1. (a)

$Y = B_0 + B_1 z + \sigma w$ and $Y = \log T$

$\Rightarrow T = e^{B_0 + B_1 z + \sigma W} = \underbrace{e^{B_0 + \sigma W}}_{T_0} e^{B_1 z} = T_0 \cdot e^{B_1 z}$

$S_T(t|z) = S_T(t_0 e^{B_1 z} | z) = S_0(t_0) = S_0(\underbrace{t_0 e^{B_1 z}}_{t} e^{-B_1 z})$

$= S_0(t e^{-B_1 z})$

covariate z control $e^{B_1 z}$ for survival function $S_T(t|z) = S(t_0 e^{B_1 z} | z)$
$= S_0(t e^{-B_1 z})$

(b) (i) $S_T(t|z) = \frac{1}{1 + \lambda t^\alpha e^{\beta z}}$

$S_0(t) = \frac{1}{1 + \lambda t^\alpha}$

$\Rightarrow S_0(t e^{-B_1 z}) = \frac{1}{1 + \lambda (t e^{-B_1 z})^\alpha} = \frac{1}{1 + \lambda t^\alpha e^{-\alpha B_1 z}} = \frac{1}{1 + \lambda t^\alpha e^{\beta z}} = S_T(t|z)$

when $-\alpha B_1 = \beta$

(ii) $Y = \log T$ is a logistic distribution $Y \sim$ logistic(u, σ)

T is a log logistic distribution

let W be the logistic distribution $W \sim$ logistic$(0,1)$

Since if Y is logistic distribution $\Rightarrow$ Y is location scale distribution

Then T is log loc scale distribution W is standard loc scale distribution

Then $Y = u(z) + \sigma W = \beta^T z + \sigma W$

So we can write $Y = B_0 + B_1 z + \sigma W$ $u(z) = B_0 + B_1 z$
$u(0) = B_0$

$$F_0(t) = F_{Y|z=0}(\log t) = \frac{e^{\frac{\log t - u(0)}{\sigma}}}{1+e^{\frac{\log t - u(0)}{\sigma}}}$$

$$S_0(t) = 1 - F_0(t) = \frac{1}{1+e^{\frac{\log t - u(0)}{\sigma}}} = \frac{1}{1+e^{\log t^{\frac{1}{\sigma}}} \cdot e^{-\frac{u(0)}{\sigma}}} = \frac{1}{1+t^{\frac{1}{\sigma}} \cdot e^{-\frac{u(0)}{\sigma}}}$$

$$S_0(t) = \frac{1}{1+\lambda t^{\alpha} e^{\beta \cdot 0}} = \frac{1}{1+\lambda t^{\alpha}} = \frac{1}{1+t^{\frac{1}{\sigma}} \cdot e^{-\frac{u(0)}{\sigma}}}$$

$\Rightarrow \lambda = e^{-\frac{u(0)}{\sigma}} = e^{-\frac{\beta_0}{\sigma}}$ $\quad \alpha = \frac{1}{\sigma}$ and from (i) $\beta = -\alpha \beta_1 = -\frac{\beta_1}{\sigma}$

(c) show T is a PH model $\Rightarrow$ show $h(t|z) = h_0(t)\psi(z)$ $\quad \psi(z) = e^{\beta z}$

$$h(t|z) = -\frac{d}{dt} \log S_T(t|z) = -\frac{d}{dt} - \log(1+\lambda t^{\alpha} e^{\beta z})$$

$$= \frac{\lambda e^{\beta z} \alpha t^{\alpha-1}}{1+\lambda t^{\alpha} e^{\beta z}}$$

$$h_0(t) = -\frac{d}{dt} \log S_0(t) = -\frac{d}{dt} - \log(1+\lambda t^{\alpha})$$

$$= \frac{\lambda \alpha t^{\alpha-1}}{1+\lambda t^{\alpha}}$$

$$h(t|z) \neq h_0(t)\, e^{\beta z}$$

So T is not proportion hazard function

2.