Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Nyquist Plot

1.1 Introduction

1.1.1. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{\pi e^{-0.25s}}{s} \quad (1.1.1.1)$$

1.1.2. Find $\mathrm{Re}\{G(\jmath\omega)\}$ and $\mathrm{Im}\{G(\jmath\omega)\}$.
Solution: From (1.1.1.1),

$$G(j\omega) = \frac{\pi}{\omega}(-\sin 0.25\omega - j\cos 0.25\omega) \quad (1.1.2.1)$$

$$\implies \mathrm{Re}\{G(\jmath\omega)\} = \frac{\pi}{\omega}(-\sin 0.25\omega) \quad (1.1.2.2)$$

$$\mathrm{Im}\{G(\jmath\omega)\} = \frac{\pi}{\omega}(-j\cos 0.25\omega) \quad (1.1.2.3)$$

1.1.3. Sketch the Nyquist plot.
Solution: The Nyquist plot is a graph of $\mathrm{Re}\{G(\jmath\omega)\}$ vs $\mathrm{Im}\{G(\jmath\omega)\}$. The following python code generates the Nyquist plot in Fig. 1.1.3

```
codes/ee18btech11007/ee18btech11007.py
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

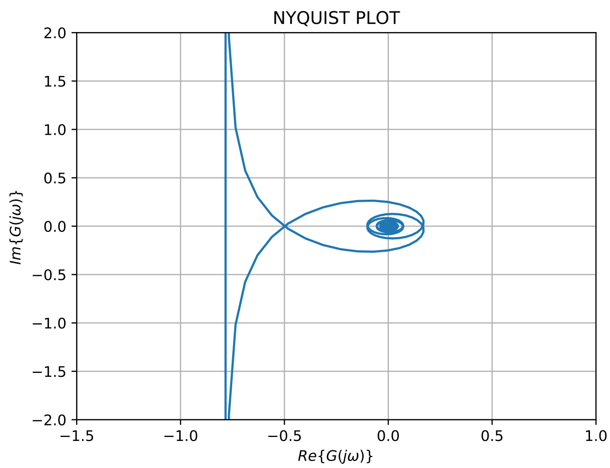

Fig. 1.1.3

1.1.4. Find the point at which the Nyquist plot of G(s) passes through the negative real axis
Solution: Nyquist plot cuts the negative real axis at ω for which

$$\angle G(\jmath\omega) = -\pi \quad (1.1.4.1)$$

From (1.1.1.1),

$$G(\jmath\omega) = \frac{\pi e^{-\frac{\jmath\omega}{4}}}{\jmath\omega} = \frac{\pi e^{-\jmath\left(\frac{\omega}{4}+\frac{\pi}{2}\right)}}{\omega} \quad (1.1.4.2)$$

$$\implies \angle G(\jmath\omega) = -\left(\frac{\omega}{4} + \frac{\pi}{2}\right) \quad (1.1.4.3)$$

From (1.1.4.3) and (1.1.4.1),

$$\frac{\omega}{4} + \frac{\pi}{2} = \pi \quad (1.1.4.4)$$

$$\implies \omega = 2\pi \quad (1.1.4.5)$$

Also, from (1.1.1.1),

$$\left|G(\jmath\omega)\right| = \frac{\pi}{|\omega|} \quad (1.1.4.6)$$

$$\implies \left|G(\jmath 2\pi)\right| = \frac{1}{2} \quad (1.1.4.7)$$

1.1.5. Use the Nyquist Stability criterion to determine if the system in (1.1.4.3) is stable.
Solution: Consider Table 1.1.5. According to

Variable	Value	Description
Z	0	Poles of $\frac{G(s)}{1+G(s)H(s)}$ in right half of s plane
P	0	Poles of $G(s)H(s)$ in right half of s plane
N	0	No of clockwise encirclements of $G(s)H(s)$ about -1+j0 in the Nyquist plot

TABLE 1.1.5

the Nyquist stability criterion,

a) If the open-loop transfer function $G(s)$ has a zero pole of multiplicity l, then the Nyquist plot has a discontinuity at ω =0. During further analysis it should be assumed that the phasor travels l times clock-wise along a semicircle of infinite radius. After applying this rule, the zero poles should be neglected, i.e. if there are no other unstable poles, then the open-loop transfer function $G(s)$ should be considered stable.
b) If the open-loop transfer function $G(s)$ is stable, then the closed-loop system is unstable for any encirclement of the point -1. If the open-loop transfer function $G(s)$ is unstable, then there must be one counter clock-wise encirclement of -1 for each pole of $G(s)$ in the right-half of the complex plane.
c) The number of surplus encirclements (N + P greater than 0) is exactly the number of unstable poles of the closed-loop system.
d) However, if the graph happens to pass through the point $-1+\jmath 0$, then deciding upon even the marginal stability of the system becomes difficult and the only conclusion that can be drawn from the graph is that there exist zeros on the $\jmath\omega$ axis.

From (1.1.1.1), $G(s)$ is stable since it has a single pole at $s = 0$. Further, from Fig. 1.1.3, the Nyquist plot does not encircle $s = -1$. From Theorem 1.1.5b, we may conclude that the system is stable.

2 Example

2.0.1. Using Nyquist criterion, find out whether the system below is stable or not.

$$G(s) = \frac{20}{s(s+1)}, H(s) = \frac{s+3}{s+4} \tag{2.0.1.1}$$

Solution: The following python code generates the Nyquist plot in Fig.2.0.1.

```
codes/ee18btech11011(1).ipynb
```

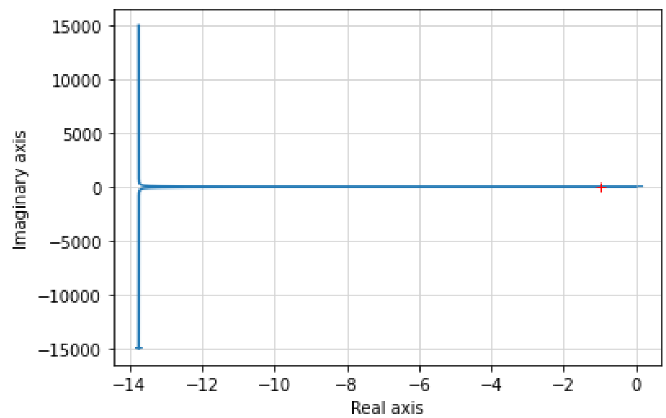

Fig. 2.0.1: Nyquist Plot

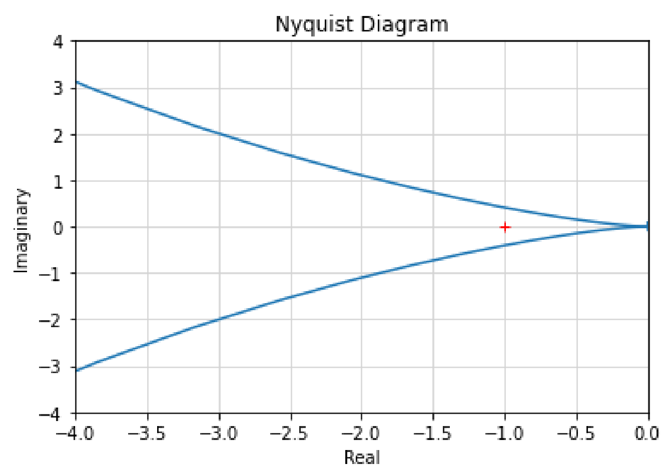

Fig. 2.0.1: Zoomed image

From Table 1.1.5 we already know the Nyquist Stability criterion so for this closed loop system the transfer function will be =

$$\frac{G(s)}{1+G(s)H(s)} \tag{2.0.1.2}$$

$$\implies G(s)H(s) = \frac{20(s+3)}{s(s+1)(s+4)} \tag{2.0.1.3}$$

So it has 3 open-loop poles 0,-1 and -4, therefore P=0.Further we know that N = Z-P , now we know Z = Poles of $\frac{G(s)}{1+G(s)H(s)}$in right half of s plane.To find the poles we can use the following Routh Hurwitz python code.Using this we get Z = 0.

```
codes/Routh.py
```

$$P = 0, Z = 0 \tag{2.0.1.4}$$

$$\implies N = 0 \tag{2.0.1.5}$$

This can also be seen from the Fig. 2.0.1 that the encirclement is counter-clockwise not clockwise.Hence the system is stable.