# Performance Analysis of a Thermal Solar Collector

**ES4E0 Renewable Energy**

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## 1 Introduction

This report details the analysis of a resilient system for the provision of hot water to a guesthouse in Leamington Spa. The f-chart method was used accordingly, to determine the fraction of the monthly heating load, met by a solar collector. A cost assessment is also provided for the system. The following sections provide the background to, and results of the analysis. Additionally, hand written calculations are appended and demonstrate the procedure for the month of January. Also appended, is the matlab script used to compute the rest of the results.

## 2 System Description and Data

The subject system is a flat-plate solar collector, positioned on the south facing roof of a guesthouse in Leamington Spa, UK. This supplies and stores hot water and supplements an existing gas boiler to meet the heating load. The analysis uses the following weather data for the system site:

Table 1: Input weather data for guesthouse at 52.3°N, 1.5°W

| Month | $\bar{K}$ | $\bar{H}$ [kWh] | $\bar{T_A}$ [$°C$] | $\bar{T}_{mains}$ [$°C$] | Av. Day Number |
|---|---|---|---|---|---|
| January | 0.25 | 20 | 3.1 | 6 | 17 |
| February | 0.30 | 33 | 3.1 | 5.7 | 47 |
| March | 0.32 | 62 | 5.2 | 6 | 75 |
| April | 0.41 | 103 | 7.6 | 7 | 105 |
| May | 0.41 | 134 | 10.6 | 9 | 135 |
| June | 0.39 | 136 | 14 | 12 | 162 |
| July | 0.40 | 137 | 15.8 | 14.5 | 198 |
| August | 0.42 | 120 | 15.4 | 15 | 228 |
| September | 0.39 | 80 | 13.2 | 14.2 | 258 |
| October | 0.33 | 46 | 10 | 12 | 288 |
| November | 0.32 | 15 | 6 | 9.2 | 318 |
| December | 0.27 | 15 | 4.2 | 7 | 344 |

**Weather Data Variables:**

- $\bar{K}$ = Monthly average clearness index ≈ the proportion of global to diffuse insolation, due to the atmospheric path of radiation to the site.
- $\bar{H}$ = Monthly average daily global horizontal insolation at site.
- $\bar{T_A}$ = Monthly average local ambient air temperature.
- $\bar{T}_{mains}$ = Monthly average local water mains temperature.

# 3 Theory

The f-chart method is one of several means of modelling the performance of a solar collector. It is based on correlations derived from experimental tests performed on select collector systems. Each solution to the f-chart model provides the result $f_m$, this is the fraction of the monthly heating load that is delivered by the collector. For liquid systems, it is given by the equation:

$$f_m = 1.029Y - 0.065X - 0.245Y^2 + 0.0018X^2 + 0.0215Y^3 \qquad (1)$$

The variables $X$ and $Y$ represent the system losses and absorbed energy, respectively, as a proportion of the monthly heating load. These are as follows:

**X Calculation**

The X parameter represents the system's losses as a proportion of the monthly heating load. These are due to heat loss from the collector as well as in the (collector-water store) heat exchanger:

$$X = \frac{A_c F_r' U_L (T_{ref} - \bar{T_A}) \Delta t}{Q_{LM}} \qquad (2)$$

- $A_c$ = Total collector area = $6m^2$ (constant)
- $U_L$ = Collector heat loss coefficient = $3.5Wm^{-2}K^{-1}$ (constant)
- $T_{ref}$ = Collector reference temperature = $100°C$ (constant)
- $\bar{T_A}$ = Monthly average ambient temperature for a given month [$°C$] → (variable, as given by column 4 of Table 1)
- $\Delta t$ = Number of seconds in a given month = 3600 × 24×N → (variable, where N is the number of days in a given month)
- $F_r'$ = DeWinter exchange factor = 0.8770 → (constant, derived and calculated in Appendix A)

- $Q_{LM}$ = Monthly heating load, [J] (variable):
  $\rightarrow [Q_{LM} = mNc\Delta T = (200N) \times 4200 \times (60 - \bar{T}_{mains})]$

- m = Mass of water (to be heated) = 200kg/day

- c = Specific heat capacity of water = 4200 J$kg^{-1}K^{-1}$

- 60 = Temperature of hot water delivery [$^{\circ}C$]

- $\bar{T}_{mains}$ = Monthly average water mains temperature
  $\rightarrow$ (variable, as given by column 5 of Table 1)

**Y Calculation**

$$Y = \frac{A_c F_r' \bar{\eta_0} \bar{H_T} N}{Q_{LM}} \tag{3}$$

- $\bar{\eta_0}$ = Monthly average optical efficiency of the collector = 0.8075
  $\rightarrow$ Normal optical efficiency (0.85) $\times$ Mean incidence angle modifier (0.95) = Proportion of absorbed to incident radiation (constant)

- $\bar{H_T}$ = Monthly average daily insolation, incident on collector
  $\rightarrow$ (calculated in Appendix B from site lattitude $\phi$, $\bar{K}$ and $\bar{H}$)

- $A_c$ , $F_r'$ , $Q_{LM}$ , N = as previously outlined

**Empirical Corrections**:

The following parameters are adjusted to fit the f-chart model, due to variation of the system from the standard specifications. These apply to X, whereby each corrected value is the product of X and the correction factor:

Storage Volume - the non standard storage volume used in this system requires a corrected value for X according to:

$$X_c = X\left(\frac{Storesize}{Standard}\right)^{-0.25} = X\left(\frac{300}{75}\right)^{-0.25} \tag{4}$$

Water Temperatures - as a solely water-based system, with no space heating components, the following further correction is applied to X:

$$X_w = X\left(\frac{11.6 + 1.18(60) + 3.86\bar{T}_{mains} - 2.32\bar{T_A}}{100 - \bar{T_A}}\right) \tag{5}$$

NB: These corrections, once sequentially applied to the calculated X values [from equation (2)] then provided the input 'X' parameter for equation (1). Errors associated with their use are highlighted in the evaluation section.

# 4 Results

The result $f_m$ is given below in Figure (1) and Table (2) for each month of the year. These values were computed numerically via the appended matlab script. The additional columns in Table (2) provide the actual heating load and solar provision, corresponding to each value for $f_m$.

Table 2: Monthly Heating Load vs Solar Provision

| **Month** | $\mathbf{Q}_{LM}$ **(kWh)** | $\mathbf{Q}_{solar}$ **(kWh)** | $f_m = \mathrm{Q}_{solar}/\mathrm{Q}_{LM}$ |
|---|---|---|---|
| January | 390 | 48.0053 | 0.1229 |
| February | 355 | 109.8827 | 0.3097 |
| March | 390 | 186.1875 | 0.4767 |
| April | 371 | 272.9481 | 0.7357 |
| May | 369 | 302.5662 | 0.8202 |
| June | 336 | 281.4311 | 0.8376 |
| July | 329 | 278.6425 | 0.8466 |
| August | 326 | 268.2840 | 0.8242 |
| September | 321 | 215.8399 | 0.6732 |
| October | 347 | 148.3138 | 0.4272 |
| November | 356 | 26.0920 | 0.0734 |
| December | 383 | 31.1557 | 0.0813 |

# 5 Cost Assessment

This section details the cost savings that result from use of the collector to supplement the gas boiler (as opposed to sole use of gas). This accounts for the renewable heat incentive (£0.2066/kWh), against the cost of gas utilisation (£0.06/kWh) versus the efficiency of the gas boiler ($\eta = 0.75$). These were used to calculate the annual savings and payback time of the system:

Annual heating cost via gas only:

$$Cg = 0.06 \times \sum_{n=1}^{12} \left[ \frac{Q_{LM}}{0.75} \right] = \pounds 341.86 \tag{6}$$

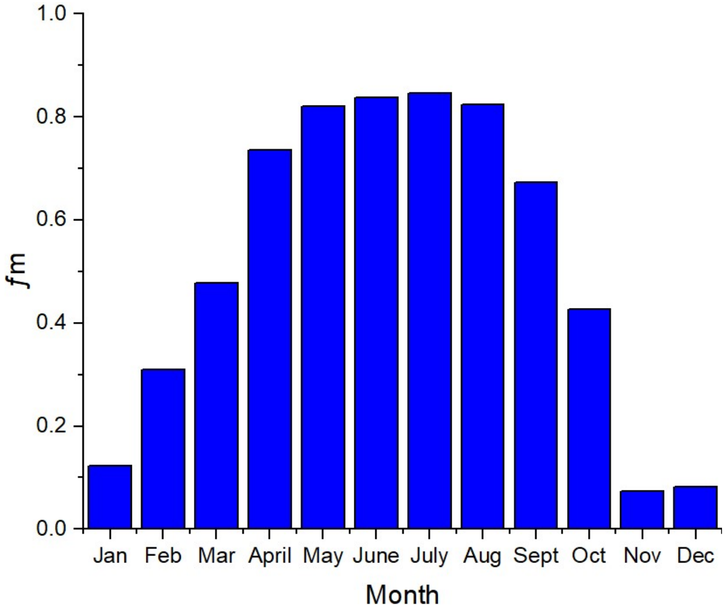


Figure 1: $f_m$ vs Month

Annual cost of gas needed to supplement the collector:

$$Cgs = 0.06 \times \sum_{n=1}^{12} \left[ \frac{(Q_{LM} - Q_{solar})}{0.75} \right] = £168.48 \tag{7}$$

Annually earned Renewable Heat Incentive (RHI):

$$Cr = 0.2066 \times \sum_{n=1}^{12} Q_{solar} = £448.19 \tag{8}$$

Annual net cost with collector:

$$Cs = Cgs - Cr = -£279.71 \ (earned) \tag{9}$$

Annual saving vs sole gas use (6):

$$S = Cg - Cs = £621.57 \tag{10}$$

Payback time on system:

$$P = \frac{Installation\, Cost}{Annual\ saving} = \frac{£4000}{£621.57} = 6\ years\ and\ 5\ months \tag{11}$$

# 6 Evaluation

### Analysis limitations

There are inherent limitations to the results due to the nature of the model. The f-chart method was devised experimentally for standard systems under certain conditions. The resulting correlations, therefore strictly apply to these exact setups only. Any real-life application will result in statistical deviation. This analysis was also subject to the following sources of error:

- Variation of weather from sample averages of input data.
- Variation of the time-load profile from the distribution assumed in 1977. This can vary unpredictably, according to occupant habit.
- The assumption that liquid systems are fully mixed, do not leak and are perfectly insulated. The former tends to under-predict performance due to an increase in collector inlet temperature and therefore, lower collector efficiency. The other two, assume the opposite.
- The above, but also applied to scaling of the store size - resulting in a higher temperature needed to supply heat from the exchanger to a larger thermal mass (accounted for by the correction factor Xc/X).
- Variation of the water mains temperature; the lower this is, the more efficient the collector (accounted for by Xw/X). However, an unintended consequence may be that more of this heat is lost to the environment in less insulated components during heat delivery.

# 7 Conclusion

The results show that this system can provide a sizeable fraction of up to nearly 85% of the monthly heating load. This is in contrast to values of down to 7% in winter. The cost assessment shows that there is considerable economic benefit to the scheme, with an effective annual cost saving of £620. This results in a payback period of 6 years and 5 months.

The evaluation highlighted that such figures are an estimate, as there is wide potential for error due to statistical variation of both input data and model processes themselves. However, for domestic feasibility studies, the f-chart method remains to be a quick and accurate (enough) tool.

The broader significance of this report is that it demonstrates the feasibility of renewable domestic heat provision, even in the U.K. Solar collectors could further decarbonise this industry - without additional strain to the national grid, as would occur with the alternative use of heat pumps.

# References

[1] S.Shire. 2018. *ES4E0, Renewable Energy - Technical Resources* School of Engineering, University of Warwick.

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[7] I. Hughes, T. Hase. 2010. *Measurements and their Uncertainties: A practical guide to modern error analysis.* Oxford

# Appendices

### Guide to Appendices

1. Handwritten procedure for January (Pages 8 - 9):
   (A) Calculation of $F_r'$ , X parameter and correction factors
   (B) Calculation of declination, irradiation, incidence ratio, Y parameter and $f_m$
2. Matlab script devised for computation (Pages 10 - 11)

## De Winter Heat Exchange Factor

$$F_R' = \frac{F_R}{\left[1 + \frac{F_R U_L A_C}{(\dot{m}\,C_p)_C}\left(\frac{(\dot{m}\,C_p)_C}{\varepsilon\,(\dot{m}\,C_p)_{min}} - 1\right)\right]}$$

$$F_R' = \frac{0.9}{\left[1 + \frac{(0.9 \times 3.5 \times 6)}{(4000 \times 0.07)}\left(\frac{4000 \times 0.07}{0.8(4200 \times 0.06)} - 1\right)\right]} = 0.877$$

$\varepsilon$ = Heat Exchanger Effectiveness = 0.8

where $(\dot{m}\,C_p)$ = higher (C) or lower (min) heat capacity rate of the Collector fluid / Water store circuits on either side of the system heat exchanger.

For January:

→ Monthly Heating load $Q_{LM} = m \times N \times C\,(\Delta T)$

$$Q_{LM} = 200 \times 31 \times 4200 \times (60 - 6)$$

$$Q_{LM} = 1.4062 \text{ GJ or } 390.6 \text{ KWh}$$

$$X = \frac{A_C\, F_R'\, U_L\, (100 - \bar{T}_A)\, \Delta t}{Q_{LM}}$$

$$X = \frac{6 \times 0.877 \times 3.5\,(100 - 3.1) \times (3600 \times 24 \times 31)}{1.4062 \times 10^9}$$

$X = 3.3992$ = (Initial Collector losses) / (Monthly Load)

Corrected for non-standard store size:

$$X_C = X\left(\frac{300}{75}\right)^{-0.25} = X\left(\frac{\text{Store size}}{\text{Standard}}\right)^{-0.25} = X_C = 3.7618$$

Corrected for Hot water System only:

$$X_W = X_C\left(\frac{11.6 + 1.18T_W + 3.86T_{mains} - 2.32\bar{T}_a}{100 - \bar{T}_a}\right) = X_W = 3.8188$$

Declination Angle $\delta = 23.45 \sin\left(\frac{360}{365} \times (284 + 17)\right)$
(Angular position of the sun WRT/ the Equator)

$\delta = -20.917^\circ$

Sunset Angles
(Angular distance of sun's exposure to the inclined collector)

$\omega_s = \cos^{-1}(-\tan\delta \tan\phi)$ where $\phi$ = latitude (52.3°)

$\omega_s$ (January) = 60.3624°

$\omega_{ss}$ = minimum of $\omega_s$ or $(\cos^{-1}(-\tan(\phi-\beta)\tan\delta))$

$\omega_{ss}$ (January) = 60.3624°

$\rho$ = Ground Reflectivity = 0.2
$\beta$ = Collector Inclination = 45°

Monthly Average Daily Diffuse Insolation at Site

If $\omega_s \leq 81.4^\circ$ = true for January

then $\bar{H}_d = 1.391 - 3.560K + 4.189K^2 - 2.137K^3 = 0.7294$

where K = local average clearness index for site = 0.25
(indicative of global: diffuse atmospheric path of radiation)

Ratio of Inclined to Horizontal radiation, incident on collector

$$\bar{R}_{bT} = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_{ss} + \frac{\pi}{180}\omega_{ss}\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \frac{\pi}{180}\omega_s\sin\phi\sin\delta} = 3.8086$$

(Daily)

Monthly Average Insolation, incident on Collector

$$\bar{H}_t = \bar{H}\left[\left(1-\frac{\bar{H}_d}{\bar{H}}\right)\times\bar{R}_{bT} + \left(\frac{\bar{H}_d}{\bar{H}}\right)\left(\frac{1+\cos\beta}{2}\right) + \rho\left(\frac{1-\cos\beta}{2}\right)\right] = 3.9076\,GJ$$

or 1.0854 KWh

$$Y = \frac{A_c F_R' \eta_o \bar{H}_t N}{Q_{LM}} = 0.3660 = \frac{\text{Absorbed Energy}}{\text{Heating Load}}$$

$$\boxed{f_m = 1.029Y - 0.065X_w - 0.245Y^2 + 0.0018X_w^2 + 0.0215Y^3}$$

$f_m$ = fraction of heating load met by collector = 0.1229

```
% Input weather data
data = [0.25 20 3.1 6 17; 0.3 33 3.1 5.7 47; 0.32 62 5.2 6 75; 0.41 103 7.6 7 105;
0.41 134 10.6 9 135; 0.39 136 14 12 162; 0.4 137 15.8 14.5 198; 0.42 120 15.4 15 228;
0.39 80 13.2 14.2 258; 0.33 46 10 12 288; 0.32 15 6 9.2 318; 0.27 15 4.2 7 344];
% Days in each month
days = [31 28 31 30 31 30 31 31 30 31 30 31];
% Further parameters
rho = 0.2; phi = 52.3; beta = 45; eta = 0.8075;

% DeWinter Exchange Factor
A_c = 6;             % Collector Area
F_r = 0.9;           % Heat Removal Factor
U_L = 3.5;           % Collector Heat Loss Coeff
m_c = 280;           % Collector Side [FlowRate*SpecificHeat]
epsilon = 0.8;       % Heat Exchanger Effectiveness
m_min = 252;         % Store Side [FlowRate*SpecificHeat]
Fr_prime = (F_r)/( 1 + ((F_r*U_L*A_c)/(m_c))*(((m_c)/((epsilon*m_min)))-1));

for n = 1:12
    % Days in each month
    N=days(n);
    % Number of seconds in a given month
    delta_t = 3600*24*N;
    % Data Column = Monthly Clearness Indexes
    K = data(n,1);
    % Data Column = Monthly Insolation
    H = data(n,2)*3.6*(10)^6/N;
    % Data Column = Ambient Temperature
    Ta = data(n,3);
    % Data Column = Mains Temperature
    Tm = data(n,4);
    % Data Column = Average Day Numbers
    Avd = data(n,5);

    % Monthly Heating Load Calculation
    c = 4200;        % Specific Heat of Water raised
    m = 200*N;       % Monthly Mass of Water Heated
    QLM = m*c*(60-Tm);

    % X calculation
    X1 = (A_c*Fr_prime*U_L*(100-Ta)*delta_t)/QLM;            % Original X
    XW = X1*(11.6 + 1.18*60 + 3.86*Tm -2.32*Ta)/(100-Ta);    % Correction 1
    X = XW*(((300)/(75*A_c))^(-0.25));                       % Correction 2

    % Declination Angle
    delta = 23.45*sind(360*((284+Avd)/365));
    % Sunset Angles
    omega_s = acosd(-tand(delta)*tand(phi));
    omega_ss = min (omega_s, acosd(-tand(phi-beta)*tand(delta)));
    % Diffuse Radiation
    if omega_s <= 81.4
        Hd = (1.391 - 3.560*K + 4.189*(K^2) - 2.137*(K^3));
    else
        Hd = (1.311 - 3.022*K + 3.427*(K^2) - 1.821*(K^3));
```

```
    end
    % Ratio of Inclined to Horizontal Radiation
    Rbt= (((cosd(phi-beta)*cosd(delta)*sind(omega_ss))+((pi/180)*omega_ss*sind(phi-
beta)*sind(delta)))/((cosd(phi)*cosd(delta)*sind(omega_s))+((pi/180)*omega_s*sind(phi)
*sind(delta))));
    % Monthly Average Incident Radiation
    Ht = H*(((1 - Hd)*Rbt) + (Hd*((1+cosd(beta))/2)) + (rho*((1-cosd(beta))/2)));
    % Y calculation
    Y = (A_c*Fr_prime*eta*Ht*N)/QLM;

    % Fm calculation
    Fm = 1.029*Y - 0.065*X - 0.245*(Y)^2 + 0.0018*(X)^2 + 0.0215*(Y)^3;

    Fm_out(n)=Fm;
    QLM_out(n)=QLM/(3.6*10^6);
    Q_solar(n)=Fm*(QLM/(3.6*10^6));
    Results = [QLM_out;Fm_out;];

    Fm_mean = mean(Fm_out);
    Fm_min = min(Fm_out);

end

    % Cost Assessment

    % Yearly Heating Cost (Gas only)
    Cg = 0.06*(sum(QLM_out)/0.75);
    % Gas energy needed to supplement Collector
    Q_extragas = [343 245 205 98 66 55 50 58 105 199 330 352]/0.75;
    % Yearly cost of Gas needed (w/solar)
    Cgs = (sum(Q_extragas))*0.06;
    % Yearly bestowed RHI
    Cr = (sum(Q_solar))*0.2066;
    % Yearly net heating cost (w/solar)
    Cs = Cgs - Cr;
    % Yearly Saving (solar vs Gas only)
    S = Cg - Cs;
    % Payback time of system
    P = 4000/S;
```