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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

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6.1. Consider a state-variable model of a system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\alpha & -2\beta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} r \tag{6.1.1}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{6.1.2}$$

where y is the output, and r is the input. Find the Damping ratio ζ and the Undamped natural frequency ω_n of the system.
Solution: The state space model is given by

$$\dot{X} = AX + BU \tag{6.1.3}$$

$$Y = CX + DU \tag{6.1.4}$$

The transfer function for the state space model is:

$$H(s) = C(sI - A)^{-1}B + D \tag{6.1.5}$$

$$\implies H(s) = \frac{\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s+2\beta & 1 \\ -\alpha & s \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}}{s(s+2\beta)+\alpha} \tag{6.1.6}$$

$$= \frac{b_1(s+2\beta)+b_2}{s^2+2s\beta+\alpha} \tag{6.1.7}$$

$$\implies H(s) = \frac{b_1 s}{s^2+2s\beta+\alpha} + \frac{2b_1\beta+b_2}{s^2+2s\beta+\alpha} \tag{6.1.8}$$

Generally for a second order system the transfer function is given by

$$H(s) = \frac{\omega_n^2}{s^2+2s\zeta\omega_n+\omega_n^2} \tag{6.1.9}$$

Now from the transfer function we got we can see that our system is bandpass and lowpass combination but the comaprison of denominator of our transfer function to the general transfer function is still valid.

$$\therefore 2\zeta\omega_n = 2\beta, \tag{6.1.10}$$

$$\omega_n^2 = \alpha \tag{6.1.11}$$

$$\implies \zeta = \frac{\beta}{\sqrt{\alpha}}, \omega_n = \sqrt{\alpha} \tag{6.1.12}$$