ES3D9 Laboratory Report
Applied Control, Instruments, Measurements & Electrical Machines
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Summary

This report details the analysis and design of a strain gauge based torsional measurement system. It consisted of two parts: a series of static torque measurements using a an amplification circuit and voltmeter, and a separate test for the dynamic response of the system. The latter was conducted by a LabVIEW interface and associated data acquisition software.

Throughout the test, it was found that the sensitivity of the torque measurement varied from $8.20 \times 10^-5VNm^-1$ to $3.57 \times 10^-4VNm^-1$ depending on the circuit configuration and whether theoretical or experimental values were used. These values related to the static torque measurement and were subject to various sources of error. These will likely have resulted from a combination of; zero offset error through manual calibration, imperfect resistance of the measurement components and a lack of optimised excitation of the bridge circuits.

This measurement system is a versatile tool for any stress-strain analysis, though consideration should be made of error sources and circuit configuration to allow for its correct application for a particular type of measurement.

Contents

1 Introduction

The aim of the experiment was to relate an applied torque to an electronic output signal. This was done with the use of strain gauges which were placed diagonally on a circular steel beam. This positioning was due to the orientation of the principal stresses that result from torsional loads. The strain gauges vary their resistance according to the shear stress and strain that result from the applied loads. With an appropriate bridge circuit, an output voltage results that is a function of the supply voltage, torsional load and the relevant beam properties (poisson ratio, young modulus, shear modulus and radius). For a discernible reading, the output signal was then amplified with a gain of ~ 1000.

2 Experimental Procedure

2.1 Mechanical Measurement System

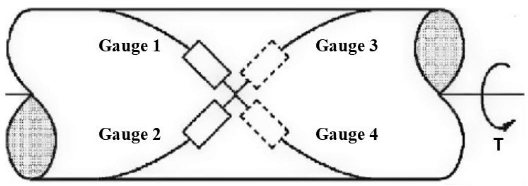


Figure 1: Torsion Beam Gauge Arrangement

The system used for the torque measurements consisted of a circular beam, a moment arm and a strain gauge circuit. The moment arm was used to apply torsional loads to a free end of the beam. Strain gauges on the beam then translated the strains resulting from these loads, into changes in resistance. A bridge circuit (of varying configurations) then converted these changes in resistance to changes in voltage and hence produced an output signal. Amplification of this signal was necessary and this was performed with the combined use of operational amplifiers. Specifically, the chosen configuration consisted of a differential amplifier cascaded with a non-inverting amplifier.

2.2 Electrical Circuit

The experiment involved variations of both the applied load and the measurement circuit used to detect these loads. The latter is explored here, with regards to the effect of circuit variations on the sensitivity of the strain gauges. Three different circuit configurations were used and their differences in sensitivity explored. These circuit configurations were comprised as follows:

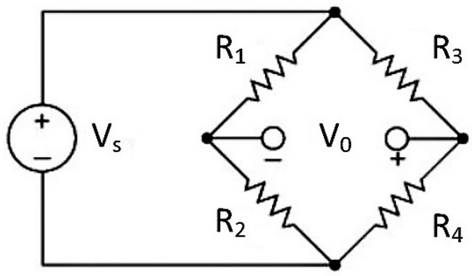


Figure 2: Wheatstone Bridge Circuit

1. Quarter bridge circuit (one of four active strain gauges)
2. Half bridge circuit (two of four active strain gauges)
3. Full bridge circuit (four of four active strain gauges)

In the quarter and half bridge configurations, the 'vacant' slots in the circuit were taken up by dummy gauges. These are specifically chosen resistors that balance the circuit when the active strain gauge(s) is unstrained. Shown in Figure 1, is a Wheatstone bridge circuit where each resistor location may represent a strain gauge or a dummy resistor depending on the configuration.

For amplification of the output signal, a gain of ~ 1000 was necessary for an output of just under 10V. The first stage (differential) amplifier had a fixed gain of 100 so a further gain of 10 was necessary in the second stage (non-inverting) amplifier. This was chosen instead to be 11, due to the available resistor values with which the amplifier was configured. This is described below, with the gain - transfer characteristic of a non-inverting amplifier:

$$A_{v2} = 1 + \frac{R_1}{R_2} \tag{1}$$

The total gain was eventually found to vary from 990-1200.

2.3 Combined Measurement System

Varying the circuit configuration can have a broad impact on the type of measurements obtainable from the system as a whole. Half and full bridge circuits possess temperature compensation along with increased sensitivity of measurement, as such they maintain accuracy with fluctuations in temperature. However these circuits are incapable of accurately measuring axial strain. There is also the caveat of the range of measurement being inversely proportional to the sensitivity of the different configurations.

Beyond the circuitry, further improvements were installed into the system to allow for appropriate signal conditioning. A low-pass filter was present so as to remove common, high-frequency noise and amplification was used, as above, to increase the resolution and improve the signal-to-noise ratios. The static measurements differed from the dynamic ones in that they relied on just the bridge circuit, conditioning and voltmeter to display readings. The dynamic response was computerised with LabVIEW and myDAQ acquisition software to allow for accurate transient analysis.

3 Derivations & Theory

The following section derives the relationships that underpinned the measurement process, from the mechanical elements to the electrical output signals:

Internal and External Beam Forces

Shear Stress, as results from the applied torque.

$$\tau = \frac{F}{A} = \frac{Tr}{J} \tag{2}$$

Where T = applied torque, F = applied force, A = cross-sectional area of the beam, r = beam radius and J = polar moment of area. Following from this, one arrives at shear strain, the deformation of the beam due to the above stress:

$$\gamma = \frac{\tau}{G} \tag{3}$$

Defining the shear modulus (4) in combination with shear strain leads to True Strain (5):

$$G = \frac{E}{2(1+\nu)} \tag{4}$$

Where G = shear modulus of the beam, ν = Poisson's ratio and E = Young's modulus of the beam:

$$\varepsilon = \frac{2T}{\pi r^3}\frac{(1+\nu)}{E} \tag{5}$$

Strain Gauge Circuit

Considering the above, and the characteristics of a strain gauge, the relation between strain and resistance is found:

$$\varepsilon = GF\frac{\Delta R}{R} \tag{6}$$

Where GF = gauge factor (property of a given strain gauge due to the type and material used). The change in resistance can then be converted into an output voltage with the use of the Wheatstone bridge circuit:

$$V_0 = \frac{R_1}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}.V_s \tag{7}$$

Where R = strain gauge resistance, V_0 = output voltage and V_s = the voltage supplied to the bridge circuit. Combining all the preceding mechanical and electrical derivations gives the total input-output relation of the measurement system as a whole:

$$T = \frac{\Delta V_0.E.\pi.r^3}{2.GF.V_s.(1+\nu)} \tag{8}$$

The following equations relate this to the various bridge configurations:

$$Quarter\ Bridge\ Output : \Delta V_0 = \frac{V_s}{4}.\frac{\Delta R}{R} \tag{9}$$

$$Half\ Bridge\ Output : \Delta V_0 = \frac{V_s}{2}.\frac{\Delta R}{R} \tag{10}$$

$$Full\ Bridge\ Output : \Delta V_0 = V_s.(\frac{\Delta R}{R}) \tag{11}$$

Sensitivity of Measurement

The sensitivity of a measurement is defined as the smallest change in an input variable that can cause a change in reading of the instrumentation. In the instance of this experiment, this refers to the smallest varied torque input that would cause a change in the output voltage. This is given by the following equation which builds on the previous relations:

$$\frac{\Delta V_0}{T} = \frac{2.GF.V_s.(1+\nu)}{E.\pi.r^3} \tag{12}$$

The sensitivity in relation to each bridge configuration was multiplied by a fraction of the supply voltage as can be seen in Equations 9 - 11. The necessary variables are as follows:

System Parameters:
Gauge factor = 2.05
Torque arm length = 0.15m
Beam Young's modulus = 207GPa
Beam Radius = 5mm ± 0.25mm
Nominal Gauge Resistance = 350Ω
Supply Voltage = 5V
Poisson Ratio = 0.302

4 Results

4.1 Static Measurements

Table 1: Strain gauge measurements

Weight (kg)	**Torque (Nm)** $\pm 10^-4$	**Strain Reading** $(\mu\varepsilon \pm 1)$/**Voltage**$(\mu V \pm 1)$					
		Quarter Bridge		Half Bridge		Full Bridge	
0	0	0	0	0	0	0	-1
0.1	0.147	5	14	5	27	5	56
0.2	0.294	9	25	10	52	10	106
0.3	0.441	15	39	15	77	15	158
0.4	0.588	20	52	18	102	20	209
0.5	0.732	24	64	20	131	25	261

Table 2: Strain gauge sensitivities

	Quarter Bridge	**Half Bridge**	**Full Bridge**
Theoretical sensitivity, B	8.20×10^-5	1.64×10^-4	3.28×10^-4
Test sensitivity, A	8.74×10^-5	1.78×10^-4	3.57×10^-4
Relative error, A/B	1.066	1.085	1.088

*Sensitivities are given in VNm^-1

Table 3: Bridge Circuit Outputs with Amplifier Gains

Weight(kg)	**Torque(Nm)**	**Input**	**Output1**	A_{v1}	**Output2**	A_{v2}	A_{vT}
0	0	0	0	N/A	0	N/A	N/A
0.1	0.147	0.053	4.77	90	54.5	11	990
0.2	0.294	0.105	10.02	95	109	11	1045
0.3	0.441	0.155	15.10	97	170	11	1067
0.4	0.588	0.210	20.50	98	235	11	1078
0.5	0.732	0.250	25.00	100	290	12	1200

*All voltage inputs and outputs are given in mV

As in the results of Table 1, the theoretical sensitivity of the torque measurements can be found with the application of Equation (12). The experimental sensitivity was then found by division of the voltage range over the corresponding torque range for each circuit configuration. These results, along with their discrepancy (relative error between the theoretical and experimental sensitivities), are given in Table 2. Further measurement was also made as to how the strain/voltage outputs corresponded to the amplifier outputs. These results are given in Table 3.

Circuit Parameters, (variables in Table 3):

A_{v2} = 1^{st} Stage (Differential) Amplifier Gain = Output1/Input

A_{v1} = 2^{nd} Stage (Non-Inverting) Gain = Output2/Output1 = $1 + \frac{R_1}{R_2}$

A_{vT} = Total (Cascade) Gain = $A_{v1}.A_{v2}$

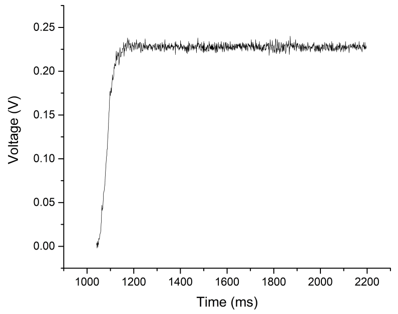
4.2 Dynamic Responses

Figure 3, overleaf, depicts the dynamic responses of the measurement system under various types of load, these ranged from slow 'normal' application of the weights to instances of shock-loading. The further instance of a sudden tap to the beam is considered in Figure 3 (d). These results differ from the static measurements by involving significant relative motion of the beam to its support, and the need for different sampling elements. All loads were due to the application of 400g weight, in the manners specified previously.

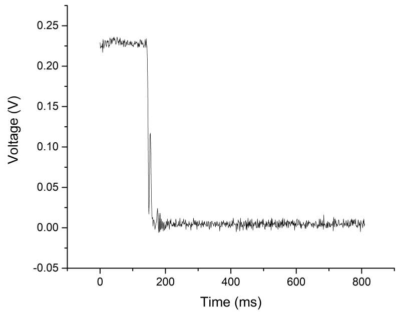
5 Discussion

As can be seen from the various results tables (4.1), there is considerable variety across all the measurements and calculations. There is an evident trend in that the test sensitivities are between 5 and 10 percent greater than the respective theoretical values for each circuit configuration. This is unexpected, given that instruments are typically theoretically more capable than their practical performance might indicate.

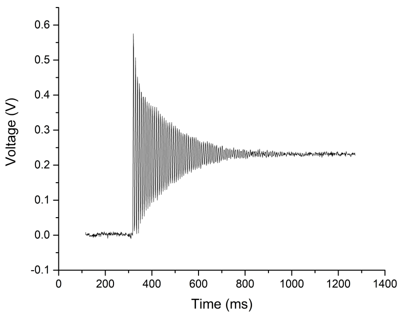
However, within this trend, the sensitivities increase with the number of active gauges. This is true to the established nature of quarter, half and full bridge circuits, whereby the sensitivity is typically doubled between each configuration. The system resolution was defined by the precision of the voltmeter used to discern the output signals, this was 0.1mV, which allowed for quite specific measurements.



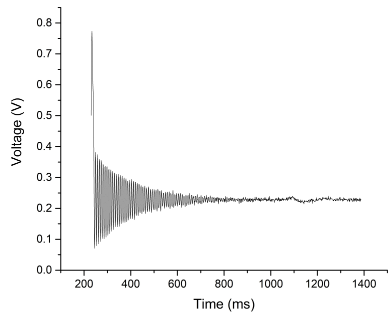
(a) Normal loading



(b) Normal unloading



(c) Weight shock-loading



(d) Tapping

Figure 3: Dynamic responses of the measurement system

There are numerous errors inherent in these measurements, though many are present from the need to have a practical system. The strain-output relationship is assumed to be linear in this system however, the full range was not encountered, so this may not be the case. Specifically, the range of measurement of torque was less than that available due to the power supply. The power supply allowed for outputs up to 5v, yet no signal resulted in an output of more than 0.8v. This would suggest that torque measurements could be made with greater weight, if desired. Beyond the circuit limitations, additional restrictions on the range of measurement would exist due to the torsional yield strength of the bar.

Error is also present through the existence of the circuit components required to display readings (i.e voltmeter, circuit board traces, switches, wires etc.) Though they may be balanced between readings, they encounter further fluctuations in temperature, noise and hysteresis. The finite resistances of these components inevitably add error and noise, even though they are necessary for the measurement process. This was also present in the tolerances of the resistors used, though this was kept to 5%. Further error will have resulted from imperfect calibration and zeroing of the display, in relation to the unloaded strain-voltage outputs. This process was done manually, yet if automated may result in reduced error.

6 Conclusion & Improvements

The measurement system has provided static and dynamic measurements, as required. The resolutions, range and sensitivities have been defined, yet alteration of the circuit may be necessary to adjust these criteria for the required applications of a given measurement.If the system was to be used for dynamic torque measurement, further considerations need to made.

When rotated further than one revolution, slip rings would be necessary to allow for continuous torque measurement. These would need to be carefully chosen to produce as little noise as possible, especially given the small magnitudes of the output signals. Furthermore, the frequency response of the gauges should have a bandwidth that is wide enough to accommodate the dynamic variations in torque.

This is important to consider if high frequency-low torque results are desired, as they are most susceptible to interference. Adherence would also need to be made to Nyquist theorem, for reliable measurement under any instances of dynamic output with a set frequency. Further improvements throughout the system could include:

- Further offset-nulling or shunt calibration for reduced zero-error.
- Use of high precision resistors and high resistance $\geq$ GΩ voltmeters.
- Highly optimised excitation of the various bridge circuits.
- Higher resistance gauges to allow for easier excitation, as above.
- Self calibration through appropriate software.

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