ES4D9 Assignment: Heat Transfer Theory and Design

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(1) Water at 10°C flows at a velocity of 3ms^{-}1 across a plate maintained at a uniform temperature of 70°C. If the length and width of the plate are 1.2m and 1m respectively, determine the convective heat transfer rate from the plate.

$$\text{Reynolds number: } Re = \frac{U_\infty L}{\nu} = \frac{3 \times 1.2}{1.307 \times 10^{-6}} = 2.75 \times 10^6 > 5 \times 10^5 = \text{turbulent flow}$$

$$\text{For turbulent flow, the local Nusselt number is given by: } Nu_x = 0.0296(re)^{0.8}(Pr)^{0.333}$$

$$\text{where } Pr = \frac{C_p \mu}{k} = \frac{4200 \times (1.307 \times 10^{-3})}{0.57864} = 9.49$$

$$\therefore Nu_x = 0.0296 \times (2.75 \times 10^6)^{0.8} \times (9.49)^{0.333} = 8875$$

$$\text{The local convective heat tranfer coefficient is: } h_x = \frac{Nu_x k}{L} = \frac{8875 \times 0.57864}{1.2} = 4280$$

$$\text{This gives the average convective heat tranfer coefficient: } h_{avg} = 1.25 h_x = 5350$$

Newton's law of cooling then gives the total convective heat transfer rate:

$$\dot{Q} = h_{avg} A (T_w - T_\infty) = 5350 \times (1.2 \times 1) \times (70 - 10) = 385 kW/m^2K$$

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(2) Consider a staggered tube bundle arrangement which has water at 70°C flowing through the tubes and air flowing over the tubes at 6m/s. The inlet air temperature is 15°C. The tube outside diameter and length are 16.4mm and 1m, respectively. The longitudinal and transverse pitches between the tubes are $S_L = 34mm$ and $S_T = 31mm$. There are seven rows of tubes in the air-flow direction and eight tubes per row ($N_L = 7, N_T = 8$). The surface temperatures of the tubes can be assumed constant and equal to the water temperature. a) calculate the outlet air temperature b) calculate the heat transfer rate from the tube bundle.

$$\text{Diagonal Pitch } S_D = \sqrt{(S_T)^2 + (S_L)^2}, \ \ S_D = 46mm \ \ \therefore \ \ 2(S_D - D) = 59.2 > S_T - D \ (14)$$

$$\text{So for this array: } v_{max} = \frac{S_T}{S_T - D} \times V = \frac{31}{31 - 16.4} \times 6 = 12.7 m/s$$

$$Re_{(D,Vmax)} = \frac{\rho V_{max} D}{\mu} = \frac{1.225 \times 12.7 \times 0.0164}{1.79 \times 10^{-5}} = 1.43 \times 10^4$$

$$\overline{Nu_D} = C_2 \left[C \times \left(Re_{(D,Vmax)}\right)^m \times Pr^{0.36} \times \left(\frac{Pr}{Pr_s}\right)^{0.25} \right]$$

$$\text{The relevant terms are: } N_L = 0.7 \ \text{so} \ C_2 = 0.95, \ \ \frac{S_T}{S_L} = \frac{31}{34} = 0.9 < 2 \ \text{so} \ m = 0.6$$

$$C = 0.35 \left(\frac{S_T}{S_L}\right)^{(1/5)} = 0.35 \left(\frac{31}{34}\right)^{(1/5)} = 0.34, \ \ Pr\ (15°C) = 0.71, \ \ Pr_s\ (70°C) = 0.701$$

$$\therefore \ \overline{Nu_D} = 0.95 \left[0.34 \times (1.43 \times 10^4)^{0.6} \times (0.71)^{0.36} \times \left(\frac{0.71}{0.701}\right)^{0.25} \right] = 105$$

$$\bar{h} = \overline{Nu_D} \left(\frac{k}{D}\right) = 105 \times \left(\frac{0.025}{0.0164}\right) = 160 W/m^2K$$

$$\frac{T_s - T_o}{T_s - T_i} = e^{\left(\frac{\pi D N \bar{h}}{\rho V N_T S_T C_p}\right)}$$

Inputting $T_s = 70,\ T_i = 15,\ D = 0.0164,\ N = (N_L)^* N_T = 56,\ \bar{h} = 160,\ \rho = 1.217,\ V = 6,\ S_T = 0.031,\ C_p = 1007$ to the equation above gives $\underline{\mathrm{T}_0 = 50.7°C}$. The logarithmic mean temperature is then required for calculation of the total heat rate:

$$\Delta T_{LM} = \frac{(T_s - T_i) - (T_s - T_o)}{ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = \frac{(70 - 15) - (70 - 70.7)}{ln\left(\frac{70-15}{70-50.7}\right)} = 34.1°C$$

Newton's law then gives the total heat transfer rate: $q = \bar{h} A_s(\Delta T_L M)$

Where A_s is the combined surface area of the tube bank, perpendicular to air flow:

$$A_s = N(\pi D L) = 56 \times \pi \times 0.0164 \times 1 = 2.885 m^2$$

Therefore: $q = 169 \times 2.885 \times 34.1 = 15.75 kW m^{-1}$

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(3) Water at 10°C flows at a rate of 12kg/hr in a tube with an internal diameter of 10mm. A heater is fabricated by a resistance wire wound uniformly over the length of the tube. The resistance element maintains a uniform surface heat flux of 1000W/m . The mean outlet water temperature is 24°C. (a) Estimate the tube length and (b) Assuming the flow is thermally and hydro-dynamically developed, estimate the average tube surface temperature.

In SI units 12kg/hr = $\frac{1}{300}$kg/s, specific heat capacity of water at T_{mean} (17°C) = 4190 J/KgK

(a) The heat rate for internal flow in a tube is the product of the heat flux and surface area:

$$q_{conv} = q_s'' \pi D L = 1000\pi \times (10 \times 10^{-3}) L = \dot{m} C_p (T_{mo} - T_{mi})$$

$$\therefore \quad L = \frac{\dot{m} C_p (T_{mo} - T_{mi})}{10\pi} = \frac{4190 \times (24 - 10)}{300 \times 10\pi} = 6.22m$$

(b) For determination of the mean surface temperature:

$$dq_{conv} = q_s'' = \bar{h}(T_s - T_{mo}), \ \ where \ \ \bar{h} = Nu\left(\frac{k}{D}\right)$$

Nu for fully-developed, laminar flows with uniform surface heat flux is constant at 4.36:

$$\bar{h} = 4.36\left(\frac{0.598}{(10 \times 10^{-3})}\right) = 261 W/m^2 K, \ \ \therefore \ \ T_s = \frac{q_s''}{\bar{h}} + 24 = \frac{1000}{261} + 24 = 27.8°C$$

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(4) Air at 20°C enters a circular tube with diameter 5mm and length 10cm, at a velocity of 5m/s. The tube wall is maintained at a constant surface temperature of 160°C. Determine the convection heat transfer coefficient and the outlet mean temperature of the air.

Flow properties were taken at a temperature of 90°C, with the outlet temperature assumed to be equivalent to the surface temperature: $T_m = (T_s + T_{mi})/2$

$$\text{Reynolds number: } Re = \frac{U_\infty L}{\nu} = \frac{5 \times 0.05}{2.19 \times 10^{-5}} = 1142 < 2300 = \text{laminar flow}$$

The Sieder-Tate correlation was then used for Nu, with $Pr = 0.695$ and μ_s at $160°C$:

$$Nu = 0.027(Re)^{\frac{4}{5}}(Pr)^{\frac{1}{3}}\left(\frac{\mu}{\mu_s}\right)^{0.14} = 0.027 \times (1142)^{\frac{4}{5}} \times (0.695)^{\frac{1}{3}} \times \left(\frac{2.13}{2.48}\right)^{0.14} = 6.54$$

The definition of Nu was then gives the average convective heat transfer coefficient:

$$Nu = \frac{\bar{h}D}{k}, \ \therefore \ \bar{h} = \frac{Nu \times k}{D} = \frac{6.54 \times 0.03}{0.005} = 40.5W/m^2K$$

$$\dot{m} = \rho\, UA = 0.973 \times 5 \times \left(\pi \times \frac{0.005^2}{4}\right) = 1.25 \times 10^{-4} kg/s, \quad A_s = \pi DL = 5\pi \times 10^{-4} m^2$$

The mean temperature distribution, given a constant surface temperature, is shown to be:

$$T_{mo} = T_s - (T_s - T_{mi}) \times \mathrm{e}^{\left(-\frac{\bar{h}A_s}{\dot{m}c_p}\right)} = 160 - 140\mathrm{e}^{\left(-\frac{40.5\times(5\pi\times10^{-4})}{(1.25\times10^{-4})\times1009}\right)} = 71.8°C$$

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(5) A thin horizontal plate, 2m long and 40cm wide is maintained at a temperature of 80°C. The plate is fully immersed in a tank with water maintained at 40°C. Determine the heat transfer rate from the plate.

$$\text{The characteristic length is: } L = \frac{\text{Plate Surface Area}}{\text{Plate Perimeter}} = \frac{2 \times 0.4}{(2 \times 2) + (2 \times 0.4)} = \frac{1}{6}$$

$$\text{Rayleigh number: } Ra = \frac{g\beta(\Delta T)L^3}{\nu\alpha} = \frac{(9.81) \times (385 \times 10^{-6}) \times (40) \times (\frac{1}{6})^3}{(0.658 \times 10^{-6}) \times (1.52 \times 10^{-7})} = 7.014 \times 10^9$$

$$\text{The resulting Nusselt number: } \overline{Nu_L} = 0.15Ra^{1/3} = 0.15(7.014 \times 10^9)^{1/3} = 287$$

$$\text{Nu gives the convective coefficient: } 287 = \frac{\overline{h}L}{k}, \ \therefore \ \overline{h} = 6 \times 0.629 \times 287 = 1084W/m^2K$$

Newton's law then gives the heat transfer rate: $\dot{Q} = \overline{h}A(\Delta T) = 1084 \times (2 \times 0.4) \times 40 = 34.69kW$

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(6) A double-pane window with width 2m and height of 1.2m, consists of two 3mm thick layers of glass separated by a 6mm of air. The indoor temperature is 25°C and the outdoor temperature is 0°C. The thermal conductivities of the glass and air can be assumed constant and equal to 0.8W$m^{-1}K^{-1}$ (glass) and 0.026W$m^{-1}K^{-1}$ (air). The average heat transfer coefficients are 10W$m^{-2}K^{-1}$ and 25W$m^{-2}K^{-1}$, inside and outside of the room, respectively. Calculate (a) the overall heat transfer coefficient U, and (b) the heat transfer rate through the window.

Modelled as a composite wall with negligible contact resistance, the total thermal resistance of the window is given by:

$$R_{total} = \frac{1}{A}\left[\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_4}\right] = \frac{1}{2.4}\left[\frac{1}{10} + 2\left(\frac{0.003}{0.8}\right) + \frac{0.006}{0.026} + \frac{1}{25}\right] = 0.1576W$$

$$R_{total} = \frac{1}{UA} \quad \therefore 0.1576 = \frac{1}{U(2 \times 1.2)} \quad so \ U = 2.64W/m^2K$$

$$\dot{Q} = UA\Delta T = 2.64 \times 2.4 \times 25 = 159W$$

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(7) A transformer at a power station experiences a short circuit, resulting in a steady power dissipation of 300W into a transmission line, which acts as a long fin emanating from the transformer. The transmission line has a diameter of 20mm and is made of an Al alloy with conductivity $k = 190Wm^{-1}K^{-1}$. The melting temperature of the alloy is 660°C. The surrounding air temperature is 20°C. The heat transfer coefficient is $40Wm^{-2}K^{-1}$. What is the temperature of the transmission line at its connection point with the transformer? Is it at risk of failure?

Assuming an infinitely long fin, the heat transfer is given by:

$$q = \sqrt{hPkA_c}\,(T_b - T_\infty), \text{ where } A_c = \pi\frac{d^2}{4} = (\pi \times 10^{-4})m^2 \text{ and } P = \pi d = \left(\frac{\pi}{50}\right)m$$

$$\frac{300}{(T_b - 20)} = \sqrt{40 \times \left(\frac{\pi}{50}\right) \times 190 \times (\pi \times 10^{-4})} \text{ rearranged, this gives } T_b = 795°C$$

As $T_b > T_{melt}$, the transmission line will melt and undergo thermal failure.

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(8) The top surface of a locomotive moving at a velocity of 95km/h is 2.8m wide and 8m long. This surface is absorbing solar radiation at a rate of 380W/m^2 and the temperature of the ambient air is 30°TpiC. Assuming the roof to be perfectly insulated and the radiation heat exchange to be relatively small relative to convection, determine the equilibrium temperature of the top surface of the locomotive.

Reynolds number to categorise flow of air at 30°C and 95km/h (26.39ms^{-1}):

$$Re = \frac{U_\infty L}{\nu} = \frac{8 \times 26.39ms^{-1}}{1.568 \times 10^{-5}} = 1.35 \times 10^7 > 5 \times 10^5 = \text{turbulent flow}$$

$$Re_{(x=0.001)} = \frac{U_\infty L}{\nu} = \frac{0.001 \times 26.39ms^{-1}}{1.568 \times 10^{-5}} = 1.68 \times 10^3 < 5 \times 10^5 = \text{combined flow}$$

$$Pr = \frac{C_p\mu}{k} = \frac{1004.9 \times (1.846 \times 10^{-5})}{0.026} = 0.71$$

Average Nusselt Number for combined flows given by:

$$Nu = Pr^{1/3}[0.037(Re)^{0.8} - 871] = (0.71)^{1/3}[0.037(1.35 \times 10^7)^{0.8} - 871] = 15931$$

$$Nu = \frac{hL}{k} \quad \therefore \quad 15931 = \frac{8h}{0.026}, \text{ so } h = 51.8W/m^2K$$

For equilibrium to occur, absorbed radiation = convection heat rate in Newton's law:

$$\dot{Q} = h(T_s - T_\infty) \quad \therefore \quad 380 = 51.8(T_s - 30) \text{ so } T_s = 37.27°C$$

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(9) A horizontal plate is experiencing uniform irradiation on both the upper and lower surfaces. The plate temperature is maintained at 390K. The ambient air temperature surrounding the plate is 290K with a convective heat transfer coefficient of 30w/m^{2}K. Both upper and lower surfaces of the plate have a radiosity of 4000W/m^2. If the plate is not opaque and has an absorptivity of 0.527, determine the irradiation and the emissivity of the plate.

Given an area of 1m^2, and the uniform temperature produces equilibrium between the incident radiation and convective heat transfer to the surrounding air, $G = J + \dot{q}_{conv}$, where $\dot{q}_{conv}$ is:

$$\dot{q}_{conv} = h(T_s - T_\infty) = 30 \times (390 - 290) = 3\,kWm^{-2} \quad \therefore \quad G = 4000 + 3000 = 7\,kWm^{-2}$$

The radiosity is then given as the sum of irradiance that is emitted, transmitted and reflected:

$$J = E + G_{reflected} + G_{transmitted} = E + (\tau \times G) + (\rho \times G) = E + (\rho + \tau)G$$

The absorptivity (known to be 0.527), transmittivity and reflectivity are then rearranged:

$$1 = \alpha + (\rho + \tau) \ \ \therefore \ \ 1 - \alpha = (\rho + \tau) \ \text{ so } \ 1 - 0.527 = (\rho + \tau) = 0.473$$

As above, the emitted radiation is given as radiosity minus that reflected and transmitted:

$$E = J - (\rho + \tau)\,G = 4000 - (0.473) \times 7000 = 689\,Wm^{-2}$$

The emissivity is then found from the Stefan-Boltzmann law, through comparison of the emitted radiation to that of a black body (E_b) at the same temperature:

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma \times (T_s)^4} = \frac{689}{(5.67 \times 10^{-8}) \times (390)^4} = 0.525$$

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(10) A furnace is shaped like a long triangular duct where the widths of the side surfaces are 0.4m and 0.5m. The width of the bottom surface is 0.3m. The temperature of the side surfaces is maintained at 1000k, while the bottom surface is maintained at 500K. The emissivity of the side surfaces is 0.5, and the emissivity of the bottom surface is 0.8. Neglecting end effects, determine the heat transfer per unit length from the side surfaces to the bottom surface. Treat this geometry as a two-surface enclosure where all of the surfaces are opaque.

Radiative transfer between two grey planar surfaces is given by adaptation of Kirchhoff's Law:

$$\dot{Q} = \frac{\varepsilon_1 \sigma {T_1}^4 \varepsilon_2 - \varepsilon_2 \sigma {T_2}^4 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \equiv \frac{\sigma({T_1}^4 - {T_2}^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

For an assumed length of 1m, the surface areas are:
(Bottom) $A_1 = W_1 = 0.3m^2$,
(Both Sides Combined) $A_{23} = (W_2 + W_3) = 0.4 + 0.5 = 0.9m^2$

For opaque surfaces, the emissivities can then be given in terms of area and view factor; this gives the heat transfer rate per unit length for the furnace, as below:

$$\frac{\dot{Q}_{23\to1}}{L} = \dot{Q}_{23\to1} = \frac{\sigma({T_{23}}^4 - {T_1}^4)}{\left(\frac{1-\varepsilon_{23}}{A_{23}\varepsilon_{23}}\right) + \left(\frac{1}{A_{23}F_{23\to1}}\right) + \left(\frac{1-\varepsilon_1}{A_1\varepsilon_1}\right)} = \frac{\sigma({T_{23}}^4 - {T_1}^4)}{\left[\frac{1-0.5}{(0.9)(0.5)}\right] + \left[\frac{1}{(0.9)F_{23\to1}}\right] + \left[\frac{1-0.8}{(0.3)(0.8)}\right]}$$

$$\dot{Q}_{23\to1} = \frac{(5.67 \times 10^{-8})(1000^4 - 500^4)}{[1.11] + [0.83] + \left[\frac{1}{(0.9)F_{23\to1}}\right]} = \frac{5.31 \times 10^4}{\left[(1.94) + \frac{1}{(0.9)F_{23\to1}}\right]}$$

The view factor $F_{23\to1}$ is given by the area ratios according to reciprocity:

$$A_1(F_{1\to23}) = A_2(F_{23\to1}) \ \ \therefore \ \ F_{23\to1} = \frac{A_1}{A_2}(F_{1\to23})$$

As A_1 is the bottom of the enclosure and $A_2 + 3$ are opaque, $F_{1\to23} = 1$

$$F_{23\to1} = \frac{W_1}{W_2 + W_3}(1) = \frac{0.3}{0.9} = 0.33$$

Substituting back into the equation for $\dot{Q}_{23\to1}$ gives the heat transfer per unit length:

$$\dot{Q}_{23\to1} = \frac{5.31\times10^4}{\left[(1.94)+\frac{1}{(0.9)(0.3)}\right]} = 10.0056kWm^{-1}$$

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(11) Two parallel black disks are coaxially separated by a gap of L = D = 0.1m. The lower disk diameter D_1 = D and the upper disk diameter D_2 = 2D. If the lower disk is heated at 20W to maintain a uniform temperature of 600K, determine the temperature of the upper disk T_2.

First, from the ES4D9 view factor table entry for coaxial discs:

$$F_{1\to2} = \frac{1}{2}\left\{S-\left[S^2-4\left(\frac{r_2}{r_1}\right)^2\right]^{1/2}\right\} \text{where } S = 1+\frac{1+R_2^2}{R_1^2}\text{ , and } R_1 = r_1/L\text{ , } R_2 = r_2/L$$

$$R_1 = 0.5\,\&\,R_2 = 1 \text{ so } S = 9\;\therefore\; F_{1\to2} = \frac{1}{2}\left\{9-\left[9^2-4\left(\frac{0.1}{0.05}\right)^2\right]^{1/2}\right\} = 0.47$$

The heat transfer rate from disk 1 to the surroundings (assumed vacuum) is therefore given by:

$$\dot{q} = A_1\times(F_{1\to2})\times\sigma\times(T_1^4-T_2^4) = (\pi\times0.5^2)\times0.53\times(5.67\times10^{-8})\times(600^4-T_2^4)$$

When rearranged, the upper disk temperature can be found:

$$T_2 = \sqrt[4]{(T_1)^2-\frac{\dot{q}}{A\times(F_{1\to2})\times\sigma}} = \sqrt[4]{(600)^4-\frac{20}{0.008\times(0.47)\times(5.67\times10^{-8})}} = 429K$$

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(12) A small aluminium ball with diameter 0.3mm, at an initial temperature of 130°C is suddenly immersed into a liquid held constant at 25°C. The convection heat transfer coefficient h = 20W/m^2K. The physical properties of aluminium are assumed constant (k = 208W/m K, Cp = 955 J/Kg K and ρ = 2700kg/m^3). Determine the time, t, for the average temperature of the ball to drop to 50°C.

First, the Biot number is found for determination of an appropriate conduction model:

$$Bi = \frac{hL_c}{k} \text{ where } L_c = \frac{V}{A}\ (sphere) = \frac{4/3\pi r^3}{4\pi r^2} = \frac{r}{3}\;\therefore\; Bi = \frac{20(4.33\times10^{-5})}{208} \approx 4\times10^{-6} << 1$$

As this final condition is true, the lumped capacitance model can be used to determine t:

$$t = \frac{\rho V C_p}{hA_s}\,ln\left(\frac{Ti-T_\infty}{T-T_\infty}\right) = \left[\frac{\rho\,C_p\left(\frac{\pi D^3}{6}\right)}{h(\pi D^2)}\right]ln\left(\frac{Ti-T_\infty}{T-T_\infty}\right)$$

$$t = \left[\frac{2700\times955\left(\frac{\pi D^3}{6}\right)}{20(9\pi\times10^{-8})}\right]ln\left(\frac{130-25}{50-25}\right) = 9.25s$$

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(13) Determine the temperature of the ball from **(12)** after 2s, 4s and 6s, if the temperature of the surrounding liquid changes with time, as follows:

$$T(t) = 130+10t-3t^2 \qquad \text{where } T \text{ is temperature in °C and } t \text{ is time in seconds}$$

To solve this, the lumped capacitance model was rearranged for ball temperature, T:

$$T = T_\infty + (Ti - T_\infty)e^{\left(\frac{-ht}{Lc\rho C_p}\right)} = T(t) + [130 - T(t)]e^{\left(\frac{-ht}{Lc\rho C_p}\right)}$$

Each value of T(t) was found according to the quadratic expression and then inputted to the equation above. The results are shown below along with the associated script:

Time/s	Liquid Temperature T(t)/°C	Ball Temperature/°C
0	130	130
2	138	132
4	122	127
6	82	108

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(14) A cylindrical shaft with radius, r_o= 4cm and length $2L$= 1m is made of AISI 1010 steel ($\rho = 7840kgm^{-3}, Cp = 460Jkg^{-1}K^{-1}, k = 43Wm^{-1}K^{-1}$). It has a uniform initial temperature of 100°C. The shaft is then suddenly exposed to an environment with a fluid temperature of 0°C and a heat transfer coefficient of $360Wm^{-1}K^{-1}$. How long will it take for a point 8mm beneath the shaft's surface to reach a temperature of 20°C?

The Biot number for this scenario is given as a product of the characteristic length:

$$L_c = \frac{V}{A}\ (Cylinder) = \frac{\pi r^2 L}{2\pi rL + 2\pi r^2} = \frac{\pi(0.04)^2(1)}{2\pi(0.04)(1) + 2\pi(0.04)^2} = \frac{1}{52}$$

$$Bi = \frac{hL_c}{k} \text{ where } Bi = \frac{360(1/52)}{43} = 0.16 \neq << 1$$

The problem cannot be approximated using the lumped capacitance model, therefore it must be assumed that transient multidimensional conduction occurs:

$$\text{As } \frac{2L}{r} = 25 >> 10\ ;\ \text{The cylinder is considered to be infinitely long (The problem is 1D)}$$

From here, the modified Bi number was found and used along with the measurement depth ($0.008m$) to determine the temperature ratio from a temperature distribution Heisler chart:

$$\frac{1}{Bi} = \frac{k}{hr_0} = \frac{43}{360 \times 0.04} = 2.986 \text{ and } \frac{r}{r_0} = \frac{0.04 - 0.008}{0.04} = 0.8 \longrightarrow \frac{\theta}{\theta_0} = 0.86$$

This was then rearranged to give θ_0 in terms of the known temperatures. The value of θ_0 was then combined with $\frac{1}{Bi}$ to give a Fo number from a Heisler chart for centerline temperatures:

$$\theta_0 = \frac{0.86(T_0 - T_\infty)}{(T_i - T_\infty)} = \frac{0.86(20 - 0)}{(100 - 0)} = 0.172 \text{ and } \frac{1}{Bi} = 2.986 \longrightarrow \text{Fo} = 3.2$$

The resulting Fo was redefined to give the time for the point at $r - 0.008m$ to reach 20°C:

$$\alpha = \frac{k}{\rho C_p} = \frac{43}{7840 \times 460} = 1.19 \times 10^{-5} \rightarrow t = \frac{(Fo)r_0^2}{\alpha} = \frac{3.2(0.04)^2}{1.19 \times 10^{-5}} = 430s$$

In reality, this will be shorter as radiative heat loss has been neglected.

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(15) A counter-flow double pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.6kgs^{-1}. The heating is to be accomplished by brine at 160°C at a mass flow rate of 2.4kgs^{-1}. The inner tube is thin walled and has a diameter of 1.5cm. The overall heat transfer coefficient of the heat exchanger is estimated to be 640Wm$^{-2}K^{-1}$. The heat capacity of water and brine are 4180 Jkg$^{-1}K^{-1}$ and 4310 Jkg$^{-1}K^{-1}$, respectively. Determine the heat exchange length required to achieve the desired heating with (a) the LMTD method and (b) the NTU method:

(a) The rate of heat, transferred to the cold water, in the heat exchanger is simply:

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{water} = (1.6)(4.18)(80 - 20) = 401kW$$

Assuming all this heat is supplied by the brine, its outlet temperature can be found similarly:

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{brine} = 401kW = (2.4)(4.31)(160 - T_{out})$$

$$\therefore \ T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p} \ \text{ so } \ T_{out} = 160 - \frac{401}{(2.4)(4.31)} = 121°C$$

As T_{out} and T_{in} is known for both the brine and cold water, the logarithmic-mean-temperature-difference (LMTD) is found as follows:

$$\Delta T_1 = T_{h,in} - T_{c,out} = 160 - 80 = 80°C \ \mid \ \Delta T_2 = T_{h,out} - T_{c,in} = 121 - 20 = 101°C$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{80 - 101}{ln\left(\frac{80}{101}\right)} = 90.1°C$$

The surface area required to perform the corresponding heat transfer rate is given by:

$$\dot{Q} = UA_s(\Delta T_{lm}) \ \therefore \ A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{401000}{(640)(90.1)} = 6.95m^2$$

The tube length required to provide this much area is:

$$A_s = \pi DL \ \therefore \ L = \frac{A_s}{\pi D} = \frac{6.95}{0.015\pi} = 147m$$

(b) For the ε-NTU method, the heat capacities of each fluid must be determined:

$$C_h = \dot{m}_h C_{ph} = 2.4 \times 4.31 = 10.34kWK^{-1} \ \mid \ C_c = \dot{m}_h C_{pc} = 1.6 \times 4.18 = 6.69kWK^{-1}$$

$$\therefore \ C_{min} = C_c = 6.69kWK^{-1} \quad \text{and} \quad C = \frac{C_{min}}{C_{max}} = \frac{6.69}{10.34} = 0.64$$

The maximum possible heat transfer rate is given by:

$$\dot{Q}_{max} = C_{min}(T_{h,in} - T_{c,in}) = 6.69(160 - 20) = 937kW$$

The actual heat transfer rate, $\dot{Q}$, is that obtained with the LMTD method (401kW):

$$\text{Therefore the exchanger's effectiveness, } \varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{401}{937} = 0.428$$

Given ε, the number of transfer units (NTU) is found with the empirical relation:

$$NTU = \frac{1}{C-1} ln\left(\frac{\varepsilon - 1}{\varepsilon C - 1}\right) = \frac{1}{0.64 - 1} ln\left[\frac{0.428 - 1}{(0.428 \times 0.64) - 1}\right] = 0.66$$

With the definition of NTU, the corresponding surface area and length are found:

$$NTU = \frac{UA_s}{C_{min}} \quad \therefore \quad A_s = \frac{NTU \times C_{min}}{U} = \frac{0.66 \times 6690}{640} = 6.90m^2$$

$$A_s = \pi DL \quad \therefore \quad L = \frac{A_s}{\pi D} = \frac{6.90}{0.015\pi} = 146m$$

This result is effectively the same as with the LMTD method.

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(16) Design Problem: A tube and shell heat exchanger for heat recovery from an industrial pilot plant is to be designed, with a view to minimise the payback time. The outdoor air is drawn into the plant while the indoor air is discharged, through the heat exchanger, to the environment. The use of a heat exchanger, however, can only be justified if the energy savings are commensurate with the capital and operational costs of the device.

Currently, the pilot plant is in operation 40 hours a week (8 hours a day, 5 days a week). The volume of the pilot plant is 600 m^3. The plant is heated by oil which is fed to a boiler (oil; Cp = 45 MJ kg^{-1}; ρ = 800 kgm^{-3}; price = £ 500 m^{-3}. The heating efficiency of the boiler is 80%. The heating season is six months (from 20 October to 20 April) with a mean outside air temperature of 6 °C during this period.

The complete system consists of the tube and shell heat-exchanger, a supply air fan and an exhaust fan. Assume equal flow rates on both sides of the heat-exchanger. The construction material is aluminium (price: £1.4/kg). Overheads of 400% apply to the costs of the aluminium used in the heat-exchanger to obtain its total price; including manufacturing, assembly and installation costs.

Design specification:

1. A temperature of 21°C should be maintained inside the pilot plant.
2. The air in the plant should be refreshed with a rate of 3600 $m^3 h^{-1}$ (6 times an hour), as recommended by industrial standard 6281.
3. The allowable pressure drop in the extract side is 300Pa. An air fan is already installed with no additional cost.
4. An electrically driven air fan is required to force fresh air across the tube bundle, with electricity costs of £0.04 per kWh.
5. Tube dimensions: 4mm inner diameter, 6mm outer diameter.

Objectives:

1. Suggest the best heat exchanger design based on this specification
2. Estimate the annual energy savings achieved by this device

Implied Constraints

As little is known about the condition of the indoor air, it is assumed that the outdoor air is more humid, polluted and therefore more corrosive. It will therefore occupy the tube side.

The heat exchanger must fit in the room. The total volume is 600m^3, so for typical dimensions of 10m×10m×6m, any given tube pass must not exceed 10m in length.

Tubeside Velocity

It is recommended that for liquids (water), the flow velocities should be kept in the range (1m/s < U < 3m/s). Using Re similarity, these velocities were converted to those for air:

$$Re(water) = \frac{Ud}{\nu} = \frac{1 \text{ or } 3 \times 0.004}{1.004 \times 10^{-6}} = \text{ (1m/s), Re} = 3984 \text{ and (3m/s) Re} = 11952$$

The corresponding, recommended air velocities were then found for these Re numbers:

$$Re(air) = 3984 \text{ or } 11952 = \frac{U \times 0.004}{1.5 \times 10^{-5}} \longrightarrow \text{ 15m/s} < \text{U} < \text{45m/s}$$

The higher Re was chosen (11952) as this allows turbulent flow for more effective heat transfer yet avoids flow switching losses.

This was compared with the max. fluid pressure and the corresponding cost for fan power:

$$\text{(Pressure Drop) } \Delta P = \lambda \times \left(\frac{L}{D}\right) \times \frac{\rho}{2} \times (U^2) \qquad \text{(Fan Power) } P = \frac{1}{2}\rho A U^3$$

As only the inlet temperatures are known, this problem demands use of the NTU method. First, the thermal capacitances of each air stream are calculated:

$$C_c(6°C) = \dot{m}C_{p,c} = 1.269 \times 1006 = 1277 \mid C_h(21°C) = \dot{m}C_{p,h} = 1.204 \times 1007 = 1212$$

The ratio of these is given as R', to find the heat exchanger effectiveness:

$$R' = \frac{(\dot{m}C_p)_{min}}{(\dot{m}C_p)_{max}} = \frac{1212}{1277} = 0.95 \approx 1$$

This allows for a simplified expression for the heat exchanger effectiveness:

$$\varepsilon = \frac{NTU}{1 + NTU} \text{ where } NTU = \frac{UA_s}{C_{min}}$$

The required heat transfer rate (heat duty) is given below.
As R' is ≈ 1, the absolute temperature difference is used instead of ΔT_{LM}:

$$Q = \bar{U}A_s\Delta T_{LM} = U[\pi(4 \times 10^{-3})L](21 - 6)$$

The term U is the overall heat transfer coefficient, given by:

$$U = \frac{1}{(1/hi) + (1/ho)}$$

Where each value for h is further given by the internal flow correlations, inside and out:

$$h_{i/o} = Nu\frac{k}{D_{i/o}} = 0.023Re^{4/5}Pr^{0.4}$$

Min. Baffle Spacing (left), Tube Pitch (right):

$$x = \frac{Shell \text{ I.D}}{5} \text{ or 50mm (whichever is greater)} \qquad y > 1.25\text{Tube O.D or (O.D + 6mm)}$$

<u>Costs</u>

Aluminium for HEX construction $\rightarrow$ CAPEX = $4\times$ (below):

$$(V_{tube} + V_{shell} + V_{baffles}) \times \rho \times price/kg = (V_{tube} + V_{shell} + V_{baffles}) \times 2700 \times 1.4$$

Oil required for shellside heat supply:

$$price(oil) \times \frac{q}{Cp(Thi - Tho)}$$