

Q) $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$ solve for $n = 2^k$

$$x(1) = 1$$

$n=2$:- $x(2) = x(2/2) + 2$

$$n = 2^1 = 2 \quad = x(1) + 2$$

$$\boxed{x(2) = 3}$$

$n=4$:-

$$n = 2^2 = 4 \quad x(4) = x(4/2) + 4$$

$$= x(2) + 4$$

$$= 3 + 4$$

$$\boxed{x(4) = 7}$$

$n=8$:-

$$n = 2^3 = 8 \quad x(8) = x(8/2) + 8$$

$$= x(4) + 8$$

$$= 7 + 8$$

$$\boxed{x(8) = 15}$$

For this recurrence relation, $2^n - 1 = 2^{2^k} - 1$

for $\boxed{n = 2^k}$, $\boxed{x(2^k) = 2^{2^k} - 1}$

Q) $x(n) = x(n/3) + 1$ for $n > 1$ $x(1) = 1$ $n = 3^k$

$$x(1) = 1$$

$n=3$:-

$$n = 3^1 = 3 \quad x(3) = x(3/3) + 1$$

$$= x(1) + 1$$

$$= 1 + 1$$

$$\boxed{x(3) = 2}$$

$n=9$:-

$$n = 3^2 = 9 \quad x(9) = x(9/3) + 1$$

$$= x(3) + 1$$

$$= 2 + 1$$

$$\boxed{x(9) = 3}$$

$n=27$:-

$$n = 3^3 = 27 \quad x(27) = x(27/3) + 1$$

$$= x(9) + 1$$

$$= 3 + 1$$

$$\boxed{x(27) = 4}$$

for this recurrence relation,

$$\boxed{x(n) = \log_3 n + 1}$$

$$\boxed{x(3^k) = \log_3 3^k + 1 = k + 1}$$

2. Evaluate the following recurrences completely

i) $T(n) = T(n/2) + 1$, where $n = 2^k$ for all $k \geq 0$.

$$T(n) = T(n/2) + 1$$

Since $n = 2^k$, we can rewrite $n/2$ as 2^{k-1} .

$$T(2^k) = T(2^{k-1}) + 1$$

$$T(2^k) = T(2^{k-2}) + 1 + 1$$

$$T(2^k) = T(2^{k-3}) + 1 + 1 + 1 \dots$$

$$T(2^k) = T(1) + k$$

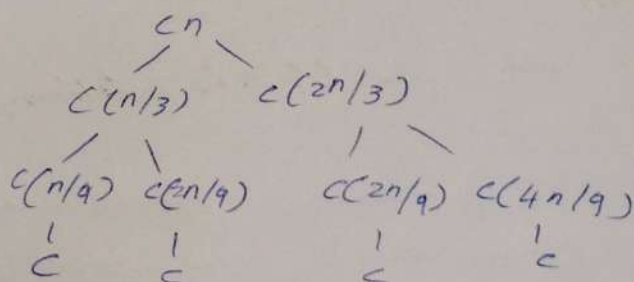
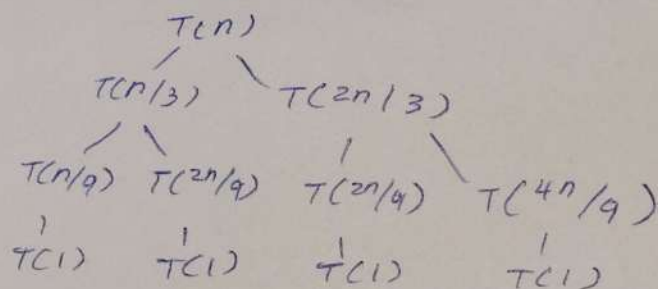
Since $T(1)$ is constant, let's call it 'a'

$$T(2^k) = a + k$$

$$\therefore T(n) = a + \log_2(n)$$

ii) $T(n) = T(n/3) + T(2n/3) + cn$

$$T(n) = T(n/3) + T(2n/3) + cn$$



$$\text{length} = \log_3 n \text{ (divided by 3)}$$

$$T(n) = cn \log_3 n$$

$$\Rightarrow \omega(n \log n)$$

3. Consider following algorithm

```

min(A[0...n-1])
if n=1 return A[0] > 1
Else temp = min(A[0...n-2])
    if temp ≤ A[n-1] return temp
    else
        return A[n-1] - n-1
    
```

- what does this algorithm compute?
- set up a recurrence relation for algorithm's basic operations count and solve it.

a) This algorithm computes minimum element in an array A of size n.

If $i < n$, $A[i]$ is smaller than all elements, then, $A[j]$, $j = i+1$ to $n-1$, then it returns $A[i]$. It also returns the left most minimal element.

b) main by comparison occurs during recursion.

So, $T(n) = T(n-1) + 1$, when $n > 1$ (one comparison at every

$T(1) = 0$ (no compare when $n=1$) step except, $n=1$)

$$T(n) = T(1) + (n-1) \times 1$$

$$= 0 + (n-1)$$

$$= n-1$$

\therefore Time complexity = $O(n)$.

4. Analyze Order of growth

i) $F(n) = 2n^2 + 5$ and $g(n) = 7n$ use $\Omega(g(n))$ notation.

$$n=1$$

$$F(1) = 2(1)^2 + 5 = 7$$

$$g(1) = 7$$

$$n=2$$

$$F(2) = 2(2)^2 + 5 = 13$$

$$g(2) = 7 \times 2 = 14$$

$$n=3$$

$$F(3) = 2(3)^2 + 5 = 23$$

$$g(3) = 21$$

$$n=1, 7=7$$

$$n=2, 13 < 14$$

$$n=3, 23 > 21$$

$$n \geq 3$$

$$F(n) \geq g(n) \cdot c$$

$F(n)$ is always greater than or equal to $c \cdot g(n)$ when n value is greater or equal to 3

$$\therefore F(n) = \Omega(g(n))$$

$F(n)$ grows more than $g(n)$ from below asymptotically.

Design and Analysis of Algorithm for polynomial problems

MANC
19232
CSA

1. solve the following recurrence relations:-

Q) $x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

$$x(1) = 0$$

substitute:

$$\begin{aligned} n=2:- x(2) &= x(2-1) + 5 \\ &= x(1) + 5 \\ &= 0 + 5 \end{aligned}$$

$$\boxed{x(2) = 5}$$

$$\begin{aligned} n=3:- x(3) &= x(3-1) + 5 \\ &= x(2) + 5 \\ &= 5 + 5 \end{aligned}$$

$$\boxed{x(3) = 10}$$

$$\begin{aligned} n=4:- x(4) &= x(4-1) + 5 \\ &= x(3) + 5 \\ &= 10 + 5 \end{aligned}$$

$$\boxed{x(4) = 15}$$

For this recurrence relation, each term is 5 more than previous

$$\boxed{\text{So, } x(n) = 5n - 5 \text{ for } n > 1}$$

Q) $x(n) = 3x(n-1)$ for $n > 1$ $x(1) = 4$

$$x(1) = 4$$

substitute:

$$\begin{aligned} n=2:- x(2) &= 3x(2-1) \\ &= 3x(1) \\ &= 3(4) \end{aligned}$$

$$\boxed{x(2) = 12}$$

$$\begin{aligned} n=3:- x(3) &= 3x(3-1) \\ &= 3x(2) \\ &= 3(12) \end{aligned}$$

$$\boxed{x(3) = 36}$$

$$\begin{aligned} n=4:- x(4) &= 3x(4-1) \\ &= 3x(3) \\ &= 3(36) \end{aligned}$$

$$\boxed{x(4) = 108}$$

For this recurrence relation, each term is 3 times previous

$$\boxed{\text{So, } x(n) = 4 \times 3^{n-1} \text{ for } n > 1}$$