

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 NYQUIST PLOT

1.1 Introduction

1.1.1. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{\pi e^{-0.25s}}{s} \quad (1.1.1.1)$$

1.1.2. Find $\text{Re}\{G(j\omega)\}$ and $\text{Im}\{G(j\omega)\}$.

Solution: From (1.1.1.1),

$$G(j\omega) = \frac{\pi}{\omega}(-\sin 0.25\omega - j \cos 0.25\omega) \quad (1.1.2.1)$$

$$\Rightarrow \text{Re}\{G(j\omega)\} = \frac{\pi}{\omega}(-\sin 0.25\omega) \quad (1.1.2.2)$$

$$\text{Im}\{G(j\omega)\} = \frac{\pi}{\omega}(-j \cos 0.25\omega) \quad (1.1.2.3)$$

1.1.3. Sketch the Nyquist plot.

Solution: The Nyquist plot is a graph of $\text{Re}\{G(j\omega)\}$ vs $\text{Im}\{G(j\omega)\}$. The following python code generates the Nyquist plot in Fig. 1.1.3

```
codes/ee18btech11007/ee18btech11007.py
```

1.1.4. Find the point at which the Nyquist plot of $G(s)$ passes through the negative real axis

1.1.5. Use the Nyquist Stability criterion to determine if the system in (1.1.4.3) is stable.

Solution: Consider Table 1.1.5. According to the Nyquist stability criterion,

a) If the open-loop transfer function $G(s)$ has a

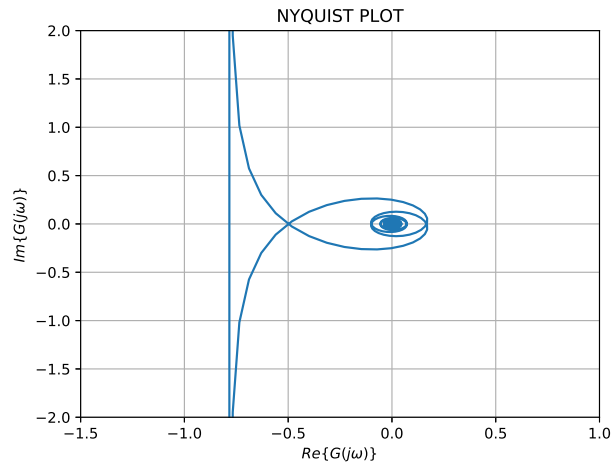


Fig. 1.1.3

Solution: Nyquist plot cuts the negative real axis at ω for which

$$\angle G(j\omega) = -\pi \quad (1.1.4.1)$$

From (1.1.1.1),

$$G(j\omega) = \frac{\pi e^{-j\frac{\omega}{4}}}{j\omega} = \frac{\pi e^{-j(\frac{\omega}{4} + \frac{\pi}{2})}}{\omega} \quad (1.1.4.2)$$

$$\Rightarrow \angle G(j\omega) = -\left(\frac{\omega}{4} + \frac{\pi}{2}\right) \quad (1.1.4.3)$$

From (1.1.4.3) and (1.1.4.1),

$$\frac{\omega}{4} + \frac{\pi}{2} = \pi \quad (1.1.4.4)$$

$$\Rightarrow \omega = 2\pi \quad (1.1.4.5)$$

Also, from (1.1.1.1),

$$|G(j\omega)| = \frac{\pi}{|\omega|} \quad (1.1.4.6)$$

$$\Rightarrow |G(j2\pi)| = \frac{1}{2} \quad (1.1.4.7)$$

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Variable	Value	Description
Z	0	Poles of $\frac{G(s)}{1+G(s)H(s)}$ in right half of s plane
P	0	Poles of $G(s)H(s)$ in right half of s plane
N	0	No of clockwise encirclements of $G(s)H(s)$ about $-1+j0$ in the Nyquist plot

TABLE 1.1.5

zero pole of multiplicity l , then the Nyquist plot has a discontinuity at $\omega = 0$. During further analysis it should be assumed that the phasor travels l times clock-wise along a semicircle of infinite radius. After applying this rule, the zero poles should be neglected, i.e. if there are no other unstable poles, then the open-loop transfer function $G(s)$ should be considered stable.

- If the open-loop transfer function $G(s)$ is stable, then the closed-loop system is unstable for any encirclement of the point -1 . If the open-loop transfer function $G(s)$ is unstable, then there must be one counter clock-wise encirclement of -1 for each pole of $G(s)$ in the right-half of the complex plane.
- The number of surplus encirclements ($N + P$ greater than 0) is exactly the number of unstable poles of the closed-loop system.
- However, if the graph happens to pass through the point $-1+j0$, then deciding upon even the marginal stability of the system becomes difficult and the only conclusion that can be drawn from the graph is that there exist zeros on the $j\omega$ axis.

From (1.1.1.1), $G(s)$ is stable since it has a single pole at $s = 0$. Further, from Fig. 1.1.3, the Nyquist plot does not encircle $s = -1$. From Theorem 1.1.5b, we may conclude that the system is stable.

1.2 Example

- Using Nyquist criterion, find out whether the system below is stable or not.

$$G(s) = \frac{20}{s(s+1)}, H(s) = \frac{s+3}{s+4} \quad (1.2.1.1)$$

Solution: The following python code generates the Nyquist plot in Fig.1.2.1.

```
codes/ee18btech11011_1.ipynb
```

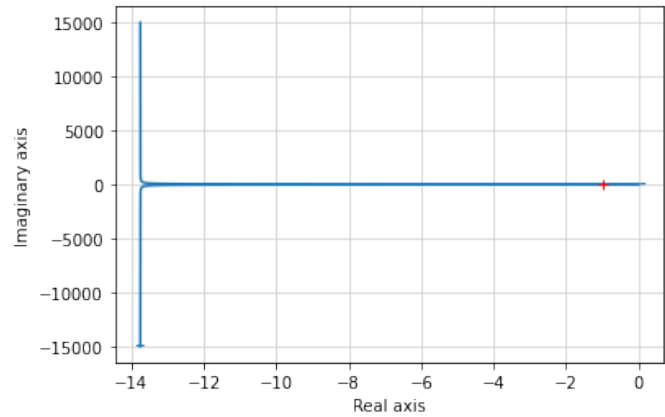


Fig. 1.2.1: Nyquist Plot

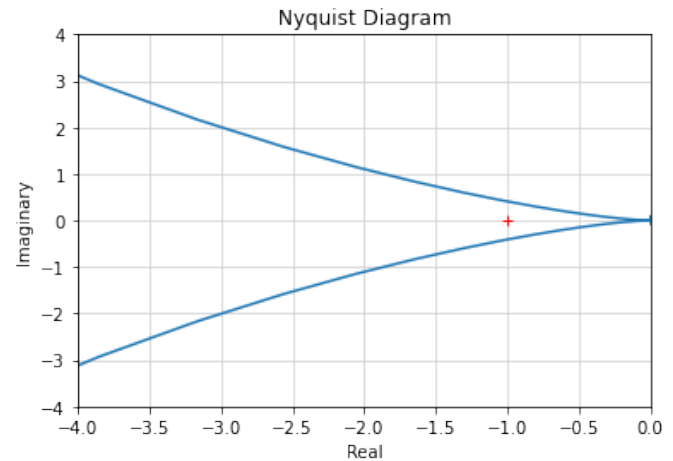


Fig. 1.2.1: Zoomed image

From Table 1.1.5 we already know the Nyquist Stability criterion so for this closed loop system the transfer function will be =

$$\frac{G(s)}{1 + G(s)H(s)} \quad (1.2.1.2)$$

$$\Rightarrow G(s)H(s) = \frac{20(s+3)}{s(s+1)(s+4)} \quad (1.2.1.3)$$

So it has 3 open-loop poles 0, -1 and -4, therefore $P=0$. Further we know that $N = Z - P$, now we know $Z = \text{Poles of } \frac{G(s)}{1+G(s)H(s)}$ in right half of s plane. To find the poles we can use the following Routh Hurwitz python code. Using this we get $Z = 0$.

codes/Routh.py

$$P = 0, Z = 0 \quad (1.2.1.4)$$

$$\implies N = 0 \quad (1.2.1.5)$$

This can also be seen from the Fig. 1.2.1 that the encirclement is counter-clockwise not clockwise. Hence the system is stable.