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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

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6.1. Consider a state-variable model of a system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\alpha & -2\beta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} r \tag{6.1.1}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{6.1.2}$$

where y is the output, and r is the input. Find the Damping ratio ζ and the Undamped natural frequency ω_n of the system.

Solution: The state space model is given by

$$\dot{X} = AX + BU \tag{6.1.3}$$

$$Y = CX + DU \tag{6.1.4}$$

The transfer function for the state space model is:

$$H(s) = C(sI - A)^{-1}B + D \tag{6.1.5}$$

$$\implies H(s) = \frac{\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s + 2\beta & 1 \\ -\alpha & s \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}}{s(s + 2\beta) + \alpha} \tag{6.1.6}$$

$$= \frac{b_1(s + 2\beta) + b_2}{s^2 + 2s\beta + \alpha} \tag{6.1.7}$$

$$\implies H(s) = \frac{b_1 s}{s^2 + 2s\beta + \alpha} + \frac{2b_1\beta + b_2}{s^2 + 2s\beta + \alpha} \tag{6.1.8}$$

Generally for a second order system the transfer function is given by

Damping ratio(ζ)	**Undamped natural frequency(ω_n)**
Damping ratio basically indicates the amount of damping present in the overall system denoted by zeta, where damping is a counter force.It is a dimensionless measure describing how oscillations in a system decay after a disturbance.	The frequency of oscillation of the system without damping.A system may or may not have an associated natural frequency.
The damping ratio is a system parameter, denoted by ζ, that can vary from undamped (ζ = 0), underdamped (ζ <1) through critically damped (ζ = 1) to overdamped (ζ >1).	Only systems with ζ <1 have a natural frequency ω and only in the case that ζ = 0 will the natural frequency ω = ω_n, the undamped natural frequency.

TABLE 6.1

$$H(s) = \frac{\omega_n^2}{s^2 + 2s\zeta\omega_n + \omega_n^2} \tag{6.1.9}$$

Now from the transfer function we got we can see that our system is bandpass and lowpass combination but the comaprison of denominator of our transfer function to the general transfer function is still valid.

$$\therefore 2\zeta\omega_n = 2\beta, \tag{6.1.10}$$

$$\omega_n^2 = \alpha \tag{6.1.11}$$

$$\implies \zeta = \frac{\beta}{\sqrt{\alpha}}, \omega_n = \sqrt{\alpha} \tag{6.1.12}$$