Control Systems

G V V Sharma*

Contents

Abstract—**This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Shunt-Shunt

1.1. The circuit in Fig. 1.1.1 utilizes a voltage amplifier with gain μ in a shunt-shunt feedback topology with the feedback network composed of resistor R_F, to use the feedback equations convert the signal source to its Norton Representation.Also draw the H-Circuit and G-Circuit.

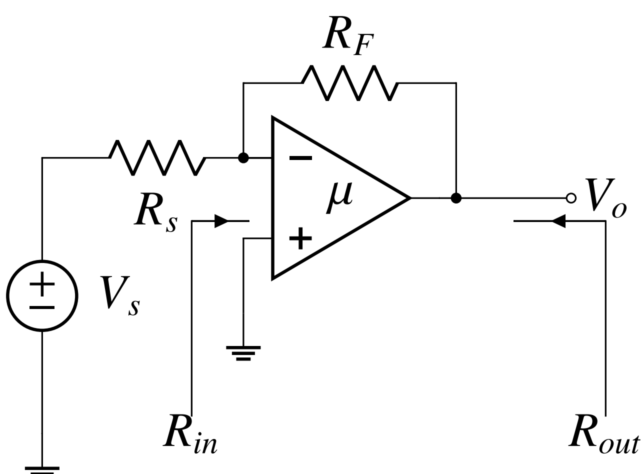

Fig. 1.1.1

Solution: See Fig. 1.1.3 for the Norton Representation, Fig. 1.1.4 for the H-Circuit , Fig.1.1.5 and Fig. 1.1.6 for the G-Circuit.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

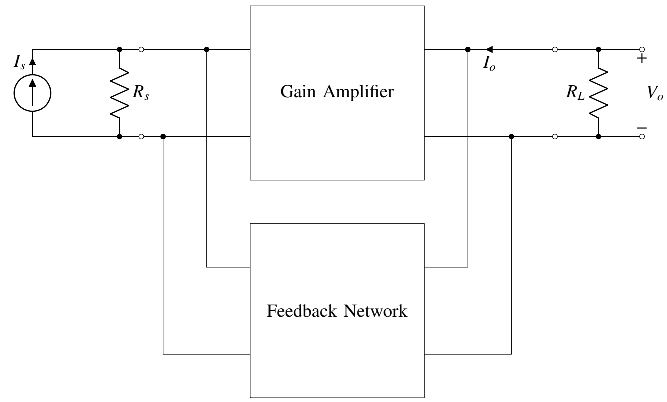

Fig. 1.1.2: Shunt Shunt Amplifier Block Diagram

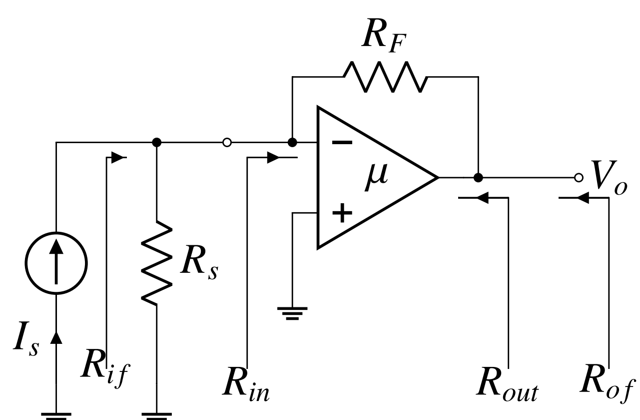

Fig. 1.1.3

1.2. Write all the feedback equations based on all the Figs. using KCL/KVL.

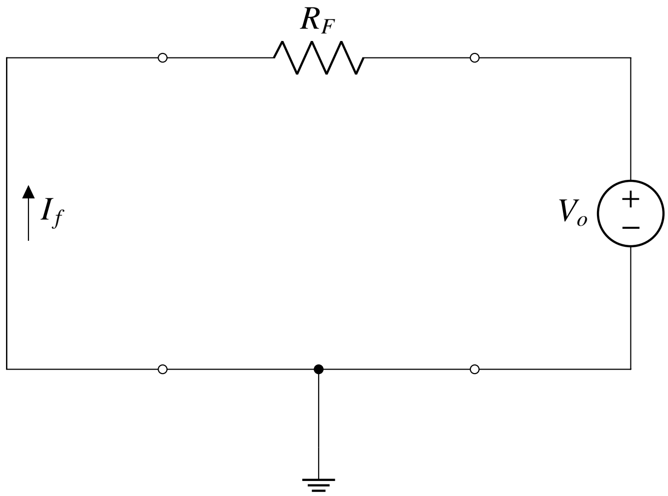

Fig. 1.1.4

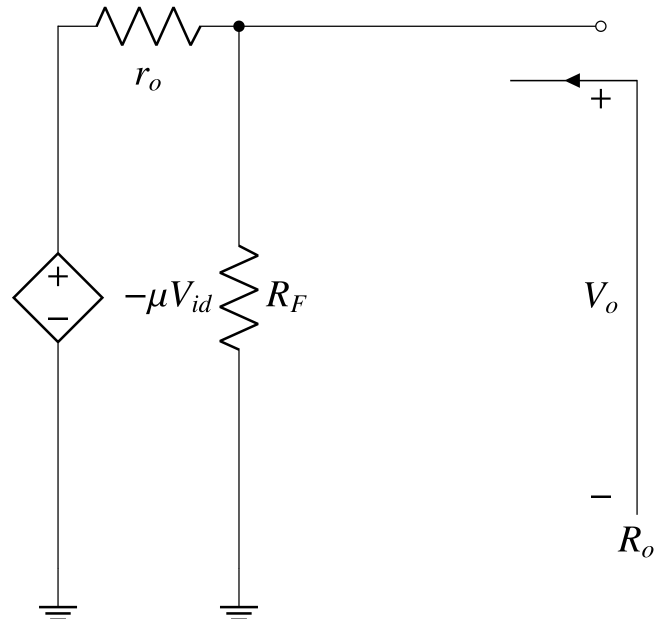

Fig. 1.1.6

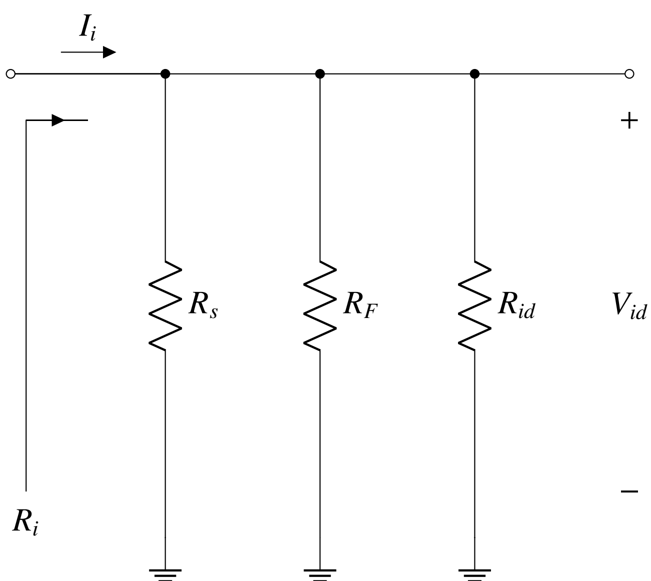

Fig. 1.1.5

Solution: The equations are as follows:

$$T = \frac{V_o}{I_s} = \frac{G}{1+GH} \tag{1.2.1}$$

$$H = \frac{I_f}{V_o} = -\frac{1}{R_F} \tag{1.2.2}$$

$$I_i = I_s - I_f = \frac{I_s}{1+GH} \tag{1.2.3}$$

$$R_{if} = \frac{V_i}{I_s} = \frac{V_i}{(1+GH)I_i} = \frac{R_i}{1+GH} \tag{1.2.4}$$

$$R_{of} = \frac{R_o}{1+GH} \tag{1.2.5}$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}}, R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}} \tag{1.2.6}$$

Parameters	**Description**
G	Open Loop Gain
H	Feedback Factor
T	Closed Loop Gain
V_0	Output Voltage
V_s	Signal Source Voltage
V_{id}	Input Voltage of Opamp
I_s	Signal Source Current
I_f	Feedback Current
R_i	Total Input Resistance
R_{out}	Total Ouput Resistance
R_{id}	Input resistance of Opamp
r_o	Output resistance of Opamp
R_i	Input resistance of Open Loop
R_o	Output resistance of Open Loop
R_{if}	Input resistance of Feedback
R_{of}	Output resistance of Feedback
R_s	Resistance of Current Source
V_f	Voltage across R_s
V_{in}	Voltage at -ve terminal of opamp
f	Closed loop 3-dB freq.

TABLE 1.2

1.3. If the loop gain is very large, what approximate closed-loop voltage gain V_o/V_s is realized?Also if R_s = 1 kΩ , give the value of R_F that will

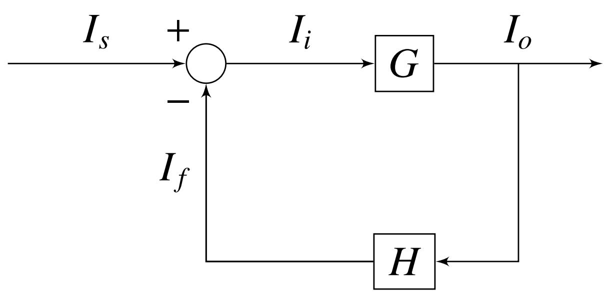

Fig. 1.2.7: Block Diagram

result in $V_o/V_s \simeq -10V/V$.

Solution: If the loop gain GH is very large then the closed loop gain is,

$$T = \frac{V_o}{I_s} = \frac{G}{1+GH} \tag{1.3.1}$$

$$\because GH >> 1 \implies T \approx \frac{1}{H} \tag{1.3.2}$$

From equation 1.2.2 and 1.3.2 we get,

$$T \approx -R_F \tag{1.3.3}$$

$$\implies \frac{V_o R_s}{V_s} \approx -R_F \tag{1.3.4}$$

$$\implies \frac{V_o}{V_s} \approx \frac{-R_F}{R_s} \tag{1.3.5}$$

$$\implies R_F = 10k\Omega \tag{1.3.6}$$

1.4. If the amplifier μ has a dc gain of 10^3 V/V, an input resistance R_{id} = 100 kΩ , and an output resistance r_o = 1 kΩ , find the actual V_o/V_s realized. Also find R_{in} and R_{out}.

Solution: To find V_o/V_s, R_{in} and R_{out} first find the other necessary parameters. From Fig. 1.1.5 we get,

$$R_i = R_{id} \| R_F \| R_s \tag{1.4.1}$$

$$R_i = 100k \| 10k \| 1k = \frac{1000}{1011}k = 0.90k\Omega \tag{1.4.2}$$

$$V_{id} = I_i R_i \tag{1.4.3}$$

From Fig. 1.1.6 we get,

$$R_o = r_o \| R_F \tag{1.4.4}$$

$$\implies R_o = 1k \| 10k = 0.91k\Omega \tag{1.4.5}$$

$$V_o = -\mu V_{id} \frac{R_F}{r_o + R_F} \tag{1.4.6}$$

From equation 1.4.3 and 1.4.6 we get the open-loop gain as,

$$G = \frac{V_o}{I_i} = -\mu R_i \frac{R_F}{r_o + R_F} \tag{1.4.7}$$

$$\implies G = -(1000)(0.90)\frac{10}{11} = -819.00k\Omega \tag{1.4.8}$$

From equation 1.2.2 and 1.4.8 we get closed loop gain T as,

$$T = \frac{G}{1+GH} = \frac{-819}{82.9} = -9.88k\Omega \tag{1.4.9}$$

From equation 1.2.1 we know,

$$T = \frac{V_o}{I_s} \tag{1.4.10}$$

$$\implies T = \frac{V_o R_s}{V_s} \tag{1.4.11}$$

$$\implies \frac{V_o}{V_s} = \frac{T}{R_s} \tag{1.4.12}$$

$$\implies \frac{V_o}{V_s} = \frac{-9.88}{1} = -9.88V/V \tag{1.4.13}$$

From equation 1.2.4 and 1.2.6 we know,

$$R_{if} = \frac{R_i}{1+GH} = \frac{0.90}{82.9} \tag{1.4.14}$$

$$\implies R_{if} = 10.87\Omega \tag{1.4.15}$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}} \tag{1.4.16}$$

$$\implies R_{in} = \frac{1}{\frac{1}{10.87} - \frac{1}{1000}} = 10.99\Omega \tag{1.4.17}$$

Because R_L is not there in the circuit so we take it's value as ∞, so from equation 1.2.5 and 1.2.6 we know,

$$R_{of} = \frac{R_o}{1+GH} = \frac{0.91}{82.9} \tag{1.4.18}$$

$$\implies R_{of} = 10.97\Omega \tag{1.4.19}$$

$$R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}} \tag{1.4.20}$$

$$\implies R_{out} = \frac{1}{\frac{1}{10.97} - \frac{1}{\infty}} = 10.97\Omega \tag{1.4.21}$$

Verify the above calculations using the following Python code.

```
codes/ee18btech11011/ee18btech11011_cal.
    ipynb
```

1.5. If the amplifier μ has an upper 3-dB frequency of 1 kHz and a uniform -20-dB/decade gain rolloff, what

is the 3-dB frequency of the gain $|V_o/V_s|$.
Solution: To find the 3-dB frequency i.e., ω_{3dB} we need to look at the Fig.1.5.8.

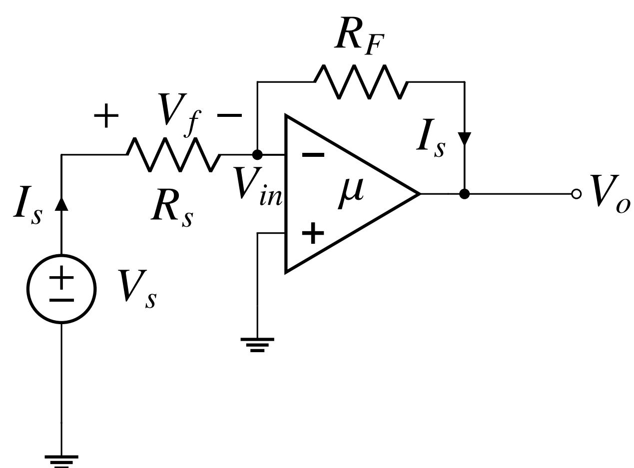

Fig. 1.5.8

The open loop gain G is given as follows in terms of frequency:

$$G = \frac{\mu}{1 + \frac{jf}{f_c}} \tag{1.5.1}$$

From Fig.1.5.8 we can say that:

$$V_{in} = V_s - V_f \tag{1.5.2}$$

$$V_o = -GV_{in} \tag{1.5.3}$$

$$\frac{V_f}{R_s} = \frac{V_{in} - V_o}{R_F} \tag{1.5.4}$$

From equation 1.5.3 and 1.5.4 we get:

$$\frac{V_f}{R_s} = \frac{-\frac{V_o}{G} - V_o}{R_F} \tag{1.5.5}$$

$$\implies \frac{V_f}{V_o} = -\frac{(1+G)}{G}\frac{(R_s)}{(R_F)} = -H \tag{1.5.6}$$

$$\because G >> 1 \implies H = \frac{R_s}{R_F} \tag{1.5.7}$$

Now from equation 1.5.2, 1.5.3 and 1.5.6 we get:

$$-\frac{V_o}{G} = V_s + HV_o \tag{1.5.8}$$

$$\implies \frac{V_o}{V_s} = -\frac{G}{1+GH} \tag{1.5.9}$$

Now, for ”f” to be 3-dB frequency given condition should be match i.e.,:

$$|\frac{V_o}{V_s}| = \frac{1}{\sqrt{2}} \tag{1.5.10}$$

$$\implies |-\frac{G}{1+GH}| = \frac{1}{\sqrt{2}} \tag{1.5.11}$$

$$\implies |-\frac{\frac{\mu}{1+\frac{jf}{f_c}}}{1 + \frac{(R_s)}{(R_F)}\frac{\mu}{1+\frac{jf}{f_c}}}| = \frac{1}{\sqrt{2}} \tag{1.5.12}$$

Now putting the appropriate values as given in

Parameters	Values
R_s	1kΩ
R_F	10kΩ
μ	1000
f_c	1kHz

TABLE 1.5

Table1.5 we get:

$$|-\frac{\frac{1000}{1+\frac{jf}{1000}}}{1 + \frac{(1)}{(10)}\frac{1000}{1+\frac{jf}{1000}}}| = \frac{1}{\sqrt{2}} \tag{1.5.13}$$

$$\frac{f^2}{10^{12}} + \frac{101^2}{10^6} = 2 \tag{1.5.14}$$

$$\implies f \approx 1.41MHz \tag{1.5.15}$$