Control Systems

Deep*

Contents

1 Feedback Voltage Amplifier: Shunt-Shunt

The circuit in Fig. 1.1 utilizes a voltage amplifier with gain μ in a shunt-shunt feedback topology with the feedback network composed of resistor R_F.In order to be able to use the feedback equations you should first convert the signal source to it's Norton Representation.

1) If the loop gain is very large, what approximate closed loop voltage gain V_o/V_s is realized? If $R_s = 1\text{k}\Omega$, give the value of R_F that will result in $V_o/V_s \simeq$ -10V/V.
2) If the amplifier μ has a dc gain of 10^3 V/V, an input resistance $R_{id} = 100\text{k}\Omega$, and an output resistance $r_o = 1\text{k}\Omega$, find the actual V_o/V_s realized. Also find R_{in} and R_{out}.
3) If the amplifier μ has an upper 3-dB frequency of 1 kHz and a uniform -20-dB/decade gain rolloff, what is the 3-dB frequency of the gain $| V_o/V_s |$.

1. Fig. 1.1 shows the original circuit. Draw the Norton Representation, A-Circuit and H-Circuit.
   Solution: See Fig. 1.3 for the Norton Representation, Fig. 1.4 and Fig. 1.5 for A-Circuit and Fig. 1.6 for the H-Circuit.
2. Refer Table 2 for the parameters.
3. Write all the feedback equations based on all the Figs. using KCL/KVL.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India.

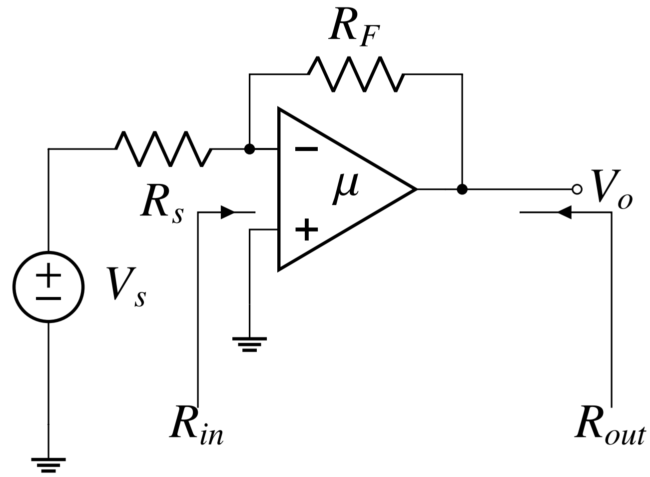

Fig. 1.1

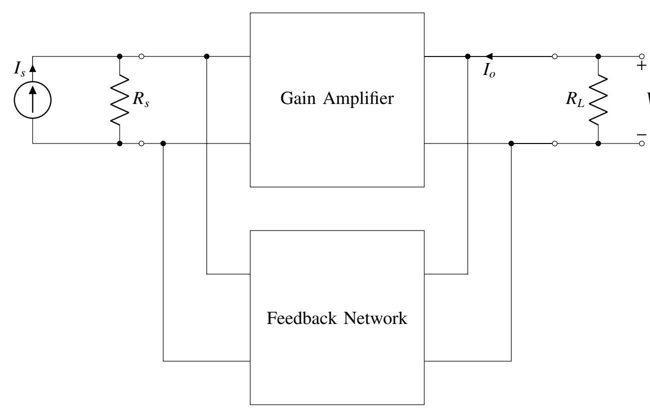

Fig. 1.2: Shunt Shunt Amplifier Block Diagram

Solution: The equations are as follows:

$$T = \frac{V_o}{I_s} = \frac{G}{1 + GH} \tag{3.1}$$

$$H = \frac{I_f}{V_o} = -\frac{1}{R_F} \tag{3.2}$$

$$I_i = I_s - I_f = \frac{I_s}{1 + GH} \tag{3.3}$$

$$R_{if} = \frac{V_i}{I_s} = \frac{V_i}{(1 + GH)I_i} = \frac{R_i}{1 + GH} \tag{3.4}$$

$$R_{of} = \frac{R_o}{1 + GH} \tag{3.5}$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}}, R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}} \tag{3.6}$$

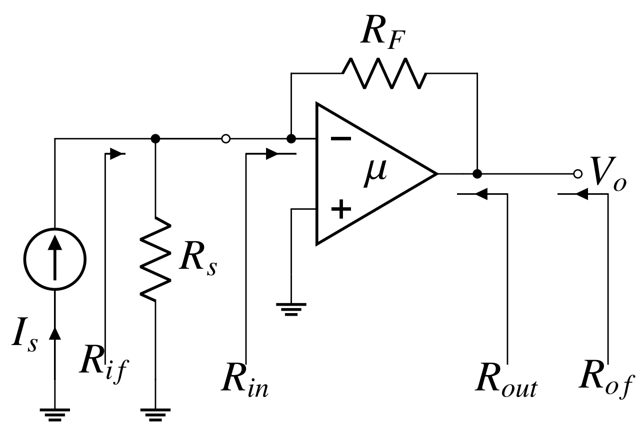

Fig. 1.3

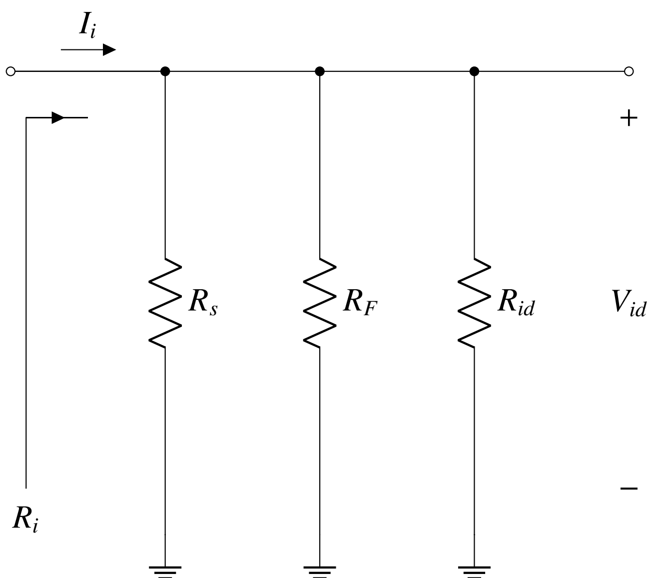

Fig. 1.4

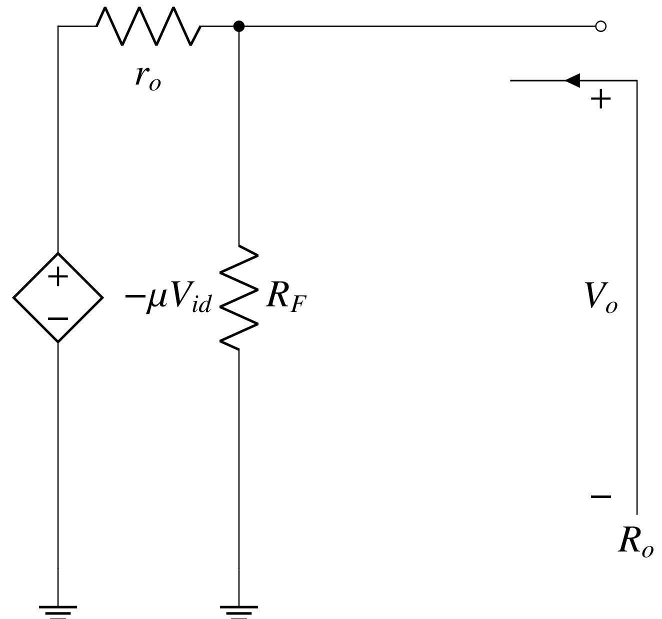

Fig. 1.5

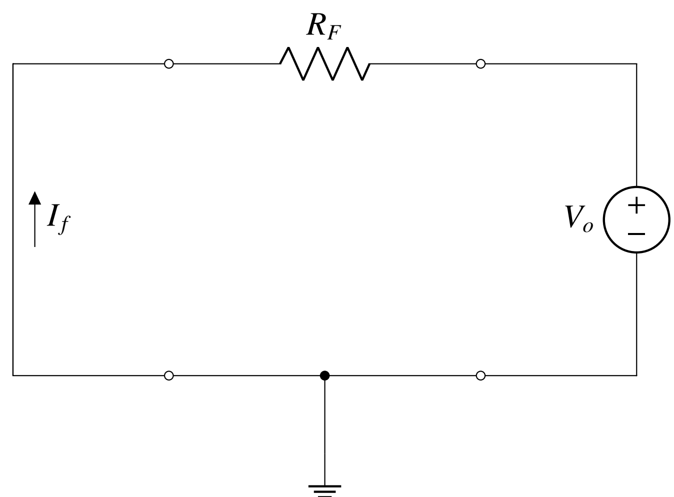

Fig. 1.6

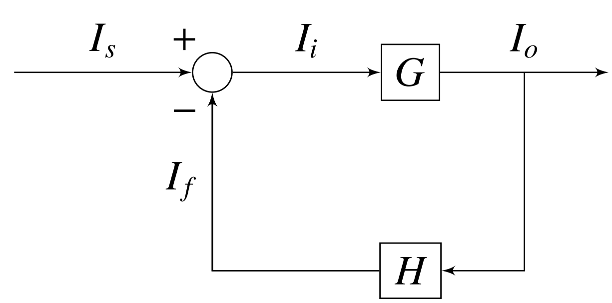

Fig. 1.7: Block Diagram

4. If the loop gain is very large, what approximate closed-loop voltage gain V_o/V_s is realized?Also if R_s = 1 kΩ , give the value of R_F that will result in $V_o/V_s \simeq$ -10 V/V.

   Solution: If the loop gain GH is very large then the closed loop gain is,

$$T = \frac{V_o}{I_s} = \frac{G}{1+GH} \tag{4.1}$$

$$\because GH >> 1 \implies T \approx \frac{1}{H} \tag{4.2}$$

From equation 3.2 and 4.2 we get,

$$T \approx -R_F \tag{4.3}$$

$$\implies \frac{V_o R_s}{V_s} \approx -R_F \tag{4.4}$$

$$\implies R_F = 10k\Omega \tag{4.5}$$

Parameters	Description
G	Open Loop Gain
H	Feedback Factor
T	Closed Loop Gain
V_0	Output Voltage
V_s	Signal Source Voltage
V_{id}	Input Voltage of Opamp
I_s	Signal Source Current
I_f	Feedback Current
R_i	Total Input Resistance
R_{out}	Total Ouput Resistance
R_{id}	Input resistance of Opamp
r_o	Output resistance of Opamp
R_i	Input resistance of Open Loop
R_o	Output resistance of Open Loop
R_{if}	Input resistance of Feedback
R_{of}	Output resistance of Feedback
R_s	Resistance of Current Source
V_f	Voltage across R_s
V_{in}	Voltage at -ve terminal of opamp
f	Closed loop 3-dB freq.

TABLE 2

5. If the amplifier μ has a dc gain of 10^3 V/V, an input resistance $R_{id} = 100\ \mathrm{k}\Omega$, and an output resistance $r_o = 1\ \mathrm{k}\Omega$, find the actual V_o/V_s realized. Also find R_{in} and R_{out}.
Solution: To find V_o/V_s, R_{in} and R_{out} first find the other necessary parameters. From Fig. 1.4 we get,

$$R_i = R_{id} \| R_F \| R_s \tag{5.1}$$
$$R_i = 100k \| 10k \| 1k = 0.90k\Omega \tag{5.2}$$
$$V_{id} = I_i R_i \tag{5.3}$$

From Fig. 1.5 we get,

$$R_o = r_o \| R_F \tag{5.4}$$
$$\implies R_o = 1k \| 10k = 0.91k\Omega \tag{5.5}$$
$$V_o = -\mu V_{id} \frac{R_F}{r_o + R_F} \tag{5.6}$$

From equation 5.3 and 5.6 we get the open-loop gain as,

$$G = \frac{V_o}{I_i} = -\mu R_i \frac{R_F}{r_o + R_F} \tag{5.7}$$
$$\implies G = -(1000)(0.90)\frac{10}{11} = -819.00k\Omega \tag{5.8}$$

From equation 3.2 and 5.8 we get closed loop gain T as,

$$T = \frac{G}{1+GH} = \frac{-819}{82.9} = -9.88k\Omega \tag{5.9}$$

From equation 3.1 we know,

$$T = \frac{V_o}{I_s} \tag{5.10}$$
$$\implies T = \frac{V_o R_s}{V_s} \tag{5.11}$$
$$\implies \frac{V_o}{V_s} = \frac{T}{R_s} \tag{5.12}$$
$$\implies \frac{V_o}{V_s} = \frac{-9.88}{1} = -9.88V/V \tag{5.13}$$

From equation 3.4 and 3.6 we know,

$$R_{if} = \frac{R_i}{1+GH} = \frac{0.90}{82.9} \tag{5.14}$$
$$\implies R_{if} = 10.87\Omega \tag{5.15}$$
$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}} \tag{5.16}$$
$$\implies R_{in} = \frac{1}{\frac{1}{10.87} - \frac{1}{1000}} = 10.99\Omega \tag{5.17}$$

Because R_L is not there in the circuit so we take it's value as ∞, so from equation 3.5 and 3.6 we know,

$$R_{of} = \frac{R_o}{1+GH} = \frac{0.91}{82.9} \tag{5.18}$$
$$\implies R_{of} = 10.97\Omega \tag{5.19}$$
$$R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}} \tag{5.20}$$
$$\implies R_{out} = \frac{1}{\frac{1}{10.97} - \frac{1}{\infty}} = 10.97\Omega \tag{5.21}$$

Verify the above calculations using the following Python code.

```
codes/ee18btech11011/ee18btech11011_cal.
```

ipynb

6. If the amplifier μ has an upper 3-dB frequency of 1 kHz and a uniform -20-dB/decade gain rolloff, what is the 3-dB frequency of the gain $| V_o/V_s |$.
Solution: To find the 3-dB frequency i.e., ω_{3dB} we need to look at the Fig.6.8.

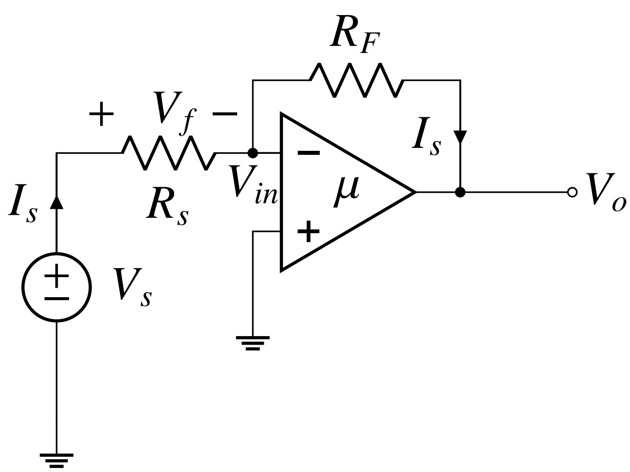

Fig. 6.8

The open loop gain G is given as follows in terms of frequency:

$$G = \frac{\mu}{1 + \frac{jf}{f_c}} \tag{6.1}$$

From Fig.6.8 we can say that:

$$V_{in} = V_s - V_f \tag{6.2}$$
$$V_o = -GV_{in} \tag{6.3}$$
$$\frac{V_f}{R_s} = \frac{V_{in} - V_o}{R_F} \tag{6.4}$$

From equation 6.3 and 6.4 we get:

$$\frac{V_f}{R_s} = \frac{-\frac{V_o}{G} - V_o}{R_F} \tag{6.5}$$
$$\implies \frac{V_f}{V_o} = -\frac{(1+G)}{G}\frac{(R_s)}{(R_F)} = -H \tag{6.6}$$
$$\because G >> 1 \implies H = \frac{R_s}{R_F} \tag{6.7}$$

Now from equation 6.2, 6.3 and 6.6 we get:

$$-\frac{V_o}{G} = V_s + HV_o \tag{6.8}$$
$$\implies \frac{V_o}{V_s} = -\frac{G}{1+GH} \tag{6.9}$$

Now, for ”f” to be 3-dB frequency given condition should be match i.e.,:

$$| \frac{V_o}{V_s} | = \frac{1}{\sqrt{2}} \tag{6.10}$$
$$\implies | -\frac{G}{1+GH} | = \frac{1}{\sqrt{2}} \tag{6.11}$$
$$\implies \frac{\frac{\mu}{1+\frac{jf}{fc}}}{1 + \frac{(R_s)}{(R_F)}\frac{\mu}{1+\frac{jf}{fc}}} = \frac{1}{\sqrt{2}} \tag{6.12}$$

Parameters	**Values**
R_s	1kΩ
R_F	10kΩ
μ	1000
f_c	1kHz

TABLE 6

Now putting the appropriate values as given in Table 6 we get:

$$\frac{\frac{1000}{1+\frac{jf}{1000}}}{1 + \frac{(1)}{(10)}\frac{1000}{1+\frac{jf}{1000}}} = \frac{1}{\sqrt{2}} \tag{6.13}$$
$$\frac{f^2}{10^{12}} + \frac{101^2}{10^6} = 2 \tag{6.14}$$
$$f \approx 1.41MHz \tag{6.15}$$

7. Using ngspice verify the Closed-Loop Transfer function or V_o/V_s.
Solution: From 5.13 we know that:

$$\frac{V_o}{V_s} = -9.88V/V \tag{7.1}$$

So, to verify this use the following spice file.

spice/ee18btech11011/ee18btech11011.net

and finally to get the result use the following python code.

spice/ee18btech11011/ee18btech11011_spice.py

Result:

figs/ee18btech11011/
ee18btech11011_spice_result.eps

figs/ee18btech11011/
ee18btech11011_spice_result.pdf

Following are the instructions to run the spice file.

spice/ee18btech11011/README.md