Assignment 1

DEEP - EE18BTECH11011

Download all python codes from

https://github.com/Deep−2903/EE3025/Assignment1/codes

and latex-tikz codes from

https://github.com/Deep−2903/EE3025/Assignment1

1 Problem

Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{1.0.1}$$

$$y(n) + \frac{1}{2} y(n-1) = x(n) + x(n-2) \tag{1.0.2}$$

Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.0.3}$$

and $H(k)$ using $h(n)$.

2 Solution

We know that Impulse response of the LTI system is the output of the system when the given input to the system is an Impulse signal. Therefore, using equation (1.0.2) we can say that the Impulse response of the system is,

$$h(n) + \frac{1}{2} h(n-1) = \delta(n) + \delta(n-2) \tag{2.0.1}$$

Now we know that DFT of a Input Signal $x(n)$ is give as :

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.0.2}$$

and similarly the DFT of Impulse Response $h(n)$ is,

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.0.3}$$

Now to compute the DFT of $x(n)$ and $h(n)$ we use the following python code.

https://github.com/Deep−2903/EE3025/Assignment1/codes

Using the above code we get the following plots.

https://github.com/Deep−2903/EE3025/Assignment1/figs

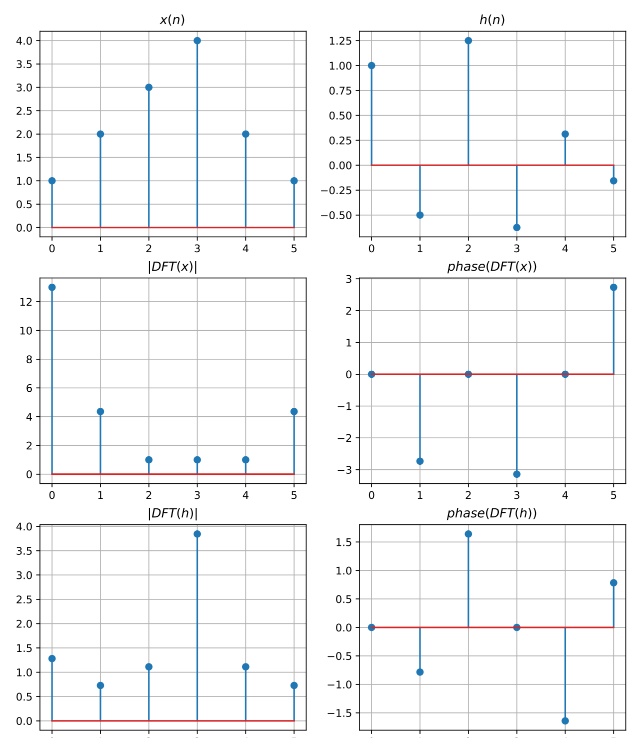

Fig. 0: x(n), h(n) and their DFT

Theoritically finding $X(k)$ and $H(k)$. From given x(n) we know that N = 6. Now on solving equation (2.0.2) for each of the values of k we get (Note: decimal values are rounded off to first 3 values),

$$X(0) = x(0) + x(1) + x(2) + x(3) + x(4) + x(5) \tag{2.0.4}$$

$$\implies X(0) = 13 + 0j \tag{2.0.5}$$

$$X(1) = x(0) + x(1)e^{-j\pi/3} + ... + x(5)e^{-j5\pi/3} \tag{2.0.6}$$

$$\implies X(1) = -4 - 1.732j \tag{2.0.7}$$

$$X(2) = x(0) + x(1)e^{-2j\pi/3} + ... + x(5)e^{-2j5\pi/3} \tag{2.0.8}$$

$$\implies X(2) = 1 + 0j \tag{2.0.9}$$

Similarly we get,

$$X(3) = -1 + 0j \tag{2.0.10}$$

$$X(4) = 1 + 0j \tag{2.0.11}$$

$$X(5) = -4 + 1.732j \tag{2.0.12}$$

Now to find $H(k)$ we need to know h(n) first. So we will first calculate h(n). For that we need to first find the Y(z) by applying Z-transform on equation (1.0.2) i.e.,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{2.0.13}$$

$$\implies Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{2.0.14}$$

Now we can find H(z) using Y(z) i.e.,

$$H(z) = \frac{Y(z)}{X(z)} \tag{2.0.15}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{2.0.16}$$

$$H(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{2.0.17}$$

From this we can say that h(n) is,

$$h(n) = Z^{-1}\left[\frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}}\right] \tag{2.0.18}$$

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{2.0.19}$$

Now for the calculations we can assume that length of h(n) is same as length of x(n) i.e., N = 6.Now on solving equation (2.0.3) for each value of k we get,

$$H(0) = h(0) + h(1) + h(2) + h(3) + h(4) + h(5) \tag{2.0.20}$$

$$\implies H(0) = 1.28125 + 0j \tag{2.0.21}$$

$$H(1) = h(0) + h(1)e^{-j\pi/3} + ... + h(5)e^{-j5\pi/3} \tag{2.0.22}$$

$$\implies H(1) = 0.51625 - 0.5141875j \tag{2.0.23}$$

Similarly we get,

$$H(2) = -0.078125 + 1.1095625j \tag{2.0.24}$$

$$H(3) = 3.84375 + 0j \tag{2.0.25}$$

$$H(4) = -0.071825 - 1.1095625j \tag{2.0.26}$$

$$H(5) = 0.515625 + 0.5141875j \tag{2.0.27}$$

The values we got from the code are approximately the same as these which we got theoritically.