Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Nyquist Plot

1.1 Introduction

1.1.1. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{\pi e^{-0.25s}}{s} \tag{1.1.1.1}$$

1.1.2. Find $\text{Re}\{G(\jmath\omega)\}$ and $\text{Im}\{G(\jmath\omega)\}$.
Solution: From (1.1.1.1),

$$G(j\omega) = \frac{\pi}{\omega}(-\sin 0.25\omega - j\cos 0.25\omega) \tag{1.1.2.1}$$

$$\implies \text{Re}\{G(\jmath\omega)\} = \frac{\pi}{\omega}(-\sin 0.25\omega) \tag{1.1.2.2}$$

$$\text{Im}\{G(\jmath\omega)\} = \frac{\pi}{\omega}(-j\cos 0.25\omega) \tag{1.1.2.3}$$

1.1.3. Sketch the Nyquist plot.
Solution: The Nyquist plot is a graph of $\text{Re}\{G(\jmath\omega)\}$ vs $\text{Im}\{G(\jmath\omega)\}$. The following python code generates the Nyquist plot in Fig. 1.1.3

```
codes/ee18btech11007/ee18btech11007.py
```

1.1.4. Find the point at which the Nyquist plot of G(s) passes through the negative real axis

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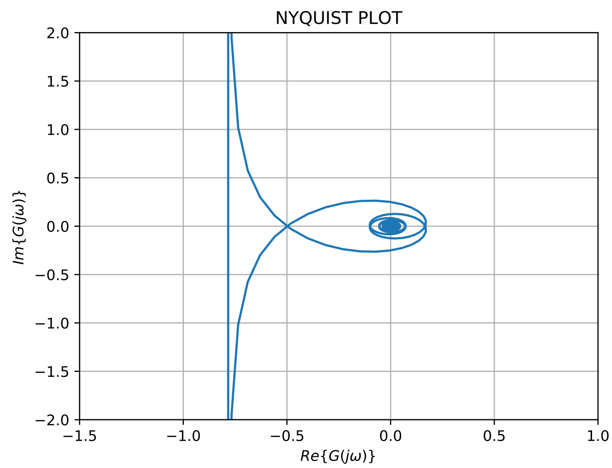

Fig. 1.1.3

Solution: Nyquist plot cuts the negative real axis at ω for which

$$\angle G(\jmath\omega) = -\pi \tag{1.1.4.1}$$

From (1.1.1.1),

$$G(\jmath\omega) = \frac{\pi e^{-\frac{\jmath\omega}{4}}}{\jmath\omega} = \frac{\pi e^{-\jmath\left(\frac{\omega}{4}+\frac{\pi}{2}\right)}}{\omega} \tag{1.1.4.2}$$

$$\implies \angle G(\jmath\omega) = -\left(\frac{\omega}{4} + \frac{\pi}{2}\right) \tag{1.1.4.3}$$

From (1.1.4.3) and (1.1.4.1),

$$\frac{\omega}{4} + \frac{\pi}{2} = \pi \tag{1.1.4.4}$$

$$\implies \omega = 2\pi \tag{1.1.4.5}$$

Also, from (1.1.1.1),

$$\left|G(\jmath\omega)\right| = \frac{\pi}{|\omega|} \tag{1.1.4.6}$$

$$\implies \left|G(\jmath 2\pi)\right| = \frac{1}{2} \tag{1.1.4.7}$$

1.1.5. Use the Nyquist Stability criterion to determine if the system in (1.1.4.3) is stable.
Solution: Consider Table 1.1.5. According to the Nyquist stability criterion,

a) If the open-loop transfer function $G(s)$ has a

Variable	Value	Description
Z	0	Poles of $\frac{G(s)}{1+G(s)H(s)}$in right half of s plane
P	0	Poles of $G(s)H(s)$ in right half of s plane
N	0	No of clockwise encirclements of $G(s)H(s)$ about -1+j0 in the Nyquist plot

TABLE 1.1.5

zero pole of multiplicity l, then the Nyquist plot has a discontinuity at ω =0. During further analysis it should be assumed that the phasor travels l times clock-wise along a semicircle of infinite radius. After applying this rule, the zero poles should be neglected, i.e. if there are no other unstable poles, then the open-loop transfer function $G(s)$ should be considered stable.

b) If the open-loop transfer function $G(s)$ is stable, then the closed-loop system is unstable for any encirclement of the point -1. If the open-loop transfer function $G(s)$ is unstable, then there must be one counter clock-wise encirclement of -1 for each pole of $G(s)$ in the right-half of the complex plane.

c) The number of surplus encirclements (N + P greater than 0) is exactly the number of unstable poles of the closed-loop system.

d) However, if the graph happens to pass through the point $-1+\jmath 0$, then deciding upon even the marginal stability of the system becomes difficult and the only conclusion that can be drawn from the graph is that there exist zeros on the $\jmath\omega$ axis.

From (1.1.1.1), $G(s)$ is stable since it has a single pole at $s = 0$. Further, from Fig. 1.1.3, the Nyquist plot does not encircle $s = -1$. From Theorem 1.1.5b, we may conclude that the system is stable.

1.2 *Example*

1.2.1. Using Nyquist criterion, find out whether the system below is stable or not.

$$G(s) = \frac{20}{s(s+1)}, H(s) = \frac{s+3}{s+4} \tag{1.2.1.1}$$

Solution: The following python code generates the Nyquist plot in Fig.1.2.1.

```
codes/ee18btech11011_1.ipynb
```

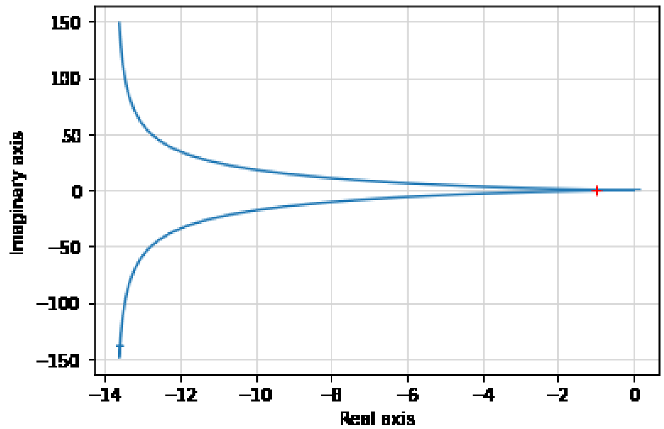

Fig. 1.2.1: Nyquist Plot

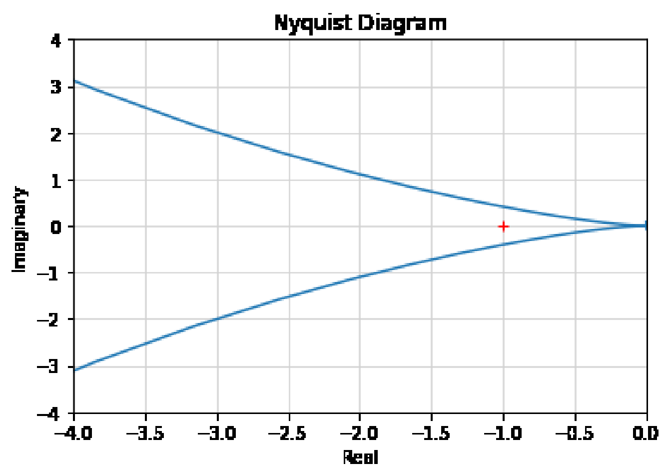

Fig. 1.2.1: Zoomed image

From Table 1.1.5 we already know the Nyquist Stability criterion so for this closed loop system the transfer function will be =

$$\frac{G(s)}{1+G(s)H(s)} \tag{1.2.1.2}$$

$$\implies G(s)H(s) = \frac{20(s+3)}{s(s+1)(s+4)} \tag{1.2.1.3}$$

So it has 3 open-loop poles 0,-1 and -4, therefore P=0.Further we know that N = Z-P , now we know Z = Poles of $\frac{G(s)}{1+G(s)H(s)}$in right half of s plane.To find the poles we can use the following Routh Hurwitz python code.Using this we get Z = 0.

codes/Routh.py

$$P = 0, Z = 0 \tag{1.2.1.4}$$

$$\implies N = 0 \tag{1.2.1.5}$$

This can also be seen from the Fig. 1.2.1 that the encirclement is counter-clockwise not clockwise.Hence the system is stable.