \ a' = {}^{6}/_{a}$, show that $\{S, +, \cdot\ , \ '1, 6\}$ is a Boolean Algebra.	**BTL -3**	Apply
5. (a)	Show that a complemented, distributive lattice is a Boolean Algebra.	**BTL -4**	Analyze
5.(b)	Show that De Morgan's laws hold in a Boolean Algebra that is show that $for \ all \ x \ and \ y$, $\overline{(x \vee y)} = \bar{x} \wedge \bar{y}$ and $\overline{(x \wedge y)} = \bar{x} \vee \bar{y}$	**BTL -2**	Understand
6. (a)	Let $\langle L, \leq \rangle$ be a lattice in which $*$ and $\oplus$ denotes the operations of meet and join respectively. For any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \oplus b = b$	**BTL -2**	Understand
6.(b)	Show that every totally ordered set is a lattice.	**BTL -3**	Apply
7. (a)	Show that in a lattice if $a \leq b \leq c$, then (1) $(a \oplus b) = b \star c$ and (2) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$.	**BTL -5**	Evaluate
7. (b)	Show that in a Boolean algebra $x \vee x = x \ and \ x \wedge x = x$	**BTL -6**	Create
8. (a)	Show that a lattice homomorphism on a Boolean Algebra which preserves 0 and 1 is a Boolean homomorphism.	**BTL -1**	Remember
8.(b)	In a Boolean Algebra. Show that $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$	**BTL -2**	Understand
9. (a)	Let L be a lattice, where $a * b = glb(a, b), a \oplus b = lub(a, b), \forall a, b \in L$. Then both binary operators $* \ and \ \oplus$ defined as in Lsatisfy	**BTL -1**	Remember

	commutative law, associative law, absorption law and idempotent law.		
9.(b)	Let $\langle L, \leq \rangle$ be a lattice. For any $a, b, c \in L$ the following properties hold good. If $b \leq c \Rightarrow i)\ a * b \leq a * c$ $ii)\ a \oplus b \leq a \oplus c$	**BTL -4**	Analyze
10.(a)	Consider the lattice D_{105} with the partial ordered relation divides then 1. Draw the Hasse diagram of D_{105} 2. Find the complement of each element of D_{105} 3. Find the set of atoms of D_{105} 4. Find the number of sub algebras of D_{105}.	**BTL -2**	Understand
10.(b)	Show that in a distributive and complemented lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$	**BTL -2**	Understand
11.(a)	If $P(S)$ is the power set of a set S and $\cup$, $\cap$ are taken as join and meet, prove that $\langle P(S), \subseteq \rangle$ is a lattice. Also, prove that the modular inequality of a lattice $(L, \leq)$, For any $a, b, c \in L, a \leq c \Leftrightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$.	**BTL -1**	Remember
11.(b)	In any Boolean algebra, show that $ab' + a'b = 0\ iff\ a = b$.	**BTL -5**	Evaluate
12.(a)	Let D_{30} = {1,2,3,5,6,10,15, 30} and Let the relation R be divisor on D_{30}. Find 1. All lower bounds of 10 and 15. 2. the GLB of 10 and 15 3. all upper bounds of 10 and 15 4. LUB of 10 and 15. 5. Draw the Hasse Diagram.	**BTL -5**	Evaluate
12.(b)	In any Boolean algebra, prove that the following statements are equivalent: $1. a + b = b$ $2. a.b = a$ $3. a' + b = 1$ $4.\ a.b' = 0$	**BTL -5**	Evaluate
13.(a)	Show that in a lattice if $a \leq b\ and\ c \leq d$, Then $a * c \leq b * d\ and\ a \oplus c \leq b \oplus d$.	**BTL -1**	Remember
13.(b)	In a distributive lattice prove that $a * b = a * c\ and\ a \oplus b = a \oplus c$ Imply$b = c$.	**BTL -1**	Remember
14.(a)	Let $\langle L, \leq \rangle$ be a Lattice. For any $a, b, c \in L$ the following inequalities known as distributive inequality hold. (i) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ (ii) $a * (b \oplus c) \leq (a * b) \oplus (a * c)$	**BTL -6**	Create
14.(b)	Prove the absorption law $x(x + y) = x$ using the the identities of Boolean algebra.	**BTL -3**	Apply

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